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INERTANCE TUBE MODELING AND THE EFFECTS OF TEMPERATURE

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Pulse tube refrigerators (PTRs) have made dramatic improvements in reliability efficiency and usage. Inertance tube PTRs have been one of the keys to these improvements. The inertance tube is the component in the PTR that most easily affects the control of the PTR fluid dynamics. In one application in multistage cryocoolers, the performance of inertance tubes at the cryogenic temperatures is of interest. One purpose of this paper is to understand how temperature and the size of the reservoir influence the phase shift between mass flow rate and pressure at the inlet of the inertance tube. Various models including a two dimensional Computational Fluid Dynamics (CFD) will be compared to understand how these models can predict the phase shift and the acoustic power.

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INERTANCE TUBE MODELING AND THE EFFECTS OF TEMPERATURE

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ABSTRACT

Pulse tube refrigerators (PTRs) have made dramatic improvements in reliability, efficiency and usage. Inertance tube PTRs have been one of the keys to these improvements. The inertance tube is the component in the PTR that most easily affects the control of the PTR fluid dynamics. In one application in multistage cryocoolers, the performance of inertance tubes at the cryogenic temperatures is of interest. One purpose of this paper is to understand how temperature and the size of the reservoir influence the phase shift between mass flow rate and pressure at the inlet of the inertance tube. Various models including a two dimensional Computational Fluid Dynamics (CFD) will be compared to understand how these models can predict the phase shift and the acoustic power.

KEYWORDS: Inertance tube, cryocoolers, pulse tube refrigerators, oscillating flow, computational fluid dynamics

INTRODUCTION

Pulse Tube Refrigerators (PTRs) are used in many different arenas requiring cryogenic cooling that require coolers with high reliability, low vibration, and high efficiency. PTRs have played an increased role as pre-coolers in the superconductor arena. These pre-coolers are typically used because they are more efficient and reliable than the coolers they replace. Multistage PTRs have been developed by engineering subsequent stages to the original single stage PTR, as shown in FIGURES 1 and 2. This typically entails adding a regenerator, pulse tube, phase shifter and reservoir for each stage. The combination of these and especially the combination of the phase shifter and the reservoir are important in controlling the efficiency of the PTR. The subsequent stages of a multistage PTR can require for the phase shifter and reservoir to be cooled to cryogenic temperatures. Usually three types of phase-shifting

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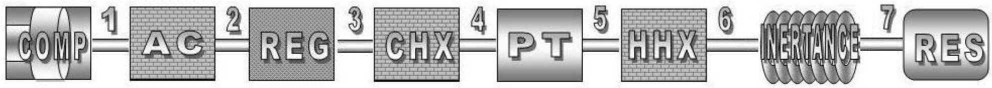

FIGURE 1. Inertance Tube Pulse Tube Refrigerator (ITPTR), showing the compressor (COMP), after cooler (AC), regenerator (REG), cold heat exchanger (CHX), pulse tube (PT), hot heat exchanger (HHX), inertance tube (INERTANCE) and reservoir (RES)

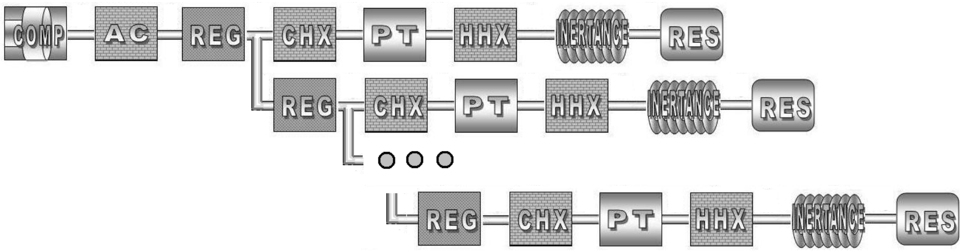

FIGURE 2. Multistage ITPTR – each subsequent stage is usually at a colder temperature than the previous stage.

processes are used to control the phase shift between the mass flow rate and pressure in PTRs. The three types of phase shifters from older to newer are the orifice valve, the double inlet and the inertance tube [1]. FIGURE 1 shows the inertance tube phase-shifting mechanism and the primary components of PTRs. The thermodynamics of PTRs have been under study by several investigators [2-4] and the importance of the phase shifter has been a key to increasing PTR efficiencies to their current level. The ability of the inertance tube to phase shift at cryogenic temperatures is the purpose of this study.

MODELING

The thermodynamic analysis has pointed out that the inertance tubes are capable of creating positive phase shifts where mass flow rate lags the pressure flow [5]. This mass flow rate is not sinusoidal typically, and thus to determine the phase shift of the mass flow rate requires the following equation, which in essence is the contribution of the first term of the Fourier series method for finding phase shift φ:

$$\varphi = \tan^{-1}\left(\frac{\int_0^\tau \dot{m}(t)\cos\omega t dt}{\int_0^\tau \dot{m}(t)\sin\omega t dt}\right) \tag{1}$$

where τ is the period of oscillation. The pressure and mass flow rate are defined by:

$$P = P_a + P_d \sin\omega t \tag{2}$$

$$\dot{m} = \dot{m}_a + \dot{m}_d \sin(\omega t - \varphi) \tag{3}$$

where ω is the angular frequency , the pressure amplitude is P_d, charge pressure is P_a, $\dot{m}_d$ is the mass amplitude and $\dot{m}_a$ is the mass average.

In previous studies a distributed model using ordinary differential equations for an inertance tube has been developed. This distributed model of an inertance tube has been compared to fluid dynamic simulations and to experiment [6,7,8]. The purpose of the ODE model is to yield a set of ODEs utilizing a friction factor based on oscillating flow. The friction factor coefficients are chosen based on experimental data from steady state laminar and turbulent flow in tubes [9].

The inertance tube is modeled by $2n+1$ ODEs using the distributed model of the inertance tube and is given as follows:

$$\begin{aligned} &\dot{m}_j = \dot{m}_{j+1} + C_j \frac{dP_j}{dt} \\ &P_j - P_{j+1} = R_j \dot{m}_{j+1} + L_j \frac{d\dot{m}_{j+1}}{dt} \qquad , j = 1 \ldots n \\ &\vdots \\ &\vdots \\ &\dot{m}_{n+1} = V_{ir} \frac{dP_{n+1}}{dt} \end{aligned} \tag{4}$$

Where the following variables are given as:

$$\begin{aligned} &P_1 = P, A = \frac{\pi}{4} d^2, C_1 = \frac{A\frac{l}{n}}{2\gamma R T_h}, C_j = \frac{A\frac{l}{n}}{\gamma R T}, R_j = \frac{2X\mu\frac{l}{n}}{\pi d^4 \rho}, \\ &Y_j = \frac{4\left|\dot{m}_j(t)\right|}{\pi d \mu}, X = \left(64^{a_1} + D_1^{a_1} Y_j^{a_1(1-D_2)}\right)^{\frac{1}{a_1}}, L_j = \frac{4l}{\pi d^2}, V_{ir} = \frac{V_r + A\frac{l}{2n}}{\gamma R T} \end{aligned} \tag{5}$$

Where the variables P_1 corresponds to the pressure that comes in from the hot heat exchanger, A is the area of the cross section of the inertance tube, d is the diameter of the inertance tube, C_j are the capacitance coefficients, l is the length of the inertance tube, n is the number of times the inertance tube is split, T_h is the hot temperature, R_j are the resistive coefficients, μ is the gas viscosity, ρ is the gas density, L_j are the inductance coefficients, V_{ir} is the volume of the reservoir (V_r) plus the last piece of the unaccounted inertance tube volume, a_1, Y_j, X, D_1 and D_2 are the laminar/turbulent coefficients described in [6]. The mass flow rate from the pulse tube is linked to the inertance tube by the conservation of mass and energy for the hot heat exchanger.

The capacitance C_j in the above equations is obtained based on the assumption of adiabatic process inside the inertance tube. The actual heat transfer process in the inertance tube is more complex. Depending on the magnitude of thermal penetration depth in helium the adiabatic assumption may not be valid. The specific heat ratio γ in the capacitance term represents the adiabatic assumption and should be replaced with a parameter between 1 and 1.667 for the helium [10]. The thermal penetration depth can be calculated from the following equation:

$$\delta_{th} = \sqrt{\frac{2RkT}{P_a C_p \omega}} \tag{6}$$

where k is thermal conductivity and ω is the angular velocity of oscillation. Thermal penetration depth is strong function of thermal conductivity and the temperature in this study. FIGURE 3 shows the thermal conductivity and the thermal penetration depth as a function of temperature for helium used in this investigation. From the figure it can be seen that for the low temperatures the thermal penetration depth is much smaller than tube radius and the assumption of adiabatic process inside the inertance tube appears to be more applicable.

One of the key metrics of interest is the phase shift, as described above and the acoustic power as follows:

$$\dot{W} = \frac{RT}{\tau}\int_0^{\tau} \dot{m} \ln \frac{P}{P_a} dt \tag{7}$$

RESULTS AND DISCUSSIONS

This set of simulations was for an inertance tube with length of 2 m, diameter of 1 mm, with a pressure ratio of 1.4 and reservoir volume of 50 cm^3. Temperatures of 30, 45, 60, 75, 90, 105, 120, 150, 180 and 210 K were simulated.

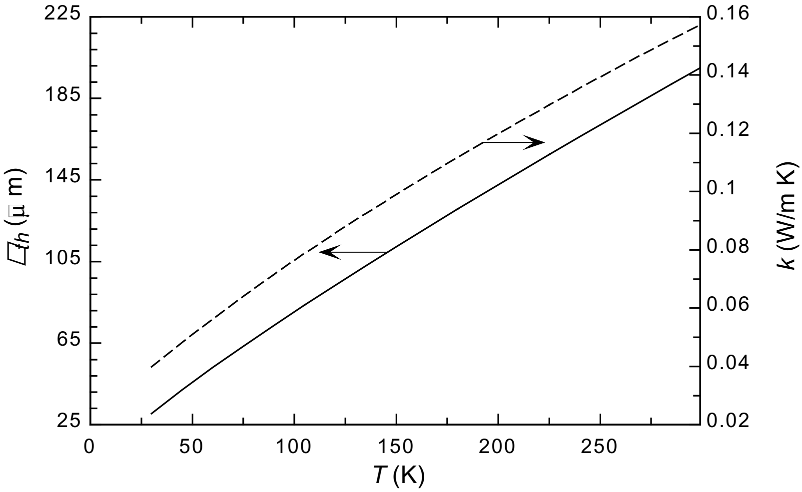

FIGURE 3. Thermal conductivity and thermal penetration depth as a function of temperature for helium at a pressure of 2.5 MPa.

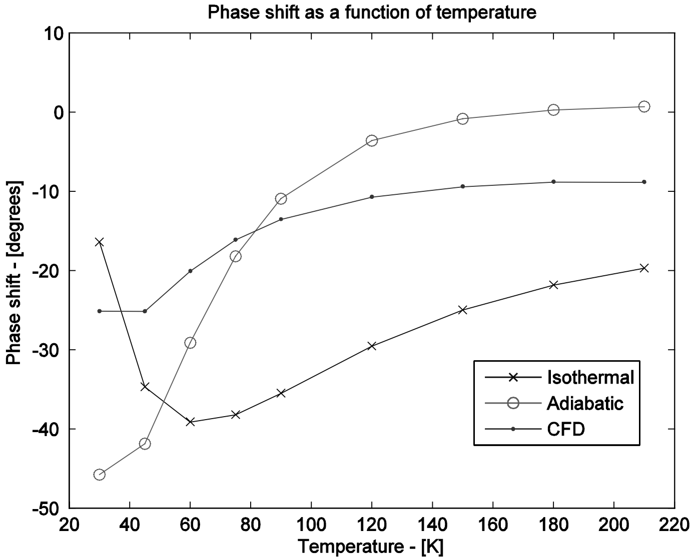

FIGURE 4. Phase shift at different temperatures in an inertance tube, as predicted by CFD and the distributed ODE model for isothermal and adiabatic cases. (L=2 m, d=1 mm, Pr=1.40, Pa=2.5 MPa, Res=50 cm^3)

As seen in FIGURE 4, the CFD simulations showed that phase shift got more negative as temperature decreased, but at low temperatures around 45 K the phase shift began to increase. The distributed ODE model showed similar results for the isothermal case, but only decreased for the adiabatic case. The acoustic power, in FIGURE 5, decreased as temperature decreased and similar to the phase shift increased at low temperatures. The distributed ODE model showed similar trends for the acoustic power for both the isothermal and adiabatic cases, except the increases for the isothermal case began at a higher temperature than the CFD and isothermal cases. The trends for the CFD and adiabatic cases were very similar, as they seem to be parallel to one another.

These results show that according to CFD and the distributed model, that this particular inertance tube operating with the given parameters does not perform like one would hope. This does not mean that all inertance tubes at these temperatures are going to be poor, and by changing the diameter and/or length of the inertance tube or changing the reservoir size that one might come up with a tube with positive phase shifts [5, 10, 11, 12].

The hope for these inertance tube models was to be able to adjust the coefficients to yield better results. The goal is to come up with empirical coefficients for the distributed ODE model that yield results that match better with experiment and CFD for design applications. From [10] and this work some of the hope has been that the experiments and in our case the simulation results would be bounded by the isothermal and adiabatic cases. This has not been the case, so the coefficients for the distributed model need to be adjusted so that the results are bounded by the isothermal and adiabatic cases. The appropriate value for gamma must be found in the distributed model to yield the correct correlation to experiment and CFD. This assumes that a good CFD model for turbulence exists for oscillating flow, which is probably not the case, as there is not an accepted turbulence model for oscillating

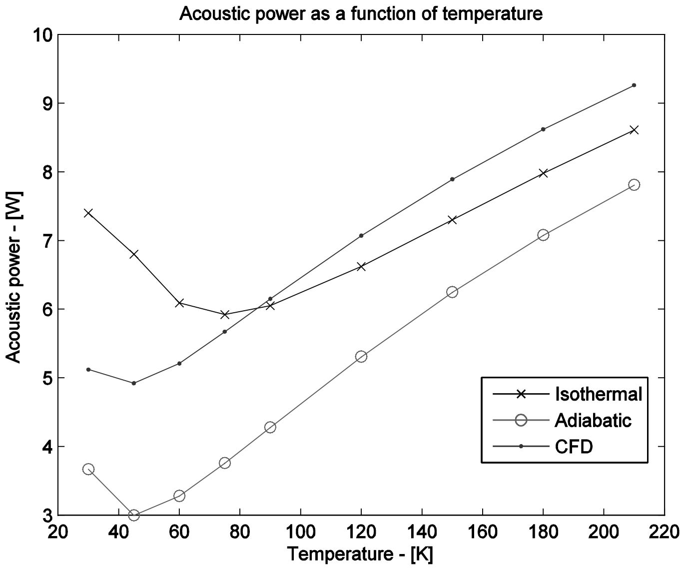

FIGURE 5. Acoustic power at different temperatures in an inertance tube as predicted by CFD and the distributed ODE model for isothermal and adiabatic cases.(L=2 m, d=1 mm, Pr=1.40,Pa=2.5 MPa, Res=50 cm^3)

flow in application to the inertance tube. Considerable research will be required to iterate over these various models and their parameters, in order to find acceptable models for oscillating fluid flow in inertance tubes and its optimization for application to PTRs.

CONCLUSIONS

The results of CFD calculations for an inertance tube are compared to the results of the distributed model of an inertance tube at cryogenic temperatures. The distributed model is used for both isothermal and adiabatic assumptions effecting the capacitance term in the model. Extensive numerical calculation is required to be able to use simple first order models with appropriate parameters to simulate complex heat and mass transfer in inertance tubes for design and optimization. CFD is the appropriate method for simulation of multidimensional effects in inertance tubes. However, comparison of CFD simulations and experimental results are needed to select appropriate turbulent models for oscillating flow in the inertance tube.

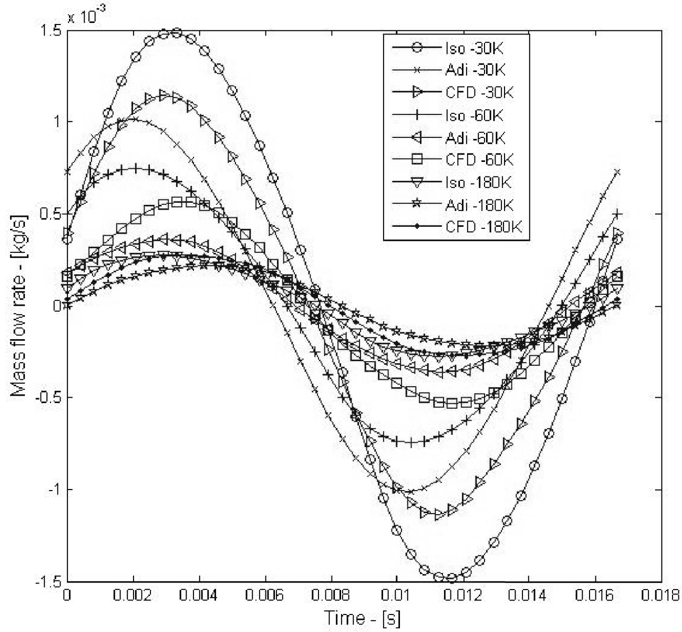

FIGURE 6. Mass flow rates in an inertance tube at different temperatures, as predicted by CFD and the distributed ODE model for isothermal and adiabatic cases.(L=2 m, d=1 mm, Pr=1.40,Pa=2.5 MPa, Res=50 cm^3)

REFERENCES

1. Radebaugh, R., "Pulse tube cryocoolers for cooling infrared sensors," *Proceedings of SPIE*, Vol. 4130, pp. 363-379, 2000.
2. Kittel, P., Kashani, A. Lee, J.M., and Roach, P.R., *Cryogenics*, Vol. 34, pp. 283-292, 1996.
3. de Boer, PCT, *Cryogenics*, Vol. 42, pp. 209-221, 2002.
4. de Waele, ATAM, Steijaert, P.P., and Gijzen, J., *Cryogenics*, Vol. 37, pp. 313-324, 1997
5. Lewis, M., Bradley, P. and Radebaugh, R., "Impedance measurements of inertance tubes", *Advances in Cryogenic Engineering*, vol. 51, pp.1557-1563, 2006.
6. Dodson, C. The comparison of CFD simulations and a first order numerical model of inertance tubes, Master's Thesis, University of New Mexico, 2007.
7. Dodson, C., Lopez, A., Roberts, T. and Razani, A., "A model for the parametric analysis and optimization of inertance tube pulse tube refrigerators", *Advances in Cryogenic Engineering*, Vol. 53A, 685-692, 2008.
8. Dodson, C., Razani, A. and Roberts, T., "Numerical Simulation of oscillating fluid flow in inertance tubes", *Cryocoolers*, Vol. 15, pp. 261-269, 2009.
9. Churchill, S. "Friction factor equation spans all fluid-flow regimes", *Chemical Engineering*, pg. 91-92, November 1977.
10. Radebaugh, R., Lewis, M., Luo, E., Pfotenhauer, J., Nellis, G. and Schunk, L., "Inertance tube optimization for pulse tube refrigerators", *Advances in Cryogenic Engineering*, Vol. 51, pp. 59-67, 2006.
11. Schunk, L., Nellis, G. and Pfotenhauer, J., "Experimental investigation of modeling inertance tubes", *Journal of Fluids Engineering*, Vol. 127, pp. 1029-1037, 2005.
12. Zhu, S., Luo, E., Wu, Z. and Dai, W., "Universal scaling law of inertance tube phase shifter", *Advances in Cryogenic Engineering*, Vol. 53, pp. 1075-1082, 2008.