Research accomplished during the period 3/2/08 - 9/2/11 in ten specific areas of study is reported. This work includes the treatment of the problems of (1) comparing coherent or mixed systems of different sizes, and obtaining representation results for system reliability under relaxed assumptions on component lifetimes (e.g., exchangeability), (2) deriving new representations of system reliability for used systems known to be working at an inspection time $t$, (3) developing extensions of the notion of system signatures to dynamic reliability settings, with...
Applications and Extensions of Signature Theory to Modeling and Inference Problems in Engineering Reliability

ABSTRACT
Research accomplished during the period 3/2/08 - 9/2/11 in ten specific areas of study is reported. This work includes the treatment of the problems of (1) comparing coherent or mixed systems of different sizes, and obtaining representation results for system reliability under relaxed assumptions on component lifetimes (e.g., exchangeability), (2) deriving new representations of system reliability for used systems known to be working at an inspection time t, (3) developing extensions of the notion of system signatures to dynamic reliability settings, with applications to nonparametric models in reliability and to the engineering practice of burn-in,(4) inference about a common component distribution F from system failure time data, (5) the derivation and application of the joint signature of pairs of systems with shared components,(6) a comparison of the Bayesian and frequentist approaches to estimation (a research monograph), (7) skewness and dispersion among convolutions of independent gamma variables, (8) signature-based representations for the reliability of systems with heterogeneous components, (9) network reliability (10) a proof of the "no internal zeros" property of system signatures.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received Paper

TOTAL:
Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:
Number of Papers published in non peer-reviewed journals:

(c) Presentations

Presentations at the Army Conference on Applied Statistics:
2009 -- On Joint Signatures of Coherent Sysytems with Shared Components
2010 -- On Network Reliability -- A Fresh look at some Basic Questions
2011 -- Estimating Component Characteristics Based on Lifetime Data from Multiple Systems
Number of Presentations: 4.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:
Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received          Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received          Paper
2010/03/12 0! 1  Z. Li, AN OVERVIEW OF THE IMMERSED INTERFACE METHOD AND ITS APPLICATIONS, (03 2010)

TOTAL:  1

Number of Manuscripts:

Books

Received          Paper

TOTAL:

Patents Submitted

N/A

Patents Awarded

N/A

Awards

F. J. Samaniego received the U. S. Army Wilks Award in October, 2008. In 2009 – 2014, he is serving on the Selection Committee for the U. S. Army Wilks Award. Dr. Samaniego served as a Distinguished Lecturer of Sigma Xi (The Scientific Research Society) in 2009 - 2011. In spring, 2010, Dr. Samaniego was honored with a first place certificate among Mathematical Scientists nominated for the “Excellence in Teaching Award” given by the Associated Students of the University of California, Davis. In 2011 – 12, he is a finalist among UC Davis faculty nominated to serve as the 2012 Faculty Research Lecturer. The Academic Senate of the University of California, Davis, selects a single faculty member each year to serve as the Faculty Research Lecturer. The distinction is the highest honor bestowed on faculty members by the Academic Senate. The 2012 Faculty Research Lecturer will be announced in February, 2012.

Graduate Students

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Names of Post Doctorates

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Names of Faculty Supported

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Names of Under Graduate students supported

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Student Metrics
This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: ...... 25.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 20.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 15.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): ...... 8.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense ...... 1.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: ...... 0.00

Names of Personnel receiving masters degrees

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Names of personnel receiving PHDs

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Total Number:

Names of other research staff

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Sub Contractors (DD882)
Specific accomplishments in ten areas of research supported by ARO grant W911NF-08-1-0077 (project number 53479-MA) are discussed below.

(1) Comparing coherent or mixed systems of different sizes, and obtaining representation results for system reliability under relaxed assumptions on component lifetimes (e.g., exchangeability).

In this work, general results are obtained which facilitate the comparison of mixed systems of different sizes. Specifically, for any given system in k i.i.d. components and for any n > k, an explicit formula is obtained for the signature of the n-component mixed system in similar independent components whose reliability function is identical to that of the original (smaller) system. This result renders applicable the preservation theorems of Kochar, Mukerjee and Samaniego (Naval Research Logistics, 1999) which apply to the comparison of mixed systems of the same size. It is then shown that these results may be extended to vectors of exchangeable random lifetimes. In this latter context (which of course includes the i.i.d. case), representation theorems for the reliability function of an arbitrary mixed system are obtained in terms of stochastic mixtures and generalized mixtures of the reliability functions of ordered component failure times. These new mixture representations are used to obtain new stochastic ordering properties for pairs of mixed systems of arbitrary sizes.

(2) Derivations of new representations of system reliability for used systems known to be working at an inspection time t.

The representation of the reliability function of the lifetime of a coherent system as a mixture of the reliability functions of order statistics associated with the lifetimes of its components is a very useful tool to study the ordering and the limiting behaviour of coherent systems. Under various conditions on the status of the components or the system, we obtain several representations of the reliability functions of residual lifetimes of used coherent systems in terms of the reliability functions of residual lifetimes of order statistics. For example, we establish new representations for the reliability function of a used but working system, i.e., for the distribution of the system lifetime T given that it is known that T > t. Our result reveals that the reliability function of the residual lifetime of the system at time t, that is, the reliability function of T - t given T > t, is a mixture of the residual lifetimes of the order statistics of the n component lifetimes at time t (that is, a mixture of the reliability functions of Xi:n - t given Xi:n > t for i = 1, ..., n). This representation applies to mixed systems and also to coherent systems with less than n components since the latter are equal in distribution to specific mixed systems with n components. Further, the representation theorem may be used to compare the residual lifetimes of a system at different ages.

(3) Extensions of the notion of system signatures to dynamic reliability settings, with applications to nonparametric models in reliability and to the engineering practice of burn-in.

System signatures are quite useful tools in the study and comparison of engineered systems whose components have i.i.d. lifetimes. In this paper, the theory of system signatures (Samaniego (1985)) is adapted to versions of signatures applicable in dynamic reliability settings. It is shown that, when a working used system is inspected at time t, the (n-k)-vector s with jth element s(j) = P(T = Xk+j:n | (T > t) and (Xk:n - t < Xk+1:n)) for j = 1, ..., n-k, for given k for which P((T > t) and (Xk:n - t < Xk+1:n)) > 0, is a distribution-free measure of the design of the residual system. Dynamic versions of known representation and preservation theorems for system signatures are then established. Two applications of dynamic signatures are studied in detail. The well-known (NBU) property of aging systems is extended to a uniform (UNBU) version which compares new systems with working used systems, conditional on the known number of failures. A theoretical result is given which provides sufficient conditions for a system to have the UNBU property. The application of dynamic signatures to a particular version of the engineering practice of "burn-in" is also treated. Specifically, we consider the comparison of new systems and working used systems burned in to a given ordered component failure time. In a reliability economics framework, we illustrate how one might compare a new system to one successfully burned in to the kth component failure, and we identify circumstances in which burn in is superior (or inferior) to the fielding of a new system.

(4) Statistical inference about a common component distribution F from system failure time data.

Suppose that failure times are available from a random sample of N systems of a given, fixed design with components which have i.i.d. lifetimes distributed according to a common distribution F. The inverse problem of estimating F from data on observed system lifetimes is considered. This is a problem of some importance in engineering reliability, as using the known relationship between the system and component lifetime distributions via signature and domination theory allows one to make precise predictions about future system performance. The nonparametric maximum likelihood estimator \( F^*(t) \) of the component distribution function F(t) is identified and shown to be accessible numerically in any application of interest. The asymptotic distribution of \( F^*(t) \) is also identified, facilitating the construction of approximate confidence intervals for F(t) when N is sufficiently large. Simulation results for samples of size N = 50 and N = 100 for a collection of nonparametric lifetime models demonstrate the efficacy of the recommended estimator is small to moderate sized samples.

(5) The derivation and application of the joint signature of pairs of systems with shared components.
The goal of the present study has been to consider extensions of the concept of system signatures to bivariate situations in which pairs of systems share some components and thus have dependent lifetimes. The problem explored here is motivated by examples of sharing of components in the design of selected computer networks. In this work, we obtain representations for the joint distribution (and joint reliability function) of pairs of coherent systems with shared components under the assumption that all components have i.i.d. lifetimes with common distribution F. The expression derived for the joint distribution G of the two system lifetimes is shown to depend on a pair of matrices S and S*, each of which has “total mass” 1. The pair (S, S*) is referred to as the joint signature, and, under the assumption i.i.d component lifetimes, it is distribution free, that is it does not depend on the underlying component distribution F. Given two pairs of such joint systems, we have studied various forms of stochastic ordering among the systems’ joint lifetimes. In particular, we provide sufficient conditions on the joint signatures of the two pairs of systems to ensure that the two joint lifetimes satisfy a specific bivariate stochastic ordering. Similar results are obtained in studying the ordering of two joint reliability functions. Applications of “joint signatures” of systems with shared components include, for example, the computation of measures of dependence between the lifetimes of two systems with shared components and the estimation of the residual lifetime distribution of one of these two systems, given that the other is known to have failed at a fixed time t.

(6) A comparison of the Bayesian and frequentist approaches to estimation (a research monograph)

In the 2010 research monograph "A Comparison of the Bayesian and Frequentist Approaches to Estimation" (New York: Springer, Inc.), the Principal Investigator, F. J. Samaniego, presents a comprehensive overview of results on Bayesian inference obtained by the PI over the past 20 years, including separate chapters on recent research on (a) combining information from ‘related’ experiments, (b) the relevance of the new concept of Bayesian self consistency in solving the Bayesian consensus problem and (c) estimating stress and strength distributions in a nonidentifiable model in reliability, all carried out under ARO support. The following is a brief summary of the monograph’s contents.

The monograph contributes to the area of comparative statistical inference. It restricts attention to the important subfield of statistical estimation. A detailed review of Decision Theory and the frequentist and Bayesian approaches to estimation is presented and carefully discussed in Chapters 1 – 3. The “threshold problem”, that is, the problem of identifying the boundary between Bayes estimators which tend to outperform standard frequentist estimators and Bayes estimators which don’t, is formulated in an analytically tractable way in Chapter 4. The formulation includes a specific (decision-theory based) criterion for comparing estimators. The centerpiece of the monograph is Chapter 5 in which, under quite general conditions, an explicit solution to the threshold problem is obtained for the case of estimating a scalar parameter under squared error loss. Chapters 6 – 11 treat a collection of other contexts in which the threshold problem can be productively addressed. Included are treatments of the Bayesian consensus problem, extensions of the univariate results obtained in Chapter 5 under a symmetric loss function to estimation problems involving of multi-dimensional parameters and/or asymmetric loss, the estimation of nonidentifiable parameters, empirical Bayes methods for combining data from ‘similar’ experiments and linear Bayes methods for combining data from ‘related’ experiments. The developments in Chapter 11 are motivated by the problem of handling data from separate developmental and operational tests in military acquisitions processes. Chapter 12 provides an overview of the monograph’s highlights and a discussion of areas and problems in need of further research.

(7) Skewness and dispersion among convolutions of independent gamma variables.

A vector x ~ R(n) reciprocallly majorizes another vector y ~ R(n) (written as x >>(rm) y) if the SUM (i = 1 to j) of terms 1/x(i) exceeds the SUM (i = 1 to j) of terms 1/y(i) for j = 1, 2,...., n, where x(1) <....< x(n) and y(1) <....< y(n).

Suppose that a = 1, and let , i = 1,..., n, be independent random variables such that {X(a, lambda(i))} have the gamma distributions with parameters a and lambda(i) for i = 1, ... , n, respectively, with common shape parameter a. Let Y = SUM (i = 1 to n) of terms X(a, lambda(i)) and Y* = SUM (i = 1 to n) of X(a,lambda*(i)). In this work, we show that if lambda >>(rm) lambda*, then Y is greater than Y* relative to the “right spread order” as well as relative to the “mean residual order”. We also prove that the vector of reciprocals of the vector lambda majorizes the vector of reciprocals of the vector lambda*, then Y is greater than Y* relative to the “new better than used order” as well as relative to the “Lorenz order”. These results generalize recent work by Koch and Xu (2009) and Zhao and Balakrishnan (2009) for convolutions of independent exponential random variables to convolutions of independent gamma variables with a common shape parameter greater than equal to 1 (that is, the “increasing failure rate” (IFR) members of this parametric family).

(8) Signature-based representations for the reliability of systems with heterogeneous components.

Signature based representations of the reliability functions of coherent systems with i.i.d. component lifetimes have proven very useful in studying the aging characteristics of such systems and in comparing the performance of different systems under varied criteria. In this paper, we consider extensions of these results to systems with heterogeneous components. New
representation theorems are established for both the case of components with independent but differing lifetimes (the so-called i.n.i.d. case) and for the case of component lifetimes under specific forms of dependence. Conditions are given under which the lifetimes of different systems with non-i.i.d. component lifetimes via order restrictions such as stochastic, hazard rate and likelihood ratio ordering. This paper contains the very useful but surprising result that the reliability function of a coherent system in n independent components can be written explicitly as a function of the signature of the system (a measure which is defined under the assumption of i.i.d. component lifetimes) and the reliability functions of the order statistics associated with a random sample drawn from a distribution G that depends solely on the individual component distributions \( F(1), F(2), \ldots, F(n) \).

(9) Network reliability.

In communication networks, "connectivity" is the quality characteristic of primary interest. A network with v vertices and n edges is denoted by the symbol \( G(v, n) \). A network with k "distinguished vertices", where \( k \in \{2, \ldots, n\} \), is connected if there is at least one set of functioning edges providing a path from any of the k distinguished terminals to any other. As is commonly assumed, vertices function with certainty, while each edge has a fixed probability of functioning at any given point in time. It is also assumed that, at a fixed point in time, edges work independently with a common probability \( p = p(t) \). Under this assumption, the reliability of a network with n edges may be written as an nth degree polynomial in p. Two networks can be compared through these polynomials, and the search for a uniformly optimal network (UON) could, at least conceptually, be based on them. In Boland, Samaniego and Vestrup (2003), a new tool, the signature of a network, is introduced. Like signatures of coherent systems, the signature of a network is the probability distribution \( s \) on the integers \( \{1, \ldots, n\} \) with \( s_i = P(T = X_i:n) \) for \( i = 1, 2, \ldots, n \), where \( X_1:n < X_2:n < \ldots < X_n:n \) are the order statistics from a random sample of the lifetimes of edges drawn from a continuous distribution \( F \), and \( T \) is the lifetime of the network. A network's signature depends on the type of connectivity that is required of the network.

Approaching the identification of uniformly optimal networks (UON) via stochastic ordering showed early promise (see Boesch et al., Networks, 1991) but was found to be yield inconclusive results (it was found that there exists infinite collection of network classes that do not contain any network that was uniformly optimal relative to a stochastic ordering criterion). Specifically, Myrvold, Cheung, Page and Perry (Networks, 1991) provided a collection of examples showing that for some classes of networks, e.g., for the network class \( G(v, C(v, 2) - v/2 - 1) \), where \( C(v, 2) \) is "v choose 2", where v is an even value exceeding 5, a UON does not exist. They demonstrated the existence of a network in each class which dominated every other network in the class for p sufficiently large, but was inferior to an alternative network for p sufficiently small. Recently, Samaniego and McAssey (2011) reexamined the class of \( G(6, 11) \) networks (the Myrvold et al. class with \( v = 6 \)), taking an alternative approach which replaces the stochastic ordering metric by the stochastic precedence metric. The \( G(6, 11) \) class contains 1365 possible network designs. The signatures of these \( G(6, 11) \) networks were shown to be totally ordered in stochastic precedence. A particular network, singled out by Myrvold et al. as optimal for sufficiently large p but suboptimal for sufficiently small p (all relative to stochastic ordering), is shown to be uniformly optimal in the \( G(6, 11) \) class relative to the stochastic precedence ordering. This striking finding breathes new life into the problem of identifying uniformly optimal networks within various classes of networks. It demonstrates that the stochastic ordering criterion is too strong to be a useful metric in comparisons of network reliability, but that such comparisons could be definitively made using the alternative criterion of stochastic precedence. Continuing research in this area is focused on determining the extent to which these findings generalize.

(10) A proof of the "no internal zeros" property of system signatures.

Consider a coherent system in n components having independent, identically distributed (i.i.d.) lifetimes. The signature of the system is an n-dimensional vector \( s = (s(1), \ldots, s(n)) \) representing the probability distribution of the index of the ordered component failure which causes the system to fail. Ross et al. (Mathematics of Operations Research, 1980) established that the number \( N \) of failed components at the time a system fails has the discrete IFRA property. This fact implies that the signature of a system of arbitrary order n cannot have internal zeros, that is, there exist no integers i in the set \( \{1, \ldots, n-2\} \) and j in the set \( \{2, \ldots, n-i\} \) for which \( s(i) > 0 \) and \( s(i+j) > 0 \) while \( s(i+1) = s(i+2) = \ldots = s(i+j-1) = 0 \). The proof given by Ross et al. (1980) is clever and elegant, but it uses rather sophisticated mathematical tools which make the intuition behind the NIZ property somewhat difficult to discern. Since the no-internal-zeros property (of system signatures) is of independent interest and has some practical utility, a direct proof of the property from basic principles and one which provides some intuitive understanding of the property would have both didactic and practical value. In this note, an elementary proof of the property is provided. The property is also shown to hold for certain mixed systems, that is, for certain stochastic mixtures of coherent systems.
Publications by Principal Investigator F. J. Samaniego in years 2008 – 2011
under the support of ARO Grant W911-08-1-0077

BOOKS:


ARTICLES AND BOOK CHAPTERS


[18] “On Universally Optimal Networks: A Reversal of Fortune?”, submitted for publication (with M. McAssey)
