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A. Stephen Morse

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14. ABSTRACT

Here is a listing some of what this project has accomplished. We've given a probabilistic explanation of why a previously proposed accelerated gossip algorithm works. We've resolved the problem of automatically localizing a mobile autonomous agent using range-only measurements. We invented new tools for analyzing convergence rates for consensus and gossiping algorithms. We derived a complete solution to the problem of controlling a directed formation of triangular shape. We proved that for communication graphs which are trees, the rate at which periodic gossiping distributed algorithms converge does not depend on the order in which the individual gossips occur. We developed a new algorithm maintaining formations of mobile autonomous agents with cycle-free graphs. We developed request-based deterministic gossip protocols which are guaranteed not to deadlock.

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The overriding objective of this project has been to develop new tools for analyzing distributed dynamical networks and for controlling them. More specifically, this research has had four interrelated thrusts. First, we have sought to devise new tools and techniques for analyzing and controlling inherently asynchronous, distributed dynamical systems. Second we've used graph rigidity theory combined with nonlinear control theory to develop techniques for autonomously maintaining the correct relative positions of sensors in a large sensor network. Maintaining correct positions is directly related to maintaining network connectivity, an issue of critical importance in almost any sensor or communication network which is to be invulnerable to component failures or attack. Third we've developed techniques for positioning a mobile sensor relative to landmarks or other sensors using only range sensing. We have addressed this highly nonlinear problem both analytically and experimentally using sophisticated, provably correct, switched adaptive control concepts. Fourth have developed new, efficient methods for accurately calculating the positions of sensors in a network from inter-sensor distance information. Since the sensor network localization problem with distance information is known to be NP hard, we have focused attention on identifying classes of networks which can be "easily localized" by either sequential methods, or by recently devised divide and conquer methods.

During the project we've made significant progress in several different directions. In the area of formation control we have developed a new method of distributed control for formations of mobile autonomous agents, which for a large class of "acyclic" formations is capable of causing any initial deployment of agents in the plane to move exponentially fast to desired positions relative to their neighbors [11]. The control exploits the concept of a "target point" and is applicable to any "directed" formation which is cycle-free and rigid, provided each agent in the formation has at most two neighbors to follow. The class of undirected rigid graphs associated with such formations can be shown to be the same as the class of rigidity graphs generated by Henneberg sequences consisting of only vertex additions. Formation control problems such as this, which are based on graph rigidity, have been under study for about a decade. Although there has been much progress, understanding the underlying issues has been challenging because the problems are highly nonlinear. Recently was it shown that the two-cycle formation stabilization problem posed in [8] is unsolvable. This has prompted us to look carefully at rigidity based formation control using undirected graphs.

By an *undirected rigid formation* of mobile autonomous agents is meant a formation based on "graph rigidity" in which each pair of "neighboring" agents i and j is responsible for maintaining the prescribed distance d_{ij} between them. Recent research by several different groups has led to the development of an elegant potential function based theory of formation control which provides gradient laws for asymptotically stabilizing a large class of rigid, undirected formations in two-dimensional space assuming all agents are described by kinematic point models. This particular methodology is perhaps the most comprehensive currently in existence for maintaining undirected formations based on graph rigidity. In our research we have been able to explain what happens if neighboring agents i and j using such gradient controls have slightly different understandings of what the desired distance d_{ij} between them is suppose to be. The question is relevant because no two positioning controls, whether they use a form of integral control or not, can be expected to move agents to precisely specified positions because of inevitable imprecision in the physical comparators used to compute the positioning errors. What one would hope for is a gradual distortion of the formation from its target shape as discrepancies is desired distances increase. While this is observed for the gradient laws in question, something else quite unexpected happens at the same time. In particular, we have been able to prove using elementary arguments that any undirected rigid formation in the plane with mismatching target distances will, almost for certain, go into a nontrivial limit cycle driven by a sinusoidal signal at a single frequency.

The robustness issues uncovered here have broader implications extending well beyond formation maintenance to the entire field of distributed control. In particular, this research illustrates that when assessing the efficacy of a particular distributed control, one must consider the consequences of distinct agents having slightly different understandings of what the values of shared data between them is suppose to be. For without the protection of exponential stability/convergence, it is likely that such discrepancies will cause significant misbehavior to occur.

In the past we have advocated the use of adaptive control to avoid the naturally arising nonlinearities one encounters in trying to control a formation using only range sensors. We have recently developed a complete analysis of a range-only formation adaptive controller for an agent tasked with the problem of moving to a position determined by given distances from three other agents [17]. The concept of "dwell-time switching" is a key component of the controller. Dwell-time switching is especially useful in its own right because it leads to adaptive controllers which are provable correct in the face of norm-bounded noise and unmodeled dynamics. To elucidate the basic features of dwell time switching we've developed in detail, a simple example application of the concept, intended for nonspecialists [13].

We've made a major effort to advance the state of the art in the area of distributed averaging. The distributed averaging problem is to devise a local protocol which will enable each agent or sensor in a network to compute the average of the initial values of the "agreement variables" acquired or sensed by each of the agents in the network. We have made significant progress in developing various protocols and algorithms, including gossiping [12], and double linear iterations [18] to solve the distributed averaging problem. In [20] we provide a probabilistic explanation of why a certain class of accelerated gossip algorithms converge. In [10] we provide a compact proof of the fact [19] that the rate of convergence of periodic multi-gossiping sequences associated with the tree does not depend on the order in which the individual multi-gossips in the sequence take place. We've also sought to understand the subtleties of aperiodic gossiping [14]. In [15] we show that the classical concept of a contraction coefficient which is important in quantifying convergence rates for nonhomogeneous Markov Chains, could be viewed as but one of a number of different types of *semi-norms* of suitably defined stochastic matrices. More directly relevant to gossiping, in [12] we've used simple examples to show that the largely ignored problem of devising a gossiping protocol which avoids *deadlock* while generating rapidly converging gossiping sequences is especially formidable [14]. Nonetheless we devised a deadlock-free protocol which solves the problem. The key idea is to stipulate at the outset that the time at which an agent can place a request to gossip cannot coincide with the times at which any of its neighbors are allowed to place requests to gossip. In some applications, this requirement may be difficult to satisfy. To deal we this, we've also devised a new, provably correct deterministic gossiping-like protocol which solves the distributed averaging problem without requiring any assumptions about the times at which agents and their neighbors place requests to gossip [14]. The protocol is deadlock-free, does not require broadcasting, and leads to consensus exponentially fast. Finally we've completed work on a new and especially promising approach to distributed averaging which uses a "double linear iteration" which is deadlock-free and completely local [18]. Moreover, we've been able to explain how to implement this protocol asynchronously.

Here is a list of our accomplishments:

1. An alternative approach to localization based on merging was derived and presented [1].

2. A partial explanation was developed of the effect of inaccurate measurements on sequential localization algorithms [2, 3].

3. A probabilistic explanation was developed of why a previously proposed accelerated gossip algorithm works [4, 20].

4. We developed the tools needed to analyze any algorithm which uses dwell-time switching [5].

5. The problem of automatically localizing a mobile autonomous agent using range-only measurements was resolved using graph rigidity [6].

6. We invented new tools for analyzing convergence rates for consensus and gossiping algorithms [7, 12, 15].

7. We derived a complete solution to the problem of controlling a directed formation of triangular shape [8].

8. We proved that for communication graphs which are trees, the rate at which periodic gossiping distributed algorithms converge does not depend on the order in which the individual gossips occur We were able to prove that the order in which individual gossips take place in a periodic gossiping process do not effect the rate of convergence if the underlying gossip graph is a tree [9, 10, 19].

9. We developed a new control law maintaining formations of mobile autonomous agents with cycle-free graphs [11].

10. With the aim of making the analysis of adaptive algorithms accessible to non-specalilists, we developed, complete analyzed and published an adaptive algorithm for controlling a simple but highly uncertain dynamical system [13].

11. We developed several request-based deterministic gossip protocol which is guaranteed not to deadlock [12, 14, 21].

12. We developed a deterministic algorithm for accelerating linear iterations such as those encountered in gossiping [16].

13. We significantly expanded our previous work on station keeping with range-only sensing [17].

14. Using a new concept, we derived two simple broadcast-free, asynchronous algorithms which avoid deadlock and solve the distributed averaging problem [18].

15. We have analyzed a previously derived model of a social network intended to explain mob uprisings such as that which occurred in Egypt during the Arab spring [22].

16. Using elementary tools we have been able to show that the most general class of algorithms for maintaining a formation of mobile autonomous agents in two dimensions, will go into a stable limit cycle driven by a sinusoidal frequency if there is either a mismatch in shared data or a bias in at least one agent's measurements [23].

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