Spectrum Opportunity Detection: How Good Is Listen-before-Talk?

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Abstract—We consider spectrum opportunity detection in cognitive radio networks for spectrum overlay. We highlight the differences between detecting primary signals and detecting spectrum opportunities. We show that besides noise and fading, the geographic distribution and traffic pattern of primary users have significant impact on the performance of spectrum opportunity detection. A necessary and sufficient condition for the equivalence between primary signal detection and spectrum opportunity detection is obtained, and the performance of listen-before-talk in a Poisson primary network with uniform traffic pattern is analyzed. Furthermore, we study the translation from the physical layer opportunity detection performance to the MAC layer performance. This issue is crucial in examining the impact of sensing errors on the design of higher layers and in choosing the optimal operating characteristics of the spectrum sensor. We demonstrate the complex dependency of the relationship between PHY and MAC on the applications and the use of MAC handshaking signaling such as RTS/CTS.

Index Terms: Cognitive radio, opportunistic spectrum access, spectrum opportunity detection.

I. INTRODUCTION

Opportunistic spectrum access (OSA), also referred to as spectrum overlay, is one of the several approaches envisioned for dynamic spectrum management [1]. The basic idea of OSA is to allow secondary users to exploit temporarily and locally unused channels without causing unacceptable interference to primary users.

One of the central issues in OSA is spectrum opportunity detection through sensing. Before transmitting over a particular channel, a secondary user needs to decide whether this channel is an opportunity. This is the so-called “Listen-before-Talk” (LBT). The general concept is that if there is no primary signal in a particular channel, the channel is an opportunity and suitable for transmission.

For the problem of spectrum opportunity detection, the focus has thus been mainly on detecting primary signals in the presence of noise and fading (see, for example, [2]). In this paper, we show that detecting primary signals is not equivalent to detecting spectrum opportunities. Even if secondary users listen to primary signals with perfect ears (i.e., perfect detection of primary signals), spectrum opportunity detection is subject to error.

The contribution of this paper is twofold. First, we highlight the differences between detecting primary signals and detecting spectrum opportunities. We show that besides noise and fading, the geographic distribution and traffic pattern of primary users have significant implications in the performance of spectrum opportunity detection. To illuminate the impact of primary users’ location and traffic pattern on opportunity detection, we consider listen-before-talk (LBT) with perfect ears. This allows us to separate detection errors caused by uncertainties in primary users’ location and traffic pattern from those caused by noise and fading. A necessary and sufficient condition for the equivalence between primary signal detection and spectrum opportunity detection is obtained, and the performance of LBT with perfect ears in a Poisson primary network with uniform traffic pattern is analyzed.

The second issue we are concerned with is the translation from the physical layer performance of spectrum opportunity detection to the MAC layer performance. This issue is crucial in examining the impact of sensing errors on the design of higher layers and in choosing the optimal operating characteristics of the spectrum sensor [3]. Figures of merit at both physical and MAC layers are defined and their relations examined. In particular, we demonstrate the complex dependency of the relationship between PHY and MAC on the application type (guaranteed delivery vs. best-effort delivery) and the use of handshaking signaling such as RTS/CTS at the MAC layer.

Throughout the paper, we use capital letters for parameters of primary users and lower-cased letters for secondary users.

II. SPECTRUM OPPORTUNITY: DEFINITION AND INTRICACIES

A rigorous study of OSA must start from a clear definition of spectrum opportunity and interference constraint. An initial attempt in defining these two central concepts can be found in [4]. To protect primary users, an interference constraint should specify at least two parameters. One is the maximum interference power level perceived by an active primary receiver; it specifies the noise floor and is inherent to the definition of spectrum opportunity. The other parameter is the maximum outage probability that the interference at an active primary receiver exceeds the noise floor. Allowing a positive outage probability is necessary due to sensing errors. This parameter is crucial to secondary users in making transmission decisions based on imperfect sensing as shown in [3].

Spectrum opportunity is a local concept defined with respect to a particular secondary transmitter and its receiver. Intuitively, a channel is an opportunity to a pair of secondary users if they can communicate successfully without violating the interference constraint imposed by the primary network\textsuperscript{\dagger}. Deceptively simple, this definition has significant complications in cognitive radio networks where primary and secondary users are geographically distributed and wireless transmissions are subject to path loss and fading.

For a simple illustration, consider a pair of secondary users (A and B) aiming to communicate in the presence of primary users as shown in Fig. 1. A channel is an opportunity to A and B if the transmission from A does not interfere with nearby primary receivers in the solid circle, and the reception at B is not affected by nearby primary transmitters in the dashed circle. The radius \( r_1 \) of the solid circle at A depends on the transmission power of A and the first

\textsuperscript{\dagger}Here we use channel in a general sense, i.e., a signal dimension (time, frequency, and code, etc.) that can be allocated to a particular user.

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Cognitive radio, opportunistic spectrum access, spectrum opportunity detection.
The use of circle to illustrate interference region is simplistic and inmaterial. This definition applies to a general signal propagation and interference model by replacing the solid and dashed circles with, respectively, the subset of primary receivers who are potential victims of \( A \)'s transmission and the subset of primary transmitters who can interfere with the reception at \( B \). The key message is that spectrum opportunities must be defined jointly at the transmitter and the receiver. It is a function of (i) the transmission powers of both primary and secondary nodes, (ii) the geographical locations of these nodes, and (iii) the interference constraint. From this definition, we arrive at the following properties of spectrum opportunity.

**Property 1: Spectrum Opportunity**

\[ P1.1 \text{ Spectrum opportunity depends on both transmitting and receiving activities of primary users.} \]

\[ P1.2 \text{ Spectrum opportunity is, in general, asymmetric: a channel that is an opportunity when } A \text{ is the transmitter and } B \text{ the receiver may not be an opportunity when } B \text{ is the transmitter and } A \text{ the receiver.} \]

\[ P1.1 \text{ determines the fundamental deficiency of LBT in detecting spectrum opportunities as detailed in Sec. IV. } P1.2 \text{ leads to a complex relationship between the opportunity detection performance at the physical layer and the link throughput and interference constraint at the MAC layer. As shown in Sec. V, this relationship varies with the application type (for example, whether acknowledgement is needed to complete a successful data transmission) and the use of handshaking signaling. In other words, it depends on whether the roles of the transmitter and receiver need to be reversed during the process of communicating a data packet.} \]

**III. SPECTRUM OPPORTUNITY DETECTION: FIGURES OF MERIT**

In this section, we specify the figures of merit for spectrum opportunity detection.

**PHY Performance.** Spectrum opportunity detection can be considered as a binary hypothesis testing problem. Let \( \mathbb{I}(A, rx) \) denote the presence of active primary receivers at which signal receptions will be corrupted by transmissions from \( A \), and \( \mathbb{I}(B, tx) \) the presence of active primary transmitters whose transmissions interfere with the reception at \( B \). Let \( \mathbb{I}^c(\cdot, \cdot) \) denote the complement of \( \mathbb{I}(\cdot, \cdot) \). The two hypotheses are given by

\[ \mathcal{H}_0 : \text{opportunity exists, i.e., } \mathbb{I}(A, rx) \cap \mathbb{I}(B, tx), \]

\[ \mathcal{H}_1 : \text{no opportunity, i.e., } \mathbb{I}(A, rx) \cup \mathbb{I}(B, tx). \]

The figures of merit at the physical layer are given by the probabilities of false alarm \( P_F \) and miss detection \( P_{MD} \):

\[ P_F \triangleq \Pr\{\mathcal{H}_1 \mid \mathcal{H}_0\}, \quad P_{MD} \triangleq \Pr\{\mathcal{H}_0 \mid \mathcal{H}_1\}. \]

The performance of the detector is specified by the receiver operating characteristic (ROC) curve, which gives \( 1 - P_{MD} \) (probability of detection or detection power denoted by \( P_D \)) as a function of \( P_F \). See Fig. 3 for an illustration. In general, reducing \( P_F \) comes at a price of increasing \( P_{MD} \) and vice versa. The tradeoff between false alarm and miss detection is thus crucial, and the operating characteristics of the spectrum sensor should be designed by considering the impact of detection errors on the MAC performance. As a consequence, the relationship between PHY and MAC needs to be carefully examined.

**MAC Performance** The MAC layer performance is measured by the throughput of the secondary user and the interference to the primary users. The design objective is to maximize the throughput under a constraint on the maximum outage probability that the interference at an active primary receiver exceeds the noise floor. We refer to such events as collisions with primary users.

The figures of merit at the MAC layer are thus given by the probability \( P_S \) of successful data transmission and the probability \( P_C \) of colliding with primary users.

\[ P_S \triangleq \Pr\{\text{successful data transmission}\}, \]

\[ P_C \triangleq \Pr\{A \text{ transmits data } | \mathbb{I}(A, rx)\}. \]

Note that \( P_C \) is conditioned on \( \mathbb{I}(A, rx) \) instead of \( \mathcal{H}_1 \). Clearly, \( \Pr[\mathbb{I}(A, rx)] \leq \Pr[\mathcal{H}_1] \). This further complicates the relationship between \( P_{MD} \) and \( P_C \) as shown in Sec. V.

**IV. PERFORMANCE OF LBT WITH PERFECT EARS**

In this section, we study the performance of LBT with perfect ears to highlight the differences between detecting primary signals and detecting spectrum opportunities. A necessary and sufficient condition for the equivalence between detecting primary signals and the presence of a spectrum opportunity is obtained. This result illuminates the impact of primary users' location and traffic pattern on the performance of LBT. As an example, the performance of LBT with perfect ears in a Poisson network with uniform traffic is analyzed.

Before we proceed, the following two definitions are in order.

**Definition 1:** Lister-before-talk with perfect ears refers to the scenario where a secondary user can detect perfectly transmissions from a subset of primary users and has the complete freedom of choosing this subset of primary users.

Note that for LBT with perfect ears, the secondary user can detect whether there are any transmissions from the chosen subset of primary users, but does not know which primary users in this subset are transmitting.

**Definition 2:** To a pair of secondary users \( A \) and \( B \), detecting spectrum opportunity is equivalent to detecting primary signals if there exists a subset \( \mathcal{P}_e \) of primary users such that an opportunity occurs if and only if no member of \( \mathcal{P}_e \) transmits.

From the above two definitions, we conclude that when detecting spectrum opportunity is equivalent to detecting primary signals, LBT with perfect ears achieves perfect opportunity detection by choosing \( \mathcal{P}_e \) as the subset of primary users to detect.
Detection errors occur even if $A$ listens to primary signals with perfect ears, and the occurrence and characteristics of detection errors depend on the geographic distribution and traffic pattern of primary users.

Fig. 2. Inferring the presence of primary receivers via LBT.  

To illustrate the basic idea, consider the disk interference model. As shown in Fig. 2, $A$ infers the presence of primary receivers within its interference range $r_I$ from the presence of primary transmitters within $r_D$, where $r_D$ is referred to as the detection range. For the scenarios shown in Fig. 2, even if $A$ can perfectly detect the presence of signals from any primary transmitters located within its detection range, the transmission from $X$ is a source for false alarm whereas the transmission from $Y$ is a source for miss detection. As illustrated in Fig. 3, adjusting the detection range $r_D \in (0, r_I + R_p]$ leads to different points on the ROC curve, where $R_p$ denote the transmission range of primary users.

The following theorem gives a necessary and sufficient condition for the equivalence between detecting primary signals and detecting spectrum opportunities under a general signal propagation and interference model.

**Theorem 1:** For a pair of secondary users $A$ (transmitter) and $B$ (receiver), let $\mathcal{P}_{tx}(A)$ denote the set of all primary users at which (potential) signal receptions can be corrupted by $A$’s transmission, and $\mathcal{P}_{tx}(B)$ the set of all primary users whose (potential) transmissions interfere with the reception at $B$. Let $\mathcal{P}_{tx}(A) \cap \mathcal{P}_{tx}(B)$, all potential receivers of $X$ are in $\mathcal{P}_{tx}(A)$.

By Definitions 1 and 2, Theorem 1 also provides a necessary and sufficient condition under which LBT with perfect ears achieves perfect opportunity detection (by choosing $\mathcal{P}_{tx}(A) \cup \mathcal{P}_{tx}(B)$ as the subset of primary transmitters to detect). Note that the above necessary and sufficient condition depends on the primary users’ geographic distribution (through $\mathcal{P}_{tx}(A)$ and $\mathcal{P}_{tx}(B)$) and traffic pattern (through the relationship between $\mathcal{P}_{tx}(A)$ and $\mathcal{P}_{tx}(B)$).

**B. LBT in Poisson Primary Networks**

In this section, we analyze the performance of LBT in a Poisson primary network with uniform traffic. Specifically, primary users are distributed according to a two-dimensional homogeneous Poisson process with density $\lambda$. Transmissions are slotted. In each slot, a primary user $X$ has a probability $p$ to become a transmitter. Its receiver is chosen with equal probability from primary users located within a distance $R_p$ (the transmission range) to $X$. Based on the Thinning Theorem and the Displacement Theorem for marked Poisson processes [5], both primary transmitters and receivers form a two-dimensional homogeneous Poisson process with density $p \lambda$. Note that these two Poisson processes are not independent.

Consider a pair of secondary users $A$ and $B$ that are distance $d$ apart, and opportunity detection is performed by the transmitter $A$ via LBT. A disk interference model is used, where the interference ranges of primary and secondary users are $R_I$ and $r_I$, respectively (see Fig. 1). We assume that $A$ can detect perfectly the presence of primary transmitters within a distance $r_D$. We can then obtain closed-form expressions for probabilities of false alarms and miss detections.
\[ P_S = \begin{cases} 
\exp(-p\lambda(\pi(r_0^2 + R_0^2) - S_I(d, r_E, R_I))) / 1, \\
\exp(-p\lambda(\pi(r_0^2 + R_0^2) - S_I(d, r_D, R_I))) / 1 
\end{cases}, \quad \text{guaranteed delivery}, \quad r_E = \max\{r_D, R_I\} \tag{4}
\]
\[ P_C = \exp(-p\lambda\pi r_0^2) [1 - \exp(-p\lambda(\pi r_0^2 - I(r_D, r_I, R_D)))] / 1 - \exp(-p\lambda\pi r_0^2), \quad I(r_D, r_I, R_D) = \int_0^{r_D} \frac{S_I(r, r_I, R_D)}{\pi r_0^2} dr \tag{5}
\]

The derivations are tedious and omitted due to space limit. Details can be found in [6].

Specifically, let \( S_I(d, r_1, r_2) \) denote the intersecting area of two circles with radius \( r_1 \) and \( r_2 \) and centered \( d \) apart. Let \( S_o(d, r_1, r_2) \) denote the complement of \( S_I(d, r_1, r_2) \) within the circle of radius \( r_1 \), i.e., the area of \( S_o(d, r_1, r_2) \) is given by the difference between \( \pi r_1^2 \) and the area of \( S_I(d, r_1, r_2) \). The probabilities of false alarms and miss detections of LBT with a detection range of \( r_D \) are given in (1)-(3) on the previous page. As shown in [6], the double integrals in (1)-(3) can be reduced to a single integral.

By varying the detection range \( r_D \in (0, r_1 + R_D) \), we obtain the ROC curve for LBT in Poisson primary networks. An example is given in Fig. 6.

V. FROM PHY TO MAC

In this section, we consider the translation from the physical layer opportunity detection performance to the MAC layer performance, i.e., the relationship between \( \{P_F, P_{MD}\} \) and \( \{P_S, P_C\} \).

A. Global Interference Model

Consider first a global interference model where the transmission from every primary user of interest affects the reception at \( B \) and the transmission from \( A \) affects the reception at every primary user. Under this condition, an opportunity occurs if and only if no primary users are transmitting. Spectrum opportunities are thus symmetric, and detecting primary signals is equivalent to detecting spectrum opportunity. Furthermore, we have the following statements, assuming that \( A \) transmits in a slot if and only if the channel is detected as an opportunity (possibly erroneously) at the beginning of this slot.

- Successful transmissions from \( A \) to \( B \) can only result from opportunities, i.e., \( H_0 \).
- Every correctly identified opportunity leads to a successful transmission.
- Every miss detection results in a collision with primary users.

The above statements lead to the following simple relationship between \( \{P_F, P_{MD}\} \) and \( \{P_S, P_C\} \).

\[ P_S = (1 - P_F) \Pr[H_0], \quad P_C = P_{MD}. \]

With this relationship, to maximize \( P_S \) under a constraint of \( P_C \leq \zeta \), we can obtain the optimal operating point \( \{P_F^*, P_{MD}^*\} \) for the spectrum sensor. The joint design of the spectrum sensor at the physical layer and the tracking and access decisions at the MAC layer is addressed in [3].

B. Local Interference Model

When the transmissions from primary and secondary users have local effect, the statements and the relationship between \( \{P_F, P_{MD}\} \) and \( \{P_S, P_C\} \) given in Sec. V-A no longer hold. The relationship between PHY and MAC has complex dependency on the applications and the use of MAC handshaking signaling.

\begin{enumerate}
\item \textbf{Impact of Application:} We illustrate here the impact of applications on the relationship between PHY and MAC. Specifically, we compare applications requiring guaranteed delivery with those relying on best effort (for example, media streaming and network gaming). For the former, we assume immediate acknowledgement is required at the end of each slot in order to complete a successful data transmission. For the latter, acknowledgements are not necessary. Due to the asymmetry of spectrum opportunities and the local effect of transmissions, we have the following relationship between \( \{P_F, P_{MD}\} \) and \( \{P_S, P_C\} \):
\begin{itemize}
\item For both types of applications, \( P_C \neq P_{MD} \).
\item For applications with guaranteed delivery, correctly detected opportunities may lead to failed data transmission, and miss detections may lead to successful data transmission, i.e.,
\[ \Pr[\text{success} | H_0] \leq 1 - P_F, \quad 0 < \Pr[\text{success} | H_1] \leq P_{MD}. \]
\item For best-effort delivery, correctly detected opportunities always result in successful data transmission, and miss detections may also lead to successful data transmission, i.e.,
\[ \Pr[\text{success} | H_0] = 1 - P_F, \quad 0 < \Pr[\text{success} | H_1] \leq P_{MD}. \]
\end{itemize}
\end{enumerate}

As given in (4) and (5) above, we can obtain closed-form expressions for the MAC layer performance \( \{P_S, P_C\} \) of LBT in a Poisson primary network with uniform traffic\(^2\). Detailed derivations can be found in [6]. Based on this result, we can study the impact of applications on the MAC layer performance, i.e., \( P_S \) (representing link throughput) under a collision constraint \( P_C \leq \zeta \). Shown in Fig. 4 is \( P_S \) as a function of the collision constraint \( \zeta \) (see [6] for parameter settings). We observe that even though the detection performance at the physical layer is the same, the MAC layer performance can be different depending on the applications.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4.png}
\caption{Success probability vs interference constraint.}
\end{figure}

As shown in Fig. 4, when the collision constraint is tight, the throughput is the same for these two types of applications. The collision constraint \( \zeta \) has a critical value \( \zeta_0 \) above which the throughput

\(^2\)We assume that collisions with primary users are caused by data transmissions. We ignore the interference from the transmission of acknowledgement and handshaking signaling such as RTS/CTS due to their short duration.
for best-effort delivery is higher than that for guaranteed delivery. Fig. 5 shows $\zeta$ as a function of the primary traffic load $p\lambda$ (or the density of active primary transmitters). We can see that $\zeta$ is a decreasing function of $p\lambda$. This suggests that primary systems with heavy traffic is more suitable for spectrum overlay with best-effort delivery applications.

2) Impact of MAC Handshaking Signaling: The fundamental deficiency of LBT resembles the hidden and exposed terminal problem in the conventional ad hoc networks of peer users. It is thus natural to consider the use of RTS/CTS handshaking signaling to enhance the detection performance of LBT. We show here that, although RTS/CTS signaling can improve the opportunity detection performance at the physical layer, it may lead to decreased throughput at the MAC layer for best-effort delivery applications.

For RTS/CTS enhanced LBT, spectrum opportunity detection is done jointly by $A$ and $B$ through the exchange of RTS/CTS signals. Specifically, the transmitter $A$ first detects a chosen set of primary transmitters. If there are no signals from this set, it transmits an RTS to $B$. Upon receiving the RTS (which automatically indicates the absence of interfering primary transmitters), $B$ replies a CTS. A successful exchange of RTS/CTS indicates an opportunity, and $A$ starts to transmit data to $B$. For this RTS/CTS enhanced LBT, we have the following relationship between $\{P_F, P_{MD}\}$ and $\{P_S, P_C\}$:

- $P_C = \frac{P_r[H_1]}{P_r[H_0] + P_{MD}}$ for $P_{MD} \geq P_{MD}$.
- Correctly detected opportunities always result in successful data transmission, as well as miss detections, i.e.,

$$P_S = (1 - P_F) Pr[H_0] + P_{MD} Pr[H_1].$$

The PHY and MAC performance of RTS/CTS enhanced LBT in a Poisson primary network with uniform traffic can be similarly analyzed [6]. An example ROC curve is shown in Fig. 6. Note that $(0,0)$ does not belong to the ROC curve of RTS/CTS enhanced LBT. This is due to the fact that the effective detection range is bounded above $R_I$, since to receive the CTS signal successfully, there cannot be primary transmitters within $R_I$ of $A$. In other words, a detection range $r_D \leq R_I$ leads to the same $(P_F, P_{MD})$ as $r_D = R_I$.

It can be shown that the ROC performance of RTS/CTS enhanced LBT is always better than or equal to that of LBT when $r_D \geq R_I$. However, at the MAC layer, RTS/CTS enhanced LBT may lead to lower throughput when the collision constraint is loose and the application relies on best-effort delivery, as shown in Fig. 7. Note that using RTS/CTS enhanced LBT, the throughput is the same for guaranteed delivery and best-effort delivery. This suggests that whether to adopt handshaking signaling at the MAC layer depends on the applications and the interference constraint.

![Fig. 5. Critical value of $\zeta$ vs. primary traffic load.](image)

![Fig. 6. ROC performance comparison.](image)

![Fig. 7. Throughput comparison.](image)

VI. CONCLUSION

We have examined the equivalence and inequivalence between detecting primary signals and detecting spectrum opportunities in cognitive radio networks for spectrum overlay. The translation from the detection performance at the physical layer to the throughput of secondary users and interference to primary users is studied, and the impact of application type and handshaking signaling on this translation demonstrated. At the system level, we show that spectrum overlay in primary systems with relatively heavy traffic is more suitable for best-effort delivery applications and using RTS/CTS signaling leads to better detection performance at the physical layer but potentially lower throughput at the MAC layer for best-effort delivery applications.

REFERENCES