

# QUANTUM LINEAR SYSTEMS THEORY

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## **Abstract**

Classical linear systems theory has a history going back some 50 years, to the birth of modern control theory with Kalman's foundational work on filtering and LQG optimal control. However, there are new imperatives arising in emerging quantum technologies that demand new concepts and tools from control theory. Based on initial work on quantum H-infinity and LQG control, and recent experimental confirmation, this project seeks to develop the fundamental principles and tools of linear systems theory that take into account the special features of quantum mechanics. The new quantum linear systems theory will provide the framework for investigating the fundamental limitations and opportunities provided by coherent quantum feedback, which need not necessarily involve measurement. Particular emphasis will be given to the investigation of coherent optimal LQG and H-infinity design methods, and the formulation and solution of entanglement enhancement problems using these tools.

## Objectives

1. Develop a new systems theory for linear quantum systems.
2. Investigate the formulation and solution of coherent optimal LQG and H-infinity approaches to control design for linear quantum systems, and where possible, develop tractable solution methodologies.
3. Investigate the new opportunities and performance limitations arising from quantum mechanics in the context of quantum linear systems using coherent feedback. This will include applications to problems of entanglement enhancement and control.

## Achievements

1. New methods for using direct physical couplings in optimal coherent feedback system design.

## Report Documentation Page

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14. ABSTRACT <b>Classical linear systems theory has a history going back some 50 years, to the birth of modern control theory with Kalman's foundational work on filtering and LQG optimal control. However, there are new imperatives arising in emerging quantum technologies that demand new concepts and tools from control theory. Based on initial work on quantum H-infinity and LQG control, and recent experimental confirmation, this project seeks to develop the fundamental principles and tools of linear systems theory that take into account the special features of quantum mechanics. The new quantum linear systems theory will provide the framework for investigating the fundamental limitations and opportunities provided by coherent quantum feedback, which need not necessarily involve measurement. Particular emphasis will be given to the investigation of coherent optimal LQG and H-infinity design methods, and the formulation and solution of entanglement enhancement problems using these tools.</b>					
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2. Differential evolution optimization methods in coherent control of linear quantum systems.
3. Decentralized coherent robust H-infinity quantum control.
4. Approximation of linear quantum system models.
5. Single photon processing by linear quantum systems.
6. Quantum physical realization of classical linear systems.
7. Robust stability of linear and nonlinear quantum systems

### **Status**

At the Australian National University, the PhD student Shi Wang and Post-doc Dr Guofeng Zhang were supported by the project during 2010 and through 2011.

At the University of New South Wales, Mr Hendra Harno, whose PhD studies were supported by the project, completed his PhD thesis and took up a one year postdoctoral position at the University of Central Florida in the US. He has recently taken up an academic position at Universiti Teknikal Malaysia Melaka in Malaysia. The project was then used to partially support a new PhD student, Mr Shanon Vuglar (who is a former Australian Airforce Officer) as well as partially supporting a Postdoctoral fellow Dr. Igor Vladimirov.

Guofeng Zhang and Matthew James have submitted a paper on the response of linear quantum stochastic systems to single-photon inputs and pulse shaping of photon wave packets. Photons are highly non-classical states of light that are of considerable importance in quantum optics and quantum information processing. At present, the majority of optical quantum information processing publications do not consider the spatial/temporal shapes of photons and the dynamics of optical components. Rather, simplified static single mode models are used. However, pulse shapes and dynamics are important, and indeed fundamental, to the operation of real physical devices. The work leading to this paper has led to further work forming the basis for a second paper which is nearing completion. Linear quantum systems with active elements produce a Gaussian contribution to the output state, and our work has identified a class of quantum optical states, which we call ‘pulsed-Gaussian’, which is preserved under the steady state action of a linear quantum system. Our results provide detailed formulas for the transfer relations for this new class of states. These results extend known classical results for transfer of Gaussian states to an important class of non-classical quantum states that includes single photon wavepackets.

Ian Petersen, Matthew James and colleague Hendra Nurdin have completed some work on characterization of entanglement infeasibility using control theory methods. This work provides some new proofs that exploit system theoretic ideas and the structure of linear quantum stochastic systems to show that classical finite dimensional linear time invariant controllers cannot generate steady state entanglement in an asymptotically stable bipartite Gaussian quantum system which is initialized in a Gaussian state. We also show that the use of such controllers cannot generate entanglement in finite time from a bipartite system initialized in a separable Gaussian state. The approach reveals some connections between systems theoretic

concepts and the fundamental inability of classical linear controllers to generate entanglement in linear quantum systems, as a consequence of the fact that these controllers can only perform local operations and classical communications (LOCC) and hence cannot generate more entanglement.

Shi Wang, Hendra Nurdn, Guofeng Zhang, and Matthew James have submitted two papers concerning the physical realization of linear classical systems using quantum devices. Quantum optical systems typically have much higher bandwidth than electronic devices, meaning faster response and processing times, and hence have a potential for providing better performance than classical systems. A complete procedure is proposed for a stable quantum linear stochastic system realizing a given stable classical linear stochastic system. This work also explains how it may be possible to realize certain measurement feedback loops fully at the quantum level, providing a simpler and faster implementation approach that is suited to emerging microchip technologies (e.g. photonic crystals).

Ian Petersen and Hendra Harno had two papers which at the 2011 IFAC World Congress on the application of evolutionary optimization methods to the design of coherent quantum feedback controllers. The first paper considers the case of passive quantum linear systems defined purely in terms of annihilation operators and for which a coherent quantum  $H$  infinity controller is sought from this same class. The second paper considers the problem decentralized coherent quantum feedback controller design to achieve an  $H$  infinity performance objective. Ian Petersen and Shanon Vuglar have a number of papers which are concerned with the number of quantum noises which need to be added to a given linear time-invariant system defined by its transfer function matrix to make it physically realizable as a quantum linear system.

Ian Petersen has completed some work on the approximation of linear quantum systems. In one approach, a method based on cascade realization of quantum systems is used and a conference and journal paper have been produced. In another approach, a method based on singular perturbation is used and a conference and journal paper have resulted. This work was extended by the graduate student Shanon Vuglar to more general linear quantum systems and a conference paper has resulted.

Ian Petersen and Matthew James have completed a series of papers together with colleague Valery Ugrinovskii on robust stability of linear quantum systems with linear and nonlinear perturbations.

Ian Petersen and Igor Vladimirov have completed research on risk sensitive analysis of perturbed linear quantum systems. This work has resulted in two conference papers.

Matthew James, Andre Carvalho and Michael Hush completed some work analyzing cross-phase modulation using single photon quantum filtering techniques. They are also collaborating with experimentalist Morgan Hedges (ANU, now Calgary) on the modeling of quantum memories using infinite dimensional linear quantum models.

Matthew James and visitor Rebing Wu (Tsinghua Univ.) worked on optimal control problems for fully coherent quantum feedback.

## **On-going research topics**

On-going research topics which have emerged from this project are as follows:

Linear quantum system realization of classical linear systems.

Canonical forms for mixed quantum-classical systems.

Physical realizability of quantum linear systems models

Single photon and pulsed-Gaussian state processing by linear quantum systems.

Estimation by interconnection.

Robust stability of linear quantum systems

Quantum memories

## **Personnel Supported During Duration of Grant**

Guofeng Zhang	April 2010 – November 2011, Postdoc, Australian National University
Shi Wang	Nov 2009 – June 2012, Graduate Student, Australian National University
Hendra Harno	April 2010 – March 2011, Graduate Student, UNSW
Shanon Vuglar	June 2011 – June 2012, Graduate Student, UNSW
Igor Vladimirov	June 2011 – December 2011, Postdoc, UNSW
Andre Carvallho	January – June, 2012, Postdoc, ANU
Michael Hush	January – June, 2012, Postdoc, ANU
Matthew R. James	Professor, Australian National University
Ian R. Petersen	Professor, University of New South Wales

## **Publications**

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I. R. Petersen. Singular perturbation approximations for a class of linear complex quantum systems. In *Proceedings of the 2010 American Control Conference*, Baltimore, MD, June 2010.

H. G. Harno and I. R. Petersen. Coherent control of linear quantum systems: A differential evolution approach. In *Proceedings of the 2010 American Control Conference*, Baltimore, MD, June 2010.

H. G. Harno and I. R. Petersen. Coherent robust H-infinity control of a class of large-scale uncertain linear complex quantum stochastic. In *Proceedings of the IEEE Multi-conference on Systems and Control*, Yokohama, Japan, 2010.

I.R. Petersen, "Low Frequency Approximation for a class of Linear Complex Quantum Systems using Cascade Cavity Realization", *Proceedings of the 2010 IEEE Conference on Decision and Control*.

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G. Zhang and M.R. James, “On the Response of Quantum Linear Systems to Single Photon Input Fields”, to appear, *IEEE Trans. Aut. Control.*, 2013.

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### **Honors & Awards Received**

Professor Petersen has been elected as a Fellow of the Australian Academy of Science during 2011.

Professor Petersen was also made an Australian Research Council Laureate Fellow in 2011.

### **AFRL Point of Contact**

Fariba Fahoo

John Seo

### **Transitions**

None

### **New Discoveries**

None