

# Stochastic Models and Robust Estimation for Broadband Acoustic Mode Signals

Kathleen E. Wage  
George Mason University  
Electrical and Computer Engineering Department  
4400 University Drive, MSN 1G5  
Fairfax, VA 22003  
phone: (703)993-1579    fax: (703)993-1601    email: [kwage@gmu.edu](mailto:kwage@gmu.edu)

Award Number: N00014-02-1-0416

<http://ece.gmu.edu/~kwage>

## LONG-TERM GOALS

The long term goals of this project are to develop stochastic models for broadband mode signals propagating in fluctuating ocean environments and to design robust signal processing techniques for these ocean environments.

## OBJECTIVES

To develop a greater understanding of broadband mode signals in fluctuating ocean environments, this project focuses on several closely-related research objectives. The first objective is to characterize mode fluctuations at megameter ranges using experimental measurements and simulations. Mode arrivals at long range are very complicated due to internal-wave-induced coupling. From a theoretical standpoint, the effects of internal waves on long-range sound propagation are not thoroughly understood, and much of the previous work in this area focused on the ray arrivals because they are amenable to analysis via the geometrical optics approximation. Characterizing the fluctuations is the first step towards the long-term goal of developing a sufficiently general model of mode propagation that explains recent experimental observations and clarifies the relationship between mode and ray representations of the field. The second objective of this project is to develop a framework for mode processing that mitigates the effects of environmental mismatch, sensor failures, and interference. The goal is to design processors that make the best use of the degrees of freedom available to them. Since mismatch is a problem that plagues many sonar applications, this work has implications beyond the current project.

## APPROACH

To address the stated objectives, this project combines experimental data analysis with simulation and theory. Over the last 10 years, a series of experiments generated rich data sets for studying normal mode signals at megameter ranges. In previous years this project studied the mode arrivals in the Acoustic Thermometry of Ocean Climate (ATOC) and Alternate Source Test (AST) experiments. This year data from the North Pacific Acoustic Laboratory (NPAL) is considered. The current work is relevant to the 2004-2005 SPICE04/LOAPEX experiment that finished in June 2005.

In addition to the PI, three Ph.D. students at George Mason University (GMU) worked on aspects of

# Report Documentation Page

Form Approved  
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE <b>30 SEP 2005</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-2005 to 00-00-2005</b>	
4. TITLE AND SUBTITLE <b>Stochastic Models and Robust Estimation for Broadband Acoustic Mode Signals</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>George Mason University, Electrical and Computer Engineering Department, 4400 University Drive, MSN 1G5, Fairfax, VA, 22003</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>code 1 only</b>					
14. ABSTRACT <b>The long term goals of this project are to develop stochastic models for broadband mode signals propagating in fluctuating ocean environments and to design robust signal processing techniques for these ocean environments.</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>11</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

this project in 2004-2005. Mr. Tarun Chandrayadula investigated the problem of designing adaptive equalizers for broadband mode signals. Mr. Aravinda Sringerapuram worked on mode estimation methods that can adapt to the changing interference characteristics of the channel. LT Eric St. Pierre attended GMU sponsored by the U.S. Coast Guard postgraduate training program. In 2004-2005, he developed a method for estimating the sound speed profile required for mode processing from sparse measurements of the temperature along a vertical array.

## **WORK COMPLETED**

*ATOC/AST Mode Analysis:* An extensive statistical analysis of the mode arrivals in the ATOC and AST experiments was completed in 2003-2004. The article by Wage *et al.* in the March 2005 issue of the *Journal of the Acoustical Society of America* describes the mode statistics, such as coherence and time spread, computed from these experiments [1]. Kathleen Wage summarized these results in an invited talk at the November 2004 meeting of the Acoustical Society of America [2].

*SPICE04 Recovery:* Kathleen Wage was a member of the science team for the SPICE04 recovery cruise, which took place June 6 through June 26, 2005. The chief scientist for that cruise was Peter Worcester (Scripps Institution of Oceanography). Two vertical line arrays and two source moorings were recovered.

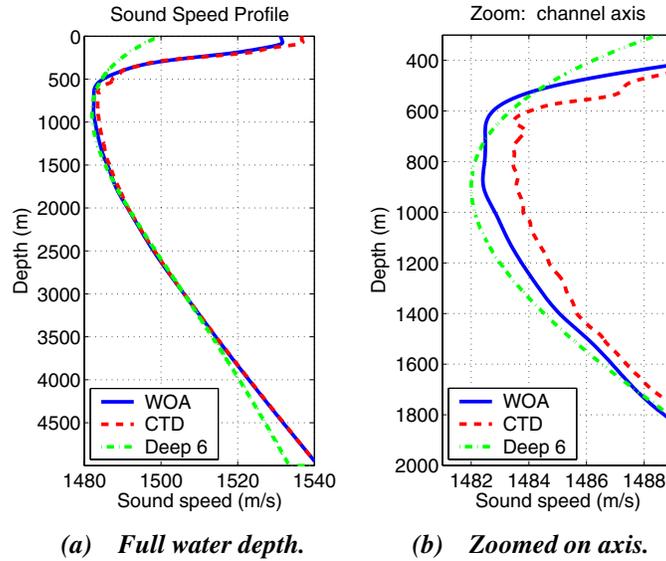
*Approximate Mode Filtering:* Mode filters are typically designed using modeshapes derived from numerical solution of the Helmholtz equation for a measured sound speed profile (SSP). In experiments such as NPAL, the temperature (hence the sound speed) is only measured across the span of the array. To facilitate mode processing, it is helpful to have an approximate solution for the modes that only requires the environmental information along the array. Kathleen Wage presented a paper at the Asilomar Conference on Signals, Systems, and Computers that explored the use of a uniform WKB-like approximation to design mode filters [3].

*Sound Speed Estimation from Sparse Environmental Measurements:* Mode filtering requires accurate knowledge of the sound speed profile at the array. For long-duration experiments, the environment is typically measured using a relatively small set of temperature sensors located along the array. In the past year a method was developed for extrapolating these sparse temperature measurements to obtain a sound speed profile for use in mode processing. The approach was applied to temperature measurements acquired during the NPAL experiment. Eric St. Pierre presented a paper describing these results at the September 2005 IEEE/MTS Oceans Conference [4].

*Mode Equalization:* At megameter ranges, the low mode arrivals are quite complicated. In the past year, the problem of equalizing these signals has been studied. Tarun Chandrayadula presented a paper on the design of mode equalizers at the September 2005 IEEE/MTS Oceans Conference [5].

## **RESULTS**

*Approximate Mode Filtering:* Spatial filtering for modes requires knowledge of the modeshapes, which requires accurate knowledge of the environment. Unfortunately, environmental information is often incomplete. For example in both the 1998 North Pacific Acoustic Laboratory (NPAL) experiment and the SPICE04 experiment, the environment was sampled along the VLA span, but not above and below

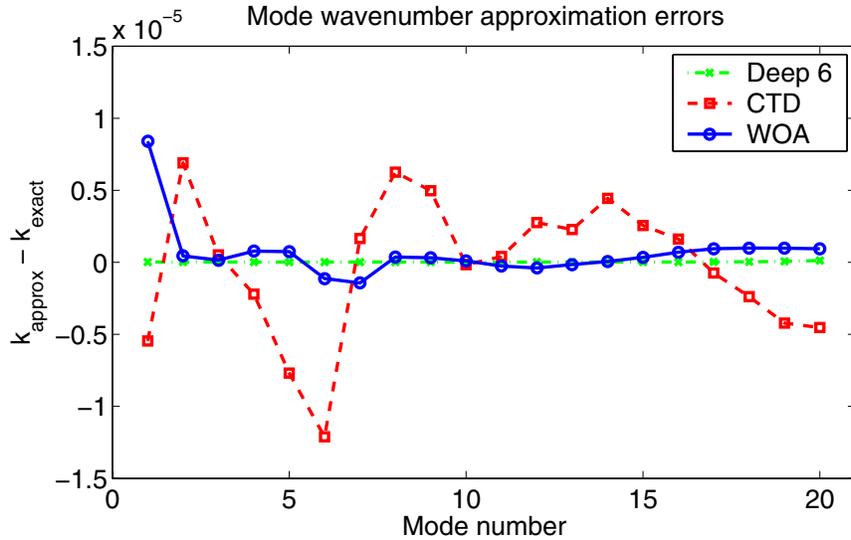


**Figure 1: Three deep water SSP's: the analytical Deep-6 profile, a measured (CTD) profile taken near Hawaii, and an archival profile from the World Ocean Atlas for a location near Hawaii.**

the array. To facilitate mode processing, an approximation to the modeshapes is needed that only requires the environmental information along the array. Ideally, the approximation should provide an analytical form for the modeshapes, rather than just a numerical solution. An analytical form is desirable because it provides more intuition about the filtering problem and may permit simpler design of robust mode filters. WKB theory provides the most common approximate solution to the mode problem. The main problem with using the standard WKB solution to design mode filters is that it is singular at the turning points. Miller and Good developed a uniform WKB-like solution to the Schrödinger equation that is continuous at the turning points [6]. A paper presented at the Asilomar Conference in 2004 describes the application of Miller and Good's method to the underwater acoustic mode problem [3]. The major findings of this paper are reviewed below.

To evaluate the accuracy of Miller and Good's solution for the acoustic mode problem, we considered a series of examples for three deep water SSP's: the "Deep-6" analytical profile [7], a profile derived from a CTD measurement near Hawaii, and an archival profile taken from the World Ocean Atlas (WOA) [8, 9] for the same Hawaii location. Figure 1 shows the three SSP's. The accuracy of the Miller/Good solution can be measured by comparing the approximate modes to the exact solutions computed using a standard mode code [10]. Fig. 2 shows the wavenumber errors for the first 20 modes of the three environments. Since the exact wavenumbers lie between 0.315 and 0.318, the maximum error is on the order of 0.004 percent. The Deep-6 profile has the smallest error, which is not surprising given that it is the type of profile that the Miller/Good method was designed for, *i.e.*, it is a smooth profile with sound speed that is strictly increasing away from the channel axis.

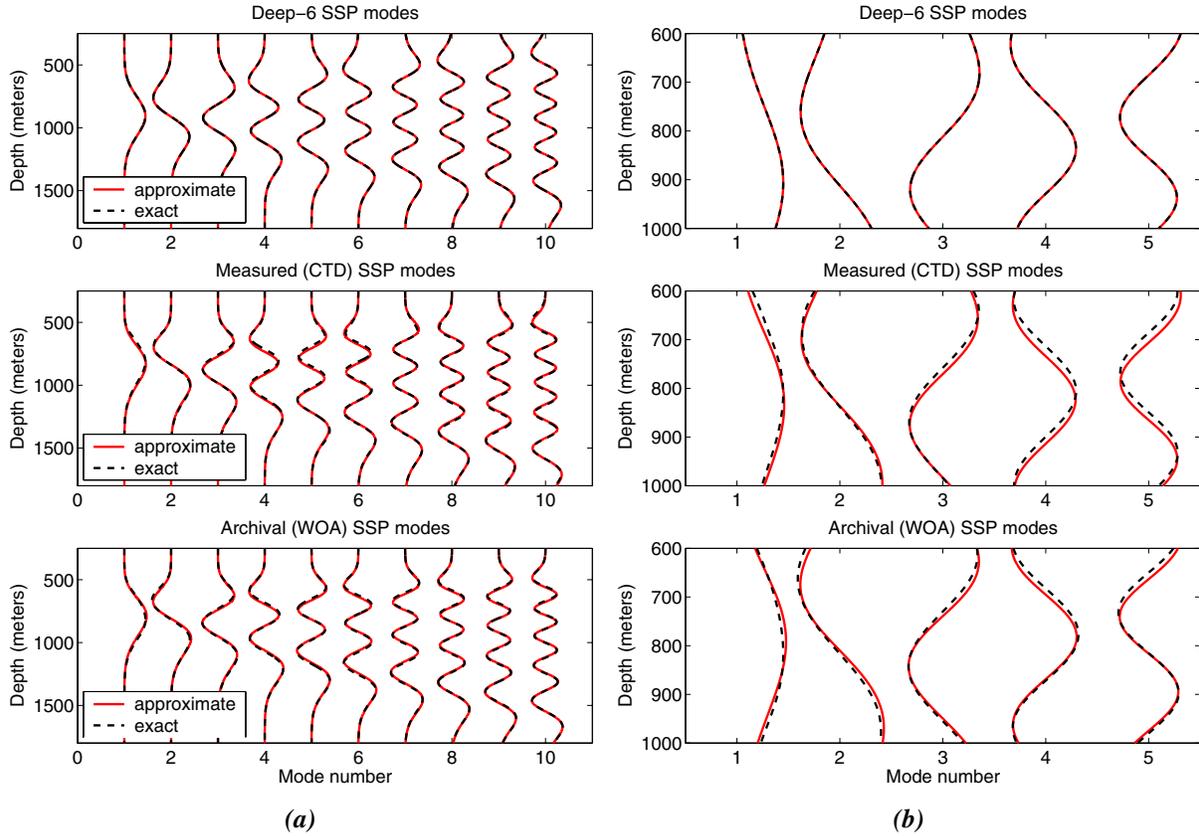
Fig. 3(a) compares the approximate modeshapes for the three environments with the corresponding exact modeshapes. The agreement is quite good. Fig. 3(b) zooms in on the first five modeshapes to illustrate the small differences. The approximate modes of the Deep-6 profile appear to line up perfectly



**Figure 2: Comparison of wavenumber approximation errors ( $k_{\text{approx}} - k_{\text{exact}}$ ) for three SSP's: the Deep-6 profile, the CTD profile, and the WOA profile.**

with the exact modes, whereas the modes for the two other profiles show small perturbations away from the exact shapes. These perturbations can lead to crosstalk problems in mode filters. As discussed in [3], the modeshape perturbations for the Deep-6 case are negligible and the resulting crosstalk is less than -48 dB. For the CTD and WOA profiles, the modeshape perturbations result in peak crosstalk levels on the order of of -18 dB and -21 dB, respectively. Ideally, mode filters should have zero crosstalk, but that cannot always be realized in practice. As demonstrated in [3], the approximate mode filter may perform better (in terms of crosstalk) than the exact filter when the approximate filter can be designed using a measurement of sound speed across the VLA and the exact filter can only be computed for an archival SSP.

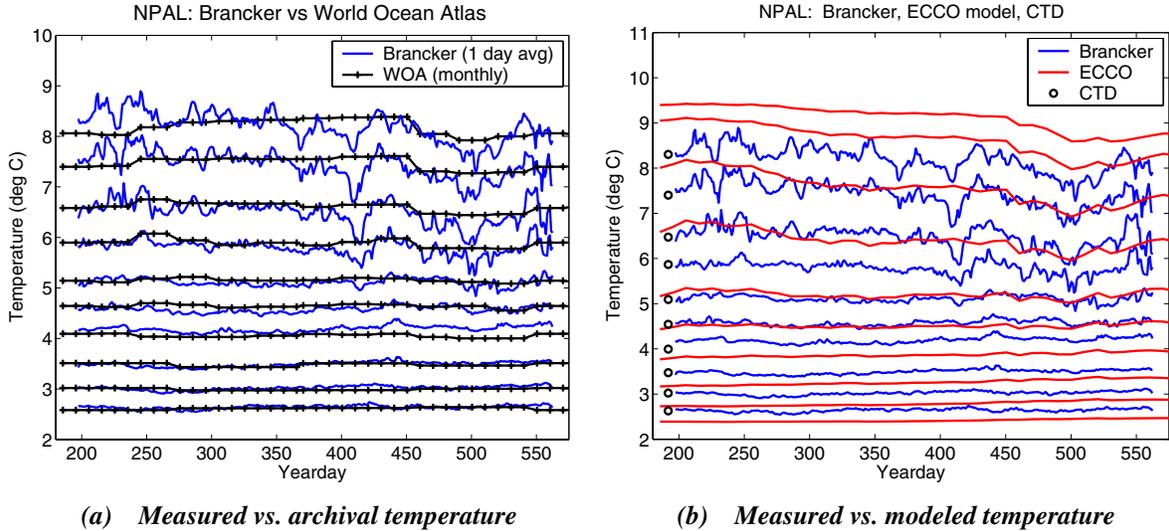
*Sound Speed Estimation for Mode Processing:* As noted in the previous section, mode filtering of underwater acoustic receptions depends on accurate knowledge of the mode shapes and wavenumbers. From an experimental standpoint, it is often impractical to measure the environmental parameters that determine the SSP on a dense grid in both time and depth. For example, in long-duration acoustic experiments such as NPAL, high resolution measurements of temperature and salinity are only possible during the deployment and recovery cruises. In the intervening months the environment is sampled by a relatively small number of sensors attached to the acoustic mooring. One approach to defining the SSP for mode filter design is to interpolate and extrapolate these environmental measurements. Other approaches would be to use archival environmental data or the output of an ocean model. To see why these two latter approaches are not appropriate for the NPAL experiment, consider the plots in Figure 4. Figure 4(a) shows the time series measured by 10 temperature sensors attached to the NPAL VLA located off the California coast in 1998-1999. The plot compares the measured temperatures to archival temperatures for the VLA location (derived from the WOA monthly database). Given that the archival data represents an average over many years, it is not surprising that the measured temperatures deviate substantially from the archival data in some places. Using the archival profile to design the mode filter could lead to significant mismatch at certain times of the year. Figure 4(b) compares the measured temperature time series to the output of the ECCO model (ECCO is a contribution of the Consortium



**Figure 3: Comparison of modeshape approximation errors for three SSP's: the Deep-6 profile, a measured profile, and an archival profile.**

for Estimating the Circulation and Climate of the Ocean funded by the National Oceanographic Partnership Program). While the overall trend is quite similar, the ECCO data is significantly biased, especially for the shallower sensors. Based on these results, it appears that neither the archival data nor the ECCO model can provide accurate enough profiles to use for mode processing.

In 2004-2005, we developed a method for interpolating and extrapolating the NPAL temperature measurements to obtain sound speed profiles. The technique, described in [4], assumes that the changes in the SSP associated with mesoscale and seasonal effects can be written in terms of the quasi-geostrophic dynamic modes. The dynamic modes are an orthonormal basis derived from historical oceanographic data for the VLA location. By fitting the observed temperature fluctuations to the dynamic modes, it is possible to obtain an estimate of the SSP required for mode processing. The Space-Time Kalman Filter (STKF) provides a convenient framework for solving the estimation problem. The Oceans paper describes the STKF implementation and examines its performance for the NPAL data set. It analyzes the sound speed errors associated with the method and considers how those errors propagate through the mode computations. Figure 5 gives an example of the STKF results for yeardays 500-503 in the NPAL data set. The plots compare the estimated temperature and sound speed profiles with the WOA annual data. Actual temperature measurements are also shown for comparison. The STKF produces very reasonable-looking estimates, though it is difficult to verify their accuracy due to lack of ground truth data. Additional work is being done to refine the approach so that it can be used

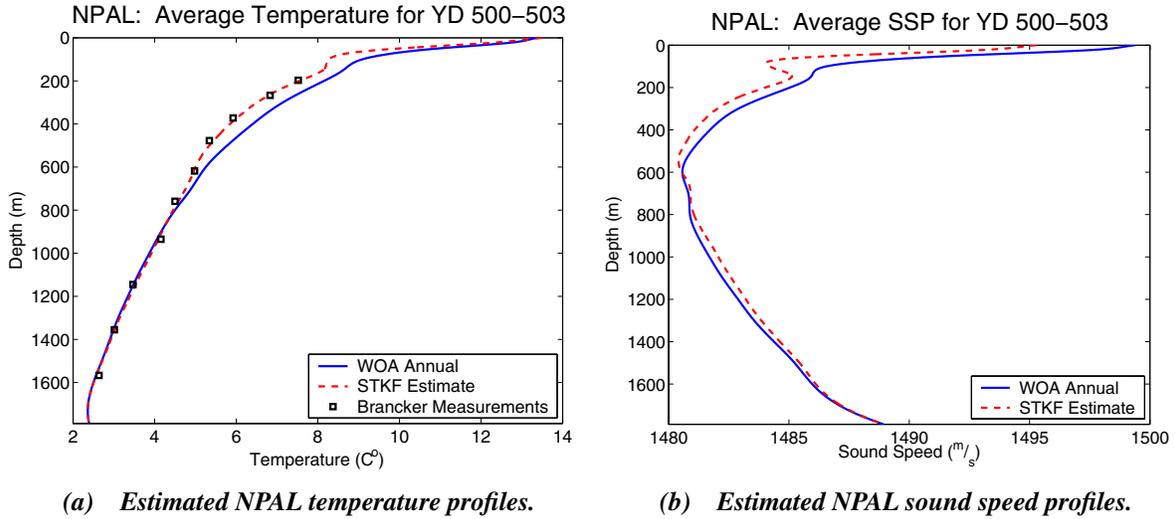


**Figure 4: Comparison of temperatures measured by the Brancker sensors on the NPAL VLA with archival temperatures from the World Ocean Atlas and temperatures derived from the ECCO model.**

to analyze the environmental measurements from the 2004-2005 SPICE/LOAPEX experiment.

*Mode Equalization:* Tomographic experiments use broadband low frequency signals such as maximal length sequences (M-sequences) to study the ocean. One problem in detecting the mode arrivals at megameter range is that multipath causes time spread in the transmitted M-sequence, leading to interference between adjacent symbols. This problem, called Inter Symbol Interference (ISI), is common in wireless channels. Equalizers are used in underwater acoustic communications to reduce ISI and provide SNR gain *e.g.*, see [11]. Working with data from the ATOC Engineering Test, Freitag and Stojanovic demonstrated that decision feedback equalizer (DFE) can be used for very long range underwater acoustic signals [12]. Freitag and Stojanovic’s equalizer operated on multiple channels of the vertical line array (VLA) data.

Motivated by this earlier work, we began exploring the design of equalizers for the low mode signals. In 2004-2005, we implemented a DFE-based equalizer for the low modes [5]. In tomography (as opposed to communications), the transmitted symbols are known. For the mode equalizer the transmitted symbols are fed back instead of the symbol decisions, hence the mode equalizer is equivalent to the DFE operating in training mode. The output SNR of the equalizer provides a useful metric for analyzing the mode arrivals. The mode equalizer works as follows. First, a spatial filter is used to estimate the desired mode time series that the equalizer will operate on. Then the equalizer is allowed to synchronize to a series of time instants and the output SNR is calculated for each. When the equalizer synchronizes to a time instant where it is unable to successfully combine multipaths, it will decode symbols with a high mean square error, thus the output SNR will be low. When the equalizer synchronizes to a time instant that enables it to combine all the multipaths in the signals, the decoded symbols will have a low mean square error, thus a high SNR. Figure 6 shows examples of SNR vs. synchronization time curves for modes 1 and 10 in one of the NPAL receptions. The plots illustrate how the results vary when equalizers of different lengths are used. Ideally the equalizer length should

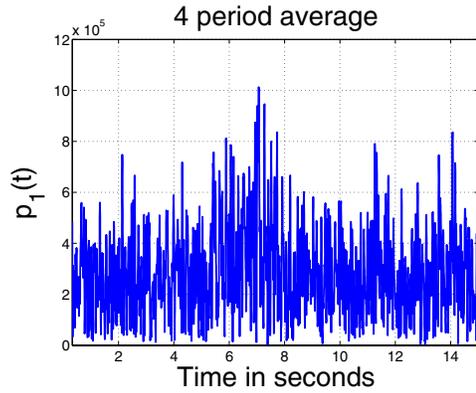


**Figure 5: Estimated temperature and sound speed profiles for the NPAL VLA compared to World Ocean Atlas annual profiles. The estimated profiles were derived from the Kalman filter algorithm described in [4].**

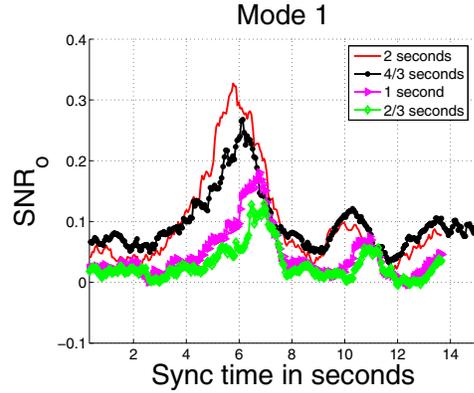
be matched to the multipath spread of the modes. Since the SNR keeps increasing for equalizers up to 2 seconds long, it appears that the multipath spread of the modes in the NPAL receptions is on the order of 2 seconds or greater. The paper presented at the 2005 IEEE/MTS Oceans Conference demonstrated that equalizers can be designed for the low mode signals in the NPAL receptions. Additional work is needed to determine whether any useful tomographic information can be derived from the equalized mode signals. One concern with the DFE-based approach is that it has relatively high computational complexity. Future work will explore less complex solutions to the problem of mode equalization.

## IMPACT/APPLICATIONS

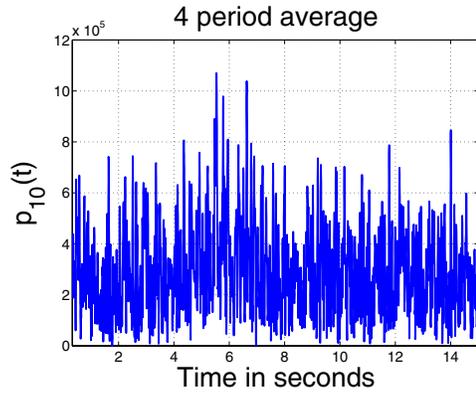
The objectives of this research have both scientific and operational applications. The statistical characterization of the acoustic channel provides crucial information to guide the design of long-range systems for tomography, communication, and surveillance. The signal processing techniques being developed are applicable in both short- and long-range propagation scenarios.



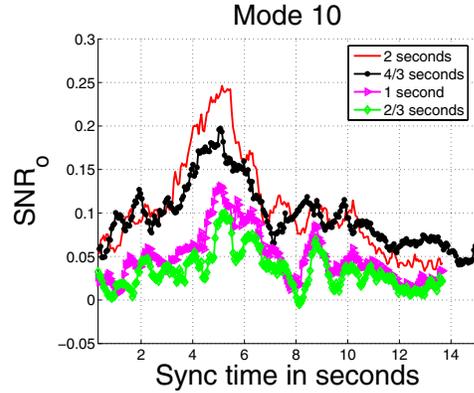
(a) Mode 1 time series (4-period avg).



(b) Mode 1 equalizer output



(c) Mode 10 time series (4-period avg).



(d) Mode 10 equalizer output.

**Figure 6: Example of mode time series measured in the NPAL experiment and the corresponding mode equalizer outputs. The plots on the left show the mode 1 and 10 time series estimated from a 4-period average of the received signal. The plots on the right show the output SNR vs. synchronization time curves obtained from the mode equalizer.**

## RELATED PROJECTS

This work is closely related to the North Pacific Acoustic Laboratory project, directed by principal investigators Peter Worcester (SIO) and James Mercer (APL-UW). A number of other ONR-sponsored researchers work on projects related to NPAL and participate in the NPAL Workshops. Kathleen Wage and Tarun Chandrayadula presented results at the May 2005 NPAL Workshop

## REFERENCES

- [1] K. E. Wage, M. A. Dzieciuch, P. F. Worcester, B. M. Howe, and J. A. Mercer, "Mode coherence at megameter ranges in the North Pacific Ocean," *J. Acoust. Soc. Am.*, vol. 117, pp. 1565–1581, March 2005.
- [2] K. E. Wage, "Experimental measurements of normal mode statistics at megameter ranges." 148<sup>th</sup> *Meeting of the Acoustical Society of America*, San Diego, CA, November 2004.
- [3] K. E. Wage, "Approximate mode filtering," in *Proceedings of the 38th Asilomar Conference on Signals, Systems, and Computers*, pp. 1436–1440, November 2004. □
- [4] E. R. S. Pierre and K. E. Wage, "Sound Speed Estimation from Sparse Environmental Measurements With Application to Mode Filtering," in *Proceedings of the 2005 IEEE/MTS Oceans Conference*, pp. 1–7, September 2005. □
- [5] T. K. Chandrayadula and K. E. Wage, "Mode Equalization at Megameter Ranges," in *Proceedings of the 2005 IEEE/MTS Oceans Conference*, pp. 1–7, September 2005.
- [6] S. C. Miller and J. R. H. Good, "A WKB-Type Approximation to the Schrödinger Equation," *Phys. Rev.*, vol. 91, pp. 174–179, July 1953. □
- [7] J. C. Miller, "Oceanic acoustic rays in the deep six sound channel," *J. Acoust. Soc. Am.*, vol. 71, pp. 859–862, April 1982.
- [8] S. Levitus and T. Boyer, *World Ocean Atlas 1994 Volume 4: Temperature*, 1994. NOAA Atlas NESDIS 4. □
- [9] S. Levitus, R. Burgett, and T. Boyer, *World Ocean Atlas 1994 Volume 3: Salinity*, 1994. NOAA Atlas NESDIS 3. □
- [10] A. B. Baggeroer, "Prüfer Transformation Methods For Determining Normal Modes of Ocean Acoustics." Unpublished memo.
- [11] M. Stojanovic, J. Catipovic, and J. G. Proakis, "Adaptive multichannel combining and equalization for underwater acoustic communications," *J. Acoust. Soc. Am.*, vol. 94, pp. 1621–1631, September 1993.
- [12] L. Freitag and M. Stojanovic, "Basin-Scale Acoustic Communication: A Feasibility Study Using Tomography M-Sequences," in *Proceedings of the 2001 IEEE/MTS Oceans Conference*, vol. 4, pp. 2256–2261, November 2001.

## **PUBLICATIONS**

K. E. Wage, M. A. Dzieciuch, P. F. Worcester, B. M. Howe, and J. A. Mercer "Mode coherence at megameter ranges in the North Pacific Ocean," *J. Acoust. Soc. Am.*, vol. 117(3), Pt. 2, pp. 1565-1581, March 2005. [published, refereed]

K. E. Wage, "Approximate mode filtering," *Proceedings of the 38th Asilomar Conference on Signals, Systems, and Computers*, pp. 1436-1440, November 2004. [published, refereed]

T. K. Chandrayadula and K. E. Wage, "Mode Equalization at Megameter Ranges," *Proceedings of the 2005 IEEE/MTS Oceans Conference*, pp. 1-7, September 2005. [published]

E. R. St. Pierre and K. E. Wage, "Sound Speed Estimation from Sparse Environmental Measurements With Application to Mode Filtering," *Proceedings of the 2005 IEEE/MTS Oceans Conference*, pp. 1-7, September 2005. [published]

## **AWARDS**

Kathleen Wage received an ONR Young Investigator Award in 2005.