2011 7th US National Combustion Meeting
Organized by the Eastern States Section of the Combustion Institute
and Hosted by the Georgia Institute of Technology, Atlanta, GA
March 20-23, 2011

Theory of Flame-Acoustic Interaction for Flame Propagation in Spherical Chamber

V'yacheslav Akkerman and Chung K. Law*

Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, New Jersey 08544-5263, USA

A simplified model describing acoustically-generated parametric instability in a spherical chamber is developed for quasi-one-dimensional, low-Mach flames. We demonstrate how sound waves generated by a centrally-ignited, outwardly-propagating accelerating flamefront can be incorporated into a theory of self-similar flame acceleration in free space developed previously. Being reflected from the chamber wall, flame-generated sound waves interact with the flamefront and the attendant hydrodynamic flame front cellular instability. This in turn affects the subsequent flame shape and propagation speed. It is shown that acoustics modifies the exponent in the self-similar power-laws flame acceleration, facilitating the transition to detonation in confinement.

1. Introduction

The dynamics and stability of expanding, globally-spherical flames is an important phenomenon in combustion science. In terrestrial situations, expanding flames are of relevance to the operation of spark-ignition engines and prevention of spark-initiated hazards, and are employed to measure the laminar flame speeds of combustible mixtures. In astrophysics, an expanding nuclear flamefront can be the precursor of a supernova event [1].

A typical evolution of an expanding flamefront includes several stages. First, an energy kernel is deposited at a point, which leads to the ignition and formation of a flame kernel. At this stage the flame evolution is controlled by stretch and mixture diffusion, i.e. the Lewis number Le. It is noted that initial sustained flame propagation is possible only for $Le > 1$ mixtures, while a $Le < 1$ flame kernel has to exceed a minimal size to acquire sustained propagation [2-4]. Due to the positive stretch effect imposed by the flame expansion, a flamefront is initially smooth. However, as the flame grows in size and the stretch intensity reduces, diffusional-thermal effects come to play, leading to the onset of cellular and pulsating instabilities for $Le < 1$ and $Le > 1$ mixtures, respectively [5]. Subsequently, as the flame thickness as compared to the global flame radius is reduced, the generation of hydrodynamic (Darrieus-Landau, DL) instability is favored, leading to cell formation over the flame surface regardless of the mixture Lewis numbers [6]. The continuous production of new cascades of cells leads to a corresponding continuous increase in the total flame surface area as compared to the globally spherical flame,

*Corresponding author. Email: akkerman@princeton.edu

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **MAR 2011**	2. REPORT TYPE	3. DATES COVERED **00-00-2011 to 00-00-2011**
4. TITLE AND SUBTITLE **Theory of Flame-Acoustic Interaction for Flame Propagation in Spherical Chamber**		5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Princeton University,Department of Mechanical and Aerospace Engineering,Princeton,NJ,08544-5263**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release; distribution unlimited**		
13. SUPPLEMENTARY NOTES **U.S. Government or Federal Rights License**		
14. ABSTRACT		
15. SUBJECT TERMS		

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**8**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

with concomitant increases in the global reactant consumption rate and flame propagation speed. Consequently, the flamefront self-accelerates [7, 8].

While the propagation and structure of initially smooth expanding flames [2-4] as well as the transition to cellularity [9-12] have been extensively studied, both theoretically and experimentally, and these stages are reasonably well understood, the subsequent development of the DL instability requires further study. In particular, studies of hydrodynamically-wrinkled expanding flames have been mostly concerned with the self-acceleration nature of the propagation, especially on the possibility that the time exponent describing the temporal variation of the flame radius could attain a constant value [7, 8, 11, 13-15]. This would then imply that the acceleration is self-similar, having a fractal nature. Furthermore, it is anticipated that flame acceleration can trigger the body-force (Rayleigh-Taylor, RT) instability and flame turbulization, and eventually the transition to detonation.

The flame dynamics depends strongly on whether combustion occurs in free space or within a confinement. A recent analysis [15] shows that the instability-induced flame acceleration in free space is quite weak, and an unconfined deflagration-to-detonation transition (DDT) in terrestrial conditions due to this mechanism is unlikely. Nevertheless, the situation can be different in the case of constant volume combustion. First of all, pressure increases during burning within a confinement, which modifies the unstretched laminar flame speed. Second, flame interaction with the acoustic dynamics of the confinement is expected to greatly facilitate flame acceleration. There are two modes through which a flame can be affected by acoustics. One is through pressure variations, which lead to modification of the laminar flame speed and to combustion instability when these variations and fluctuations of the heat release rate obey the Rayleigh criterion. We focus however on the other mode of acoustic interaction, i.e. on modification of the velocity field, which in turn affects the flamefront surface. In particular, we are interested in the interaction between the DL instability and the flame-generated sound waves. We aim to clarify whether such an interaction intensifies or weakens flame propagation.

In view of these considerations, we shall extend the formulation of Akkerman et al. [15] to analyze the phenomenon of flame-acoustic interaction in a chamber, incorporating the effects of the parametric instability [16, 17]. It is shown that acoustics modifies the exponent in the self-similar power-laws acceleration of the flame.

2. Formulation

We consider a flamefront with the normal propagation speed with respect to the unburned mixture S_L and the density expansion factor $\Theta \equiv \rho_u / \rho_b$, which propagates outwardly from an ignition point in the center of a spherical chamber of radius R_C as illustrated in Fig. 1. The radial velocity of the flame front in the laboratory reference frame, U_L, and that with respect to the reactive mixture, U_w, and the average radial velocity of the flow immediately upstream of the front U_F are respectively given by

$$U_L = \frac{dR_F}{dt}, \qquad U_w = \frac{U_L}{\Theta} = \frac{1}{\Theta}\frac{dR_F}{dt}, \qquad U_F = (\Theta - 1)U_w = \frac{\Theta - 1}{\Theta}\frac{dR_F}{dt}, \tag{1}$$

where $R_F(t)$ is the instantaneous global flame radius. During the early stage of burning, the flame expands with a constant speed, $dR_F / dt = \Theta S_L$. Subsequently, when the flame radius exceeds some critical value R_0, the front becomes corrugated due to the onset of flame front instability and the flame accelerates. An outwardly-accelerating flamefront generates sound waves, which intensify with an increase in the global flame radius, and at a certain stage these acoustics can modify the flame dynamics. Upon reflection from the chamber wall, the flame-generated sound waves interact with the hydrodynamic flame instability and influence the subsequent flame shape and propagation speed. In particular, this can initiate the onset of the flamefront parametric instability.

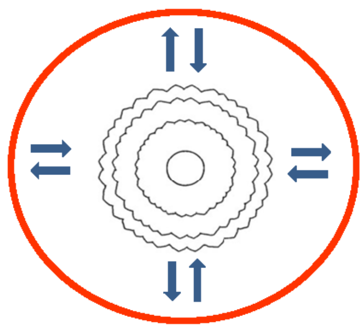

Figure 1: Schematic of an outwardly-propagating, accelerating flame within a confinement

In this work we start a systematic study of flame-acoustic interaction by using a simplified quasi-one-dimensional (1D) model. Our basic assumptions are:

- The Landau limit of an infinitely thin flamefront.
- Only radial waves are under consideration, which renders the problem quasi-1D.
- The characteristic acoustic time (a time needed for sound to travel from the flame to the chamber wall and back) is assumed to be much smaller than the characteristic DL time (a time needed for a new cascade of DL cells to develop), which basically corresponds to the requirement of a low Mach number.
- Consideration limited to the intermediate stage of flame expansion within a confinement, ideally with $R_0 << R_F << R_C$.

The first inequality, $R_0 << R_F$, denotes that self-similar flame acceleration is attained, and sound waves are strong enough. The second inequality, $R_F << R_C$, is of primary importance for the analysis. First of all, it means that an increase in the overall pressure inside the chamber due to combustion is small. Consequently we neglect pressure variations, and assume S_L to be constant and neglect the pressure-related aspects of the acoustics. Hence we focus on the sound-generated velocity field in the vicinity of the flame. Furthermore, as demonstrated below, with the condition $R_F << R_C$ we can neglect the time-dependence of the acoustic frequency (but not the amplitude), and subsequently develop a self-similar formulation. It is noted that with the above assumptions a simplified model of flame-acoustic interaction can be developed, which provides us with a basic qualitative understanding of the process and allows further development of the analysis.

Within the quasi-1D formulation, the flamefront can be described as

$$R(\mathbf{x},t) = R_F(t) + \tilde{F}(\mathbf{x},t), \tag{2}$$

where $\tilde{F}$ represents small deviations of the flame shape from the globally-spherical one, $|\tilde{F}| << R_F$. In the linear approach, the small perturbations can be described as $\tilde{F}(\mathbf{x},t) = f(t)\exp(i\,\mathbf{k}\cdot\mathbf{x})$, and the dispersion relation for the perturbation amplitude takes the form [16]

$$\frac{\partial^2 f}{\partial t^2} + \frac{2\Theta}{\Theta+1} S_L k \frac{\partial f}{\partial t} - A\Theta S_L^2 k^2 f = Ak\tilde{g}_a f \ , \tag{3}$$

where $A = (\Theta - 1)/(\Theta + 1)$ is the Atwood number and $\tilde{g}_a$ the general acceleration field. Equation (3) describes the linear stage of the DL and RT instabilities in the acceleration field. Without an external acceleration, $\tilde{g}_a = 0$, it reduces to the DL dispersion relation [6]. Hereafter we assume that the acceleration is only through sound waves, and the acoustical field is represented by a dominant mode $\tilde{u}_r = U\sin(\omega t)$. Consequently, $\tilde{g}_a = d\tilde{u}_r / dt = U\omega\cos(\omega t)$, and Eq. (3) becomes

$$\frac{\partial^2 f}{\partial t^2} + \frac{2\Theta}{\Theta+1} S_L k \frac{\partial f}{\partial t} - A\Theta S_L^2 k^2 f = AkU\omega\cos(\omega t) f \ . \tag{4}$$

First, we shall investigate an arbitrary coherent acoustic field. Then a specific case of flame-generated acoustics within a spherical confinement will be considered.

3. Solution

Here we employ the approach of Bychkov [17] on the parametric instability of a premixed flamefront in an oscillating field, which in turn is based on the classical theory of parametric instability of an oscillator [18]. The parametric instability is related to the mode of frequency half as that of the oscillating field, and the solution to Eq. (4) takes the form

$$f = \left[f_1 \cos\left(\frac{\omega t}{2}\right) + f_2 \sin\left(\frac{\omega t}{2}\right) \right] \exp(\sigma t), \tag{5}$$

where the coefficients f_1 and f_2 are free parameters of the problem, and σ is the growth rate of parametric instability, which is generally complex. Substituting Eq. (5) into Eq. (4), collecting the terms containing $\cos(\omega t/2)$ and $\sin(\omega t/2)$, and omitting terms of higher frequencies, we find

$$f_1\left(\sigma^2+\frac{2\Theta}{\Theta+1}S_L k\sigma - A\Theta S_L^2 k^2 - \frac{\omega^2}{4} - \frac{A}{2}U\omega k\right) + f_2\left(\sigma\omega + \frac{\Theta}{\Theta+1}S_L k\omega\right) = 0\,, \qquad (6)$$

$$f_2\left(\sigma^2+\frac{2\Theta}{\Theta+1}S_L k\sigma - A\Theta S_L^2 k^2 - \frac{\omega^2}{4} + \frac{A}{2}U\omega k\right) + f_1\left(\sigma\omega + \frac{\Theta}{\Theta+1}S_L k\omega\right) = 0\,. \qquad (7)$$

We emphasize that this approach is rigorous for high-frequency oscillations and large-scale perturbations, and it works as an evaluation of the parametric stability limits otherwise. Eliminating f_1 and f_2 from Eqs. (6) and (7) we obtain the dispersion relation

$$\left(\sigma^2+\frac{2\Theta}{\Theta+1}S_L k\sigma - A\Theta S_L^2 k^2 - \frac{\omega^2}{4}\right)^2 + \left(\sigma\omega + \frac{\Theta}{\Theta+1}S_L k\omega\right)^2 = \left(\frac{AU\omega k}{2}\right)^2. \qquad (8)$$

The parametric instability is triggered if $\mathrm{Re}\{\sigma\} > 0$, i.e. when

$$\left(\frac{U}{S_L}\right)^2 - \left(\frac{2\Theta}{\Theta-1}\right)^2 > \left(\frac{\omega^2+4A\Theta S_L^2 k^2}{2AS_L\omega k}\right)^2. \qquad (9)$$

For an asymptotic solution, only the leading terms in (9) are retained. Consequently two limits exist. For $\omega >> 2(A\Theta)^{1/2}S_L k$ parametric instability is triggered at large wavenumbers (small scales) satisfying

$$k > k_0 = \omega / 2AU\,, \qquad (10)$$

while in the opposite limit of $\omega << 2(A\Theta)^{1/2}S_L k$, the parametric instability is triggered at small wavenumbers satisfying

$$k < k_1 = U\omega / 2\Theta S_L^2\,. \qquad (11)$$

Obviously, in the present work we focus on the high-frequency case, Eq. (10).

We now analyze flame-generated acoustics within a spherical confinement in this limit. Then the instantaneous acoustic amplitude can be estimated as $U \sim U_F = (\Theta-1)U_w$, Eq. (1), while the acoustic frequency is given by

$$\omega = 2\pi/\tau_a = \pi c_s / (R_C - R_F) \approx \pi c_s / R_C\,, \qquad (12)$$

where c_s is the sound speed. Equations (1), (10) and (12) constitute the cut-off wavenumber for the parametric instability in the form

$$k_0 = \frac{\pi}{2}\frac{\Theta+1}{(\Theta-1)^2}\frac{c_s}{R_C U_w}\,. \qquad (13)$$

The parametric instability is triggered only at small scales, less than $2\pi/k_0$, while the large-scale flame evolution is still controlled by the DL instability. Consequently, we can adopt the self-similarity formulation on large scales,

$$R_F(t) = Ct^{\alpha}\,, \qquad (14)$$

which can also be expressed in the form $U_w \propto R_F^D$, i.e.

$$U_w / \tilde{S}_L = (k_0 R_F)^D , \tag{15}$$

where $D = (\alpha - 1)/\alpha$ is the fractal excess, and k_0, given by Eq. (13), assumes the role of the largest self-similarity wavenumber. It is related to the smallest fractal cascade, and $\tilde{S}_L$ is the flame speed at the scale $2\pi / k_0$, so it describes the pure effect of the parametric instability. We emphasize that unlike the situation of flame expansion in free space, the parameter C of Eq. (14), and k_0 of Eq. (15), depend on t (and R_F) because of U_w in Eq. (13), which modifies the exponent in the general self-similar power law. Indeed, combining Eqs. (1), (13) and (15), we obtain

$$\frac{dR_F}{dt} = \Theta \tilde{S}_L^{\frac{1}{D+1}} \left[\frac{\pi}{2} \frac{\Theta + 1}{(\Theta - 1)^2} \frac{c_s}{R_C} R_F \right]^{\frac{D}{D+1}} , \tag{16}$$

which can be integrated over any interval (t_1, t_2) as

$$R_F^{\frac{1}{D+1}}(t_2) - R_F^{\frac{1}{D+1}}(t_1) = \frac{\Theta}{D+1} \tilde{S}_L^{\frac{1}{D+1}} \left[\frac{\pi}{2} \frac{\Theta + 1}{(\Theta - 1)^2} \frac{c_s}{R_C} \right]^{\frac{D}{D+1}} (t_2 - t_1) \ . \tag{17}$$

Consequently, Eq. (14) is now replaced by a new asymptotic

$$R_F(t) \approx \underbrace{\tilde{S}_L \left(\frac{\Theta\alpha}{2\alpha - 1} \right)^{\frac{2\alpha-1}{\alpha}} \left[\frac{\pi}{2} \frac{\Theta + 1}{(\Theta - 1)^2} \frac{c_s}{R_C} \right]^{\frac{\alpha-1}{\alpha}}}_{C_1} t^{\frac{2\alpha-1}{\alpha}} . \tag{18}$$

It is noted that the quantity $\tilde{S}_L$ may also include dependence on time, but we omit this possibility in the present work.

4. Discussion

As a result, based on a simple model we have obtained a modified power law, Eq. (18), which accounts for flame-sound interaction in a spherical confinement. According to various experimental measurements, the exponent α falls in the range of $\alpha = 1.25 - 1.5$ [7, 8, 11]. For instance, Gostintsev et al. [11] suggested $\alpha = 3/2$. In contrast, the recent, well-controlled experiments of Jomaas and Law [8], conducted in a quiescent, confined environment, showed that α is about $4/3$. The formulation above allows reconciliation of such a discrepancy, at least qualitatively. That is, having $R_F \propto t^{\alpha}$ in free space, we can predict $R_F \propto t^{(2\alpha-1)/\alpha}$ for the same combustible mixture within a confinement. Furthermore, even within the same confinement, one can find $R_F \propto t^{\alpha}$ when the radial sound waves are too weak, which will evolve to $R_F \propto t^{(2\alpha-1)/\alpha}$ later, when the effect of acoustics becomes stronger. Taking $\alpha = 3/2$ as in [11], we find $(2\alpha - 1)/\alpha = 4/3$ in agreement with [8]. Since $(2\alpha - 1)/\alpha < \alpha$ for any $\alpha > 1$, Eq. (18) exhibits a lower power dependence as compared to Eq. (14). However, this does not necessarily mean that the acceleration proceeds slower within a confinement because the process depends strongly on the pre-exponential constant C_1 in Eq. (18). In fact, we can readily extend our recent formulation on self-similar flame acceleration and the detonation triggering in free space [15] to Eq. (18) instead of Eq. (14). In particular, the instant and position of the detonation triggering can be estimated as

$$t_{\mathrm{expl}} \approx \varphi\, \Omega^{\frac{\alpha}{2(\alpha-1)}}, \qquad R_{\mathrm{expl}} \approx \psi\, \Omega^{\frac{2\alpha-1}{2(\alpha-1)}}, \tag{19}$$

where φ and ψ are the characteristic time and length scales, determined by

$$\varphi = (c_s / C_1)^{\frac{\alpha}{\alpha-1}}, \qquad \psi = c_s^{\frac{2\alpha-1}{\alpha-1}} \Big/ C_1^{\frac{\alpha}{\alpha-1}}, \tag{20}$$

$$\Omega = \frac{\alpha^2}{(\gamma-1)(2\alpha-1)^2}\left[\left(B_1 + \frac{5\alpha-3}{2\alpha-1}\right)B_2 - \frac{B_2^2}{2} - \frac{B_1}{2}\left(B_1 + \frac{\alpha}{2\alpha-1}\right) - \frac{3}{2}\frac{3\alpha-2}{2\alpha-1}B_1^{1/3}B_2^{2/3}\right]^{-1}\left(\frac{T_i}{T_0}-1\right), \tag{21}$$

with the adiabatic exponent γ, the ignition temperature T_i scaled by the room temperature T_0,

$$B_1 = \frac{2}{3}\frac{\alpha-1}{2\alpha-1}\frac{1}{\gamma-1}, \qquad B_2 = B_1 + \frac{\Theta-1}{\Theta}. \tag{22}$$

The result is shown in Fig. 2 for $\Theta = 8$, $\gamma = 7/5$, and $\alpha = 3/2$. For typical $T_i \approx 3T_0$ we have $t_{\mathrm{expl}} \approx 10\varphi$ and $R_{\mathrm{expl}} \approx 25\psi$, which is definitely far from terrestrial conditions.

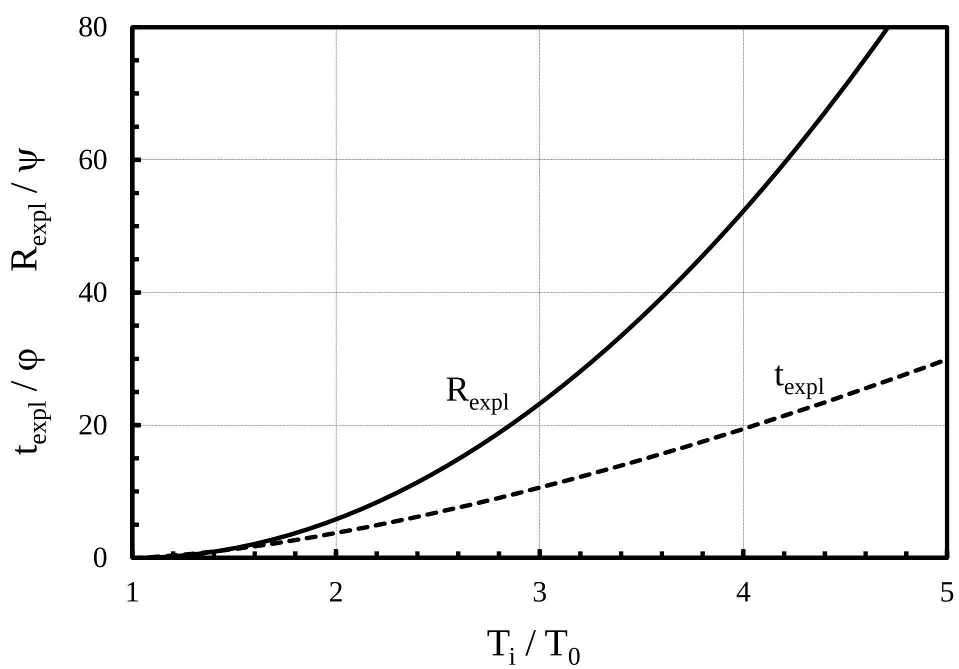

Figure 2: Scaled instant and position of explosion for $\Theta = 8$, $\gamma = 7/5$, and $\alpha = 3/2$

Concluding remarks

In this paper we have developed a simple, quasi-1D model for interaction of an outwardly-expanding accelerating flamefront with acoustics in a spherical confinement. The formulation is based on the assumptions of zero flame thickness, low-Mach flows and self-similar flame dynamics. The model accounts for triggering of the parametric instability, and it describes the flame evolution at the intermediate stage, when the flamefront is far from the ignition point as well as the chamber wall. This model is then incorporated into a previous formulation on self-accelerating flames in free space, demonstrating the potential of facilitated transition to detonation due to confinement.

Acknowledgement

This work was supported by the Air Force Office of Scientific Research under the technical monitoring of Dr. Mitat Birkan.

References

[1] E. S. Oran, *Proceedings of the Combustion Institute* 30 (2005) 1823–1840.
[2] Z. Chen, Y. Ju, *Combustion Theory and Modelling* 11 (2007) 427–453.
[3] M. P. Burke, Z. Chen, Y. Ju, F. L. Dryer, *Combustion and Flame* 156 (2009) 771–779.
[4] A. P. Kelley, C. K. Law, *Combustion and Flame* 156 (2009) 1844–1851.
[5] C. K. Law, *Combustion Physics*, Cambridge University Press, New York, NY, 2006.
[6] Ya. B. Zel'dovich, G. I. Barenblatt, V. B. Librovich, G. M. Makhviladze, *The Mathematical Theory of Combustion and Explosion*, Consultants Bureau, New York, NY, 1985.
[7] D. Bradley, C. G. W. Sheppard, R. Woolley, D. A. Greenhalgh, R. D. Lockett, *Combustion and Flame* 122 (2000) 195–209.
[8] G. Jomaas, C. K. Law, *Proceedings of the 47th AIAA Aerospace Sciences Meeting and Exhibit*, paper 1185, 2009.
[9] A. G. Istratov, V. B. Librovich, Astrounautica Acta 14 (1969) 453.
[10] J. Bechtold, M. Matalon, *Combustion and Flame* 67 (1987) 77–90.
[11] Y. Gostintsev, A. Istratov, Y. Shulenin, *Combustion Explosion and Shock Waves* 24 (1988) 563–569.
[12] G. Jomaas, C. K. Law, J. K. Bechtold, *Journal of Fluid Mechanics* 583 (2007) 1–26.
[13] V. V. Bychkov, M. A. Liberman, *Physical Review Letters* 76 (1996) 2814–2817.
[14] K. L. Pan, R. Fursenko, *Physics of Fluids* 20 (2008) 094107.
[15] V. Akkerman, C. K. Law, V. Bychkov, *Physical Review* E (2011) in press.
[16] G. Searby, D. Rochwerger, *Journal of Fluid Mechanics* 231 (1991) 529–543.
[17] V. Bychkov, *Physics of Fluids* 11 (1999) 3168–3173.
[18] L. D. Landau, E. M. Lifshitz, *Fluid Mechanics*, Pergamon Press, Oxford, UK, 1989.