

# JOINT PHY-MAC DESIGN FOR OPPORTUNISTIC SPECTRUM ACCESS WITH MULTI-CHANNEL SENSING

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## ABSTRACT

This paper addresses the design of opportunistic spectrum access (OSA) where secondary users are allowed to sense and access multiple channels in the spectrum without causing unacceptable interference to primary users. Integrated in OSA design are a spectrum sensor at the physical (PHY) layer and a sensing and an access strategy at the MAC layer. Within the framework of partially observable Markov decision process, we develop a separation principle for the joint OSA design, leading to an explicit optimal design of the spectrum sensor and a closed-form optimal access strategy when spectrum sensor and access strategy are designed independently across channels. We also propose two heuristic approaches that exploit the correlation among channel occupancies, one at the PHY layer and the other at the MAC layer. Simulation results indicate that the exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer, and that the detection capability of the spectrum sensor can be improved by exploiting MAC layer information.

## 1. INTRODUCTION

Opportunistic spectrum access (OSA) exploits temporal and spatial spectrum opportunities resulting from the bursty traffic of primary users and guard bands in space. It has captured increasing attention recently due to its potential in improving spectrum efficiency [1]. As shown in [2, 3], basic design components of OSA include (i) a spectrum sensor at the physical (PHY) layer, which identifies spectrum opportunities; (ii) a sensing strategy at the MAC layer, which specifies which channels in the spectrum to sense; and (iii) an access strategy, also at the MAC layer, which determines whether to access based on potentially erroneous sensing outcomes. The design objective is to maximize the throughput of secondary users under the constraint that the probability of colliding with primary users is capped below a certain threshold.

Most existing work on OSA focuses on the cognitive MAC design under the ideal assumption that the spectrum sensor is perfect [4, 5]. Recently, within the framework of partially observable Markov decision process (POMDP), the design of

the PHY layer spectrum sensor has been integrated into the MAC design for optimal OSA in the presence of sensing errors [2, 3]. In [2], a separation principle is established for OSA with single-channel sensing, which leads to an explicit optimal design of the spectrum sensor and a closed-form optimal access strategy.

This paper extends the results in [2] to the scenario where secondary users can sense and access multiple channels simultaneously. We show that when the spectrum sensor and the access strategy are designed independently across channels, the separation principle developed in [2] still holds for OSA with multi-channel sensing. We, however, note that such independent design is suboptimal since it ignores the correlation among channel occupancies. We thus propose two heuristic approaches to exploit channel correlation, one at the PHY layer and the other at the MAC layer. Simulation results indicate that the exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer. We also find that the performance of the PHY layer spectrum sensor improves over time by incorporating the MAC layer sensing and access decisions. These observations illustrate the two-way interaction between the PHY and the MAC layers: the necessity of incorporating the sensor operating characteristics into the MAC design and the benefit of exploiting the MAC layer information in the PHY layer design.

## 2. PROBLEM STATEMENT AND FORMULATION

### 2.1. Network Model

Consider a spectrum of  $N$  orthogonal channels licensed to a slotted primary network. Let  $S_n(t) \in \{0 \text{ (busy)}, 1 \text{ (idle)}\}$  denote the occupancy of channel  $n$  in slot  $t$ . We assume that the spectrum occupancy  $\mathbf{S}(t) \triangleq [S_1(t), \dots, S_N(t)]$  follows a discrete-time homogeneous Markov process with finite state space  $\mathbb{S} \triangleq \{0, 1\}^N$ . The transition probabilities are denoted as  $\{P_{\mathbf{s}, \mathbf{s}'}\}_{\mathbf{s}, \mathbf{s}' \in \mathbb{S}}$ , where  $P_{\mathbf{s}, \mathbf{s}'} \triangleq \Pr\{\mathbf{S}(t+1) = \mathbf{s}' \mid \mathbf{S}(t) = \mathbf{s}\}$  is the probability that the spectrum occupancy state transits from  $\mathbf{s} \in \mathbb{S}$  to  $\mathbf{s}' \in \mathbb{S}$ . We assume that the transition probabilities are known and remain unchanged in  $T$  slots<sup>1</sup>.

This work was supported in part by the Army Research Laboratory CTA on Communication and Networks under Grant DAAD19-01-2-0011 and by the National Science Foundation under Grants CNS-0627090 and ECS-0622200.

<sup>1</sup>The robustness of the optimal OSA design to inaccurate transition probabilities has been demonstrated in [2]. When the transition probabilities are unknown, formulations and algorithms for POMDP with an unknown model exist in the literature [6] and can be applied to this problem.

## Report Documentation Page

*Form Approved*  
*OMB No. 0704-0188*

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1. REPORT DATE <b>JUN 2007</b>	2. REPORT TYPE	3. DATES COVERED <b>00-00-2007 to 00-00-2007</b>	
4. TITLE AND SUBTITLE <b>Joint PHY-MAC Design for Opportunistic Spectrum Access with Multi-Channel Sensing</b>		5a. CONTRACT NUMBER	
		5b. GRANT NUMBER	
		5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)		5d. PROJECT NUMBER	
		5e. TASK NUMBER	
		5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>University of California, Department of Electrical and Computer Engineering, Davis, CA, 95616</b>		8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S)	
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>			
13. SUPPLEMENTARY NOTES <b>in Proceedings of the 8th IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), 17-20 June 2007, Helsinki, Finland</b>			
14. ABSTRACT <b>This paper addresses the design of opportunistic spectrum access (OSA) where secondary users are allowed to sense and access multiple channels in the spectrum without causing unacceptable interference to primary users. Integrated in OSA design are a spectrum sensor at the physical (PHY) layer and a sensing and an access strategy at the MAC layer. Within the framework of partially observable Markov decision process we develop a separation principle for the joint OSA design leading to an explicit optimal design of the spectrum sensor and a closed-form optimal access strategy when spectrum sensor and access strategy are designed independently across channels. We also propose two heuristic approaches that exploit the correlation among channel occupancies, one at the PHY layer and the other at the MAC layer. Simulation results indicate that the exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer and that the detection capability of the spectrum sensor can be improved by exploiting MAC layer information.</b>			
15. SUBJECT TERMS			
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>	
			18. NUMBER OF PAGES <b>21</b>
			19a. NAME OF RESPONSIBLE PERSON

We consider a secondary ad hoc network whose users independently search for and exploit instantaneous spectrum opportunities in these  $N$  channels<sup>2</sup>. Specifically, at the beginning of slot  $t$ , a secondary user with data to transmit chooses a set  $\mathcal{A}(t) \subset \{1, \dots, N\}$  of channels to sense, where  $|\mathcal{A}(t)| = L$  and  $1 \leq L \leq N$ . Based on the imperfect sensing outcomes  $\Theta_{\mathcal{A}}(t) \triangleq \{\Theta_n(t)\}_{n \in \mathcal{A}(t)} \in \{0 \text{ (busy)}, 1 \text{ (idle)}\}^L$ , the secondary user decides whether to access each sensed channel:  $\Phi_{\mathcal{A}}(t) \triangleq \{\Phi_n(t)\}_{n \in \mathcal{A}(t)} \in \{0 \text{ (no access)}, 1 \text{ (access)}\}^L$ . A collision with primary users happens when the secondary user accesses a busy channel. Collisions among secondary users are resolved via carrier sensing, *i.e.*, the secondary user with the smallest backoff time transmits. At the end of this slot, the receiver acknowledges every successful transmission. We let  $\mathbf{K}_{\mathcal{A}}(t) \triangleq \{K_n(t)\}_{n \in \mathcal{A}(t)} \in \{0 \text{ (no ACK)}, 1 \text{ (ACK)}\}^L$ . We assume that the acknowledgement is error-free. That is, acknowledgement  $K_n(t) = 1$  is received if and only if the secondary user accesses an idle channel. Our goal is to design an OSA strategy for the secondary user, which sequentially specifies which channels in the spectrum to sense, which spectrum sensor to use, and which sensed channels to access.

## 2.2. Basic Components of OSA

Integrated in the design of OSA are three basic components: a spectrum sensor, a sensing strategy, and an access strategy. *Spectrum Sensor* Suppose that a set  $\mathcal{A}(t)$  of channels are chosen at the beginning of slot  $t$ , where  $|\mathcal{A}(t)| = L \geq 1$ . The spectrum sensor detects the occupancies of chosen channels by performing a  $2^L$ -ary hypothesis test:

$$\mathcal{H}_0 : \mathbf{S}_{\mathcal{A}}(t) = [1, \dots, 1], \dots, \mathcal{H}_{2^L-1} : \mathbf{S}_{\mathcal{A}}(t) = [0, \dots, 0],$$

where  $\mathbf{S}_{\mathcal{A}}(t) \triangleq \{S_n(t)\}_{n \in \mathcal{A}(t)} \in \{0, 1\}^L$  denotes the occupancies of the chosen channels  $\mathcal{A}(t)$  in the current slot. The probabilities of these hypotheses are learned from the entire decision and observation history of the secondary user.

Sensing errors occur if the spectrum sensor mistakes one hypothesis for another. Since there are a total of  $2^L$  hypotheses, the performance of the spectrum sensor can be specified by a set of  $2^L \times (2^L - 1)$  error probabilities. In the presence of noise and fading, perfect sensing cannot be attained. Hence, the optimal design of the spectrum sensor should achieve a tradeoff among these  $2^L \times (2^L - 1)$  error probabilities.

*Sensing and Access Strategies* A sensing strategy specifies, in each slot  $t$ , a set  $\mathcal{A}(t)$  of channels to be sensed, where  $\mathcal{A}(t) \subset \{1, \dots, N\}$  and  $|\mathcal{A}(t)| = L$ . An access strategy decides a set of transmission probabilities  $\{f_n(\boldsymbol{\theta}_{\mathcal{A}}, t)\}$ , where

$$f_n(\boldsymbol{\theta}_{\mathcal{A}}, t) \triangleq \Pr\{\Phi_n(t) = 1 | \Theta_{\mathcal{A}}(t) = \boldsymbol{\theta}_{\mathcal{A}}\}$$

is the probability of accessing sensed channel  $n \in \mathcal{A}(t)$  when the sensing outcomes are given by  $\boldsymbol{\theta}_{\mathcal{A}}(t) = \{0, 1\}^L$  in slot  $t$ .

<sup>2</sup>The design of cooperative OSA strategies for secondary users can be formulated by a decentralized POMDP problem [7]. We point out that while cooperation among secondary users might improve the overall network throughput, such cooperative design is much more complex.

## 2.3. POMDP Formulation

The joint design of OSA with multi-channel sensing can be formulated as a POMDP problem.

*Reward* Assume that the expected number of bits that can be delivered by the secondary user in a slot is proportional to the channel bandwidth. Given sensing action  $\mathcal{A}(t)$ , we define the immediate reward  $R_{\mathbf{K}_{\mathcal{A}}(t)}^{(\mathcal{A}(t))}$  as

$$R_{\mathbf{K}_{\mathcal{A}}(t)}^{(\mathcal{A}(t))} = \sum_{n \in \mathcal{A}(t)} K_n(t) B_n. \quad (1)$$

Hence, the total expected reward represents overall throughput, the total expected number of bits that can be delivered by the secondary user in  $T$  slots.

*Observation* As detailed in [9], the secondary user and its desired receiver must have the same history of observations so that they make the same channel selection decisions without exchanging extra control message. Since sensing errors may cause different sensing outcomes at the transmitter and the receiver, the acknowledgement  $\mathbf{K}_{\mathcal{A}}(t)$  should be used as the common observation in each slot.

*Belief Vector* Within the framework of POMDP, the secondary user's knowledge of the spectrum occupancy based on its decision and observation history can be encoded in a belief vector  $\boldsymbol{\Lambda}(t) \triangleq \{\lambda_{\mathbf{s}}(t)\}_{\mathbf{s} \in \mathbb{S}}$  [8], where  $\lambda_{\mathbf{s}}(t)$  is the conditional probability (given the entire observation history) that the spectrum occupancy is in state  $\mathbf{s} \in \mathbb{S}$  at the beginning of slot  $t$  prior to the state transition. We point out here that based on the belief vector  $\boldsymbol{\Lambda}(t)$  at the beginning of slot  $t$ , the secondary user can calculate the distribution of the current spectrum occupancy  $\mathbf{S}(t)$  as

$$\Pr\{\mathbf{S}(t) = \mathbf{s}\} = \sum_{\mathbf{s}' \in \mathbb{S}} \lambda_{\mathbf{s}'}(t) P_{\mathbf{s}', \mathbf{s}}, \quad \forall \mathbf{s} \in \mathbb{S}. \quad (2)$$

The probabilities of the hypothesis used in the design of the spectrum sensor can thus be determined from (2).

*Objective* The design objective is to maximize the total expected reward in  $T$  slots under the constraint that the probability  $P_n(t)$  of collision perceived by the primary network in any channel  $n$  and any slot  $t$  is capped below a threshold  $\zeta$ :

$$\begin{aligned} \max \mathbb{E} \left[ \sum_{t=1}^T R_{\mathbf{K}_{\mathcal{A}}(t)}^{(\mathcal{A}(t))} | \boldsymbol{\Lambda}(1) \right] \\ \text{s.t. } P_n(t) \triangleq \Pr\{\Phi_n(t) = 1 | S_n(t) = 0\} \leq \zeta, \quad \forall t, n, \end{aligned} \quad (3)$$

where  $\boldsymbol{\Lambda}(1)$  is the initial belief vector, which represents the information on the initial spectrum occupancy. Note that when  $\Pr\{S_n(t) = 0\} = 0$ , no collision will occur and the optimal access decision is straightforward:  $\Phi_n(t) = 1$ .

## 3. SEPARATION PRINCIPLE

The constrained POMDP given in (3) appears to be intractable due to the high-dimensional and uncountable action space. In this section, we show that under certain conditions, there exists a separation principle for the optimal joint design of OSA, which enables us to obtain an explicit optimal design of the spectrum sensor and a closed-form optimal access strategy.

*Theorem 1: When the spectrum sensor and the access strategy are designed independently across channels, the optimal joint design of OSA with multi-channel sensing can be carried out in two steps:*

1. Choose the spectrum sensor and the access strategy to maximize the expected immediate reward subject to the collision constraint.
2. Choose the sensing strategy to maximize the expected total reward.

*In this case, the optimal spectrum sensor is given by the optimal Neyman-Pearson (NP) detector with probability of miss detection equal to  $\zeta$ , which detects the channel occupancy by using only the measurements from this channel, and the optimal access decision is to trust the sensing outcome from this channel. The optimal sensing strategy can be obtained by solving an unconstrained POMDP.*

*Proof:* See [9].  $\square\square\square$

We emphasize that the extension of the separation principle developed in [2] to the multi-channel sensing scenarios is based on the assumption that the spectrum sensor and the access strategy are designed independently across channels. Specifically, we assume that the occupancy of a channel is detected without taking into account the measurements of other channels and the access decision on a channel is made independently of the sensing outcomes from other channels. Intuitively, under the above assumption, the design of the spectrum sensor and the access strategy for the multi-channel sensing case can be treated as  $L$  independent design problems, one for each chosen channel. Hence, the optimal design for the single-channel  $L = 1$  sensing case can be extended to  $L > 1$ .

Theorem 1 provides sufficient conditions under which the design given by the separation principle (referred to as the SP approach for simplicity) is optimal. In Proposition 1, we show that the SP approach is locally optimal (*i.e.*, maximizes the expected immediate reward) under certain relaxed conditions.

*Proposition 1: Suppose that the spectrum sensor is designed independently across channels while the access strategy jointly exploits the sensing outcomes from all channels. The SP approach is locally optimal when channel occupancies are independent.*

*Proof:* See [9].  $\square\square\square$

#### 4. HEURISTIC APPROACHES

While simplifying the joint design of OSA with multi-channel sensing, the condition that the spectrum sensor and the access strategy are designed independently across channels result in throughput degradation since the correlation among channel occupancies is ignored. We propose two heuristic approaches to exploit the channel correlation: the PHY layer and the MAC layer approaches.

##### 4.1. The PHY Layer Approach

When the spectrum occupancy states are correlated across channels, we have correlated channel measurements at the

PHY layer. Hence, the channel correlation can be exploited by using the measurements of all chosen channels in occupancy detection. With this in mind, we propose a heuristic design of the spectrum sensor: it adopts the optimal NP detector with probability of miss detection equal to  $\zeta$ , which is designed to uses all channel measurements. We note that the structure of the optimal NP detector adopted by this sensor relies on the joint distribution of the spectrum occupancy states, which is given by the belief vector (see Section 5 for an example). That is, this heuristic sensor design is affected by the observation and decision history of the secondary user and thus improves over time due to accumulated observations (see Figure 1).

Based on the sensing outcomes given by the above spectrum sensor, the secondary user can adopt the access strategy of the SP approach: to access if and only if the channel is sensed as idle. We refer this approach as the PHY layer approach. Proposition 2 provides a sufficient condition under which this PHY layer approach is locally optimal.

*Proposition 2: Suppose that the access strategy is designed independently across channels while the spectrum sensor jointly exploits the measurements taken from all chosen channels. The PHY layer approach is locally optimal. When channel occupancies are independent, the PHY layer approach reduces to the SP approach.*

*Proof:* See [9].  $\square\square\square$

##### 4.2. The MAC Layer Approach

When channel occupancies are correlated, so are the sensing outcomes given by the spectrum sensor. Hence, the channel correlation can also be exploited at the MAC layer by making access decisions jointly across channels. A heuristic MAC layer approach is to adopt the SP sensor, which detects the channel occupancy by using only the measurements of this channel, and then choose the access decisions that exploit sensing outcomes from all chosen channels to maximize the expected immediate reward. Specifically, for any chosen channels  $\mathcal{A}(t)$  and any belief vector  $\mathbf{\Lambda}(t)$  in slot  $t$ , we choose the set of transmission probabilities as follows

$$\begin{aligned} \{\hat{f}_n(\boldsymbol{\theta}_{\mathcal{A}}, t)\} &= \arg \max_{f_n(\boldsymbol{\theta}_{\mathcal{A}}) \in [0,1]} \mathbb{E}[R_{\mathbf{K}_{\mathcal{A}}(t)}^{\mathcal{A}(t)} | \mathbf{\Lambda}(t)] \\ &= \arg \max_{f_n(\boldsymbol{\theta}_{\mathcal{A}}) \in [0,1]} \sum_{n \in \mathcal{A}(t)} B_n \Pr\{S_n(t) = 1\} \\ &\quad \times \sum_{\boldsymbol{\theta}_{\mathcal{A}}, \mathbf{s}_{\mathcal{A}}} h_{\mathbf{S}_{\mathcal{A}} | S_n}(\mathbf{s}_{\mathcal{A}} | 1) l_{\boldsymbol{\Theta}_{\mathcal{A}} | \mathbf{S}_{\mathcal{A}}}(\boldsymbol{\theta}_{\mathcal{A}} | \mathbf{s}_{\mathcal{A}}) f_n(\boldsymbol{\theta}_{\mathcal{A}}), \\ \text{s.t. } P_n(t) &= \sum_{\boldsymbol{\theta}_{\mathcal{A}}, \mathbf{s}_{\mathcal{A}}} h_{\mathbf{S}_{\mathcal{A}} | S_n}(\mathbf{s}_{\mathcal{A}} | 0) \\ &\quad \times l_{\boldsymbol{\Theta}_{\mathcal{A}} | \mathbf{S}_{\mathcal{A}}}(\boldsymbol{\theta}_{\mathcal{A}} | \mathbf{s}_{\mathcal{A}}) f_n(\boldsymbol{\theta}_{\mathcal{A}}) \leq \zeta, \quad \forall n \in \mathcal{A}(t), \end{aligned}$$

where  $h_{\mathbf{S}_{\mathcal{A}} | S_n}(\mathbf{s}_{\mathcal{A}} | i) \triangleq \Pr\{\mathbf{S}_{\mathcal{A}}(t) = \mathbf{s}_{\mathcal{A}} | S_n(t) = i\}$ ,  $i = 0, 1$ , is the conditional distribution of the channel occupancy states  $\mathbf{S}_{\mathcal{A}}(t)$ , which can be obtained from the belief vector via (2), and  $l_{\boldsymbol{\Theta}_{\mathcal{A}} | \mathbf{S}_{\mathcal{A}}}(\boldsymbol{\theta}_{\mathcal{A}} | \mathbf{s}_{\mathcal{A}}) \triangleq \Pr\{\boldsymbol{\Theta}_{\mathcal{A}}(t) = \boldsymbol{\theta}_{\mathcal{A}} | \mathbf{S}_{\mathcal{A}}(t) = \mathbf{s}_{\mathcal{A}}\}$

is the sensing error probability determined by the operating characteristics of the SP sensor.

We can obtain the above access strategy via linear programming. Proposition 3 shows that this MAC layer approach is equivalent to the SP approach when channel occupancies are independent. This agrees with our intuition that when channels are independent, so are the sensing outcomes from the chosen channels. Hence, independent access decision-making performs as well as the joint one in terms of immediate reward.

*Proposition 3: Suppose that the spectrum sensor is designed independently across channels while the access strategy jointly exploits the sensing outcomes from all chosen channels. When channel occupancies are independent, the MAC layer approach reduces to the SP approach and hence is locally optimal.*

*Proof:* See [9].  $\square\square\square$

## 5. SIMULATION EXAMPLES

In this section, we compare the performance of the SP, the PHY layer, and the MAC layer approaches. Since these three approaches differ in the spectrum sensor and the access strategy, we can employ any sensing strategy to compare their performance. For simplicity, we adopt a myopic sensing strategy that chooses, in each slot, the set  $\mathcal{A}$  of channels that maximizes the expected immediate reward under perfect sensing,

$$\mathcal{A} = \arg \max_{\mathcal{A} \in \mathcal{A}_s} \sum_{n \in \mathcal{A}} B_n \Pr\{S_n(t) = 1\}. \quad (4)$$

We model both the background noise and the primary signal as white Gaussian processes. Let  $\sigma_0^2$  and  $\sigma_1^2$  denote the noise power and the primary signal power, respectively. At the beginning of each slot, the spectrum sensor takes  $M$  independent measurements  $\mathbf{Y}_n \triangleq [Y_{n,1}, \dots, Y_{n,M}]$  from each chosen channel  $n \in \mathcal{A}$ .

For each chosen channel  $n \in \mathcal{A}$ , the SP sensor uses the corresponding measurements  $\mathbf{Y}_n$  and performs the following hypothesis test:

$$\begin{aligned} \mathcal{H}_0(S_n = 1) : \quad & \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}_M, \sigma_0^2 \mathbf{I}_M), \\ \mathcal{H}_1(S_n = 0) : \quad & \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}_M, (\sigma_1^2 + \sigma_0^2) \mathbf{I}_M), \end{aligned} \quad (5)$$

where  $\mathcal{N}(\mathbf{0}_M, \sigma^2 \mathbf{I}_M)$  denotes an  $M$ -dimensional Gaussian distribution with identical mean 0 and variance  $\sigma^2$  in each dimension. It can be readily shown that the optimal NP detector is given by an energy detector [10, Sec. 2.6.2]:

$$\|\mathbf{Y}_n\|_2 = \sum_{i=1}^M Y_{n,i}^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \eta_n. \quad (6)$$

The probabilities of false alarm  $\epsilon_n$  and miss detection  $\delta_n$  of the energy detector can be calculated by [10, Sec. 2.6.2]:

$$\delta_n = \gamma\left(\frac{M}{2}, \frac{\eta_n}{2(\sigma_0^2 + \sigma_1^2)}\right), \quad \epsilon_n = 1 - \gamma\left(\frac{M}{2}, \frac{\eta_n}{2\sigma_0^2}\right),$$

where  $\gamma(m, a) = \frac{1}{\Gamma(m)} \int_0^a t^{m-1} e^{-t} dt$  is the incomplete gamma function. The optimal decision threshold  $\eta_n^*$  of the energy detector is chosen so that the probability of miss detection is fixed at  $\delta_n = \zeta$ .

On the other hand, the sensor of the PHY layer approach uses all channel measurements  $\{\mathbf{Y}_n\}_{n \in \mathcal{A}}$  and performs a composite hypothesis test for each chosen channel  $n \in \mathcal{A}$ :

$$\begin{aligned} \mathcal{H}_0(S_n(t) = 1) : \quad & \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}_M, \sigma_0^2 \mathbf{I}_M), \\ \mathbf{Y}_m & \sim \mathcal{N}(\mathbf{0}_M, (\sigma_0^2 + 1_{[s_m=0]} \sigma_1^2) \mathbf{I}_M), \quad \forall m \in \mathcal{A} \setminus \{n\} \\ \mathcal{H}_1(S_n(t) = 0) : \quad & \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}_M, (\sigma_0^2 + \sigma_1^2) \mathbf{I}_M), \\ \mathbf{Y}_m & \sim \mathcal{N}(\mathbf{0}_M, (\sigma_0^2 + 1_{[s_m=0]} \sigma_1^2) \mathbf{I}_M), \quad \forall m \in \mathcal{A} \setminus \{n\}, \end{aligned} \quad (7)$$

The distribution of the channel occupancy states  $\mathbf{S}_{\mathcal{A}}(t)$  under each hypothesis is given by  $h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} | i)$ . The optimal NP detector for (7) is a likelihood ratio test [10, Sec. 2.5]:

$$\frac{\sum_{\mathbf{s}_{\mathcal{A}}} h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} | 0) \prod_{m \in \mathcal{A}} p_m(\mathbf{Y}_m | s_m)}{\sum_{\mathbf{s}_{\mathcal{A}}} h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} | 1) \prod_{m \in \mathcal{A}} p_m(\mathbf{Y}_m | s_m)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \tau_n, \quad (8)$$

where  $p_n(\mathbf{Y}_n | s_n)$  is the probability density function of independent Gaussian channel measurements  $\mathbf{Y}_n$ :

$$p_n(\mathbf{Y}_n | s_n) = \prod_{i=1}^M \frac{1}{\sqrt{2\pi}(\sigma_0^2 + 1_{[s_n=0]} \sigma_1^2)} e^{-\frac{Y_{n,i}^2}{2(\sigma_0^2 + 1_{[s_n=0]} \sigma_1^2)}}.$$

Note that when channel occupancies are independent, the sensor employed by the PHY layer approach is equivalent to the SP sensor since  $h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} | 0) = h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}'_{\mathcal{A}} | 1)$  for any  $\mathbf{s}_{\mathcal{A}}$  and  $\mathbf{s}'_{\mathcal{A}}$  such that  $s_m = s'_m$  when  $m \neq n$ . The error probabilities of this sensor can be evaluated via simulation. In each slot, the optimal detection threshold  $\tau_n^*$  is chosen according to the belief vector so that the resulting probability of miss detection is fixed at  $\zeta$ .

As proven in Propositions 2 and 3, the PHY layer and the MAC layer approaches are equivalent to the SP approach when channel occupancies are independent. We thus compare below the performance of these three approaches in correlated channels. Specifically, we consider  $N = 4$  correlated channels, each with bandwidth  $B_n = 1$ . The transition probabilities of the spectrum occupancy are given by

$$\begin{aligned} P_{[0000],[0111]} &= 0.6, \quad P_{[0000],[0000]} = 0.4, \\ P_{[0111],[0000]} &= P_{[1011],[0000]} = P_{[1101],[0000]} = P_{[1110],[0000]} = 0.2, \\ P_{[0111],[1011]} &= P_{[1011],[1101]} = P_{[1101],[1110]} = P_{[1110],[0111]} = 0.8. \end{aligned}$$

For simplicity, we assume that there is only one secondary user seeking instantaneous spectrum availability. The initial belief vector is set to the stationary distribution of the underlying Markov process. The maximum allowable probability of collision is  $\zeta = 0.05$ . In each slot,  $L = 3$  channels are chosen. The spectrum sensor takes  $M = 1$  measurement of each chosen channel and the noise and the primary signal powers are given by  $\sigma_0^2 = 0$  dB and  $\sigma_1^2 = 10$  dB.

### 5.1. Comparison of Sensor Performance

In Figure 1, we plot the receiver operating characteristic (ROC) curves (probability of false alarm vs. probability of detection) of the SP sensor and the sensor employed by the PHY layer approach. Note that the sensor of the MAC layer approach is the same as the SP sensor. We see that the sensor of the PHY layer approach outperforms the SP sensor. Specifically, for a fixed probability of miss detection, the probability of false alarm of the sensor of the PHY layer approach is much smaller than that of the SP sensor. This is because

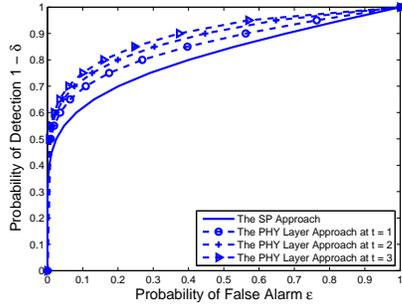


Fig. 1. Comparison of ROC curves.

the PHY layer approach exploits the correlation among channel measurements in occupancy detection while the SP sensor does not. We also observe that the ROC curve of the sensor employed by the PHY layer approach improves in each slot while that of the SP sensor remains the same. This observation can be explained by comparing the optimal detectors (6) and (8). Clearly, the energy detector (6) used by the SP approach is static and so is its performance. However, as seen from (8), the decision variable of the sensor employed by the PHY layer approach depends on the conditional distribution  $h_{S_A|S_n}(s_A | i)$  of the channel occupancies. Hence, its performance varies over time according to the belief vector. As time  $t$  increases, the belief vector and hence the sensor performance improves due to the accumulated observations. Figure 1 demonstrates that the performance of the PHY layer sensor can be improved by incorporating the MAC layer sensing and access decisions encoded in the belief vector.

## 5.2. Comparison of Throughput Performance

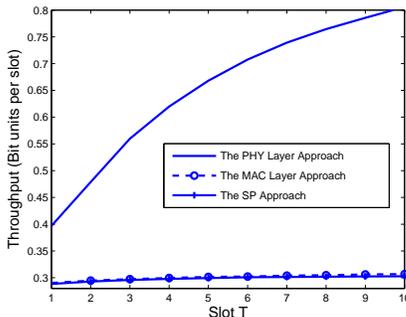


Fig. 2. Comparison of normalized throughput.

In Figure 2, we compare the throughput, measured by the total expected reward per slot, of these three approaches. As expected, the SP approach, which ignores the channel correlation, performs the worst. By jointly exploiting the sensing outcomes in access decision-making, the MAC layer approach can improve throughput performance. A much larger performance gain is achieved by the PHY layer approach which jointly exploits the channel measurements in occupancy detection. We can thus see that the exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer. In other words, independent channel occupancy detection at the PHY layer hurts throughput more than inde-

pendent access decision-making at MAC layer. This agrees with our intuition because the spectrum sensor of the MAC layer approach makes a hard-decision on whether the channel is idle or not. The resulting sensing outcomes are thus less informative than the original channel measurements, leading to throughput degradation.

## 6. CONCLUSION

In this paper, we provided sufficient conditions for the existence of separation principle for optimal design of OSA with multi-channel sensing. We also proposed two heuristic approaches that exploit the correlation among channel occupancies. Simulation examples demonstrated the performance improvement of the PHY layer spectrum sensor over time resulting from the incorporation of the MAC layer sensing and access decisions. We also found that the exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer in improving throughput.

## 7. REFERENCES

- [1] Q. Zhao and B. Sadler, "A survey of dynamic spectrum access: signal processing, networking, and regulatory policy," in *IEEE Signal Processing Magazine*, May 2007; available at <http://arxiv.org/abs/cs/0609149>.
- [2] Y. Chen, Q. Zhao, and A. Swami, "Joint Design and Separation Principle for Opportunistic Spectrum Access," in *Proc. of 39th Annual Asilomar Conference on Signal, Systems, and Computers*, Asilomar, CA, Oct. – Nov. 2006.
- [3] Q. Zhao, L. Tong, A. Swami, and Y. Chen "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *IEEE Journal on Selected Areas in Communications*, Apr. 2007.
- [4] W. Wang and X. Liu, "List-coloring based channel allocation for open-spectrum wireless networks," in *Proc. of the 62nd IEEE Vehicular Technology Conference*, vol. 1, pp. 690-694, Sept. 2005.
- [5] P. Papadimitratos, S. Sankaranarayanan, and A. Mishra, "A bandwidth sharing approach to improve licensed spectrum utilization," *IEEE Communications Magazine*, vol. 43, pp. 10-14, Dec. 2005.
- [6] D. Aberdeen, "A survey of approximate methods for solving partially observable Markov decision processes," Technical Report, National ICT Australia, Dec. 2003. <http://users.rsise.anu.edu.au/~daa/papers.html>.
- [7] D. S. Bernstein, "Complexity analysis and optimal algorithms for decentralized decision making," Ph.D. Dissertation, University of Massachusetts Amherst, Sept. 2005.
- [8] R. Smallwood and E. Sondik, "The optimal control of partially observable Markov processes over a finite horizon," *Operations Research*, vol. 21, pp. 1071–1088, 1973.
- [9] Y. Chen, Q. Zhao, and A. Swami, "Joint design and separation principle for opportunistic spectrum access in the presence of sensing errors," submitted to *IEEE Transactions on Information Theory* in Feb. 2007; available at <http://arxiv.org/pdf/cs/0702158>.
- [10] H. L. V. Trees, "Detection, Estimation, and Modulation Theory, Part I," Wiley-Interscience, Sept., 2001.

# Joint PHY-MAC Design for Opportunistic Spectrum Access with Multi-Channel Sensing

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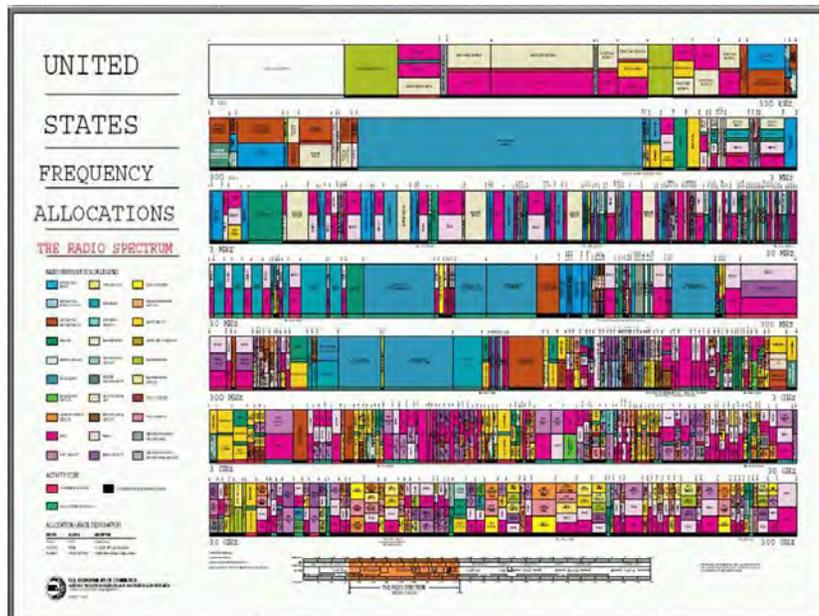
*June 19, 2007*

Supported by NSF and ARL-CTA.

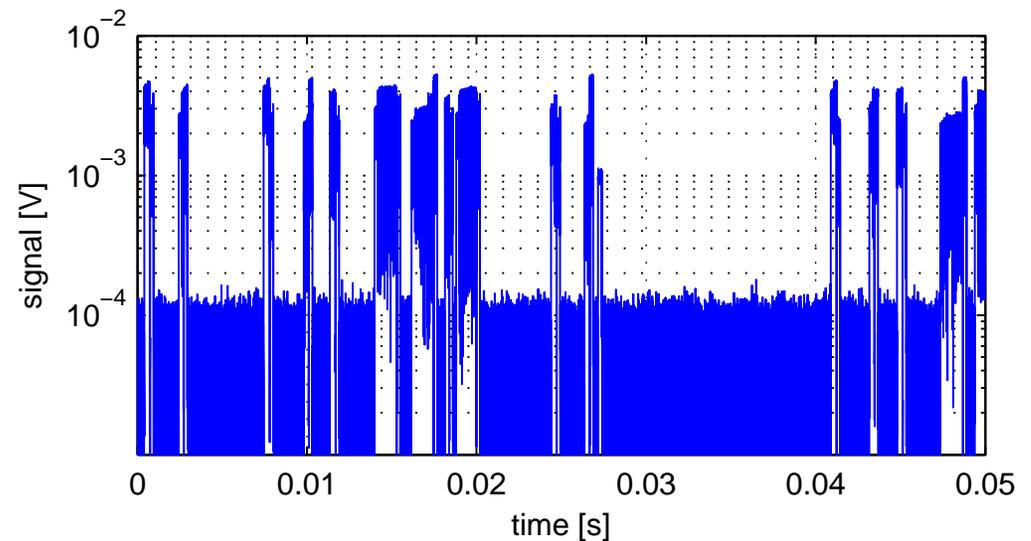
SPAWC'07 SESSION T2B: Cross-Layer Issues: From Physical to Networking Layers.

# Spectrum Scarcity vs. Spectrum Opportunity

## Overly Crowded Spectrum



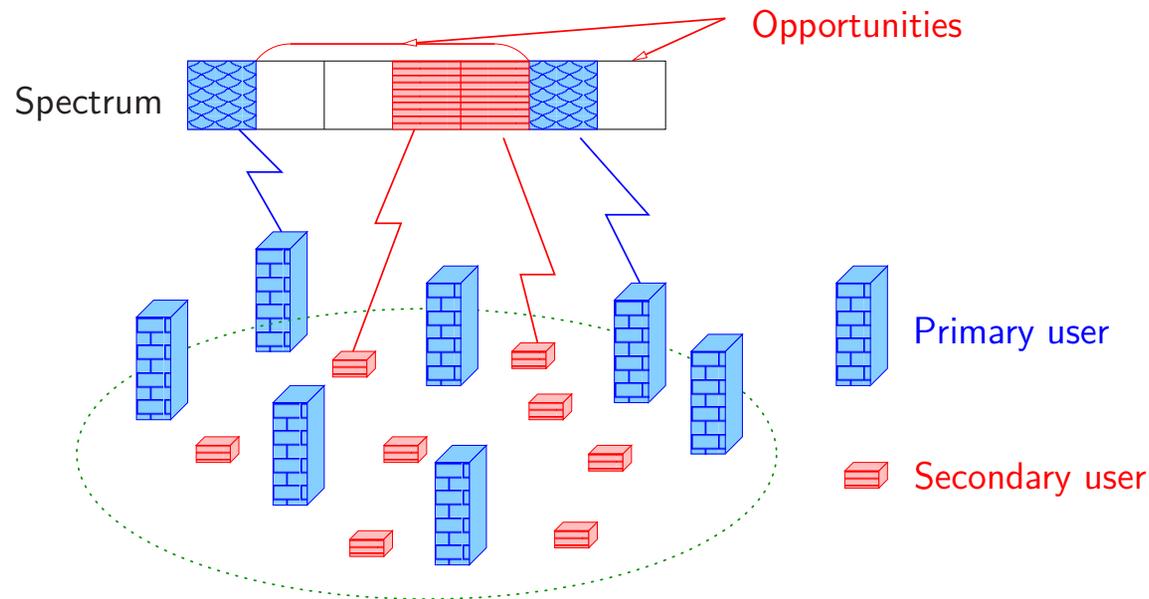
## Pervasive Spectrum Opportunities



Spectrum usage of an active FTP session in a WLAN (ACSP at Cornell).

- ▶ Almost all usable radio frequencies have already been licensed.
- ▶ At any given time and location, a large portion of licensed spectrum lies unused.
  - Over 62% white space exists in the spectrum under 3GHz.
  - About 75% idle time during an active FTP session in WLAN.
  - Up to 90% idle time during voice-over-IP applications such as Skype.

# Opportunistic Spectrum Access (OSA)



## Basic idea:

Allow secondary users to exploit spectrum opportunities.

## Design objective:

Maximize secondary users' throughput while limiting their interference to primary users (licensees).

## Three basic design components:

- ▶ **Spectrum sensor:** opportunity identification (PHY)
- ▶ **Sensing policy:** where in the spectrum to sense (MAC)
- ▶ **Access policy:** whether to tx given sensing errors may occur (MAC)

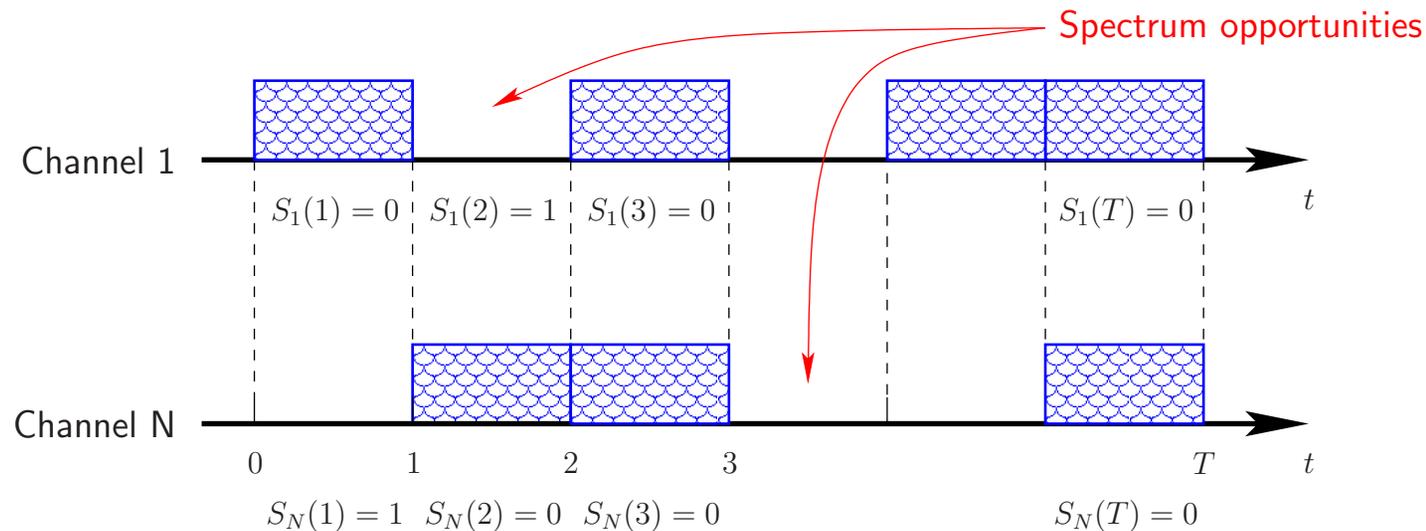
**A decision-theoretic framework for joint PHY-MAC design.**

## Main Results

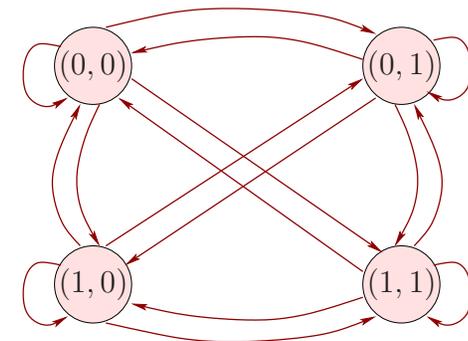
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- ▶ A decision-theoretic framework based on partially observable Markov decision process (POMDP)
  - jointly optimizes the three basic design components
  - captures fundamental design tradeoffs
    - Spectrum sensor:** false alarm vs. miss detection
    - Access Policy:** overlooked opportunity vs. collision
    - Sensing Policy:** gaining access vs. gaining information
- ▶ Structural policies for the joint design
  - Separation principle for single-channel sensing and its extension to multi-channel sensing
  - Explicit optimal sensor design and closed-form optimal access policy.
  - Sensing design reduced from a constrained POMDP to an unconstrained one.
- ▶ Quantitative characterization of the interaction between PHY and MAC
  - Impact of the operating characteristics of spectrum sensor on MAC
  - Exploiting MAC information at PHY for improved sensor performance.

## Network Model



- ▶ A spectrum of  $N$  channels, each with bandwidth  $B_n$ .
- ▶ A slotted primary network
  - Markovian spectrum usage with  $2^N$  states:
 
$$\mathbf{S}(t) \triangleq [S_1(t), \dots, S_N(t)] \in \{0 \text{ (busy), } 1 \text{ (idle)}\}^N.$$
  - Known transition probabilities.
- ▶ An ad hoc secondary network without dedicated control channel
  - Independent users, each can sense and access  $L$  channels in each slot.
  - User obtains a reward  $R(t) = B_n$  for each successful access of idle channels.
  - User collides with primary users if access a busy channel.



## Basic Components and Design Tradeoffs

### Spectrum Sensor: false alarm vs. miss detection

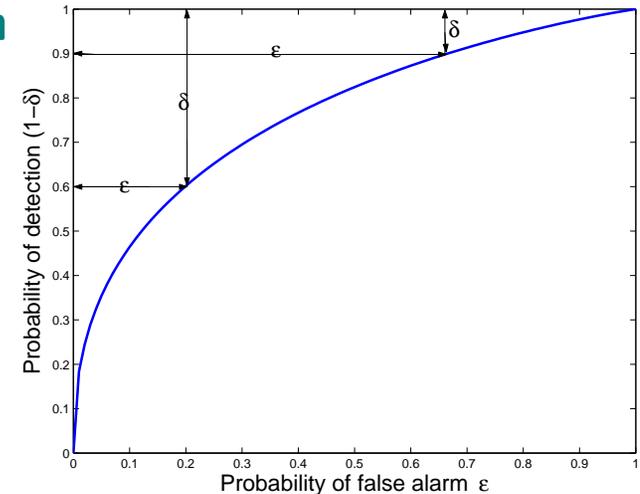
► Binary hypotheses test (for  $L = 1$ ):

$\mathcal{H}_0$  : channel is idle vs.  $\mathcal{H}_1$  : channel is busy

► Two Types of sensing errors:

□ false alarm ( $\epsilon$ ):  $\mathcal{H}_0 \rightarrow \mathcal{H}_1$  (overlook)

□ miss detection ( $\delta$ ):  $\mathcal{H}_1 \rightarrow \mathcal{H}_0$  (misidentification)



► Which point  $(\epsilon, \delta)$  on the ROC curve should the sensor operate? (decision rule)

### Access Policy: overlooked opportunity vs. collision

► Consequences of trusting sensing outcome:

□ false alarm (idle sensed as busy)  $\Rightarrow$  overlooked opportunity

□ miss detection (busy sensed as idle)  $\Rightarrow$  collision

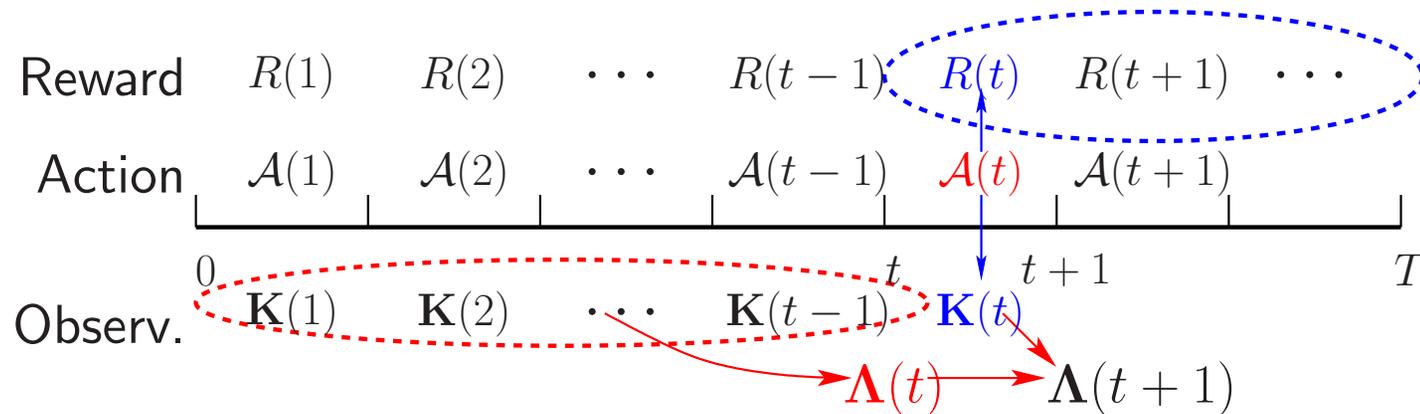
► When and how much to trust the sensor?

$$\text{For } L = 1, \text{ tx prob.} = \begin{cases} p_0 & \text{if idle} \\ p_1 & \text{if busy} \end{cases} \quad \begin{array}{l} p_0 < 1 : \text{conservative} \\ p_1 > 0 : \text{aggressive} \end{array} \Leftrightarrow \text{sensor } \{\epsilon, \delta\}$$

**Joint design of access policy at MAC and spectrum sensor at PHY**

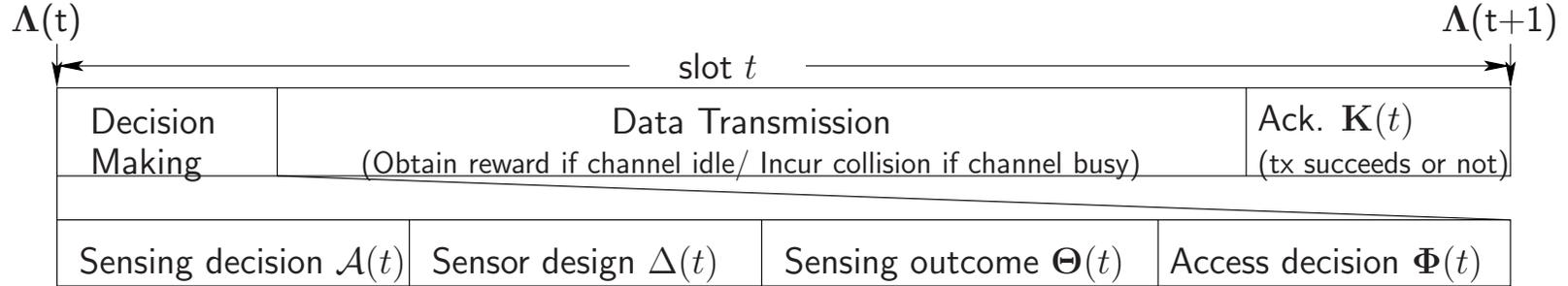
## Basic Components and Design Tradeoffs

### Sensing Policy: gaining access vs. gaining information



- ▶ Each observation  $\mathbf{K}(t) \in \{0(\text{unsuccessful}), 1(\text{successful access})\}^L$  provides partial information on the spectrum usage state.
- ▶ Sensing action  $\mathcal{A}(t)$  should be based on the **conditional distribution**  $\Lambda(t)$  that exploits the entire decision and observation history.
- ▶  $\mathcal{A}(t)$  results in **immediate reward**  $R(t)$  and **observation**  $\mathbf{K}(t)$  that affects future reward.
- ▶ Optimal  $\mathcal{A}(t)$  achieves the best tradeoff between gaining immediate reward and gaining spectrum information.

## A Constrained POMDP Formulation



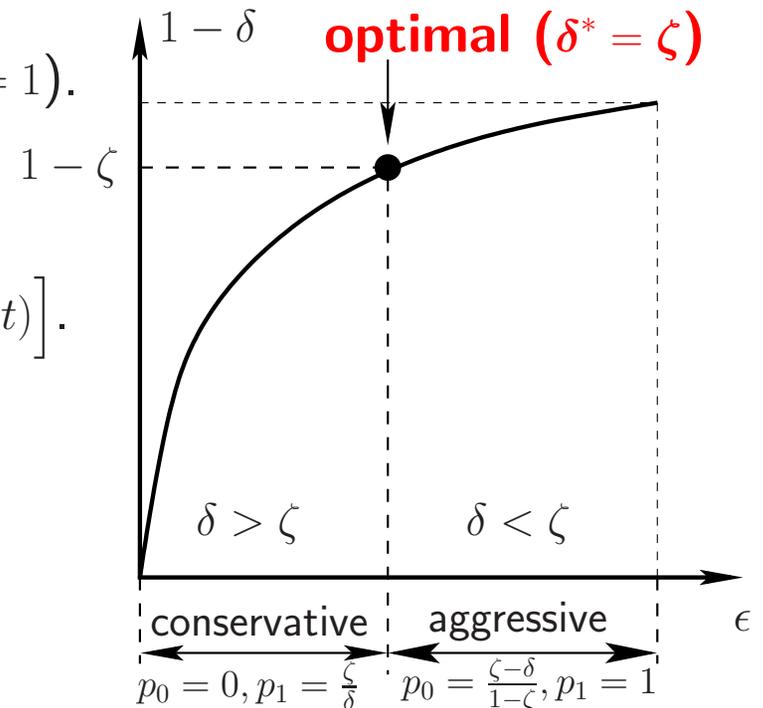
- ▶ **Belief vector**  $\Lambda(t) = \{\Lambda_s(t)\}_{s \in \{0,1\}^N}$ , where  $\Lambda_s(t)$  is the conditional probability that the spectrum is in state  $s$ :  $\{\Lambda(t), \mathcal{A}(t), \mathbf{K}(t)\} \rightarrow \Lambda(t+1)$
- ▶ **Sensing policy**  $\pi_s$ 
  - deterministic:  $\Lambda(t) \rightarrow$  a set  $\mathcal{A}(t)$  of  $L$  channels to be sensed in slot  $t$ .
  - randomized:  $\Lambda(t) \rightarrow$  PMF of  $\mathcal{A}(t)$ .
- ▶ **Spectrum sensor**  $\pi_\delta$ :  $\Lambda(t) \rightarrow$  a decision rule  $\Delta(t)$  used for occupancy detection:
 
$$\{\Delta(t), \text{channel measurements}\} \rightarrow \Theta(t) = \{\Theta_n(t)\}_{n \in \mathcal{A}(t)} \in \{0(\text{busy}), 1(\text{idle})\}^L.$$
- ▶ **Access policy**  $\pi_c$ 
  - deterministic:  $\{\Lambda(t), \Theta(t)\} \rightarrow \Phi(t) \in \{0(\text{no access}), 1(\text{access})\}^L$ .
  - randomized:  $\{\Lambda(t), \Theta(t)\} \rightarrow$  tx probabilities.
- ▶ **Objective:**

$$\{\pi_\delta^*, \pi_s^*, \pi_c^*\} = \arg \max_{\pi_\delta, \pi_s, \pi_c} \mathbb{E} \left[ \sum_{t=1}^T R(t) \right] \quad \text{s.t. collision prob. } P_n(t) \leq \zeta, \quad \forall n \in \mathcal{A}(t) \quad (*)$$

# The Separation Principle for Single-Channel Sensing

**Theorem:**  $\pi_\delta$  and  $\pi_c$  can be decoupled from  $\pi_s$  without losing optimality

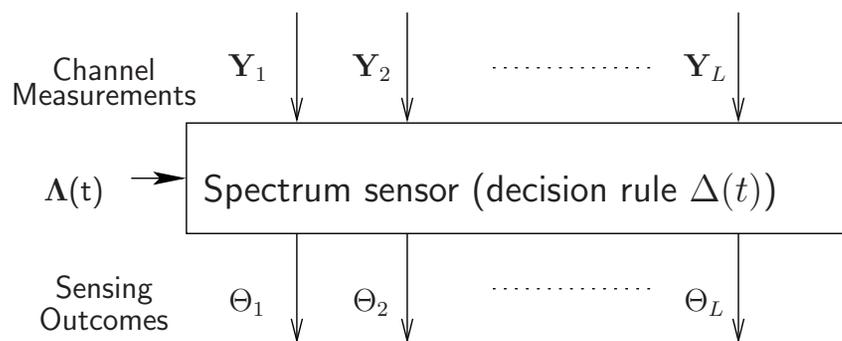
- Choose  $\pi_\delta$  and  $\pi_c$  to max. immediate reward  $R(t)$  and ensure constraint  $P_n(t) = \zeta$ .
  - ⇒ A static optimization problem.
  - ⇒ Explicit optimal design of spectrum sensor:
    - optimal Neyman-Pearson (NP) detector with prob. of missing (PM)  $\delta = \zeta$ .
  - ⇒ Closed-form optimal access policy:
    - trust sensing outcome (tx prob.  $p_0 = 0, p_1 = 1$ ).
- Choose  $\pi_s$  to maximize total reward  $\mathbb{E} \left[ \sum_{t=1}^T R(t) \right]$ .
  - ⇒ An unconstrained POMDP.
  - ⇒ Deterministic sensing policy.



# Two Spectrum Sensor Structures for Multi-Channel Sensing

**Goal:** dynamically choose decision rules  $\Delta(t)$  for spectrum opp. identification.

## Joint opportunity identification



- ▶ Perform a  $2^L$ -ary hypothesis test:

$$\mathcal{H}_0 : \mathbf{S}_{\mathcal{A}}(t) = [1, 1, \dots, 1],$$

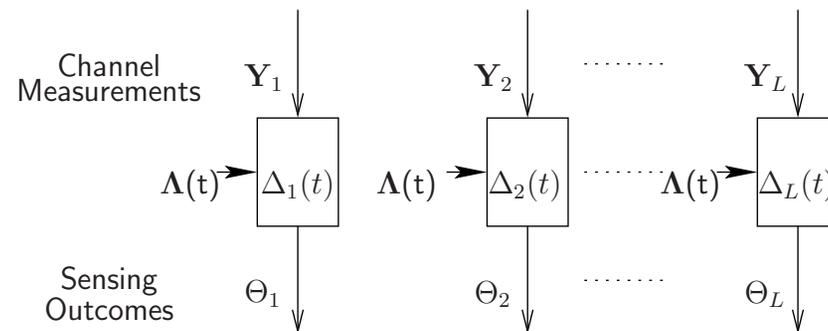
$$\mathcal{H}_1 : \mathbf{S}_{\mathcal{A}}(t) = [0, 1, \dots, 1],$$

...

$$\mathcal{H}_{2^L-1} : \mathbf{S}_{\mathcal{A}}(t) = [0, 0, \dots, 0].$$

- ▶ Decision rule:  $\{\{\mathbf{Y}_n\}_{n=1}^L\} \rightarrow \{\mathcal{H}_0, \dots, \mathcal{H}_{2^L-1}\}$   
jointly exploits channel measurements.
- ▶ Performance is specified by a set of  $2^L \times (2^L - 1)$  error probabilities.

## Independent opportunity identification

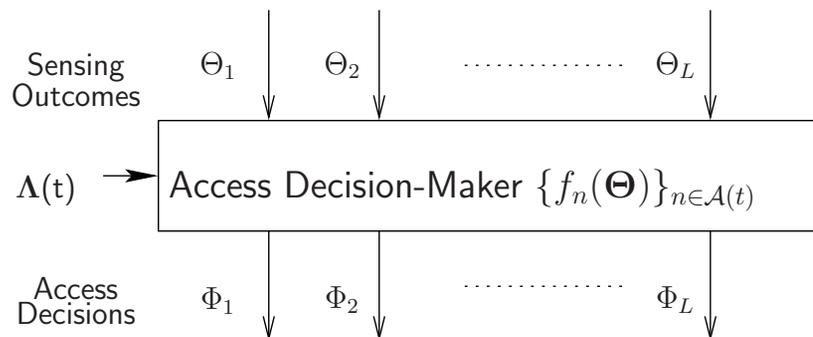


- ▶ Performs  $L$  independent hypothesis tests:  $\mathcal{H}_0 : S_n(t) = 1,$   
 $\mathcal{H}_1 : S_n(t) = 0, \quad n \in \mathcal{A}(t).$
- ▶ Decision rule  $\Delta_n(t) : \{\mathbf{Y}_n\} \rightarrow \{\mathcal{H}_0, \mathcal{H}_1\}$   
ignores correlation among channel measurements.
- ▶ Performance is specified by  $L$  pairs of false alarm and miss detection rates.
- ▶ Less complex than joint identification.

## Two Access Policy Structures for Multi-Channel Sensing

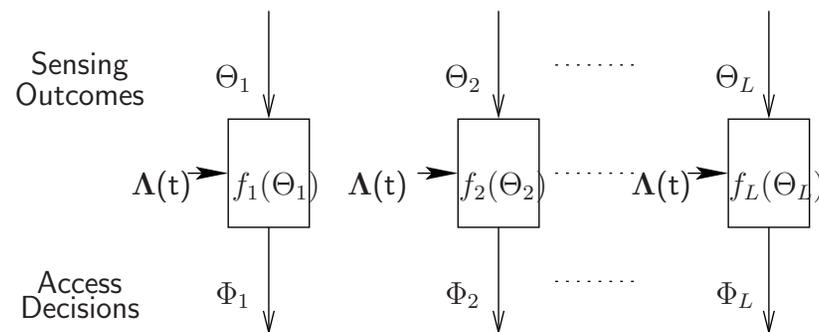
**Goal:** dynamically choose access decisions or transmission probabilities.

### Joint access decision-making



- ▶ Tx. probability  $f_n(\Theta) \triangleq \Pr\{\Phi_n = 1 | \Theta\}$  governs access decision  $\Phi_n, \forall n \in \mathcal{A}(t)$ .
- ▶ Access decision **jointly exploits sensing outcomes from all sensed channels:**  
 $\Theta = \{\Theta_n\}_{n \in \mathcal{A}(t)}$ .
- ▶ # of tx. probabilities to be designed =  $2^L$  (possible sensing outcomes)  $\times L$  (chosen channels).

### Independent access decision-making



- ▶ Tx. probability  $f_n(\Theta_n) \triangleq \Pr\{\Phi_n = 1 | \Theta_n\}$  independent of sensing outcomes from other channels.
- ▶ Access decision **ignores correlation among sensing outcomes.**
- ▶ # of tx. probabilities to be designed =  $2L$  (chosen channels).
- ▶ Less complex than joint identification.

## Extension of the Separation Principle

$$\{\pi_\delta^*, \pi_s^*, \pi_c^*\} = \arg \max_{\pi_\delta, \pi_s, \pi_c} \mathbb{E} \left[ \sum_{t=1}^T R(t) \right] \quad \text{s.t. collision prob. } P_n(t) \leq \zeta, \quad \forall n \in \mathcal{A}(t) \quad (*)$$

### Joint sensor & joint access structure

- ▶ Provides globally optimal solution.
- ▶ Requires **randomized** policies for optimality.
- ▶ Optimal but computationally prohibitive.

### Independent sensor & independent access structure

- ▶ The **separation principle** holds.
- ▶ Optimal solution under this structure (the SP approach):
  - Spectrum sensor: optimal NP detector with PM =  $\zeta$ .



- Access policy: trust the sensing outcome.



- ▶ Caveat: ignores correlation among channel occupancies.

# Exploiting Correlation: The PHY Layer Approach

## Joint sensor & independent access structure

- Sensor: optimal NP detector, using all channel measurements, with  $PM = \zeta$ .



- Access policy: trust the sensing outcome (using one sensing outcome).



- ▶ Exploits all channel measurements  $\{\mathbf{Y}_n\}_{n \in \mathcal{A}(t)}$  in occupancy detection for each chosen channel.
- ▶ Uses MAC layer information at PHY layer: the *a priori* joint distribution of channel measurements is obtained from the **belief vector**  $\Lambda(t)$ .
- ▶ **Locally optimal** (maximizes instantaneous throughput).
- ▶ Reduces to the SP approach when channels evolve independently.

## Exploiting Correlation: The MAC Layer Approach

### Independent sensor & joint access structure

- Sensor: optimal NP detector, using single channel measurements, with  $PM = \zeta$ .



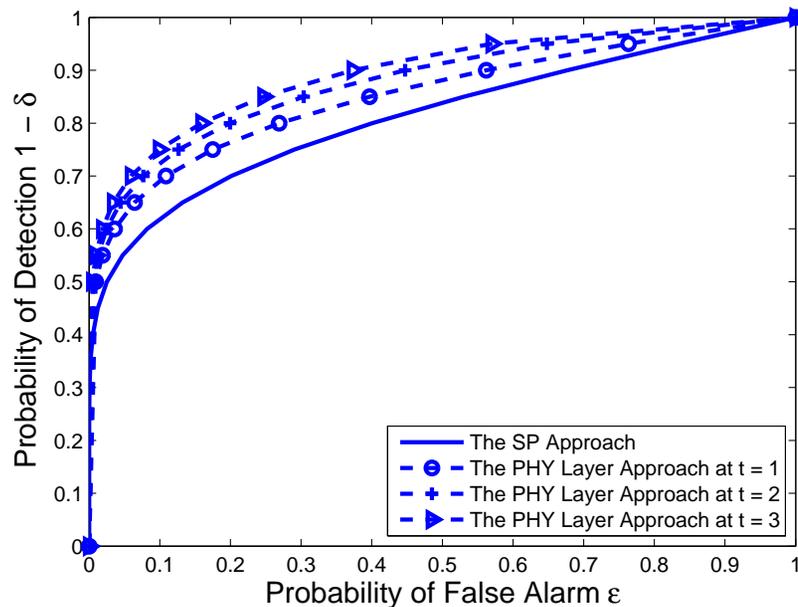
- Access policy: myopic tx probabilities  $f_n(\Theta)$  (maximizes the instantaneous throughput) obtained via linear programming.



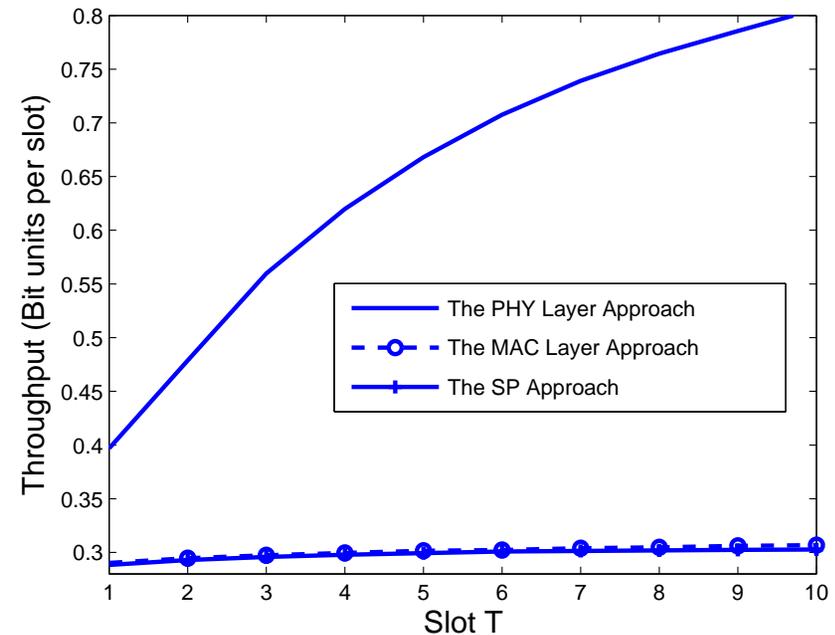
- ▶ Exploits all sensing outcomes  $\Theta = \{\Theta_n\}_{n \in \mathcal{A}(t)}$  in making access decision for each chosen channel.
- ▶ **Locally optimal** when channels evolve independently.
- ▶ Reduces to the SP approach when channels evolve independently.

## Performance Comparison

### ROC of spectrum sensor



### Normalized Throughput



- ▶ Performance of the PHY layer spectrum sensor is improved by exploiting the MAC layer sensing and access decisions (characterized by the belief vector).
- ▶ Exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer.

## Conclusions

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- ▶ Optimal OSA with multi-channel sensing formulated as a constrained POMDP.
- ▶ **Separation Principle**
  - holds for  $L = 1$ ; extends to  $L > 1$  under the independent sensor & independent access structure.
  - leads to explicit optimal sensor design and closed-form optimal access policy.
  - reduces sensing design from a constrained POMDP to an unconstrained one.
- ▶ **Exploiting Channel Correlation**
  - PHY layer approach: improved sensor performance by exploiting MAC information (belief vector).
  - MAC layer approach: infers channel correlation from sensing outcomes.

### Limitations

- ▶ Known transition probabilities of the underlying Markov process (robustness to model mismatch can be found in [1]).
- ▶ Interaction among secondary users not taken into account (exploited in [2]).

[1] Y. Chen, Q. Zhao, and A. Swami, "Joint Design and Separation Principle for Opportunistic Spectrum Access," in *Proc. of IEEE Asilomar Conference*, Nov. 2006.

[2] Y. Chen, Q. Zhao, and K. Liu, "Distributed Spectrum Sharing Among Competing Secondary Users," submitted to 45th Annual Allerton Conference on Communication, Control, and Computing, 2007.