LONG-TERM GOALS

Radio tracking, communications and guidance at sea rely on interpreting of radar signals. This has motivated extensive studies of the physical factors governing the pattern of signal propagation over the ocean. Although over the last several decades the models of electromagnetic propagation have been substantially advanced (Kukushkin (2004); Levy (2000)), measurements and modeling results still show differences that can distort signal interpretation. In particular, signal intensity at the receiver is often over-predicted by models (Barrios and Patterson (2002)), which suggests that mechanisms of signal energy loss are still incompletely understood and/or are not correctly built into the propagation models. The long-term goal of this work is to identify the source of discrepancies between observations and propagation model predictions and update the models so that the discrepancies are reduced.

OBJECTIVES

An essential element in models of over-the-ocean propagation is the scattering from the wavy water surface, a problem we specifically focus on in this work. Although the understanding of rough surface scattering has been gradually expanded (Elfouhaily and Guerin (2004); Voronovich (1999)) and the exact solution of the scattering problem can be obtained numerically, computational efficiency requires that we use a simplified description of the scattering. For that purpose one specific scattering model (Miller, Brown and Vegh (1984)) has been considered to be more realistic (Levy (2000)) when compared to earlier models (Ament (1953); Eckart (1953b)) and has been adopted in propagation models employed by the US Navy. However, an analysis of that model determines that it is inconsistent with preexisting both theoretical predictions and observational results regarding the statistics of the ocean surface.
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14. ABSTRACT

Radio tracking, communications and guidance at sea rely on interpreting of radar signals. This has motivated extensive studies of the physical factors governing the pattern of signal propagation over the ocean. Although over the last several decades the models of electromagnetic propagation have been substantially advanced (Kukushkin (2004); Levy (2000)), measurements and modeling results still show differences that can distort signal interpretation. In particular, signal intensity at the receiver is often over-predicted by models (Barrios and Patterson (2002)), which suggests that mechanisms of signal energy loss are still incompletely understood and/or are not correctly built into the propagation models. The long-term goal of this work is to identify the source of discrepancies between observations and propagation model predictions and update the models so that the discrepancies are reduced.

15. SUBJECT TERMS
Figure 1. Probability density distributions of surface elevation (upper left), of wave amplitude (lower left), and of wave phase (upper right. The experimental data were obtained over 1 hour of surface elevation measurements during the Rough Evaporation Duct experiment. The vertical lines in the upper left plot mark the waves' standard deviation. The filled circles indicate experimental data, the continuous line corresponds to the case of Gaussian distribution for the surface elevations, and dash-dot line shows the probability distributions as assumed in the Miller-Brown-Vegh model. The lower right plot presents the magnitude of the scattered field according to Ament (1953), for a Gaussian random surface (continuous line) and according to Miller-Brown-Vegh, dash-dot line.

The model of Miller, Brown and Vegh (1984) is based on the Kirchhoff approximation (i.e. assuming the curvature of the rough surface to be much greater than the wavelength of the electromagnetic signal). The model also prescribes some statistical properties of the sea surface, as summarized in Figure 1. Specifically, the model of Miller, Brown and Vegh (1984) considers the sea surface elevation as described by

\[ \eta = A \sin \frac{2\pi}{\Lambda} x, \]

where \( A \) is a Gaussian random variable, \( \Lambda \) is a characteristic wavelength, and \( x \) is uniformly distributed in the interval \((-\Lambda/4, \Lambda/4)\).
Given that, surface elevation $\eta$ is a random variable with a distribution

$$p(\eta) = \frac{1}{2\pi^{3/2} \sigma_\eta} \exp\left(-\frac{\eta^2}{8\pi\sigma_\eta^2}\right) K_0\left(\frac{\eta^2}{8\pi\sigma_\eta^2}\right),$$

where $K_0(\ )$ is a modified Bessel function of the second kind. Within the Kirchhoff approximation the scattered field intensity is determined by the characteristic function corresponding to the distribution $p(\eta)$, which in this case becomes

$$R = \exp(-2\sigma_\eta^2 q^2) I_0(2\sigma_\eta^2 q^2).$$

Figure 1 shows the probability density distributions of surface elevations, wave amplitudes, and wave phases as assumed in the model of Miller, Brown and Vegh (1984) and for the case of Gaussian sea surface, as in Ament (1953) and Eckart (1953b). Sea surface statistics obtained from measurements during the Rough Evaporation Duct experiment is shown in Figure 1 as well. Clearly, sea surface statistics proposed by Miller, Brown and Vegh (1984) is not supported by the observations. The narrow peak in the distribution of elevations appears unphysical and has never been reported from measurements. The amplitude must remain non-negative to be consistent with its definition, thus a Gaussian distribution cannot be valid for it. The phase distribution in the model of Miller, Brown and Vegh (1984) does not cover the phase’s range of variation. The surface statistics inconsistencies make the model of Miller, Brown and Vegh (1984) to predict enhanced intensity of the scattered signal and therefore could be, at least partially, responsible for the tendency of propagation models to overpredict the signal intensity at the receiver.

In this context, our objective was to examine the model of Miller-Brown-Vegh and propose a revision that is free of that model’s deficiencies.

**APPROACH**

For a Gaussian random surface $\eta(x, y)$ ($\langle \eta \rangle = 0$), all the central moments $\mu_n \equiv \langle \eta^n \rangle$ can be expressed through the surface’s standard deviation $\sigma^2 \equiv \mu_2 \equiv \langle \eta^2 \rangle$ as

$$\mu_{2n} = \frac{(2n)!}{2^n n!} \sigma_\eta^{2n} \quad \text{and} \quad \mu_{2n+1} = 0.$$

Specifically, $\mu_4 = 3\sigma^4$, $\mu_6 = 15\sigma^6$, $\mu_8 = 105\sigma^8$, $\mu_{10} = 945\sigma^{10}$, etc.

We used data collected during the two weeks of the Rough Evaporation duct experiment to obtain the moments of the sea surface. Figure 2 presents the probability density function for the moments of orders between 2 and 16. It was observed that the moments generally cluster about their Gaussian values although do not follow them exactly, i.e. the surface statistics shows a detectable deviation from Gaussianity.
The analysis of cumulants is a well established statistical approach that has not been applied to the problem of sea surface scattering. We chose the cumulant analysis as an adequate tool for describing the statistics of non-Gaussian processes. Such approach should let us produce an approximation for the scattering cross-section of the sea surface, based on surface’s statistical moments, which are measurable. Such approximation will be free of the inconsistencies inherent to the model of Miller, Brown and Vegh (1984), where the cumulants $\kappa_n$ are defined by $\kappa_n = K^{(n)}(0) = i^{-n} \frac{d^n}{dq^n} \log \varphi(q) \bigg|_{q=0}$.

Figure 2. Experimental probability density distributions of the sea surface statistical moments $\mu_2, \mu_3 / \sigma^3, \ldots, \mu_{16} / \sigma^{16}$. The dashed vertical line marks the value of these moments for a Gaussian variable, i.e. $(2n)! / (2^n n!)$.
They can be expressed through the measurable moments as

\[ \kappa_2 = \mu_2 \]
\[ \kappa_3 = \mu_3 \]
\[ \kappa_4 = \mu_4 - 3\mu_2^2 \]
\[ \kappa_5 = \mu_5 - 10\mu_2\mu_3 \]
\[ \kappa_6 = \mu_6 - 15\mu_2\mu_4 - 10\mu_3^2 + 30\mu_2^3 \]
\[ \kappa_7 = \mu_7 - 21\mu_2\mu_5 - 35\mu_2\mu_3 + 210\mu_2^2 \mu_3 \]

The explicit form for \( \kappa_n \) was obtained up to the order of 16. For a strictly Gaussian random variable, the cumulants of order greater than 2 should be zero. Figure 3 presents the distribution density of the sea surface cumulants calculated from Rough Evaporation Duct measurements. The sea surface’s deviation from Gaussianity is also apparent from these distributions. The field intensity as a result of the incoherent scattering from the sea surface is determined by the joint characteristic function

\[ \varphi(q_1, q_2) = \int e^{iq_1\eta_1 + iq_2\eta_2} d\eta_1 d\eta_2 = \sum_{n=0}^{\infty} \sum_{l=0}^{n} \frac{\mu_{l,n-l}}{l!(n-l)!}(i q_1)^l (i q_2)^{n-l} \]

which can be treated similarly. The joint cumulant generating function,

\[ K(q_1, q_2) \equiv \log \varphi(q_1, q_2) = \sum_{n=1}^{\infty} \sum_{l=0}^{n} \frac{\kappa_{l,n-l}}{l!(n-l)!}(i q_1)^l (i q_2)^{n-l} \]

where \( \kappa_{l,n-l} \) are the joint cumulants, can be used for faster convergence and \( \kappa_{l,n-l} \) can be expressed through the joint moments \( \mu_{l,n-l} = \left\langle \eta_1^l, \eta_2^{n-l} \right\rangle \)

\[ \kappa_{1,1} = \mu_{1,1} = \left\langle \eta(r_1), \eta(r_2) \right\rangle = \sigma_q^2 C(\vec{r}_1 - \vec{r}_2) \]
\[ \kappa_{2,0} = \kappa_{0,2} = \sigma_q^2 \]
\[ \kappa_{3,0} = \mu_{3,0}, \kappa_{2,1} = \mu_{2,1} \]
\[ \kappa_{4,0} = \mu_{4,0} - 3\mu_{2,0}^2 \]
\[ \kappa_{3,1} = \mu_{3,1} - 3\mu_{1,1}\mu_{2,0} \]
\[ \kappa_{2,2} = \mu_{2,2} - 2\mu_{1,1}^2 - \mu_{2,0}\mu_{0,2} \]

where \( C(\vec{r}_1 - \vec{r}_2) \) stands for a two-point correlation function.
The cumulants of a Gaussian variable have a value of 0, indicated by the dashed vertical line.

RESULTS

The cumulant expansion leads to the following form of the sea surface scattering cross-section.

\[
R = \exp(-2\sigma_q^2 q^2) \times \exp \left[ \frac{4}{3} (i\sigma_q q)^3 \left( \frac{\mu_3}{\sigma_q^3} \right) \right] \times \exp \left[ \frac{2}{3} (i\sigma_q q)^4 \left( \frac{\mu_4}{\sigma_q^4} - 3 \right) \right] \times \\
\exp \left[ -\frac{4}{15} (i\sigma_q q)^5 \left( \frac{\mu_5}{\sigma_q^5} - 10 \frac{\mu_3^2}{\sigma_q^6} \right) \right] \times ...
\]

The first factor \( \exp(-2\sigma_q^2 q^2) \) corresponds to the case of strictly Gaussian ocean surface, as considered by Ament (1953) and Eckart (1953b). The factors that follow, represent correction to the scattering cross-section associated with the surface’s deviation from Gaussianity. The signal amplitude (and intensity) are determined by the factors corresponding to even-order cumulants. The factors corresponding to odd-order cumulants affect only the phase of the scattered signal. Nevertheless, these

Figure 3. Probability density distributions of the sea surface cumulants \( \kappa_2, \kappa_3 / \sigma^3, ..., \kappa_{16} / \sigma^{16} \).
odd-order factors are important for determining the nulls, i.e. the locations where the scattered signal interferes destructively with the signal from the source and thus produces “blind spots”.

For the case of incoherent scattering, the cumulant expansion predicts the following intensity

\[
R_{inc} = \exp\left[-\sigma_q^2 q^2 (1 - C)\right] \times \exp\left[\frac{q^4}{12} (\kappa_{4,0} - 4\kappa_{3,1} + 3\kappa_{2,2})\right] \times \exp\left[\frac{q^6}{360} (\kappa_{6,0} - 6\kappa_{5,1} + 15\kappa_{4,2} - 10\kappa_{3,3})\right] \times ...
\]

Again, the first factor, \(\exp\left[-\sigma_q^2 q^2 (1 - C)\right]\), corresponds to the case of strictly Gaussian surface, and the factors that follow are corrections needed to account for the deviations from Gaussianity.

**IMPACT/APPLICATIONS**

The corrections for non-Gaussian sea surface statistics become more pronounced with the increase of the parameter \((\sigma_q q)\). For small \((\sigma_q q)\) the cumulant analysis predicts changes in the phase of the scattered signal, while changes in the intensity remain negligible. Nevertheless, it is reasonable to assume that a radar signal propagating in strong refractive duct will encounter increased (as compared to a weaker duct) number of scatterings from the surface and even small non-Gaussian corrections could accumulate (over multiple acts of scattering) to a substantial change in the signal’s intensity.

I am working in close collaboration with Amalia Barrios of SPAWAR to explore the effect of non-Gaussian sea surface statistics on the predictions from the Advanced Propagation Model (APM). Very preliminary results indicate that for S-band signals (i.e. longer wavelengths and \((\sigma_q q) \ll 1\)) the cumulant corrections are insignificant, for intermediate wavelengths (X-band signals) the cumulant corrections visibly improve the agreement between model predictions and direct observations (Figure 4). For shorter wavelengths (Ku-band signals) the discrepancy between model and observations is also reduced, but with some increased standard deviation of the difference between them.
Figure 4. Comparison of direct observations from RED experiment with predictions from the Advanced Propagation Model with different scattering cross-sections (Ament (1953), Miller, Brown, Vegh (1984), and cumulant analysis) for X-band signals. Calculations were performed and the figure was provided by Amalia Barrios of SPAWAR.

TRANSITIONS

A logical transition from this work is an experimental testing of the available scattering models. Data from direct measurements suitable for such testing so far are scarce. Karasawa and Shiokawa (1988) have reported measurements intended to examine the predictions of Miller, Brown, Vegh (1984), but do not reach a definitive conclusion (Figure 4 in Karasawa and Shiokawa (1988)).

REFERENCES


**PUBLICATIONS**