THE PRANDTL PLUS SCALING PARAMETERS AND VELOCITY PROFILE SIMILARITY: PART 2

David Weyburne

Optoelectronic Technology Branch
Electromagnetics Technology Division

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*//Signature//   *//Signature//

DAVID W. WEYBURNE, Program Manager   JONAHIRA R. ARNOLD, Chief

Optoelectronic Technology Branch   Optoelectronic Technology Branch

Electromagnetics Technology Division   Electromagnetics Technology Division

*//Signature//

DANE F. FULLER, Lt Col, USAF

Deputy Division Chief

Electromagnetics Technology Division

Sensors Directorate

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14. ABSTRACT

In a previous report (Weyburne, AFRL-RY-WP-TR-2012-0227) it was pointed out that the momentum balance type approach to velocity profile similarity cannot always properly identify scaling variables that result in similar velocity profiles. Example datasets were found for which the Prandtl's "Plus" scaling variables satisfied all of the momentum balance requirements for similarity yet the velocity profile plots indicate the profiles were NOT similar. It was concluded that the scaling variables discovered by the momentum balance type approach as presently constituted are a necessary but not sufficient condition for velocity profile similarity. In that report, it was speculated that there is some other factor that is not being captured with the momentum balance type approach that would explain the failure. In this report we use the results from another report (Weyburne, AFRL-RY-HS-TR-2010-0014) to show the nature of the failure. In this earlier report, Weyburne described some new results for similarity of the velocity profile of the 2-D wall-bound boundary layer flow. By looking at the area under a set of scaled velocity profile curves, it was shown that if similarity exists, then: 1) the similarity velocity and length scaling parameters cannot be independent, 2) the displacement thickness must be a similar length scaling parameter, and 3) the velocity scaling parameter must be proportional to the free stream value of the stream-wise velocity above the boundary layer edge. By using these new results, it becomes possible to predict which datasets will show similar behavior and which will not.

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SUMMARY

In a previous report (Weyburne, AFRL-RY-WP-TR-2012-0227) it was pointed out that the momentum balance type approach to velocity profile similarity cannot always properly identify scaling variables that result in similar velocity profiles. Example datasets were found for which the Prandtl's "Plus" scaling variables satisfied all of the momentum balance requirements for similarity yet the velocity profile plots indicate the profiles were NOT similar. It was concluded that the scaling variables discovered by the momentum balance type approach as presently constituted are a necessary but not sufficient condition for velocity profile similarity. In that report, it was speculated that there is some other factor that is not being captured with the momentum balance type approach that would explain the failure. In this report we use the results from another report (Weyburne, AFRL-RY-HS-TR-2010-0014) to show the nature of the failure. In this earlier report, Weyburne described some new results for similarity of the velocity profile of the 2-D wall-bound boundary layer flow. By looking at the area under a set of scaled velocity profile curves, it was shown that if similarity exists, then: 1) the similarity velocity and length scaling parameters cannot be independent, 2) the displacement thickness must be a similar length scaling parameter, and 3) the velocity scaling parameter must be proportional to the free stream value of the stream-wise velocity above the boundary layer edge. By using these new results, it becomes possible to predict which datasets will show similar behavior and which will not using scaling variables other than the displacement thickness and the velocity at the boundary layer edge.

1. INTRODUCTION

Beginning with the pioneering work of Reynolds [1], there has been a concerted effort to find coordinate scaling variables that make plots of the scaled velocity profiles and shear-stress profiles taken at different stations along the wall in the flow direction to appear to be identical or, in the flow community vernacular, "similar". The standard approach for studying velocity profile similarity is to use the dimensionless momentum balance equation. The search for similarity scaling behavior for the turbulent boundary layer using this approach began with the experimental and theoretical work of Clauser [2] and subsequently expanded by others [3-6]. Recently Weyburne [7] pointed out that the momentum balance type approach to velocity profile similarity cannot always properly identify scaling variables that result in similar velocity profiles. This conclusion was based on finding datasets which satisfied all of the conditions for similarity yet the velocity profiles did not show similarity. In particular, example datasets were found for which the Prandtl's "Plus" scaling variables satisfied all of the momentum balance requirements for similarity yet the velocity profile plots indicate the profiles were NOT always similar. It was concluded that the scaling variables discovered by the momentum balance type approach as presently constituted are a necessary but not sufficient condition for velocity profile similarity. In that report, it was speculated that there is some other factor that is not being captured with the momentum balance type approach that would explain the failure.

In this report we use the results from a different earlier report [8] to show the nature of the failure. In the earlier report Weyburne presented some new results for similarity of the velocity profile of the 2-D wall-bound boundary layer flow. The results were based on a simple concept; the area under a set of scaled velocity profile curves that show similarity behavior must be equal. By taking certain integrals (equivalent to the area) of the scaled velocity profile, a number of new results were obtained. For example, it was shown that if similarity exists, then: 1) the similarity velocity and length scaling parameters are not independent, 2) the displacement thickness must be a similar length scaling parameter, and 3) the velocity scaling parameter must be proportional to the free stream value of the stream-wise velocity above the boundary layer edge. These results are used in this report to show that these additional requirements for similarity explains why certain datasets that satisfy the momentum balance type approach requirements do not show similarity of the velocity profiles. That is, looking at the seven previous datasets which were used to show the failure of the momentum balance approach, we can now correctly predict which of these datasets will show similarity and which do not using the additional similarity requirements from Weyburne [8].

2. THE MATHEMATICS OF SIMILARITY

As a first step we review the important aspects of Weyburne's new results dealing with the mathematics of similarity [8]. The following is a summary of the most relevant details from that report. In the analysis below, no assumptions are necessary as to the functional form of the velocity profile.

Consider a 2-D flow along a wall. The formal definition of similarity for this case is that two velocity profiles taken at different stations along the flow are similar if they differ only by a scaling constant in y and $u(x,y)$, where y is the normal direction to the wall, x is parallel to the wall in the flow direction, and $u(x,y)$ is the velocity parallel to the wall in the flow direction. We take the length scaling variable as δ and velocity scaling variable as u_s. These scaling variables can vary with the flow direction but not in the y-direction. The scaled velocity profile at a station x_1 will be similar to the scaled profile at x_2 if

$$\frac{u\left(x_1, y/\delta(x_1)\right)}{u_s(x_1)} = \frac{u\left(x_2, y/\delta(x_2)\right)}{u_s(x_2)} \quad \text{for all } y. \tag{1}$$

Starting with this formal definition of similarity then it is self-evident that for the profiles to be similar, the area under these scaled velocity profiles plotted versus the scaled y-coordinate must be equal. Furthermore, if similarity is present in a set of velocity profiles then it is self-evident that the scaled first derivative profiles (derivative with respect to the scaled y-coordinate) must also be similar. It is also self-evident that the area under the scaled first derivative profiles must be equal for similarity. In mathematical terms, area under the scaled first derivative profile is expressed by

$$d(x) = \int_0^{h/\delta} d\left\{\frac{y}{\delta}\right\} \frac{d\left\{u(x,y/\delta)/u_s\right\}}{d\left\{\frac{y}{\delta}\right\}} \quad , \tag{2}$$

where $d(x)$ is in general a non-zero numerical constant, and $y = h$ is deep into the free stream above the wall. Using a simple variable switch, Eq. 2 can be shown to reduce to

$$d(x) = \frac{u_e}{u_s} \quad . \tag{3}$$

where u_e is value of the stream-wise velocity $u(x,y)$ at the edge of the boundary layer. Similarity requires that $d(x_1) = d(x_2)$. Therefore, for similarity of the velocity profiles, the scaling velocity must be proportional to the free-stream velocity above the boundary layer edge. Note this is the same condition for similarity determined by

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Castillo and George [5] for flows with a pressure gradient but in this case the constraint applies to all 2-D wall flows.

Now let us consider the area under the scaled profiles themselves. In mathematical terms, the area under the scaled profile is given by

$$c(x) = \int_0^{h/\delta} d\left\{\frac{y}{\delta}\right\} \frac{u_e - u(x, y/\delta)}{u_s} \quad , \tag{4}$$

where $c(x)$ is in general a nonzero numerical constant. Note that the integral is written using the velocity difference rather than just the scaled velocity. It is simple to show that the area under the defect profile $u_e - u$ and the velocity profile is equivalent so long as Eq. 3 holds (note the use of Eq. 3 was not explicit but subsumed in the original derivation given in reference 8). The use of the defect profile has the advantage that the integral value is not dependent on the numerical value of h as long as h is located deep in the free stream. Using a simple variable switch ($d\{y/\delta\} \Rightarrow (1/\delta)dy$) and simple algebra, Eq. 4 can be shown to reduce to

$$c(x) = \frac{\delta_1 u_e}{\delta u_s} \tag{5}$$

where the δ_1 is the displacement thickness given by

$$\delta_1(x) = \int_0^h dy \left\{1 - u(x,y)/u_e\right\} \quad . \tag{6}$$

Eq. 5 is an exact equation that applies whether the profiles are similar or not. Similarity requires that $c(x_1) = c(x_2)$. Note that if we solve for u_s in Eq. 5, then u_s becomes the empirically derived velocity scale successfully used by Zagarola and Smits [2] to scale turbulent boundary flows over wedges, in channels, and in pipes. The importance of Eq. 5 in regards to similar profiles is that it means that the thickness scaling and the velocity scaling variables are not independent for 2-D wall-bounded similarity flows.

Now we turn to the task of considering what else can be learned from this equal area approach to similarity. Using the results given by Eq. 3 and Eq. 5, then it is evident that

$$b(x) = \frac{\delta_1}{\delta} \quad , \tag{7}$$

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where $b(x)$ is in general a non-zero numerical constant. Similarity requires that $b(x_1) = b(x_2)$. Eq. 7 is important in that it states that if similarity exists, then the displacement thickness must be a length scale that results in similarity.

3. PRANDTL PLUS SCALING SIMILARITY

Now that we have explored the new requirements for similarity that can be developed from the equal area approach, we now turn to the similarity and the momentum balance approach. In a recent report Weyburne [7] pointed out that the momentum balance type approach to velocity profile similarity cannot always properly identify scaling variables that result in similar velocity profiles. This conclusion was based on finding datasets for which the Prandtl's "Plus" scaling variables satisfied all of the momentum balance requirements for similarity yet the velocity profile plots indicate the profiles were NOT similar. Substituting Prandtl's length scale $\delta \propto v/u_\tau$ and the velocity scale $u_s \propto u_\tau$ into the reduced momentum balance terms, then it is possible to show that similarity in the near-wall region requires that the friction velocity must be a function of the type

$$u_\tau(x) = \frac{a}{x - x_0} \tag{8}$$

where a and x_0 are constants. It is also necessary that for flows with a pressure gradient, we must have $u_\tau \propto u_e$. Under these conditions then the Prandtl Plus scaling variables will insure that all momentum balance similarity requirements are satisfied [7].

Now consider the new requirements for similarity summarized in Section 2. In particular, substituting $\delta \propto v/u_\tau$ and the velocity scale $u_s \propto u_\tau$ into Eq. 3, then for similarity of the velocity profiles using the Prandtl Plus scaling variables we must have

$$\delta_1(x_1)\, u_e(x_1) = \delta_1(x_2)\, u_e(x_2) \ . \tag{9}$$

That is for a set of profiles to be similar using the Prandtl Plus scaling variables, it is required that $\delta_1 u_e$ must be a constant from one measurement station to the next. Furthermore, from Eqs. 6-7, if similarity is present in a set of profiles then δ_1 must be a length scaling variable and u_e must be a velocity scaling variable. This means that if similarity exists in set of profiles and "if" the Prandtl Plus scaling variables are also similarity scaling variables, then we must have the ratio of the thickness variables $\delta_1/(v/u_\tau)$ be a constant or equivalently

$$\delta_1(x_1)\, u_\tau(x_1) = \delta_1(x_2)\, u_\tau(x_2) \ , \tag{10}$$

and velocity ratio u_e/u_τ must also be a constant so that

$$\frac{u_e(x_1)}{u_\tau(x_1)} = \frac{u_e(x_2)}{u_\tau(x_2)} \quad . \tag{11}$$

This is the so-called Rotta constraint for similarity. It is necessary to reemphasize that Eqs. 9-11 only apply when one is considering whether the Prandtl Plus scaling variables result in similar velocity profiles.

4. EXPERIMENTAL VERIFICATION

The datasets used herein are the same datasets used previously to illustrate the problem with the momentum balance equation and similarity [7]. Recall that in the earlier report [7] it was shown that for these datasets the Prandtl's friction velocity did behave as $u_\tau \propto 1/(x - x_0)$ and $u_\tau \propto u_e$. This should have guaranteed similar profiles using the Prandtl Plus scaling variables since it insured that ALL of the *x*-grouping ratios in the scaled momentum balance equation would be constant at each measurement station along the wall. However, most of the datasets did not show similar behavior. Therefore satisfying the momentum balance approach must not be sufficient to insure finding similar profiles. Some other consideration is evidently necessary. In Section 3 above we showed that the mathematics of similar profiles imposes additional constraints on similarity. In particular, when one is considering the Prantl Plus scaling variables for similarity, Eq. 9 requires that $Re_{\delta_1} = \delta_1 u_e / \nu$ must be a constant at each measurement station along the wall.

In Fig. 1 we plot $(\delta_1 u_e)/(\delta_1 u_e)_{avg}$ versus $(x_2 - x_1)/\delta_1(x_1)$ for the seven datasets from the previous report [7]. Notice that $(\delta_1 u_e)/(\delta_1 u_e)_{avg}$ is plotted instead of Re_{δ_1} or $\delta_1 u_e$. This is to facilitate comparison between the datasets. What we would expect is that if the Re_{δ_1} values are equal at each measurement station then $(\delta_1 u_e)/(\delta_1 u_e)_{avg}$ should be a constant equal to one. Also notice that we used $(x_2 - x_1)/\delta_1(x_1)$ to facilitate comparison. In this case we arbitrarily choose the first measurement station as the reference point x_1 and used the boundary layer thickness at this station ($\delta_1(x_1)$) as the normalizing constant. What this gives then is a measure of the distance between measurement stations in terms of the boundary layer thickness measured at station one. Thus, in Fig. 1, similarity using Prandtl's Plus scaling variables would be indicated if the $(\delta_1 u_\tau)/(\delta_1 u_\tau)_{avg}$ is a constant (equal to one) at each measurement station or in this case as a function of the reduced distance between the measurement stations.

The other requirement for similarity using Prandtl's Plus scaling variables discussed in Section 3 is that the quantities given by Eqs. 10 and 11 must be constant. In Figs. 2 and 3 we plot these quantities as a function of the reduced distance between measurement stations. As mentioned in the previous report, the seven datasets were chosen because they satisfied the Rotta constraint (Eq. 11). Thus it is not unexpected that Fig. 3 shows

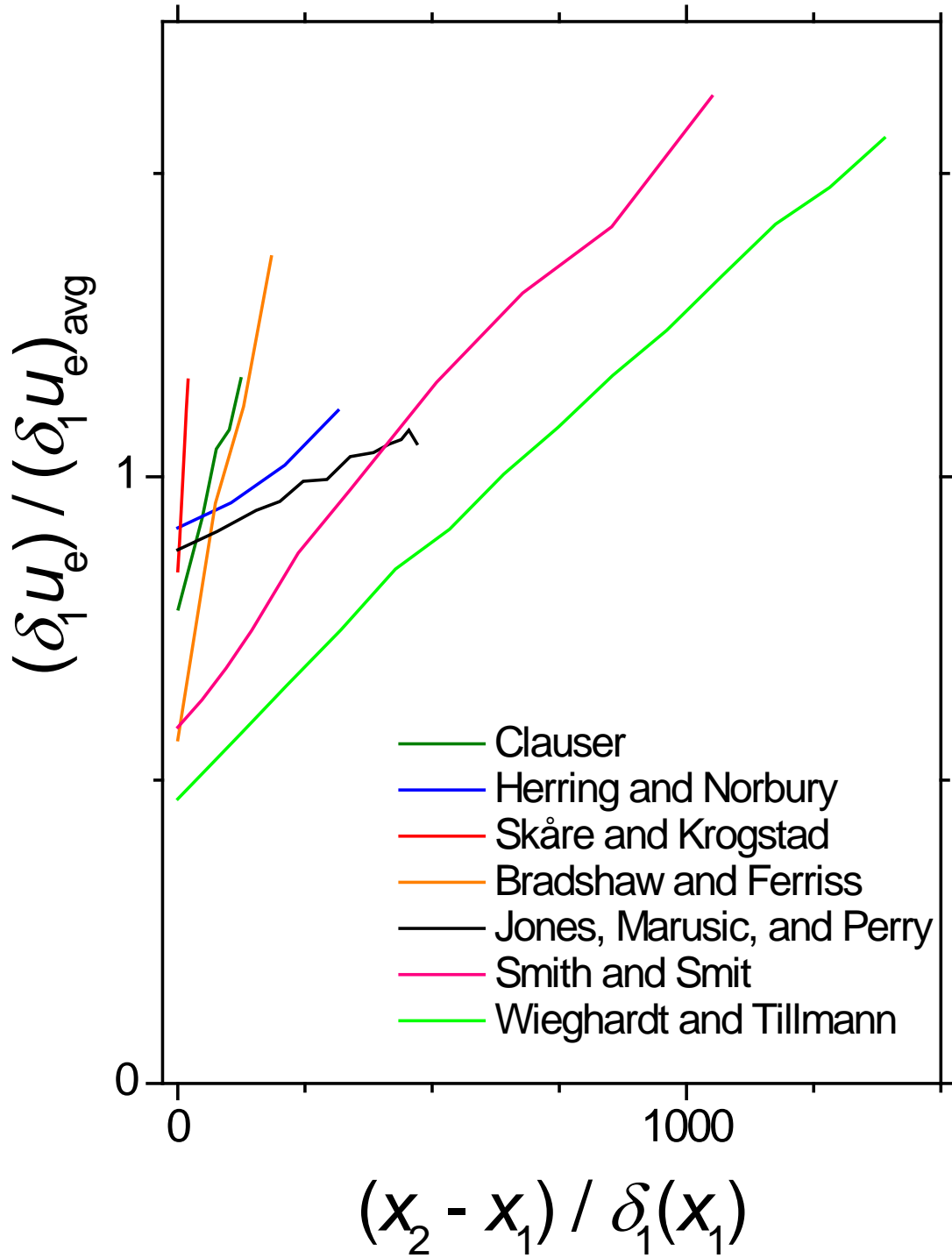Figure 1: The $(\delta_1 u_e)/(\delta_1 u_e)_{avg}$ values plotted versus the reduced station separation for the seven listed datasets.
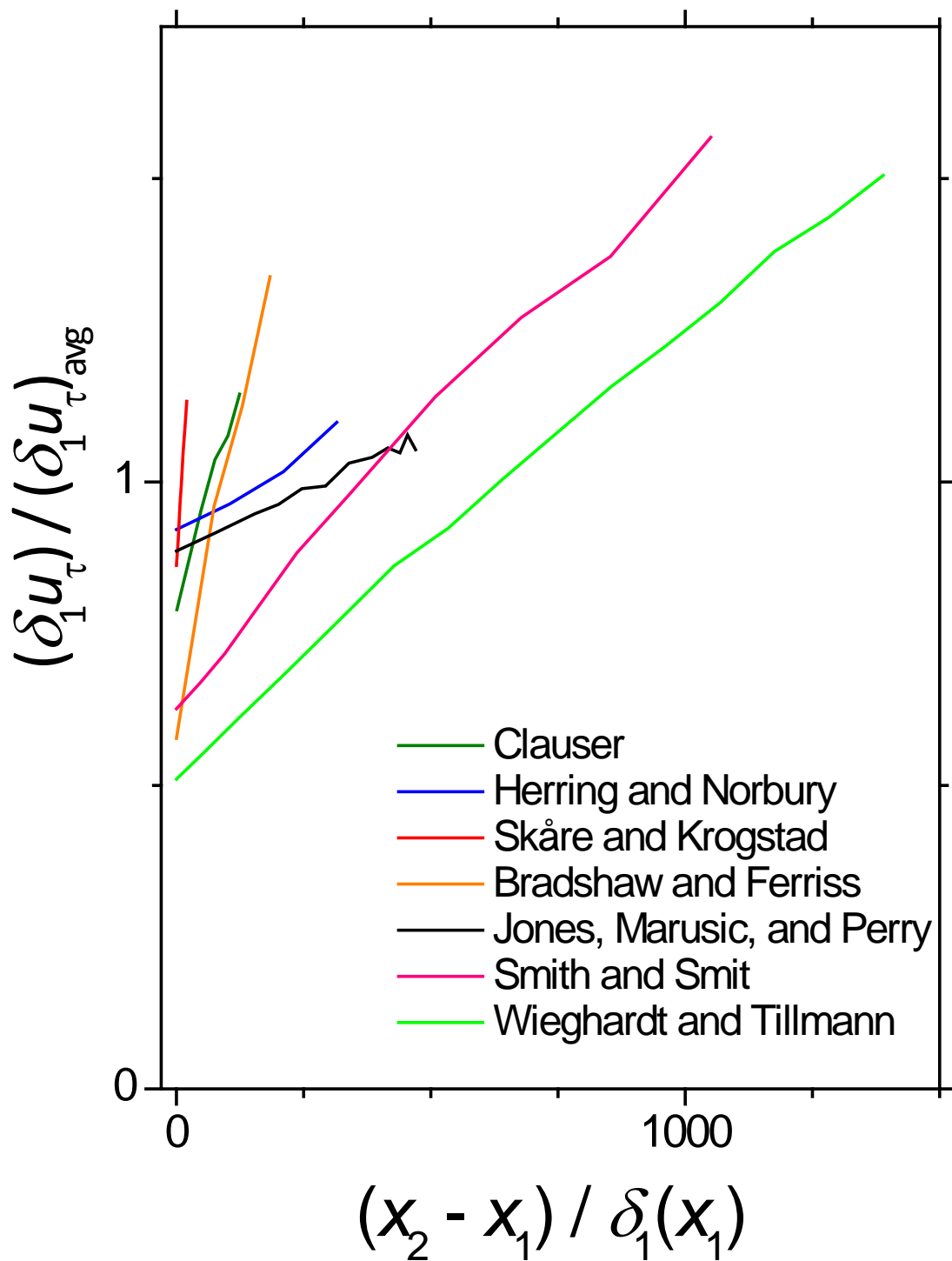
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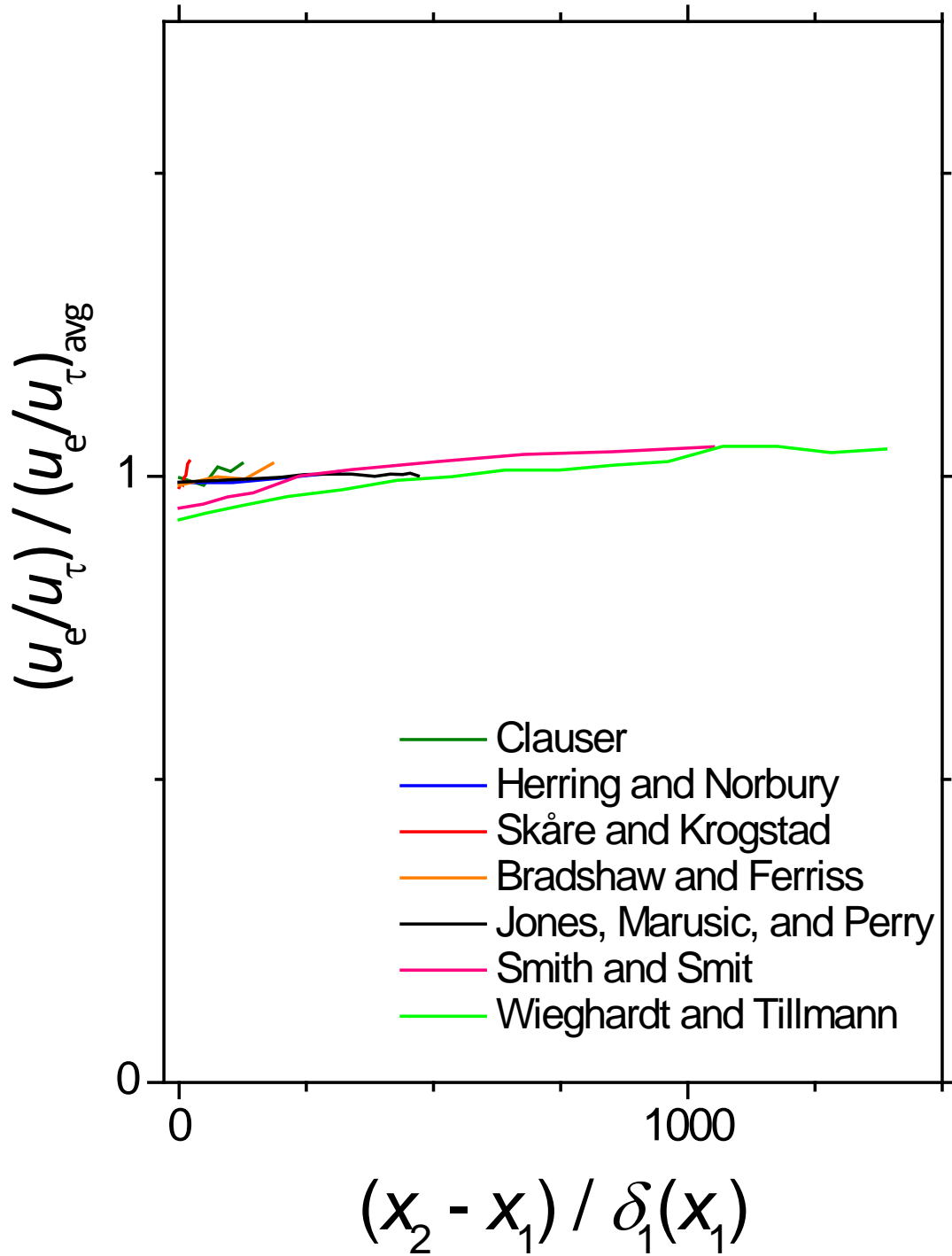
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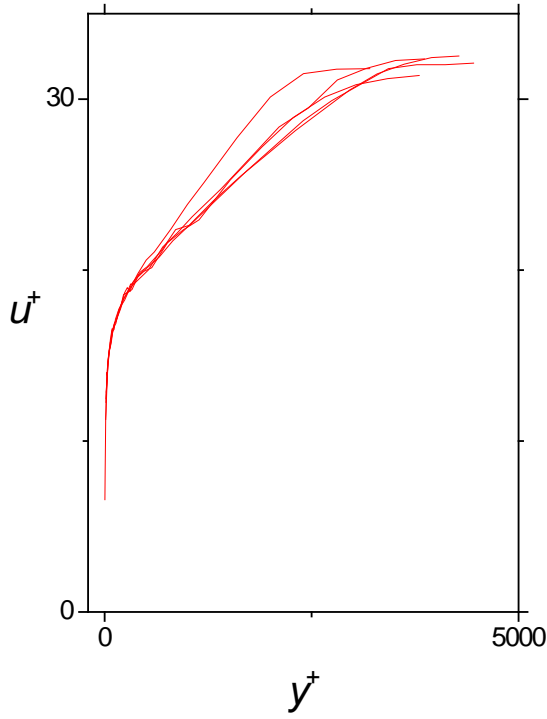
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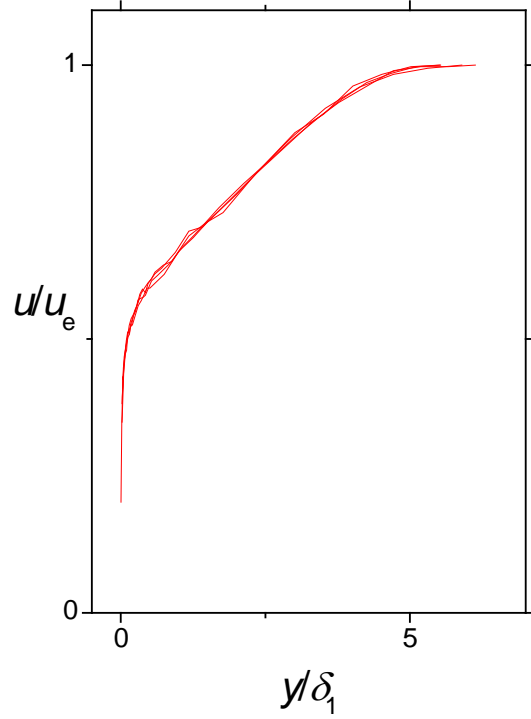
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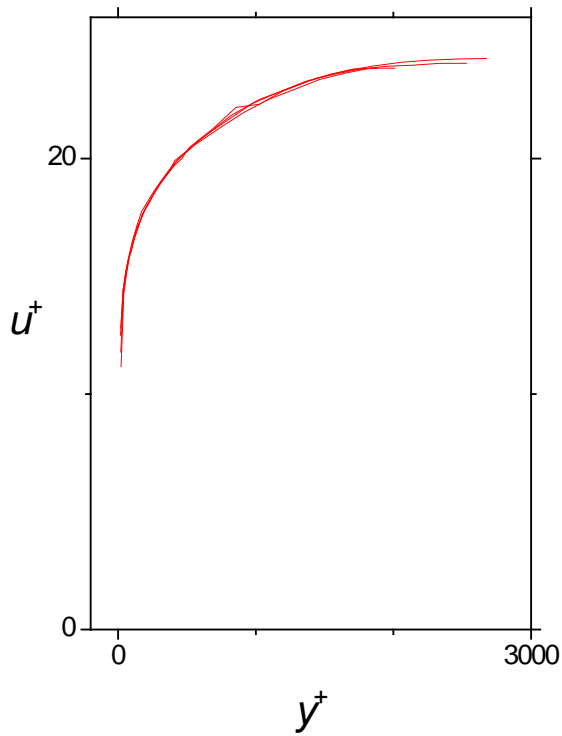
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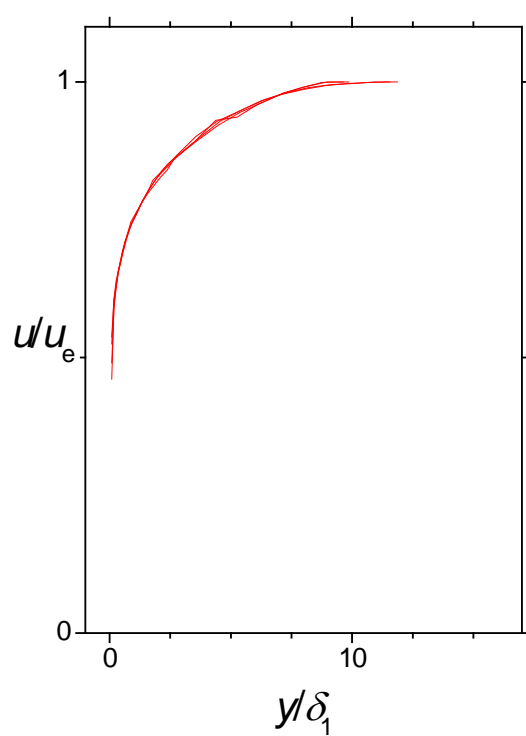
Figure 5b: The same four Herring and Norbury [11] profiles plotted using δ_1 and u_e.

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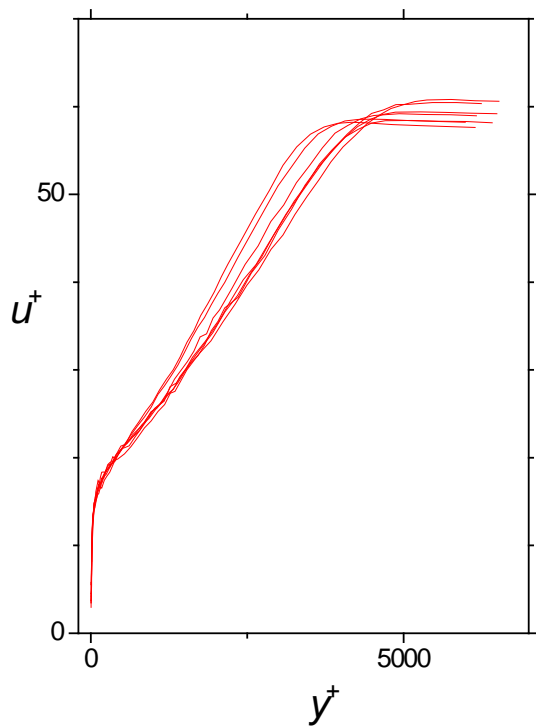
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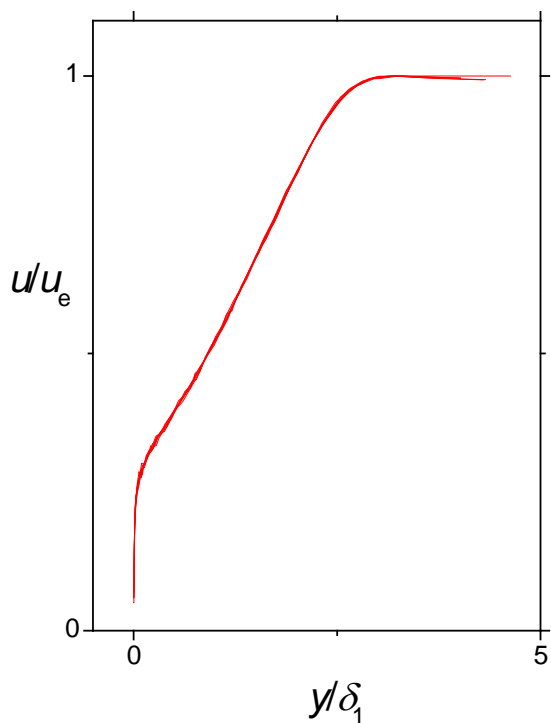
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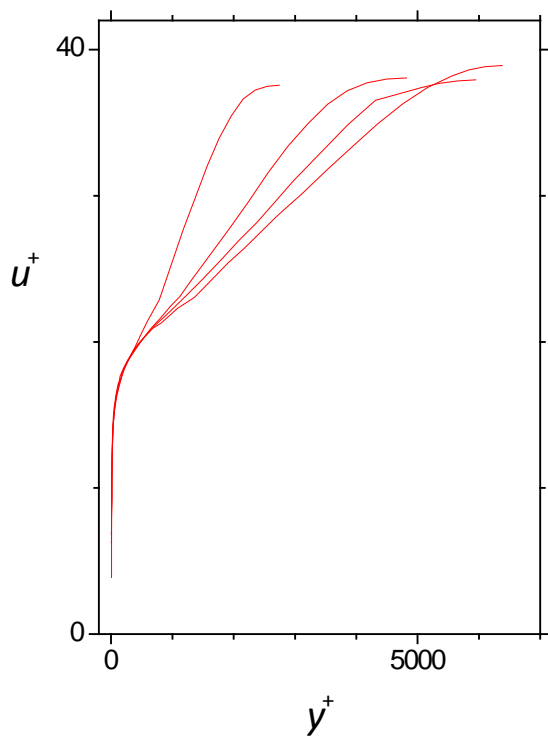
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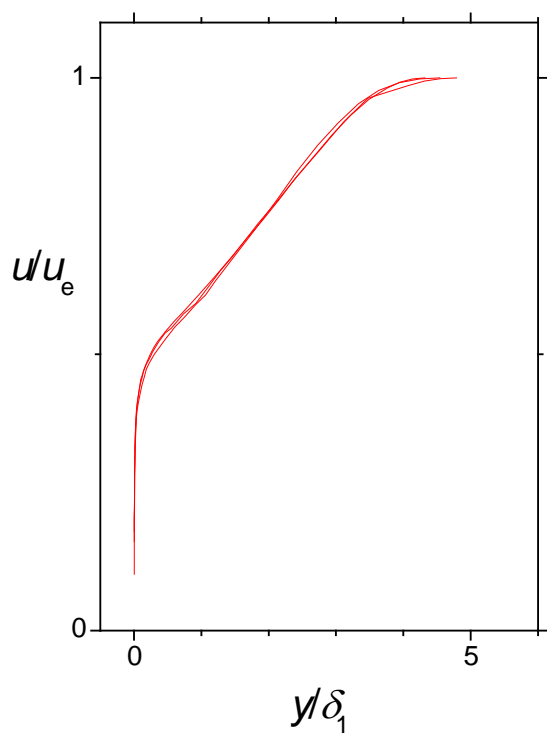
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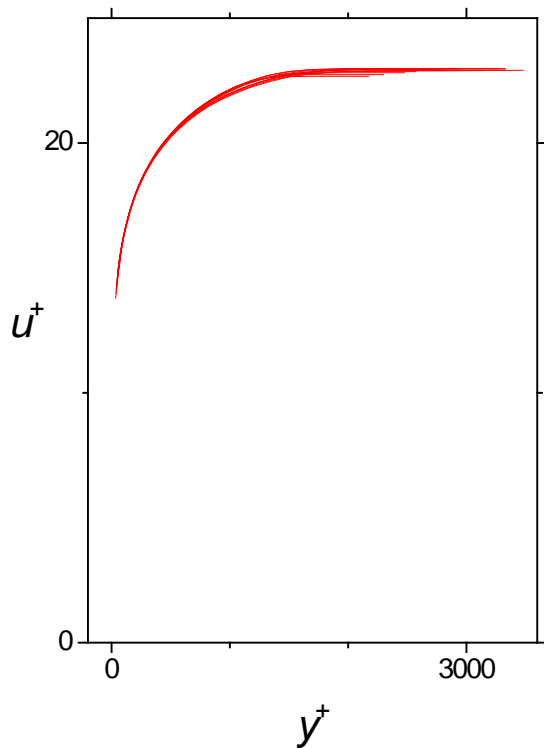
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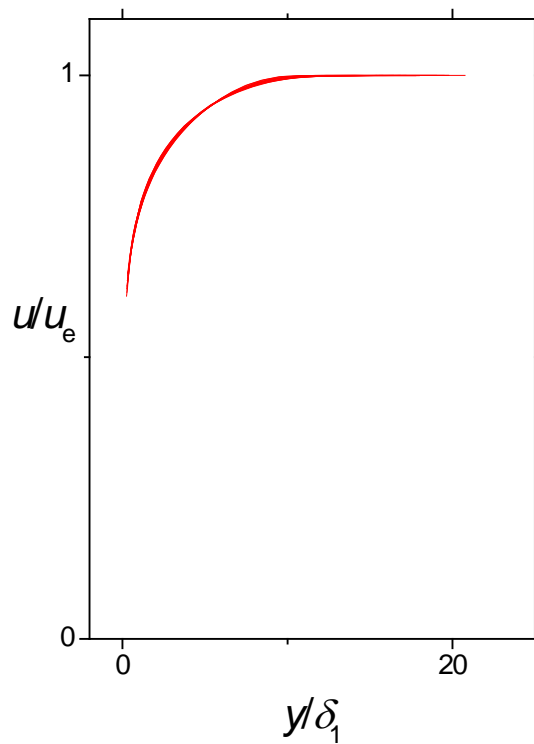
Figure 8b: The same twelve Jones, Marusic, and Perry [14] profiles plotted using δ_1 and u_e.
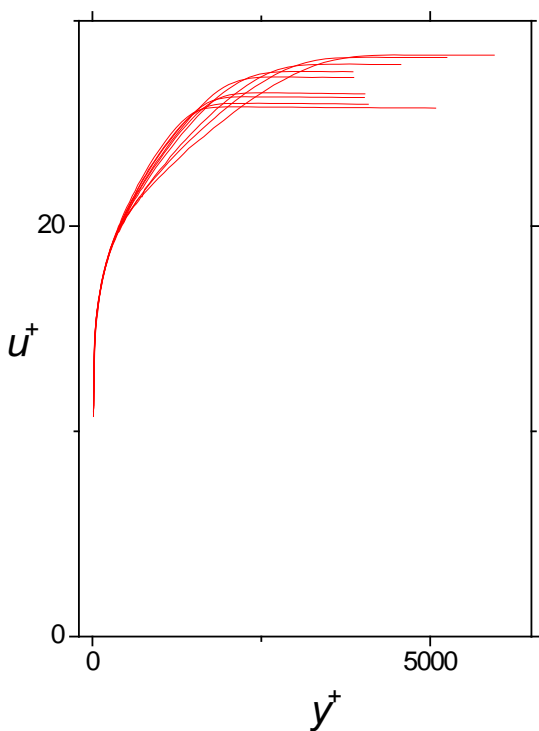
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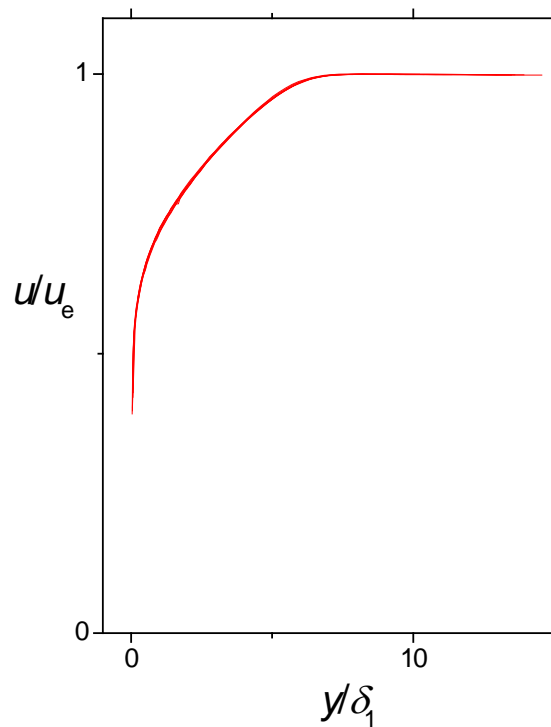
Figure 9b: Seven Smith and Smit [14] scaled velocity profiles plotted using δ_1 and u_e.
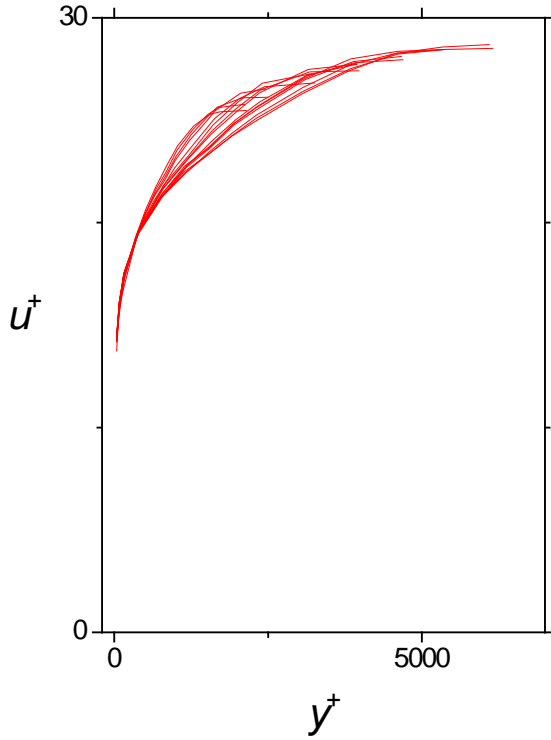
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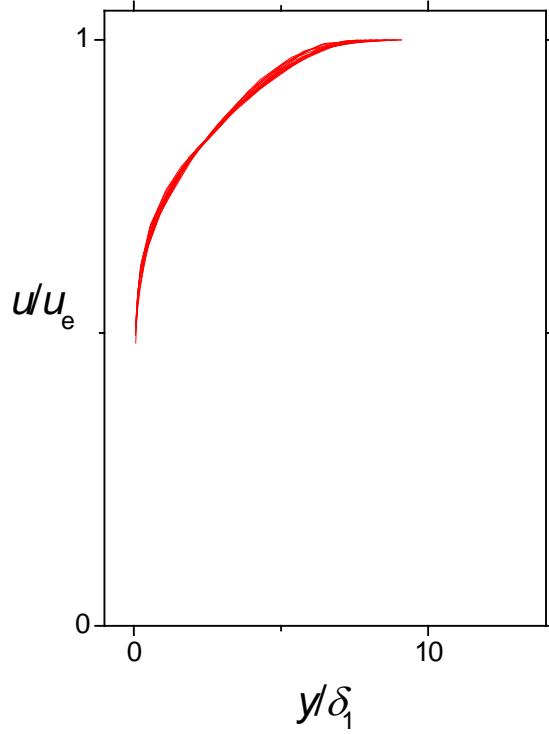
Figure 10b: The same fifteen Wieghardt and Tillmann [15] scaled velocity profiles plotted using δ_1 and u_e.

essentially constant results. In contrast, it is evident from Figs. 1 and 2 that the new similarity constraints show much more variation.

The purpose of the work herein is to show that the new similarity requirements allow one to correctly predict which datasets will show similarity using Prandtl's Plus scaling variables. In order to allow verification, we replot the velocity profile datasets in Plus units in Figs. 4a-10a. For comparison purposes, we also plot the same data using δ_1 and u_e as scaling variables in Figs. 4b-10b. It is evident from these plots that some of the datasets show whole profile similarity and some do not. In the following Section we discuss the implication of these results shown in the above figures in the context of predicating which datasets show whole profile similarity.

5. DISCUSSION

In the previous report [7], we observed that the Prandtl Plus scaling variables satisfied all of the conditions for similarity using the momentum balance type approach but not all of the datasets showed similarity. It was speculated that there was some other factor that the momentum balance approach was missing that would explain the results. Herein, we make the claim that the new similarity constraints presented by Weyburne [8] are the missing factors. In particular, in Section 3 it is pointed out that if similarity is present in a set of profiles using the Prantl's Plus scaling variables, then Eqs. 9-11 must hold. For the seven datasets used previously [7], the results for Eqs. 9-11 are displayed in Figs. 1-3.

First consider Fig. 3. The datasets chosen previously and used herein were in most cases intentionally set up to obtain similar profiles. In fact the search for datasets used herein was initiated by looking for datasets that satisfied the Rotta constraint given by Eq. 11. Therefore it is not surprising that Fig. 3 shows that the datasets used herein satisfy the Rotta constraint and that some datasets actually show whole profile similarity.

Now consider the results shown in Figs. 1 and 2. None of the datasets show exactly constant behavior, not like in Fig. 3. The datasets with the smallest variation, those of Herring and Norbury [11] and Jones, Marusic, and Perry [14], show variations of about ±10% from the average. The datasets showing the most variation showed variations of more than ±50% from the average. Now consider the datasets that actually show similarity using the Prandtl Plus scaling variables. By reviewing Figs. 4a-10a, it is evident that the Herring and Norbury [11] and Jones, Marusic, and Perry [14] datasets show a set of similar velocity profiles. Note that these two datasets also show the smallest variation in Figs. 1 and 2. We therefore make the following prediction: datasets which satisfy the Rotta constraint within ±5% and show a ±10% or smaller variation from the average for $\delta_1 u_e$ will display similarity behavior when plotted using Prandtl's Plus scaling variables.

The above results are based on using the new similarity results of Weyburne [8]. For similarity of the velocity profiles, Weyburne [8] showed that the scaling velocity must be proportional to u_e and the thickness scaling variable must scale as δ_1. Indirect support for the results presented herein is given in Figs. 4b-10b. In these Figures the velocity profile data for the seven datasets is plotted using u_e and δ_1 as the scaling variables. Recall that these datasets were chosen because they were experimentally designed, either intentionally or unintentionally, to produce similar flows. The results are quite remarkable in that most of the flows using u_e and δ_1 as the scaling variables show similarity, including the Herring and Norbury [11] and Jones, Marusic, and Perry [14] datasets which also show similar velocity profiles using the Prandtl Plus scaling. The fact that u_e and δ_1 as the scaling variables show similarity in datasets designed to produce similar profiles supports the theoretical results of Weyburne [8] and therefore the

results produced herein in regards to the similarity criteria developed for the Prandtl's Plus scaling variables.

The new similarity results of Weyburne [8] need to be compared to the traditional momentum balance approach to similarity. Although Weyburne [8] did not present the results formally, it is worth pointing out that the results are mathematically rigorous and are easily substantiated with mathematical proofs. In contrast the momentum balance approach to similarity is not so rigorous. In fact the experimental results herein and the results earlier [7] would indicate that the momentum balance approach does not guarantee similarity so a mathematical proof would not be possible.

It is also worth pointing out that there is another problem with the momentum balance equation approach which has been ignored in the past. In particular, the momentum balance equation in general applies to a point location in the flow. For the case of the 2-D wall bounded turbulent flows, one finds that the restrictions on the similarity scaling variables changes depending on whether one chooses to look at a point close to the wall where viscous forces are important or a point in the upper region of the boundary layer where viscous forces are absent [7]. We are interested in velocity profile similarity over the whole profile, not just at one point. Which restrictions should be applied? There does not appear to be any convincing arguments for choosing one set of restrictions over the other. It therefore leaves us with some serious concerns in regards to the application of the momentum balance equation to study similarity of the velocity profile.

The results above cast serious doubt on the use of the momentum boundary layer equation approach to discover similarity in velocity profile datasets. However, this is in direct conflict with the prevailing model of the boundary layer similarity found in the literature. Not only is the momentum boundary layer equation approach widely accepted, but Castillo and George [5] have used it to make the contention that similarity in turbulent boundary layers is not rare, as claimed by Clauser [2], but is in fact fairly common. The claim is based on discovering many datasets which they claim satisfies the momentum balance equation and whose scaled velocity profile plots indicate similarity. However, in the Appendix we point out that there is a serious flaw in the way Castillo and George [5] present their evidence. By replotting the data, we show that many of the datasets are actually not similar after all. Thus, their corrected data supports our previous observation [7]. That is it is possible to find datasets which satisfy the momentum balance type approach to similarity but whose scaled velocity profile plots reveal there is NO similarity of the velocity profiles.

Finally, it is also relevant to discuss the use of the measurement station difference $(x_2 - x_1)/\delta_1(x_1)$ in Figs. 1-3. This factor is a measure of the distance between measurement stations in terms of the boundary layer thickness measured at station one. We note that one way to insure similarity is to make the distance between the

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measurement stations small. For example, if one is making measurements in a 10 meter wind tunnel, the velocity profiles taken a few millimeters apart would be similar-like under almost any flow conditions. The scaled station difference or something similar should therefore be an important factor in evaluating and designing experiments for studying similarity of velocity profiles.

6. CONCLUSION

In a previous report Weyburne [7] concluded that the scaling variables discovered by the momentum balance type approach are a necessary but not sufficient condition for velocity profile similarity. In that report, it was speculated that there is some other factor that is not being captured with the momentum balance type approach. Herein we made the assertion that the missing factor was the new similarity results presented by Weyburne [8]. It was shown that the new similarity constraints were successful in predicting which datasets will show similarity and which will not.

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APPENDIX

The results above cast serious doubt on the use of the momentum boundary layer equation approach to discover similarity in velocity profile datasets. However, this is in direct conflict with the prevailing model of the boundary layer similarity found in the literature. Not only is the momentum boundary layer equation approach widely accepted, but Castillo and George [5] have used it to make the contention that similarity in turbulent boundary layers is not rare, as claimed by Clauser [2], but is in fact fairly common. The claim is based on discovering many datasets which they claim satisfies the momentum balance equation and whose scaled velocity profile plots indicate similarity. What they have shown is that the reduced pressure term of the momentum boundary layer equation given by

$$\Lambda = -\frac{\delta \, du_s/dx}{u_s \, d\delta/dx} \tag{A1}$$

is a constant for many datasets as required by the momentum boundary layer equation approach. They then plot up these datasets and show that the scaled defect velocity profiles do indeed show similarity.

However, as pointed out previously [6], we believe there is a serious flaw in the way Castillo and George [5] present their evidence. By replotting the data as a standard velocity profile rather than as a defect profile, we can show that many of the datasets are actually not similar after all. Thus, their corrected data supports our previous observation [7]. That is it is possible to find datasets which satisfy the momentum balance type approach to similarity but whose scaled velocity profile plots reveal there is NO similarity.

Consider a number of specific examples which shows how the data of Castillo and George [5] and co-workers is being misinterpreted. First consider Castillo and George's [5] claim that using δ_{99} and the velocity scale $u_{zs} = u_e \delta_1 / \delta_{99}$ results in similarity collapse of the profile data for Clauser's [2] mild APG case. We reproduce their Fig. 8a here as our Fig. 11a. In Fig. 11b we replot the same exact data using the *y*-axis scale u/u_{zs} instead of $(u_e - u)/u_{zs}$. Contrary to Castillo and George's claim, the Clauser data scaled with δ_{99} and $u_e \delta_1 / \delta_{99}$ does not result in similarity-like behavior according to the formal definition of similarity given by Eq. 1.

Consider another example given by Castillo and Walker [17] in which they claim the δ_{95} and the velocity scale $u_{zs} = u_e \delta_1 / \delta_{95}$ results in similarity collapse of the data for some
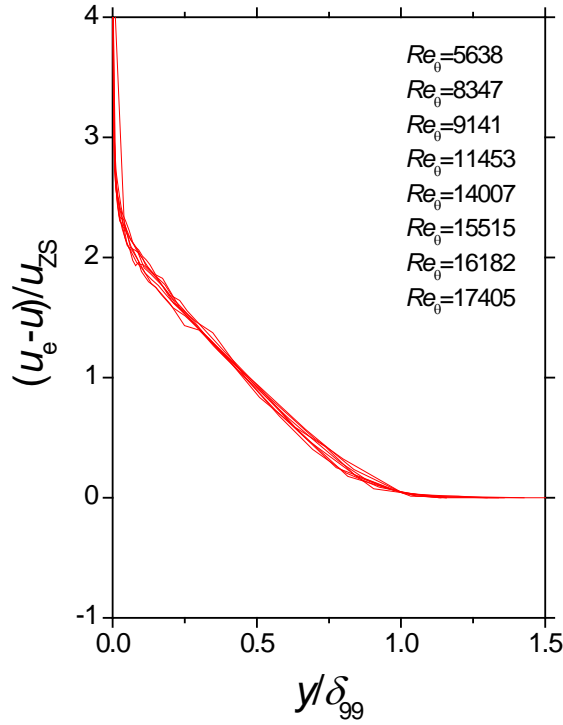
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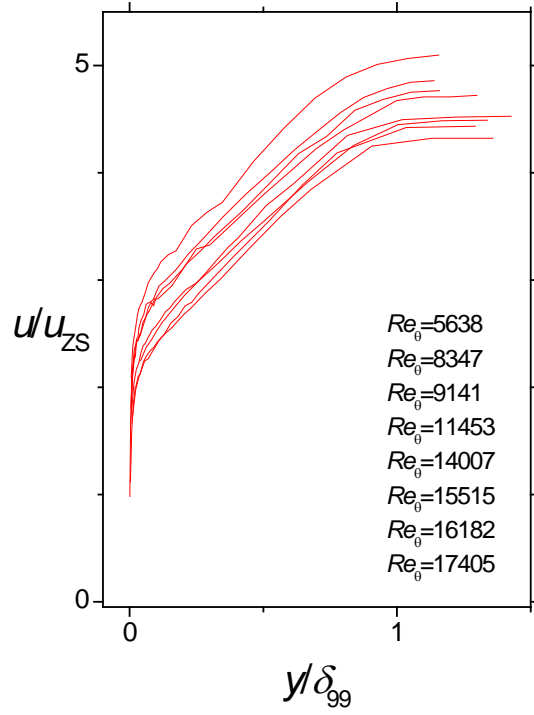Figure 11a: Eight Causer [2] scaled velocity profiles as plotted by Castillo and George [5].

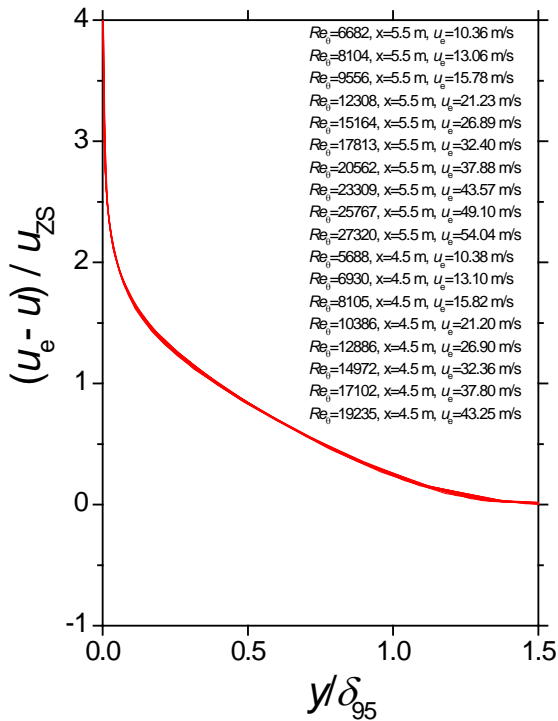Figure 11b: The same eight Causer [2] scaled profiles plotted using the standard *y*-scale.

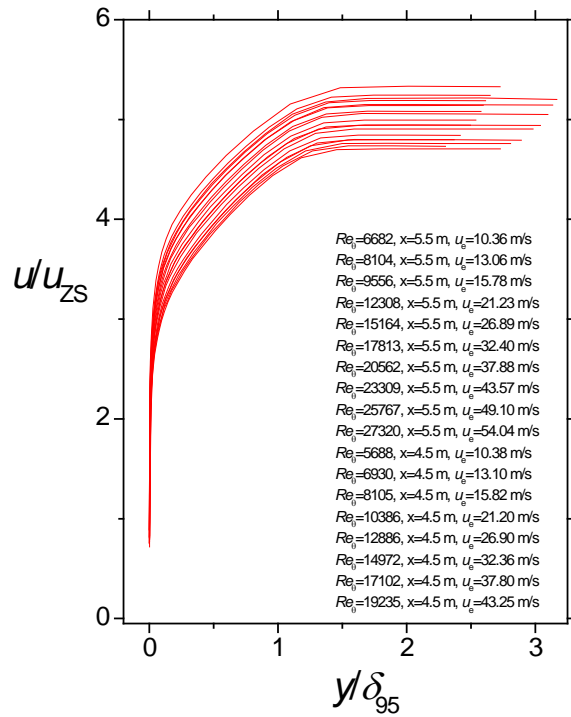Figure 12a: Eighteen Österlund [18] profiles plotted according to Castillo and Walker [17].

Figure 12b: The same eighteen Österlund [18] profiles plotted using the standard *y*-scale.

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of Österlund's [18] ZPG datasets. We reproduce their Fig. 3 here as Fig. 12a. In Fig. 12b we replot the same exact data using the y-axis scale u/u_{zs} instead of $(u_e - u)/u_{zs}$. It is apparent that this dataset does not show similarity using the standard definition of similarity given by Eq. 1.

In yet another publication, Brzek, *et*. *al*. [19] plot ten defect profiles from Smith [15] using the δ_{99} and u_{zs} scaling advocated by George and Castillo. In Fig. 13a we reproduce the Smith plot from their Fig. 5b.   The ten defect profiles in Fig. 13a do appear to show similarity behavior. However, looking at Fig. 13b, it is clear that although there are a few profiles that show nice collapse, overall the profiles of Smith do not collapse to a single profile and are therefore not similar by the formal definition of similarity given by Eq. 1.

For our fourth example we consider the Castillo and George [5] claim that the Herring and Norbury [11] mild and strong FPG datasets show equilibrium behavior. In Fig. 14a we reproduce part of their Fig. 8b. The eleven defect profiles show very nice collapse to a single profile. Now consider the same dataset plotted as standard velocity profiles. In Fig. 14b we replot the data using the same length and velocity scaling parameters. For this case, it does appear that a couple of the profiles still show similarity when plotted in this way. However, as in the previous examples, overall one would have to conclude most of the profiles do not satisfy the formal definition of similarity.

The formal definition of similarity is that two velocity profiles taken at different stations along the flow are similar if they differ only by a scaling constant in y and u. Castillo and George and co-workers assumed that similarity of deficit profile $u_e - u$ and similarity of the standard velocity profile u would be equivalent. The fact that this is not the case is surprising and is under study at this time to try to understand the reasons for this behavior. However, the bottom line is that they are NOT equivalent at least for the subset of datasets that we have been able to verify. Hence, it is necessary to point out that the momentum balance equation based model of similarity advocated by Castillo and George [5] and co-workers needs to be carefully revisited.

What are the implications of the data presented here in the Appendix? It means that another group has identified a number of examples from the literature in which datasets satisfy the momentum balance equation conditions for similarity but do not show similarity. Hence, the scaling variables discovered by the momentum balance type approach as presently constituted are a necessary but not sufficient condition for velocity profile similarity (as previously claimed [7]).
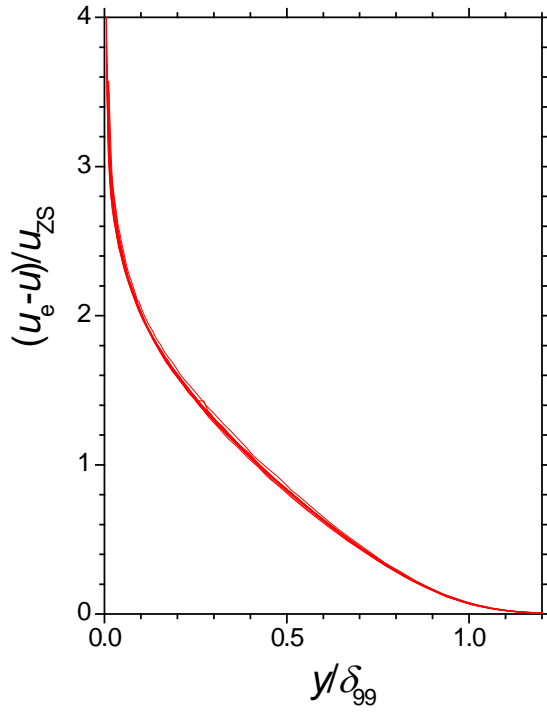
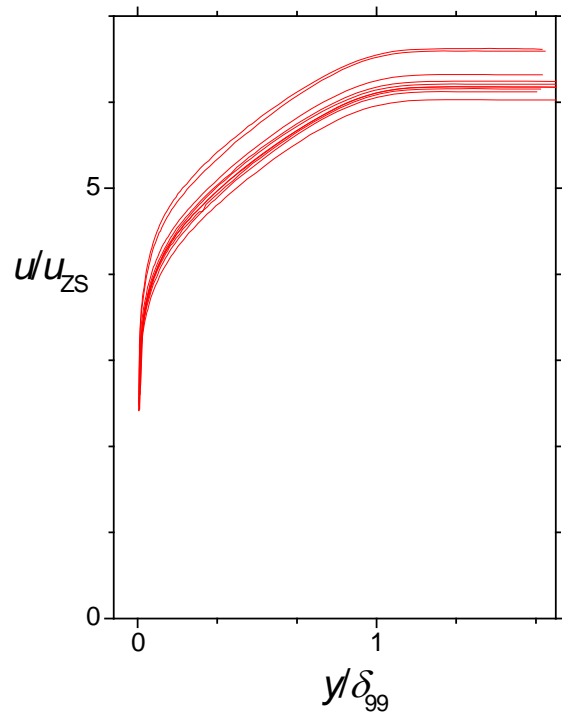Figure 13a : Ten Smith [15] profiles plotted according to Brzek, *et*. *al*. [19].

Figure 13b : The same Smith [15] data plotted using the standard y-axis scale.
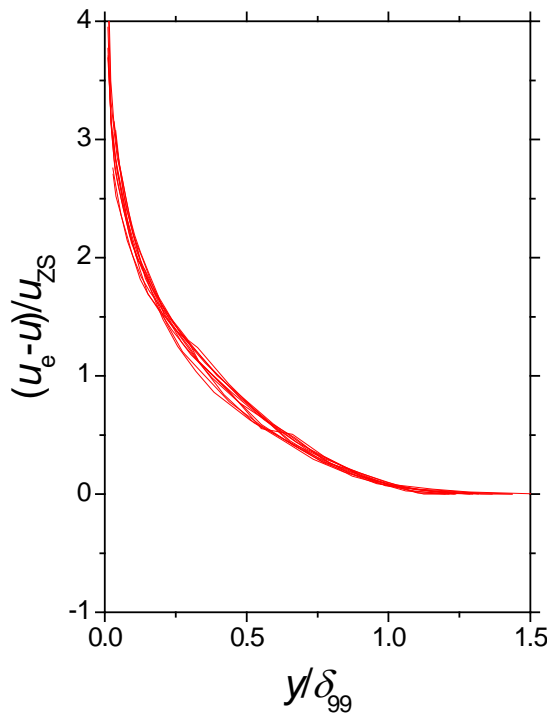

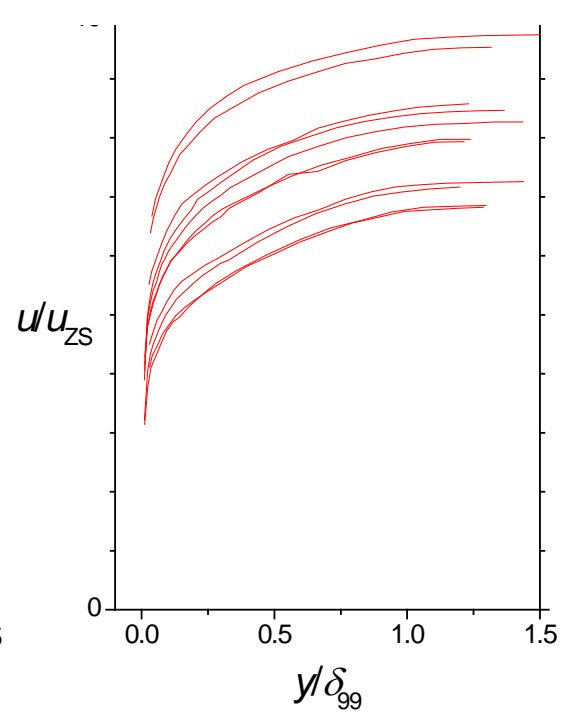Figure 14a: Eleven Herring and Norbury [11] plotted according to Castillo and George [5].

Figure 14b: The same Herring and Norbury [11] plotted using the standard y-axis scale.

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