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A major focus of the research of this grant has been the control of stochastic systems with a large family of noise processes that are important for modeling and that have not previously been used or solved for control problems. Among these noise processes are the family of fractional Brownian motions. While Brownian motion is included in this family, all of the other members are neither semimartingales nor Markov processes.

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Optimal and Adaptive Control of Stochastic Systems
AFOSR Grant FA9550-09-1-0554
Final Technical Report

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A major focus of the research of this grant has been the control of stochastic systems with a large family of noise processes that are important for modeling and that have not previously been used or solved for control problems. Among these noise processes are the family of fractional Brownian motions. While Brownian motion is included in this family, all of the other members are neither semimartingales nor Markov processes. Thus the usual stochastic calculus or the methods from Markov processes cannot be used. However, empirical data in a wide variety of physical phenomena, such as turbulence and telecommunications, provides evidence of the necessity for using these fractional Brownian motions in mathematical models. Thus it is important to consider control systems that are driven by an arbitrary fractional Brownian motion.

In [6,7,9,22] a control problem is solved for a finite dimensional linear stochastic system with an arbitrary fractional Brownian motion and a quadratic cost functional. The methods of proof in [22] demonstrate that the form of the explicit control is valid for an arbitrary square integrable process with continuous sample paths. This optimal control result is extended to linear stochastic partial differential equations with a fractional Brownian motion and a control that is restricted to the boundary or to discrete points in the domain [11,12,13]. While the optimal control has the same formal structure as for finite dimensional linear systems, the analysis is significantly more difficult. A unified view of these finite dimensional problems is given in [15] including the solution of an infinite time horizon problem. The significant problem of nonlinear filtering for systems with fractional Brownian motions is addressed in [1]. Mutual information is a basis notion in information theory. The mutual information for stochastic signals and Levy processes is given explicitly in terms of filtering and smoothing errors in [2]. Some results for consistency for Bayesian information criterion estimators for ergodic processes are given in [3,10]. A growth optimal portfolio with transaction costs and a required diversification of the portfolio is solved in [4], which is a stochastic control problem for Markov processes. Some general results for stochastic control are given in [5]. Since computations of optimal controls is important, a control problem for a discrete time linear stochastic system with an arbitrary correlated noise and a quadratic cost functional is explicitly solved in [19]. A risk sensitive control problem with an exponential quadratic cost is important for its explicit optimal control and its relation to differential games. A control problem for a linear system and an exponential quadratic cost functional is solved in [18]. This solution does not require the use of a Hamilton-Jacobi-Bellman equation and explains the additional term in the Riccati equation solution as compared with the Riccati equation for a quadratic cost functional. Stochastic control problems for linear stochastic partial differential equations with Brownian motions and exponential quadratic cost functionals are solved in [16,20]. These solutions do not require the use of a Hamilton-Jacobi-Bellman equation and provide an explanation for the terms in the infinite dimenstional Riccati equation solution. Some general methods in stochastic adaptive control are given in [8]. The control of some nonlinear stochastic systems that evolve in manifolds are explicitly solved in [17,21] without the use of a Hamilton-Jacobi-Bellman equation. The manifolds are compact and noncompact symmetric spaces of rank one. These results should also provide insight into solving stochastic control problems for nonlinear systems.

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