A cross-shore model of barrier island migration over a compressible substrate

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ABSTRACT

Barrier islands that overlie a compressible substrate, such as islands in deltaic environments or those that overlay mud or peat deposits, load and consolidate the underlying subsurface. Through time, the elevation and aerial extent of these islands are reduced, making them more susceptible to future inundation and overwash. Sand washed over the island and onto back-barrier marsh or into the bay or estuary begins the consolidation process on a previously non-loaded substrate, with time-dependent consolidation as a function of the magnitude of the load, duration of load, and characteristics of the substrate. The result is an increase in the overwash, migration, breaching, and segmentation of these islands.

This research developed a two-dimensional (cross-shore) numerical model for evolution of a sandy barrier island that spans durations of years to decades as a function of erosion, runup, overwash, migration, and time-dependent consolidation of the underlying substrate as a function of loading by the island. The model was tested with field data and then applied to evaluate the effects of a compressible substrate on long-term barrier island evolution. Results illustrate that barrier islands overlying a compressible substrate are more likely to have reduced dune elevation due to consolidation, incur overall volumetric adjustment of the profile to fill in compressed regions outside the immediate footprint of the island, and experience increased overwash and migration when the dune reaches a critical elevation with respect to the prevalent storm conditions.

1. Introduction

Barrier islands form a dynamic coastal boundary for bays, estuaries, and mainland shores. They buffer these more fragile environments from coastal winds, waves, and storm surges (Stone and McBride, 1998), provide habitat for static and migrant populations (Moore et al., 2000), and foster quiescent, reduced salinity habitat for evolution of juvenile species (Courrat et al., 2009). Estuaries, particularly those on deltaic coasts, represent the most productive ecosystems in the world yet they are the most threatened by anthropogenic activities (Edgar et al., 2000), an inability to expand with relative sea level rise (so-called “coastal squeeze,” French, 2006), and disintegration of protective barrier islands (e.g., McBride et al., 1999; McBride and Byrnes, 1997; Penland et al., 2005). Approximately 12% of the world’s open-ocean coast is fronted by barrier islands, and 28% of these islands occur in deltaic systems (Pilkey and Fraser, 2003). The benefits and functioning of barrier islands, especially those in deltaic settings, are threatened by reduced sources of sand, relative sea level rise, and anthropogenic activities. Intervention to restore barrier islands through placement of beach-quality sand from an external source has been conducted since the 1930s (Farley, 1923; Marine Board, 1995) and continues to be considered in increasingly large-scale, regional applications (van Heerden and DeRouen, 1997).

Barriers evolve in form and migrate in response to coastal processes, sediment availability, and geologic setting on time scales ranging from hours to decades to centuries. On time scales of hours to days, cross-shore processes during storms can erode the foreshore and overwash the island and deposit sediment on the back-barrier or into the bay (e.g., Leatherman, 1979; Kahn and Roberts, 1982; Dingler and Reiss, 1990; Doughty et al., 2006). Over seasons, years, and decades, a gradient in longshore sand transport, change in regional sediment supply, and adjacent inlet processes can erode, accrete, and migrate a barrier island (e.g., Penland et al., 2005; Morton, 2008). On geologic time scales ranging from decades to centuries, processes in the vertical dimension such as eustatic sea level change, regional down-warping or uplift, and consolidation of sediment may contribute to the long-term evolution of coastal morphology (e.g., Storms et al., 2002; Stolper et al., 2005; FitzGerald et al., 2008; Moore et al., in press). For barrier islands overlying poorly-consolidated sediment, such as deltaic, bay, estuarine, and peat deposits, consolidation of the underlying substrate...
Barrier islands that overlie a compressible substrate, such as islands in deltaic environments or those that overlay mud or peat deposits, load and consolidate the underlying subsurface. Through time, the elevation and aerial extent of these islands are reduced, making them more susceptible to future inundation and overwash. Sand washed over the island and onto back-barrier marsh or into the bay or estuary begins the consolidation process on a previously non-loaded substrate, with time-dependent consolidation as a function of the magnitude of the load, duration of load, and characteristics of the substrate. The result is an increase in the overwash, migration, breaching, and segmentation of these islands. This research developed a two-dimensional (cross-shore) numerical model for evolution of a sandy barrier island that spans durations of years to decades as a function of erosion, runup, overwash, migration, and time-dependent consolidation of the underlying substrate as a function of loading by the island. The model was tested with field data and then applied to evaluate the effects of a compressible substrate on long-term barrier island evolution. Results illustrate that barrier islands overlying a compressible substrate are more likely to have reduced dune elevation due to consolidation, incur overall volumetric adjustment of the profile to fill in compressed regions outside the immediate footprint of the island, and experience increased overwash and migration when the dune reaches a critical elevation with respect to the prevalent storm conditions.
due to the weight of the island can accelerate long-term morphologic response (Guber and Slingerland, 1981; Gayes, 1983; Dean, 1997; Bourman et al., 2000; Rosati, 2009).

Deltaic, bay, estuarine, and peat deposits compress, or consolidate as a function of the load that is applied, duration of loading, and characteristics of the substrate itself. River deltas experience consolidation wherever the river deposits organics and fine sediments, such as silt and clay. Deltaic systems that experience accelerated subsidence include the Mississippi River, U.S.A. (Coleman et al., 1998); Rhine–Meuse River, The Netherlands (Berendsen, 1998); Ebro River, Spain (Sánchez-Arcilla et al., 1998); Nile River, Egypt (Stanley and Warne, 1998), the Ganges–Brahmaputra Rivers, Bangladesh, India (Allison, 1998); and the Yangtze River, China (Xiqing, 1998). Compaction of the subsurface can also occur with fine sediment and peat deposited in estuaries and bays (e.g., Bloom, 1964; Kaye and Barghoorn, 1964; Cahoon et al., 1995; Long et al., 2006; Meckel et al., 2007). For barrier islands overlying a soft substrate, whether the subsurface is riverine, estuarine, or organic in origin, the weight of the island compresses the subsurface resulting in a time-dependent reduction of the island elevation. The net effect is an increase in the likelihood for overwash of the island and subsequent migration. New washover deposits begin to consolidate previously non-loaded sediment, thus perpetuating the morphologic change process.

In this paper, we develop a two-dimensional (2D) model for barrier island erosion, overwash, and washover, including time-dependent consolidation of the subsurface as a function of local loading by the barrier island (Fig. 1). The model is applied to test the hypothesis that cross-shore barrier island migration and volumetric losses are as a function of the weight and duration of loading. First, we review the state-of-modeling for barrier island morphologic change process.

2. Behavior and modeling of barrier islands that overlie compressible sediment

Barrier islands overlying compressible sediment or peat deposits have been observed to rapidly erode, rollover, breach, breakup, and possibly become submerged (e.g., Penland and Boyd, 1981; Leatherman et al., 1982; McBride et al., 1995). Penland and Boyd (1981) defined three stages of deltaic barrier island formation based on evolution of barrier islands associated with the Mississippi River Delta. After a mature active delta was abandoned by the river, Stage 1 began with an erosional headland that fed flanking barrier islands with sand that had been reworked from the mixed deltaic deposit. Over time (millennia), subsidence and wave-induced erosion deplete the source of deltaic sediment. Stage 2 consisted of a transgressive (retreating) sandy barrier island arc. Finally, Stage 3 occurred when erosion and subsidence reduce the barrier island to a subaqueous inner shelf shoal. Until human intervention began in the early 1900s (through levee construction and river diversion), this cycle repeated as the river occupied new locations or former deltas and provided a new source of sediment.

Because of this cycle of delta formation and abandonment, the Louisiana barrier islands are comprised of a relatively thin layer of fine sand that was reworked from the abandoned delta. The islands overlie a thick deltaic sequence of clay and silt that was deposited during the mid-to-late Holocene by the river, and eventually transgressed over back-barrier estuarine deposits (Coleman et al., 1998). Penland et al. (2005) documented long-term (greater than 100 years) and short-term (less than 30 years) shoreline change in Louisiana as −6.1 and −9.4 m/yr, respectively. The rapid erosion of Louisiana’s coast is attributed to the predominance of muddy sediment, thickness of peaty marsh soils, rapid rates of subsidence, and frequency of hurricanes (Kuecher, 1994; Penland et al., 2005). Without a source of littoral sand in the regional coastal system, and with rapid subsidence, barrier islands in Louisiana have ultimately drowned (e.g., Ship Shoal, Penland and Boyd, 1981). In the region of the abandoned LaFourche Delta, Kuecher (1994) correlated thicker deltaic sediment with the highest rates of land loss as compared to thinner deposits. Similarly, Penland and Ramsey (1990) found that local rates of relative sea level rise were related to the thickness of Holocene sediment for the Mississippi River Delta and Chenier plains.

Examples of barrier islands with similar cross-sections occur along the Delaware–Maryland–Virginia (“Delmarva”) Atlantic coast. Some of the barrier islands in this region, for example in southwestern Delaware Bay, and Wallips, Assawoman, and Metompkin Islands, Virginia, are comparable to the deltaic barriers in Louisiana in that they are composed of a thin veneer of sand over a compressible substrate. However, in this region the origin of the substrate is lagoonal mud and marsh deposits, over which the islands have overwashed and transgressed through time (Kraft et al., 1979; Leatherman et al., 1982; Oertel and Kraft, 1994). The Delmarva lagoonal and mud

![Fig. 1. Increase in overwash and migration of sandy barrier island with consolidation of underlying compressible substrate.](image-url)
deposits are of a similar thickness [order of 15 m or less, Halsey, 1978] as the deltaic sediment sequences in the vicinity of modern Louisiana barrier islands (approximately 10 to 30 m; Kulp et al., 2002, their Fig. 7). Conceptual, analytical, and numerical studies incorporating sand and mud fractions in long-term barrier island evolution and migration are being developed and improved (e.g., Cowell et al., 1995; van Maren, 2005; Stolper et al., 2005; Alfageome and Cañizares, 2005; Campbell, 2005; Campbell et al., 2005a,b; de Sonneville, 2006; Campbell et al., 2006, 2007; Thomson et al., 2007). Many studies have discussed compaction of peat deposits, either due to autocompaction (compaction due to self-weight) or compaction due to subsequent loading by estuarine sediments, in lagoons, estuaries, and deltaic settings (e.g., Bloom, 1964; Kaye and Barghoorn, 1964; Cahoon et al., 1995; Long et al., 2006; Meckel et al., 2007). Peat is much more compressible than other types of substrate such as sand, clay, silt, or mud. However, similar to sediment substrate, the magnitude of peat consolidation is related to the thickness of the deposit (Kuecher, 1994; Meckel et al., 2007). For example, Bloom (1964) measured 13 to 44% compaction of a sedge-peat deposit due to loading by a 10 m deposit of estuarine mud in Clinton, Connecticut. Long et al. (2006) determined that rapid compaction was a primary mechanism driving coastal change for a coastal marsh in southeast England, United Kingdom, where the peat surface compacted at least 3 m due to loading by 4 m of intertidal mudflat and tidal channel sediments. Meckel et al. (2007) developed a compaction model for deltaic settings and concluded with a statement pertinent to this study: “high density, permeable sediments such as sand, at the surface (typically considered relatively stable) can be associated with high compaction rates, especially if they overlie thick peat deposits.”

Guber and Slingerland (1981; see also Leatherman, 1987) and Gayes (1983) considered the compaction of back-barrier and estuarine sediment through placement of dredged material and by washover sand. Data from two dredged sediment disposal sites placed on the back marsh of Assateague Island, Maryland, indicated a linear relationship between the effective pressure (overburden) and subsidence of the marsh surface, with the older site having greater subsidence due to the longer loading time. Lateral plastic flow and diapirism (extrusion of sediment from the substrate such as “mudlumps” of the Mississippi River Delta system, Morgan, 1951) are possible with loading by barrier island sands and tidal deltas, for cases in which the pore water pressure is the same order of magnitude as cohesion in the substrate (both in units of kPa; Guber and Slingerland, 1981). Compaction and lateral flow of sediments in the vicinity of barrier islands include the following potential consequences. (1) Washover removes sand from the foreshore or dune system and, with settlement into a compressible marsh or bay subsurface, will induce more losses from the barrier island system rather than increasing elevation. Thus, the island will be susceptible to additional overwash in the same region and this additional overwash would increase migration and breakup of the island, suggesting that compaction may also be a factor in migration. (2) Barrier island cross-section and resulting geomorphology may be influenced by the subsurface characteristics, especially if the subsurface were non-homogeneous. Variable retreat rates for barrier island systems might be related to subsurface characteristics (e.g., void ratio, permeability, yield criteria). Observations of washover fans on Assateague Island, Virginia and Maryland showed lower elevations at the distal ends of the fan as evidenced by ponding as compared to the adjacent marsh surface which was at the mean water level. These fans also indicated that lateral flow of the subsurface sediments might have occurred, as evidenced by arcuate ridges paralleling the distal portions of the fans. (3) The potential for settlement must be known for characterization of the sediment budget of the barrier island system and washover fans, because sequestration of sand into the substrate would represent a net loss to the budget.

Roberts et al. (1994) linked consolidation of Holocene sediments in the Mississippi River deltaic system to the thickness of the deposits. In turn, the thickness of these deposits is correlated with the location of previous fluvial entrenchment by the Mississippi River system. Thus, with knowledge of the sediment type and former locations of fluvial entrenchment, Kuecher’s (1994) and Roberts et al.’s (1994) work lend information with which to estimate potential future compaction of the substrate as a function of the magnitude of the loading.

Bourman et al. (2000) discussed rapid geomorphologic changes that have been observed at the River Murray Estuary, Australia due to eolian, riverine, tectonic, tidal, and wave processes, as well as changes in eustatic sea levels over the past 125,000 years and recent human activity. Of pertinence to this review is the observation that, as barrier islands fronting the River Murray Estuary have migrated landward over the past 3000 years, they have differentially loaded plastic mud in the lagoon resulting in an increase in height up to 10 m above present sea level. The authors discussed that differential loading of these lagoonal sediments was sufficient to explain their elevation, but that seismic events may have also played a role.

This review of experiences demonstrates that loading of the substrate through barrier island migration can influence the resulting morphology change. In particular, subsurface characteristics can contribute to future morphology change of a barrier island system through three mechanisms: the cross-section of the island as a function of subsurface characteristics and loading; the feedback between consolidation, barrier island elevation, and subsequent washover; and the apparent non-conservation of sediment due to compaction of the substrate. None of the conceptual, analytical, or numerical models that were reviewed is capable of calculating these processes. The links between the loading by the islands on a compressible substrate, the magnitude of time-dependent consolidation as a function of the load, and the subsequent morphologic change and evolution of a barrier island has not been previously quantified.

3. Model development

The two-dimensional Migration, Consolidation, and Overwash (2D MCO) model was developed to investigate the role of consolidation on migration of sandy barrier islands that overlie compressible sediments. The paradigm in model development is that storm waves and elevated water levels provide the primary forcing for island erosion, washover, and cross-shore migration during an annual storm season (e.g., see Ritchie and Penland, 1988; Dingler and Reiss, 1990, 1995; Rosati and Stone, 2009). During the remainder of the year, sand transport processes are assumed to be relatively minor in the cross-shore morphologic evolution and migration of the island. Although longshore transport processes during this time could translate the island laterally, it is assumed that the cross-shore profile is not significantly modified. Time-dependent consolidation of the underlying substrate is calculated as a function of the loading by the island as it evolves over an annual storm season. The island evolves over years to decades as a function of storm surge, wave height and period, consolidation of the substrate, and the rate of eustatic sea level change.

Initial conditions are defined by a sandy barrier island with a given cross-shore profile that overlies a sediment substrate of specified characteristics. The 2D MCO model calculates erosion, runup overwash, or inundation overwash depending on the storm conditions and relative elevation of the barrier island. If washover of the island occurs, the barrier migrates into the bay and consolidation occurs due to the existing and any new loading (if migration occurred onto partially-consolidated sediments) (Fig. 2).

3.1. Wave transformation

The 2D MCO model transforms storm waves from deep to shallow water using a time series of wave height, period, and direction; or, the
model can randomly generate wave and surge conditions from user-specified averages. Deep-water waves are transformed from offshore measurements to breaking conditions using linear wave theory, in which time-dependent measurements of deep-water wave height, $H_o(t)$, are related to wave height at breaking, $H_b(t)$, by (Dean and Dalrymple, 1984, p. 115)

$$H_b(t) = \frac{\kappa}{g} \frac{1}{\sqrt{C_{18}/C_{19}}} \left( \frac{L_o(t)}{T(t)} \cos \theta_o(t) \right)^{2/5}. \tag{1}$$

The depth-limited breaking criterion is $\kappa = H_b(t)/d_b(t) = 0.78$, in which $d_b(t)$ is the depth at breaking, and the deep-water wave height and direction are given by $H_o(t)$ and $\theta_o(t)$, respectively. The deep-water wave speed is given by $C_o(t)$,

$$C_o(t) = \frac{L_o(t)}{T(t)} \tag{2}$$

where $T(t)$ is the wave period and $L_o(t)$ is the deep-water wave length equal to

$$L_o(t) = \frac{gT(t)^2}{2\pi}. \tag{3}$$

### 3.2. Erosion

Barrier island erosion and deposition offshore occur if the storm surge plus wave runup do not exceed the barrier island elevation (Fig. 3a).

The time-dependent recession of the berm through erosion, $E(t)$, is calculated based on the duration and magnitude of the storm as well as beach profile morphology using the Convolution Storm Erosion Method (Kriebel and Dean, 1993),

$$E(t) = E_\infty \left\{ 1 - \beta_t \exp \left( -\frac{2\sigma t}{\beta_t} \right) - \frac{1}{1 + \beta_t^2} \right\} \left( \cos 2\sigma t + \beta_t \sin 2\sigma t \right) \tag{4}$$

in which the maximum potential recession through erosion, $E_\infty$, is given by a form of the Bruun Rule that incorporates wave setup,

$$E_\infty = (S(t) + 0.068H_b(t)) \left( \frac{W_b(t)}{B + d_b(t)} \right). \tag{5}$$

In Eq. (5), the storm surge is $S(t)$, $W_b(t)$ is the width of the surf zone out to $d_b$, and $B$ is the berm elevation. In Eq. (4), $\beta_t$ represents the percentage of time the storm is effective in eroding the beach and $\sigma = \pi/T_D$, where $T_D$ is the total storm duration. The percentage of effective erosion time, $\beta_t$, is calculated by

$$\beta_t = \frac{2\pi}{T_D} \tag{6}$$

in which the duration of effective erosion, $T_\epsilon$, is given by,

$$T_\epsilon = 320 \frac{H_b(t)^{1.5}}{g^{0.35}A^2} \left( 1 + \frac{d_b(t)}{B} + \tan \beta(t) \frac{W_b(t)}{d_b(t)} \right)^{-1} \tag{7}$$

in which $g$ is the acceleration due to gravity, and $\beta(t)$ is the beach slope. The width of the surf zone is calculated as

$$W_o(t) = \left( \frac{d_b(t)}{A} \right)^{1.5} \tag{8}$$
in which $A$ is the equilibrium beach profile parameter and can be related to median grain size or sand fall speed. The equilibrium beach profile concept was first developed by Bruun in 1954 (Bruun, 1962; Komar, 1998) based on beaches in Monterey Bay, California, and the exponential value confirmed by Dean (1977) in analysis of more than 500 beach profiles along the U.S. Atlantic and Gulf coasts and has since been applied to beaches around the world (e.g., Dean et al., 1993; Larson et al., 1999; Dean and Dalrymple, 2002). The equilibrium beach profile relates the long-term shape of the profile elevation of the beach profile, $y$, to distance offshore, $x$.

$$x = Ay^{2/3}. \quad (9)$$

During an erosion event, the eroded sand is transported offshore and deposited, thereby decreasing depths offshore such that sediment volume is conserved. Avalanching of the profile is initiated if the slope is greater or equal to a user-specified avalanching angle with a default value of 30°.

### 3.3. Runup overwash and inundation overwash

Overwash is any wave uprush which passes over the “crown,” or crest of the barrier beach (Leatherman, 1979, p. 3). Of relevance to this study is the magnitude of the morphologic feature created by overwash and deposited on the bayside of the crest, called “washover,” “washover deposit” or “washover fan” (Leatherman, 1979, p. 2). The frequency and magnitude of overwash depend on long-term conditions, such as storm climatology, relative sea level rise, and sediment supply. Overwash and the resulting washover deposit are one of the mechanisms through which the barrier island migrates towards the bay (across shore). Two modes of overwash are simulated in the model: runup overwash and inundation overwash. Runup overwash occurs if the island is not submerged, and washover is caused by the uprushing wave bore. Inundation overwash occurs when the storm surge level and wave setup exceed the elevation of the barrier island crest, and the barrier island is submerged (Donnelly et al., 2009).

The overwash transport rate over the beach crest due to runup overwash per unit length of beach, $q_{\text{DR}}(t)$, can be described as (Donnelly et al., 2009),

$$q_{\text{DR}}(t) = 2K_{R}\sqrt{2g\frac{z_{R}(t)^{3/2}}{R(t)} \left(1 - \frac{b_{h}(t)}{R(t)} \right) \text{for } 0 < z_{g}(t) \text{ and } S(t) + d(t) < b_{h}(t)} \quad (10)$$

where $K_{R}$ is a calibration coefficient that accounts for sediment concentration and properties of the wave bore, $z_{g}(t)$ is the elevation of

Fig. 3. Terminology for erosion and overwash calculations.
the runup, \( R(t) \), relative to the dune crest elevation, \( h_d(t) \), and \( d(t) \) is the local water depth (Fig. 3b). For calculations herein, \( K_s \) was set equal to 0.005 as recommended by Donnelly et al. (2009).

The two-percent runup, \( R_{ux2}(t) \), is calculated as (Hughes, 2004),

\[
R_{ux2}(t) = 4.4\,\frac{A_0(t)}{\rho g (S(t) + d(t))^0.70} \frac{M_t(t)}{(S(t) + d(t))^2} \left[ \frac{M_t(t)}{\rho g (S(t) + d(t))^2} \right]^{1/2}
\]

for \( \frac{1}{5} \leq \tan \beta(t) \leq \frac{1}{3} \)

(11)

in which \( \rho \) is the density of water, and the maximum dimensionless depth-integrated wave momentum flux per unit width, \( M_t(t) \), is

\[
\left[ \frac{M_t(t)}{\rho g (S(t) + d(t))^2} \right]_{\text{max}} = A_0(t) \left[ \frac{(S(t) + d(t))}{g T_p(t)^2} \right]^{0.5256}
\]

where \( A_0(t) = 0.6392 \left( \frac{H_{mx}(t)}{S(t) + d(t)} \right)^{0.2056} \)

(12)

and \( A_1(t) = 0.1804 \left( \frac{H_{mx}(t)}{S(t) + d(t)} \right)^{0.391} \).

The transport rate over the beach crest per unit width of beach due to inundation overwash, \( q_{sw}(t) \), is given by (Donnelly et al., 2009) as,

\[
q_{sw}(t) = 2K_i \sqrt{2g} \frac{h_d(t)}{\tan \beta_{eq}} \left( \tan \beta_{eq} - \tan \beta_{rew} \right)
\]

where \( K_i \) is an empirical coefficient, and \( \beta_{rew} \) is as defined previously (Fig. 3c). For calculations herein, \( K_i \) was set to 0.005 (Donnelly et al., 2009).

Transport in the swash zone is calculated as (Larson et al., 2004),

\[
q_{sw}(t) = K_{sw} 2 \sqrt{2g} R_{ux2}(t)^{3/2} \left( \tan \beta_{eq} - \tan \beta_{rew} \right)
\]

where \( K_{sw} \) is an empirical coefficient, set equal to 0.0016 for calculations herein, \( \beta_{rew} \) is the local slope in the swash zone, and \( \beta_{eq} \) is the equilibrium slope calculated using equilibrium profile concepts,

\[
\beta_{eq} = \frac{\frac{x_{dune}^{2/3} - x_{sw}^{2/3}}{x_{dune} - x_{sw}}} {A}
\]

Location of the dune and swash are given by \( x_{dune} \) and \( x_{sw} \), respectively.

3.4. Consolidation

The original relationship for calculating soil consolidation was derived by Terzaghi (1925, 1943) and Terzaghi and Peck (1967). Primary consolidation is defined as the process during which excess pore water pressure is dissipated from a soil particle matrix, and is derived from fundamental hydraulic principles. The assumptions for one-dimensional consolidation theory are: (1) a fully-saturated sediment system; (2) unidirectional flow of water; (3) one-dimensional compaction occurring in the opposite direction of flow; (4) a linear relationship between the change in sediment volume and the applied pressure (linear small-strain theory); and (5) validity of Darcy’s Law, which states that the specific discharge (flow rate per area) through a porous medium is equal to the hydraulic gradient times the hydraulic conductivity (Yang and Warkentin, 1966; Hornberger et al., 1998). For one-dimensional vertical flow, the maximum consolidation, \( z_c \), for a given loading \( p \) greater than an initial loading, \( p_o \), can be calculated as,

\[
z_c = z_0 \left( \frac{C_s}{1 + e_0 \log \frac{p}{p_o}} \right)
\]

where \( z_0 \) is the thickness of compressible sediment; \( C_s \) is the value of the compression index that is determined experimentally from a Casagrande consolidation test (Casagrande and Fadum, 1940; Hornberger et al., 1998); and \( e_0 \) is the initial void ratio, equal to the volume of voids divided by the volume of solids as averaged over the thickness \( z_0 \). The parameter \( z_0 \) can be estimated from sediment core data or depositional maps that represent the thickness of soft sediment. For the Mississippi River system, Kulup et al. (2002) analyzed data from more than 800 boreholes and mapped the topstratum lithosome, which represents fine-grained deposition in fluvial, deltaic, and shelf environments that overlies coarser-grained substratum. Kulup et al. determined that the maximum thickness of topstratum sediment is approximately 120 m in vicinity of the modern Balize depocenter, and ranges from approximately 10 to 30 m in vicinity of the modern barrier islands (Kulp et al., 2002, their Fig. 7). In the applications discussed herein, \( z_0 \) did not vary spatially, but non-homogeneous variation in the subsurface characteristics could be incorporated in site-specific application of the model.

The magnitudes of \( C_s \) and \( p_o \) vary depending on whether \( p \) is greater or less than the pre-consolidation loading, \( p_o \), which can be determined from a Casagrande consolidation test. The magnitude of the pre-consolidation stress is important because it separates soils that are over-consolidated (i.e., these soils have experienced a greater load at some time in their past) from those that are under-consolidated (i.e., the present loading is the maximum that has occurred). Loading greater than the pre-consolidation stress will result in greater rates of consolidation than have previously occurred. Fig. 4 shows results of a Casagrande consolidation test conducted for a sediment sample at 12.5–13.1 m depth from Chaland Headland, Louisiana.

For the example shown in Fig. 4, if the loading \( p \) is greater than the pre-consolidation stress, \( p_o \), then \( p = p_o \) and \( C_s = C_s = 0.4 \) in Eq. (17). If \( p \) is less than the pre-consolidation stress, then \( p \) is equal to the initial stress, \( p_o = 660 \) kg/m\(^2\), and \( C_s = C_s = 0.125 \) in Eq. (17).

Terzaghi’s (1943) time-dependent relationship for consolidation is

\[
\frac{\partial u}{\partial t} = C_{cvu} \frac{\partial^2 u}{\partial z^2}
\]

where \( u \) is pore water pressure in excess of hydrostatic pressure, \( t \) is elapsed time, \( C_{cvu} \) is a property of the compressible sediment, called the coefficient of consolidation and also determined during consolidation testing, and \( z \) is the vertical coordinate with the origin at the initial sediment surface (Fig. 5). If the loading is greater than the pre-consolidation stress, then \( C_{cvu} = C_{cv} \) otherwise \( C_{cvu} = C_{cv} \) (Fig. 4).

The proportion of the initial pore water pressure remaining at any time, \( M(t) \), can be expressed as,

\[
M(t) = \frac{1}{z_0} \int_0^{z_0} u \, dz = \frac{e(t) - e_t}{e_0 - e_t}
\]
in which \( u_0 \) is the initial pore water pressure, \( e(t) \) is the average void ratio at any time, and \( e_f \) is the final average void ratio corresponding to the consolidation test results for the portion of the curve less than or greater than the pre-consolidation stress. The variable \( M(t) \) varies between 1 and 0, at time \( t = 0 \) and infinity, respectively. The proportion of vertical consolidation that occurs at any time can also be expressed as,

\[
z(t) = z_c(z_c - e(t) - e_0). \tag{20}
\]

Combining Eqs. (19) and (20) gives,

\[
z(x, t) = z_c(x, t)(1 - M(t)) \tag{21}
\]

where \( M(t) \) can be expressed as (Dean, 2002, p. 119)

\[
M(t) = 8 \sum_{n=1}^{\infty} e^{-\frac{(2n-1)^2}{4t}} \frac{(2n-1)^2\pi^2}{(2n-1)^2\pi^2}. \tag{22}
\]

Fig. 6 shows a flowchart for the consolidation routine in the 2D MCO model.

The consolidation routine was evaluated with information presented by Blum et al. (2008) in which they discussed uplift and subsidence of the Mississippi River Delta over the past 30,000 years. Although the spatial and temporal scales applied by Blum et al. for the Mississippi River Delta are much greater than for a barrier island, the magnitude of consolidation as a function of the thickness and associated weight of sediment are commensurate. Specific to this study, Blum et al. discuss the gradual deposition of approximately 40 m of deltaic sediment in the vicinity of New Orleans (90.5° longitude) from 11,500 to 4,000 years ago, and no additional deposition from 4000 years ago to the present as the river’s depocenter had moved further downstream. Blum et al.’s calculations with a three-dimensional visco-elastic consolidation model indicated that approximately 5.9 m surface deflection (the net of consolidation plus uplift) occurred over the past 10,000 years (see their Fig. 3b and c, reproduced below in Fig. 7b and c). As the river incised the Lower Mississippi River Valley with meltwater, the removal of sediment created an uplift of the surface from 30,000 to 9500 years ago. This loading cycle (linear deposition of 40 m thickness from 11,500 to 4000 years, then no deposition from 4000 years to present) was programmed into the consolidation code with the same sediment–water density as discussed by Blum et al. (1800 kg/m³) and indicated a value of 6.7 m consolidation over the past 11,500 years (Fig. 7). The value calculated by the consolidation routine developed herein is 15%
greater than Blum et al.’s results. However, Blum et al.’s value includes approximately 0.5 m that occurred at the end of the uplift period. Adjusting the consolidation calculations for the uplift results in an estimate of 6.2 m, within 5% of Blum et al.’s results. This comparison is considered a validation of the consolidation routine.

4. Model comparison with field data

To fully calibrate and validate 2D MCO, an initial beach profile and subsurface elevation, similar data several years to decades later, sediment characteristics of the overlying sand beach and compressible subsurface, and the forcing hydrodynamics during that time period would be required. A complete search of existing databases and literature was conducted and identified only a data set from Virginia as sufficient for comparison with model applications. The term “comparison” is introduced here to describe application of the model to measurements in which several unknowns were estimated. In this comparison, the model input conditions were likely only one of many sets that could be developed and executed in the model to adequately reproduce the measured response. Although the comparison to Virginia data is not an ideal calibration and validation of the model, it lends credibility to 2D MCO applications.

4.1. Virginia data

In a study of Virginia barrier islands, Gayes (1983) surveyed the barrier and beach profile and collected sediment cores across three migrating barrier island systems that overlie a compressible peat and
bay sediment substrate: Assawoman Island, Metomkin Island, and Wallops Island, Virginia, U.S.A (Figs. 8–11). Wallops Island represents a site disturbed by construction and adjacent inlet processes and is not discussed further (see Gayes, 1983 for more detail). When adjusted for eustatic sea level rise of 2 mm/yr (Douglas, 1992; Peltier, 1998) over a 40-year period, these barrier island systems experienced consolidation of the substrate between 0.9 and 1.1 m as they migrated landward at rates ranging from 3.8 to 4.8 m/yr. The elevations of these islands were approximately 1.2 to 2.6 m relative to Mean High Water (MHW), with the maximum thickness of sand overlying the substrate ranging from 1.4 to 3.5 m. If consolidation of the underlying substrate had not occurred, the sand–clay/silt interface would lie at approximately the zero MHW line. Void ratios of the back-barrier sediment are greater than those of the clay and silt underlying the sandy barrier island, reinforcing the interpretation of consolidation due to the loading of the island. Measurements are summarized in Table 1.

The goals of model comparisons with the Virginia data were to test whether 2DMCO could reproduce the cross-shore shape and elevation of the profiles and magnitude and shape of the consolidated surface. Unknowns in this process were the initial profile shape, storm wave and surge conditions during the simulation period, the magnitude of $z_0$ in the consolidation calculations (Eqs. (17), (19), and (22)), and results from a
Casagrande consolidation test of the subsurface or back-barrier sediment cores ($e_o, c_o,$ and $p_s$ in Eq. 17). Values of $c_v$ (Eq. 22) were available for Metomkin Island at two locations, and the average value of a back-barrier sample equal to 2.5 m$^2$/yr was applied for the Metomkin and Assawoman sites. In the absence of Casagrande consolidation test data, values for Chaland Headland, Louisiana (Fig. 4) were adopted for the Virginia barrier islands.

All islands have marsh on the back-barrier, and the 2D MCO model was modified to allow the back-barrier marsh surface to maintain elevation with MHW during the simulation period. The mean, spring, and neap tidal amplitudes for Metomkin are 0.55, 0.65, and 0.43 m, respectively (Byrnes and Gingerich, 1987). Based on Gayes’ description that overwash was “extensive” for the Metomkin site (at 1.2 m MHW dune elevation) and “infrequent” for the Assawoman sites (at...
and 2.6 m MHW dune elevations), it is speculated that a dominant storm water level elevation (including surge, tide, wave setup and runup) has been between 1.2 and 2.6 m MHW for this region of the coast during the period of measurement. Wave Information Study (WIS) data for Station 179, located in 18-m water depth at latitude 37.75° and longitude 75.33°, indicated that the mean $H_{mo} = 0.9 \pm 0.6$ m, with associated peak period $T_p = 6 \pm 2.8$ s (WIS, 2008). From these discussions and in the absence of other data, it is assumed that a reasonable value for total storm water elevation at the site ranges from 1.2 to 2.6 m MHW, with maximum deep-water significant wave height approximately 1.5 m.

For comparison with 2D MCO, variations in storm surge, offshore wave height, and tidal elevation (randomly generated within the mean tide amplitude $= +/−0.55$ m) were applied to force the model.

Table 1
Data from Virginia. Adapted from Gayes, 1983.

<table>
<thead>
<tr>
<th>Site</th>
<th>Dune elev (m MHW)</th>
<th>Max sand below (m MHW)</th>
<th>Sand vol $(m^3/m)$</th>
<th>Mig rate (m/yr)</th>
<th>Relative back-barrier sed</th>
<th>Relative freq of overwash</th>
<th>Relative sand supply</th>
<th>Max consol due to weight of sand $(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 (Fig. 8)</td>
<td>1.4</td>
<td>2.2</td>
<td>225</td>
<td>3.75</td>
<td>Finest; organic-rich</td>
<td>Infrequent</td>
<td>Greatest</td>
<td>1.1</td>
</tr>
<tr>
<td>A2 (Fig. 9)</td>
<td>2.6</td>
<td>3.5</td>
<td>343</td>
<td>4.05</td>
<td>Coarsest; greatest energy</td>
<td>Infrequent</td>
<td>Intermediate</td>
<td>1.0</td>
</tr>
<tr>
<td>M (Fig. 10)</td>
<td>1.2</td>
<td>1.4</td>
<td>194</td>
<td>4.79</td>
<td>Medium</td>
<td>Extensive</td>
<td>Lowest</td>
<td>1.2</td>
</tr>
</tbody>
</table>

| 1 | Al=Assawoman 1, A2=Assawoman 2, M=Metomkin. |
| 2 | Over a 40-year period. |
| 3 | Adjusted to remove sequestering of sand due to eustatic sea level rise $= +/−0.55$ m (revised from original calculation by Gayes (1983) in which he applied 1 mm/yr). |

Table 2
Summary of 2D MCO applications to Virginia Barrier Islands (100 year calculation).

<table>
<thead>
<tr>
<th>2D MCO input parameters</th>
<th>Calculations and measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial dune elevation (m MHW)</td>
<td>Island width (m)</td>
</tr>
<tr>
<td>Assawoman cross-section 1</td>
<td>5.6</td>
</tr>
<tr>
<td>Assawoman cross-section 2</td>
<td>7.6</td>
</tr>
<tr>
<td>Metomkin</td>
<td>7.6</td>
</tr>
</tbody>
</table>

| 1 | Average total storm water level includes storm surge, wave runup, tide, and relative sea level rise over 100-year simulation period. |
over a 100-year simulation period, which was a sufficient duration to allow the barrier island and subsurface to equilibrate to the forcing and loading conditions. In the model simulations, eustatic sea level rise was 2 mm/yr (Douglas, 1992; Peltier, 1998) and water level increased each year with sea level rise.

An iterative process was applied in the modeling exercise, modifying the starting dune crest elevation, barrier island width, wave and surge conditions, and thickness of compressible sediment $z_0$ such that the measured profile cross-sections and cross-shore magnitude of consolidation were approximated after the simulation period. Eight thousand 1-m cross-shore calculation cells, and up to thirty 1-m vertical cells were simulated with the Matlab© code, which took approximately 2 to 5 min for 100-year simulation on a 1.86 GHz personal computer. Input variables and a comparison between measured and calculated output for simulations that agreed best with the known information about the sites are summarized in Table 2. Fig. 12 summarizes results from the model simulations as measured at the dune. Simulation profiles calculated after 100 years, as well as the subsurface at 25, 50, and 100 years are shown for each site in Figs. 13–15.

Model calculations approximate the general shape of the beach and subsurface elevations, with the maximum dune crest elevation and minimum consolidated subsurface elevation agreeing within 0.2-m of measured values. The calculated profile poorly represents the measured beach profile seaward of the dune, likely because erosion and overwash processes are only represented during storm events when water levels

---

**Fig. 12.** Summary of 2D MCO calculations for Assawoman cross-section 1 simulation as measured at the dune crest.

**Fig. 13.** Comparison of 2D MCO calculations with measured beach and subsurface profiles for Assawoman Island cross-section 1.
are near the dune crest elevation. The shape of the calculated subsurface agrees with the general trend of the measured subsurface at the 25 and 50-year durations. The shape of the calculated 100-year subsurface indicates additional compression from washover sand on the back-barrier marsh, which is not evident in the measured subsurface but becomes more likely over longer time periods.

5. Implications of compressible substrate

In this section, three sets of 2D MCO simulations are presented to compare evolution of barrier islands overlying a compressible substrate (BIC) to those overlying a stable substrate (BIS). The first series evaluated how compressibility of the substrate modified cross-shore barrier island migration and morphology change over a 50-year simulation period. The analysis varied the thickness of compressible sediment \( z_0 \) for a barrier island with initial dune elevation of 3.5 m relative to mean sea level (MSL), 2200 m width at MSL, 1-m average storm wave height, and +1 m average surge relative to MSL. Five magnitudes of \( z_0 \) were evaluated: 0 (stable substrate), 5, 10, 20, and 25 m, with consolidation parameters \( c_{vi} = c_{vc} = 2.5 \text{ m}^2/\text{yr} \). Simulations terminated if the crest of the barrier island became submerged below MSL. Results are shown in Fig. 16.

Larger values of \( z_0 \) increased the magnitude of maximum consolidation, the rate of dune lowering, and decreased the duration of the simulation. The migration rate increased as substrate thickness increased from 0 to 10 m then was constant for greater substrate thicknesses. The constant migration for substrate thickness greater than 20 m reflects the tendency for BIC to drown in place rather than migrate, as the consolidation process dominates the island evolution. This process of island lowering, followed by breakup and submergence has been observed for barrier islands with a limited sand supply that overlie thick deltaic deposits in Louisiana (Penland and Boyd, 1981; Kuecher, 1994; Roberts et al., 1994; McBride et al., 1995; Kulp et al., 2002).

The next series of simulations evaluated whether a sufficient supply of littoral sand would increase the migration rates for BIC. A longshore sand transport (LST) sub-module was coupled with 2D...
MCO to provide a net source of sand to the island (arbitrarily set to a maximum of 75 m$^3$/m/yr) if it was submerged below MSL during the previous storm season. The source of LST implies that the rate of sand transport provided to the island is greater than that removed from the island; if LST increases with distance along the island or diverges, then LST represents a net sink (or loss) to the island. In the simulations, if the source of LST was insufficient to maintain the island above MSL, the simulation terminated. Simulations were set for a 99-year duration with the same island and storm parameters as in the previous simulations, and the thickness of compressible substrate $z_0 = 0, 10, 15,$ and 20 m with consolidation parameters $c_v = c_w = 2.5$ m$^2$/yr. Results of these simulations are shown in Fig. 17.

Net LST rates required to maintain the islands through the migration process were equal to 4 m$^3$/m/yr for BIS and ranged between 31 and 73 m$^3$/m/yr for BIC. Migration rates of BIC were 3 to 11 times greater than for BIS, if a sufficient source of net LST was available to replenish the island. Nevertheless, the LST rate was not sufficient to maintain BIC with $z_0 = 15$ and 20 m for the full 99 years; simulations terminated after 60 and 43 years, respectively. Thus, for this set of simulation parameters, deltaic deposits greater than 15 m thickness continue to dominate morphologic change and cause island submergence even with a net source of LST.

The final series evaluated whether, in the absence of a sufficient LST sand source, the lifetime of BICs with varying dune crest elevations was reduced as compared to BIS. Simulations varied initial dune crest elevation from 2 to 4.5 m MSL for a 50-year simulation period, and are summarized in Table 3. All BIC durations were less than those of BIS except for the largest barrier island (dune crest elevation = 4.5 m MSL). For these cases, the island provided a sufficient elevation to reduce washover as evidenced by similar migration rates for both BIC and BIS. The simulations suggest that the lifespan of BIC are reduced as compared to BIS, except for BIC that are of sufficient elevation to reduce washover over the time periods of consideration and for prevailing storm conditions at the site.

6. Conclusions

This study developed a cross-shore model of sandy barrier island erosion and overwash, including time-dependent consolidation of the underlying compressible substrate as a function of the thickness and...
sediment characteristics of the underlying substrate, and weight of the barrier island. The model is intended for applications spanning years to decades and was applied herein for simulations of 100 years in duration to test the hypothesis that substrate characteristics modify the cross-shore evolution (migration and volumetric losses) of a barrier island. The hypothesis was formulated to evaluate whether the consolidation process accelerates long-term morphologic response for sandy barrier islands overlying poorly-consolidated sediment, such as deltaic, bay, estuarine, and peat deposits. Data to validate the model were limited to profile and subsurface elevations for three cross-shore transects of two barrier islands in Virginia. Reasonable values for other input parameters were estimated to compare with model calculations. After calibration, the model was able to reproduce the maximum dune crest and minimum subsurface elevation within 0.2 m of the measured values in a 100-year simulation.

Application of the model validated the hypothesis through simulations of cross-shore barrier island morphologic change. For a compressible substrate thickness from 10 to 30 m, such as beneath the modern Louisiana barrier islands (Kulp et al., 2002), model simulations indicated that substrate thickness was non-linearly related to the consolidation rate and loss of dune crest elevation. For substrate thickness greater than 10 m, model calculations indicated a tendency for barrier islands to drown in place as the process of subsurface consolidation dominated the morphologic response. With a sufficient source of sand to the island, migration rates for islands overlying a compressible substrate were 3 to 11 times greater than for the same island over a stable substrate. Without a source of sand, islands overlying a compressible substrate became submerged more rapidly than within a stable substrate. The exception was for a barrier island with sufficient elevation such that overwash and inundation did not occur.

Barrier islands overlying a compressible substrate incur an additional volumetric loss due to the consolidation process as a function of the magnitude of loading applied to the substrate and duration of the loading. Because of consolidation of the substrate, islands that migrate and overwash experience additional losses due to consolidation, as compared to islands that are stable. Barrier islands overlying a consolidating substrate are more likely to have (1) reduced dune elevations because of consolidation, (2) overall volumetric loss to fill in compressed regions outside the footprint of the island, and (3) increased overwash and migration after the dune reaches a critical elevation with respect to the total water elevation of the prevalent storm conditions. In effect, the consolidation process decreases the return period (increases the frequency) of the prevailing storm conditions. Numerical calculations with the model illustrated how consolidation modifies profile response through lowering of the dune elevation and increasing the potential for overwash and migration.

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<table>
<thead>
<tr>
<th>Initial dune elevation (m MSL)</th>
<th>Duration (yr)</th>
<th>Migration (m/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BIC</td>
<td>BIS</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>2.5</td>
<td>34</td>
<td>50</td>
</tr>
<tr>
<td>3.5</td>
<td>46</td>
<td>50</td>
</tr>
<tr>
<td>4.5</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

References


