REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
31-05-2011

2. REPORT TYPE
Final Performance Report

3. DATES COVERED *(From - To)*
January 1, 2008 - March 31, 2011

4. TITLE AND SUBTITLE
Detection of Hostile Objects

5a. CONTRACT NUMBER

5b. GRANT NUMBER
AFOSR FA9550-08-1-0138

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Dr. David Colton
Dr. Peter Monk
Dr. Fioralba Cakoni

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Delaware
Newark, Delaware 19716

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Dr. Arje Nachman
AFOSR
875 North Randolph Road
Ste 325, Room 3112
Arlington, VA 22203

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-OSR-VA-TR-2012-0640

12. DISTRIBUTION/AVAILABILITY STATEMENT
Distribution A

13. SUPPLEMENTARY NOTES
Final Performance Report

14. ABSTRACT
During the period of this grant we 1) showed that transmission eigenvalues exist and can be determined from the far field pattern of the scattered wave, 2) obtained a formula showing how accurate estimates of the permittivity can be obtained from a knowledge of the first (real) transmission eigenvalue and 3) extended the linear sampling method from the frequency domain to the time domain. In addition, we continued our study of the target identification problem for partially coated dielectric objects.

15. SUBJECT TERMS
Inverse scattering, hostile objects, transmission eigenvalues, anisotropic materials,

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON / 19b. TELEPHONE NUMBER *(Include area code)*
U	U	U	UU	13	Prof. David Colton / 302-831-1863

AFOSR Grant FA9550-08-1-0138
Detection of Hostile Objects

Final Performance Report

January 1, 2008 - March 31, 2011

Principal Investigators

- Dr. David Colton
  University of Delaware
  Newark, Delaware 19716
  Phone: 302-831-1863
  Fax: 302-831-4511
  Email: colton@math.udel.edu

- Dr. Peter Monk
  University of Delaware
  Newark, Delaware 19716
  Phone: 302-831-2652
  Fax: 302-831-4511
  Email: monk@math.udel.edu

- Dr. Fioralba Cakoni
  University of Delaware
  Newark, Delaware 19716
  Phone: 302-831-0592
  Fax: 302-831-4511
  Email: cakoni@math.udel.edu

Objectives

This grant is concerned with the problem of the detection of hostile objects whether they be missile in space or tunnels in the earth where the possibility exists that these objects have complicated anisotropic coatings in order to avoid detection. This investigation also includes the study of a new class of spectral problems called transmission eigenvalue problems.

Status of Effort

During the period of this report particular attention was paid to the transmission eigenvalue problem and in particular the role that transmission eigenvalues can play in determining information about the permittivity tensor for anisotropic media from a knowledge of the scattering data due to plane wave interrogation. In particular, it was shown for the first time that under mild assumptions such eigenvalues exist [8], [12], [13], [15], [18], [19], [20] and, in addition, a method was given for determining these eigenvalues from a knowledge of the far field pattern of the scattered wave [14], [16]. A major breakthrough was achieved in showing how the first (real) transmission eigenvalue can provide information about the permittivity of the scattering object [2], [7], [15], [24]. In particular, for anisotropic media the first transmission eigenvalue provides information on the eigenvalues of the permittivity tensor, thus providing a new method for the nondestructive testing of anisotropic materials such as airplane canopies [3], [7], [16], [24]. We have initiated a collaboration with researchers at Wright-Patterson Air Force Base in order to put these new theoretical results into practical use.

A second problem that we have investigated is a continuation of our study of the scattering by a dielectric that may be partially coated by a thin highly conducting layer. Such problems arise when a benign object is coated in an effort to make it appear hostile to radar interrogation. The corresponding target identification problem is rendered particularly difficult since neither the nature nor the extent of the coating is known a priori. Continuing the research initiated in our previous grant, we have used the linear sampling method to develop numerical algorithms for solving target identification problems such as these [11], [17], [21], [22]. In these algorithms no a priori assumption is made on the material properties of the dielectric, the connectivity of the scattering obstacle nor on the extent of the coating. We have also used Newton's method to investigate a new class of inverse scattering problems involving a source placed in a cavity with unknown boundary [29].

A third problem that we have considered during the period of this report is to extend the linear sampling method to problems involving fluid-solid interaction [9] as well as to inverse scattering problems in the time domain [23] and for multifrequencies data [25]. The linear sampling method is the oldest and most developed of a new qualitative approach to inverse scattering problems that avoids many of the difficulties inherent in the use of weak scattering approximations and nonlinear optimization techniques. This approach opens up new vistas in the field of target identification problems and inverse scattering theory. For a survey of linear sampling methods in inverse electromagnetic scattering theory, we refer the reader to the recent monograph written by the principal investigators of this grant [27] as well as the review article [28].

Central to all of the above investigations is the ability to numerically solve the direct scattering problem in order to generate data for testing our inversion algorithms.This has been done through the continuing development of the ultra-weak method for the numerical solution of electromagnetic scattering problems [1] as well as developing the ultra-weak method for problems in fluid-solid interactions [5]. We have also investigated the use of integral equations and variational methods to solve the direct scattering problem in both the frequency and time domain [4], [6], [10], [26], [30].

Accomplishments/New Findings

The main accomplishments during the period of this report were:

1. Showing the existence of transmission eigenvalues and how they can be determined from the far field pattern of the scattered wave.

2. The derivation of a formula showing how accurate estimates of the permittivity can be obtained from a knowledge of the first (real) transmission eigenvalue.

3. The extension of the linear sampling method from the frequency domain to the time domain.

Personnel Supported

1. Faculty
   D. Colton, P. Monk and F. Cakoni (Principal Investigators)

2. Short- Term Visitors
   Tilo Arens, Karlsruhe Institute of Technology, Germany
   Anne Cossoniere, Ecole Polytechique, France
   Andreas Kirsch, Karlsruhe Institute of Technology, Germany
   Houssem Haddar, Ecole Polytechique, France
   Bojan Guzina, University of Minnesota, USA
   Tomi Huttunen, University of Eastern Finland, Finland
   Armin Lechleiter, Ecole Polytechique, France
   Teemu Luostari, University of Eastern Finland, Finland
   Roland Potthast, University of Reading, England
   Virginia Selgas, University of Coruna, Spain
   Jon Trevelyan, Durham University, England.

Interactions/Transitions

Professors Colton, Monk and Cakoni have attended numerous conferences and seminars as invited speakers both in this country and in Europe and Asia.

Extension Granted

A no-cost extension of this grant was given to extend the end date from November 30, 2010 to March 31, 2011.

Publications (with abstract)

2008

1. A. Buffa and P. Monk, Error estimates for the ultra weak variational formulation of the Helmhotz equation, *ESAIM: Mathematical Modeling and Numerical Analysis* **42** (2008), 925-940.

   Abstract: The Ultra Weak Variational Formulation (UWVF) of the Helmholtz equation provides a variational framework suitable for discretization using plane wave solutions of an appropriate adjoint equation. Currently convergence of the method is only proved on the boundary of the domain. However substantial computational evidence exists showing that the method also converges throughout the domain of the Helmholtz equation. In this paper we exploit the fact that the UWVF is essentially an upwind discontinuous Galerkin method to prove convergence of the solution in the special case where there is no absorbing medium present. We also provide some other estimates in the case when absorption is present, and give some simple numerical results to test the estimates. We expect that similar techniques can be used to prove error estimates for the UWVF applied to Maxwell's equations and elasticity.

2. F. Cakoni, M. Cayoren and D. Colton, Transmission eigenvalues and the nondestructive testing of dielectrics, *Inverse Problems* **24** (2008), 066016.

   Abstract: We show how transmission eigenvalues can be determined from electromagnetic scattering data and used to determine the presence of cavities in a dielectric.

3. F. Cakoni and D. Colton, The inverse scattering problem for anisotropic media, *Advanced Topics in Scattering and Biomedical Engineering, World Scientic Publishing* (2008), 321-31.

   Abstract: We give a survey of recent results on the inverse scattering problem for anisotropic media including both uniqueness theorems and reconstruction algorithms.

4. S. Chandler-Wilde and P. Monk, Wave-number-explicit bounds in time-harmonic scattering, *SIAM J. Math. Anal.* **39** (2008), 1428-1455.

   Abstract: In this paper we consider the problem of scattering of time-harmonic acoustic waves by a bounded, sound soft obstacle in two and three dimensions, studying dependence on the wave number in two classical formulations of this problem. The first is the standard weak formulation in the part of the exterior domain contained in a large sphere, with an exact Dirichlet-to-Neumann map applied on the boundary. The second formulation is as a second kind boundary integral equation in which the solution is sought as a combined single- and double-layer potential. For the variational formulation we obtain, in the case when the obstacle is starlike, explicit upper and lower bounds which show that the inf-sup constant decreases like k^{-1} as the wave number k increases. We also give an example where the obstacle is not starlike and the inf-sup constant decreases at least as fast as k^{-2}. For the boundary integral equation formulation, if the boundary is also Lipschitz and piecewise smooth, we show that the norm of the inverse boundary integral operator is bounded independently of k if the coupling parameter is chosen correctly. The methods we use also lead to explicit bounds on the solution of the scattering problem in the energy norm when the obstacle is starlike. The dependence of these bounds on the wave number and on the geometry is made explicit.

5. T. Huttunen, J. Kaipio and P. Monk, An ultra-weak method for acoustic fluid-solid interaction, *J. Comput. Appl. Math.* **213** (2008), 166-185.

   Abstract: We introduce the ultra-weak variational formulation (UWVF) for fluid-solid vibration problems. In particular, we consider the scattering of time-harmonic acoustic pressure waves from solid, elastic objects. The problem is modeled using a coupled system of the Helmholtz and Navier equations. The transmission conditions on the fluid-solid interface are represented in an impedance-type form after which we can employ the well known ultra-weak formulations for the Helmholtz and Navier equations. The UWVF approximation for both equations is computed using a superposition of propagating plane waves. A condition number based criterion is used to define the plane wave basis dimension for each element. As a model problem we investigate the scattering of sound from an infinite elastic cylinder immersed in a fluid. A comparison of the UWVF approximation with the analytical solution shows that the method provides a means for solving wave problems on relatively coarse meshes. However, particular care is needed when the method is used for problems at frequencies near the resonance frequencies of the fluid-solid system.

6. X. Wang, R. Wildman, D. Weile and P. Monk, A finite difference delay modeling approach to the discretization of the time domain integral equations of electromagnetics, *IEEE Trans. Antennas and Propagation*, **56** (2008), 2442-2452.

   Abstract: A new method for solving the time-domain integral equations of electromagnetic scattering from conductors is introduced. This method, called finite difference delay modeling, appears to be completely stable and accurate when applied to arbitrary structures. The temporal discretization used is based on finite differences. Specifically, based on a mapping from the Laplace domain to the z-transform domain, first- and second-order unconditionally stable methods are derived. Spatial convergence is achieved using the higher-order divergence-conforming vector bases of Graglia et al.. Low frequency instability problems are avoided with the loop-tree decomposition approach. Numerical results will illustrate the accuracy and stability of the technique.

2009

7. F. Cakoni, D. Colton and H. Haddar, The computation of lower bounds for the norm of the index of refraction in an anisotropic media, *J. Int. Eqns. Appl*, **21** (2009), 203-227.

   Abstract: We consider the scattering of time harmonic electromagnetic plane waves by a bounded, inhomogeneous, anisotropic dielectric medium and show that under certain assumptions a lower bound on the norm of the (matrix) index of refraction can be obtained from a knowledge of the smallest transmission eigenvalue corresponding to the medium. Numerical examples are given showing the efficaciousness of our estimates.

8. F. Cakoni and H. Haddar, On the existence of transmission eigenvalues in an inhomogenous medium, *Applicable Analysis* **88**, (2009), 475-493.

   Abstract: We prove the existence of transmission eigenvalues corresponding to the inverse scattering problem for isotropic and anisotropic media for both the scalar problem and Maxwell's equations. Considering a generalized abstract eigenvalue problem, we are able to extend the ideas of Päivärinta and Sylvester to prove the existence of transmission eigenvalues for a larger class of interior transmission problems. Our analysis includes both the case of a medium with positive contrast and of a medium with negative contrast provided that the contrasts are large enough.

9. P. Monk and V. Selgas, An inverse fluid-solid interaction problem, *Inverse Problems and Imaging* **3** (2009), 173-198.

   Abstract: This paper is devoted to studying the Linear Sampling Method (LSM) applied to the inverse problem for the fluid-solid interaction problem of determining the shape of the solid from far field measurements of the fluid pressure field. We provide a simplified proof of the uniqueness problem in this case, an analysis of the appropriate interior transmission problem, and the existence of a solution to the LSM that can be used as an indicator for the shape of the solid. The analysis of uniqueness rests on a new technical result concerning regularity. Finally we present some numerical results for the method.

10. S. Chandler-Wilde and P. Monk, The PML for rough surface scattering, *Applied Num. Math.* **59** (2009), 3131-2154.

Abstract: In this paper we investigate the use of the perfectly matched layer (PML) to truncate a time harmonic rough surface scattering problem in the direction away from the scatterer. We prove existence and uniqueness of the solution of the truncated problem as well as an error estimate depending on the thickness and composition of the layer. This global error estimate predicts a linear rate of convergence (under some conditions on the relative size of the real and imaginary parts of the PML function) rather than the usual exponential rate. We then consider scattering by a half-space and show that the solution of the PML truncated problem converges globally at most quadratically (up to logarithmic factors), providing Support for our general theory. However we also prove exponential convergence on compact subsets. We continue by proposing an iterative correction method for the PML truncated problem and, using our estimate for the PML approximation, prove convergence of this method. Finally we provide some numerical results in 2D.

11. N. Zeev and F. Cakoni, The identification of thin dielectric objects from far field and near field scattering data, *SIAM J. Appl. Math.*, **69** (2009), 1024-1042.

    Abstract: We consider the inverse scattering problem of determining the shape and the material properties of a thin dielectric infinite cylinder having an open arc as cross section from knowledge of the TM-polarized scattered electromagnetic field at a fixed frequency. We investigate two reconstruction approaches, namely the linear sampling method and the reciprocity gap functional method, using far field or near field data, respectively. Numerical examples are given showing the efficaciousness of our algorithms.

2010

12. F. Cakoni, D. Colton and D. Gintides, The interior transmission eigenvalue problem, *SIAM J. Math. Analysis*, **42** (2010), 2912-2921.

    Abstract: We consider the inverse problem of determining the spherically symmetric index of refraction $n(r)$ from a knowledge of the corresponding transmission eigenvalues (which can be determined from field pattern of the scattered wave). We also show that for constant index of refraction $n(r) = n$, the smallest transmission eigenvalue suffices to determine n, complex eigenvalues exist for n sufficiently small and, for homogeneous media of general shape, determine a region in the complex plane where complex eigenvalues must lie.

13. F. Cakoni, D. Colton and D. Gintides, The transmission eigenvalue problem, *Advanced Topics in Scattering and Biomedical Engineering, World Scientific Publishing* (2010), 368-380.

    Abstract: This paper provides a survey of recent developments on the transmission eigenvalue problem.

14. F. Cakoni, D. Colton and H. Haddar, On the determination of Dirichlet and transmission eigenvalues from far field data, *Comptes Rendus Mathematique*, **348** (2010), 379-383.

    Abstract: We show that the Herglotz wave function with kernel the Tikhonov regularized solution of the far-field equation becomes unbounded as the regularization parameter tends to zero if the wavenumber k belongs to a discrete set of values. When the scatterer is such that the total field vanishes on the boundary, these values correspond to the square root of Dirichlet eigenvalues for Δ. When the scatterer is a non absorbing inhomogeneous medium these values correspond to so-called transmission eigenvalues.

15. F. Cakoni, D. Colton and H. Haddar, The interior transmission problem for regions with cavities, *SIAM J. Math. Analysis*, **42** (2010), 145-162.

    Abstract: We consider the interior transmission problem in the case when the inhomogeneous medium has cavities, i.e. regions in which the index of refraction is the same as the host medium. In this case we establish the Fredholm property for this problem and show that transmission eigenvalues exist and form a discrete set. We also derive Faber-Krahn type inequalities for the transmission eigenvalues.

16. F. Cakoni, D. Colton, P. Monk and J. Sun, The inverse electromagnetic scattering problem for anisotropic media, *Inverse Problems*, **26** (2010), 074004.

    Abstract: The interior transmission problem is a boundary value problem that arises in the scattering of time-harmonic waves by an inhomogeneous medium of compact support. The associated transmission eigenvalue problem has important applications in qualitative methods in inverse scattering theory. In this paper, we first establish optimal conditions for the existence of transmission eigenvalues for a spherically stratified medium and give numerical examples of the existence of both real and complex transmission eigenvalues in this case. We then propose three finite element methods for the computation of the transmission eigenvalues for the cases of a general non-stratified medium and use these methods to investigate the accuracy of recently established inequalities for transmission eigenvalues.

17. F. Cakoni, D. Colton and P. Monk, The determination of boundary coefficients from far field measurements, *J. Int. Eqns Appl.* **42**, (2010) 167-191.

    Abstract: We consider the problem of determining either the surface impedance $\lambda = \lambda(x)$ or surface conductivity $\eta = \eta(x)$ from far field data corresponding to time-harmonic incident plane waves scattered by a coated infinite cylinder. We show that λ and η are uniquely determined from the far field data and provide a numerical algorithm for determining these quantities.

18. F. Cakoni and D. Gintides New results on transmission eigenvalues, *Inverse Problems and Imaging*, **4** (2010), 39-48.

Abstract: We consider the interior transmission eigenvalue problem corresponding to the inverse scattering problem for an isotropic inhomogeneous medium. We first prove that transmission eigenvalues exist for media with index of refraction greater or less than one without assuming that the contrast is sufficiently large. Then we show that for an arbitrary Lipshitz domain with constant index of refraction there exists an infinite discrete set of transmission eigenvalues that accumulate at infinity. Finally, for the general case of non constant index of refraction we provide a lower and an upper bound for the first transmission eigenvalue in terms of the first transmission eigenvalue for appropriate balls with constant index of refraction.

19. F. Cakoni, D. Gintides and H. Haddar, *The existence of an infinite discrete set of transmission eigenvalues*, SIAM J. Math Anal., 42 (2010), 237-255.

    Abstract: We prove the existence of an infinite discrete set of transmission eigenvalues corre- sponding to the scattering problem for isotropic and anisotropic inhomogeneous media for both the Helmholtz and Maxwell's equations. Our discussion includes the case of the interior transmission problem for an inhomogeneous medium with cavities, i.e. subregions with contrast zero.

20. F. Cakoni and A. Kirsch, On the interior transmission eigenvalue problem, *Int. Jour. Comp. Sci. Math*, **3** (2010), 142-16.

    Abstract: We consider the transmission eigenvalue problem corresponding to the scattering problem for anisotropic media for both the scalar Helmholtz equation and Maxwell's equations in the case when the contrast in the scattering media occurs in two independent functions. We prove the existence of an infinite discrete set of transmission eigenvalues provided that the two contrasts are of opposite signs. In this case we provide bounds for the first transmission eigenvalue in terms of the ratio of refractive indices. In the case of the same sign contrasts for the scalar case we show the existence of a finite number of transmission eigenvalues under restrictive assumptions on the strength of the scattering media.

21. F. Cakoni and P. Monk, The determination of anisotropic surface impedance in electromagnetic scattering, *J. Methods and Applications of Analysis* **17** (2010), 379-394.

    Abstract: We consider the inverse scattering problem of determining the anisotropic surface impedance of a bounded obstacle from far field measurements of the electromagnetic scattered field due to incident plane waves. Such an anisotropic boundary condition can arise from surfaces covered with patterns of conducting and insulating patches. We show that the anisotropic impedance is uniquely determined if sufficient data is available, and characterize the non-uniqueness present if a single incoming wave is used. We derive an integral equation for the surface impedance in terms of solutions of a certain interior impedance boundary value problem. These solutions can be reconstructed from far field data using the Herglotz theory underlying the Linear Sampling Method. We complete the paper with preliminary numerical results.

22. F. Cakoni, G. Nakamura, M. Sini and N. Zeev, The identification of a partially coated dielectric medium from far field measurements, *Applicable Analysis*, **89** (2010), 29-47.

Abstract: We consider the two-dimensional electromagnetic inverse scattering problem for a dielectric medium partially coated with a thin layer of highly conductive material. Using the linear sampling method, we show that the approximate solution of the far field equation can be used to reconstruct the support of the coating in addition to the shape of the scattering obstacle. We also deduce formulaes providing point-wise reconstruction of the surface conductivity on the coated portion and the real index of refraction on the uncoated portion of the boundary. Numerical examples are given for the case with constant surface conductivity and index of refraction showing the viability of our reconstruction procedure.

23. Q. Chen, H. Haddar, A. Lechleiter and P. Monk, A sampling method for inverse scattering in the time domain, *Inverse Problems*, **26** (2010), 085001.

Abstract: We consider a near-field inverse scattering problem for the wave equation: find the shape of a Dirichlet scattering object from time domain measurements of scattered waves. For this time-domain inverse problem, we propose a linear sampling method, a well-known technique for corresponding frequency domain inverse scattering problems. The problem setting and the algorithm incorporate two basic features. First, the data for the method consist of measurements of causal waves, that is, of waves that vanish before some moment in time. Second, the inversion algorithm directly works on the time-domain data without using a Fourier transformation. The first point is related to the applications we have in mind, which include for instance ground-penetrating radar imaging. The second feature allows us to naturally incorporate multiple (in fact, a continuum of) frequencies in the inversion algorithm. Consequently, it offers the potential of improving the quality of the reconstruction compared to frequency domain methods working with a single frequency. We demonstrate this potential by several numerical examples.

24. D. Colton, P. Monk, and J. Sun, Analytical and computational methods for transmission eigenvalues, Inverse Problems, **26** (2010), 045011.

Abstract: The interior transmission problem is a boundary value problem that arises in the scattering of time-harmonic waves by an inhomogeneous medium of compact support. The associated transmission eigenvalue problem has important applications in qualitative methods in inverse scattering theory. In this paper, we first establish optimal conditions for the existence of transmission eigenvalues for a spherically stratified medium and give numerical examples of the existence of both real and complex transmission eigenvalues in this case. We then propose three finite element methods for the computation of the transmission eigenvalues for the cases of a general non-stratified medium and use these methods to investigate the accuracy of recently established inequalities for transmission eigenvalues.

25. B. Guzina, F. Cakoni and C. Bellis, On multi-frequency obstacle reconstruction via linear sampling method, *Inverse Problems* **26** (2010),125005.

Abstract: This paper investigates the possibility of multi-frequency reconstruction of sound-soft and penetrable obstacles via the linear sampling method involving either far-field or near-field observations of the scattered field. On establishing a suitable approximate solution to the linear sampling equation and making an assumption of continuous frequency sweep, two possible choices for a cumulative multi-frequency indicator function of the scatterer's support are proposed. The first alternative, termed the "serial" indicator, is taken as a natural extension of its monochromatic companion in the sense that its computation entails *space-frequency* (as opposed to space) L^2-norm of a solution to the linear sampling equation. Under a set of assumptions that include experimental observations down to zero frequency and compact frequency support of the wavelet used to illuminate the obstacle, this indicator function is further related to its time-domain counterpart. As a second possibility, the so-called "parallel" indicator is alternatively proposed as an L^2-norm, in the frequency domain, of the monochromatic indicator function. On the basis of a perturbation analysis which demonstrates that the monochromatic solution of the linear sampling equation behaves as $O(|k^2-k_*^2|^{-m}),\ m\geqslant 1$ in the neighborhood of an isolated eigenvalue, k_*^2, of the associated interior (Dirichlet or transmission) problem, it is found that the "serial" indicator is unable to distinguish the interior from the exterior of a scatterer in situations when the prescribed frequency band traverses at least one such eigenvalue. In contrast the "parallel" indicator is, due to its particular structure, shown to be insensitive to the presence of pertinent interior eigenvalues (unknown beforehand and typically belonging to a countable set), and thus to be robust in a generic scattering configuration. A set of numerical results, including both "fine" and "coarse" frequency sampling, is included to illustrate the performance of the competing (multi-frequency) indicator functions, demonstrating behavior that is consistent with the theoretical results.

26. P. Monk, J. Schoeberl and A. Sinwel, Hybridizing Raviart-Thomas Elements for the Helmholtz Equation, *Electromagnetics* **30** (2010), 149-176.

Abstract: This article deals with the application of hybridized mixed methods for discretizing the Helmholtz problem, which allows for a fast, iterative solution, from a mixed formulation using Raviart-Thomas finite elements. Two ways of hybridizing the problem are presented, which means breaking the normal continuity of the fluxes and imposing it weakly via functions supported on the element interfaces. The first method is the ultra-weak variational formulation, first introduced by Cessenat and Després (1998); the second one uses Lagrange multipliers. Good behavior of iterative solvers is observed for both methods, as well as for high wave numbers.

2011

27. F. Cakoni, D. Colton and P. Monk, *The Linear Sampling Method in Inverse Electromagnetic Scattering* CBMS-NSF, **80**, SIAM Publications, 2011.

    Abstract: The linear sampling method is the oldest and most developed of the qualitative methods in inverse scattering theory. It is based on solving a linear integral equation and then using the equations solution as an indicator function for the determination of the support of the scattering object. This book describes the linear sampling method for a variety of electromagnetic scattering problems. It presents uniqueness theorems and the derivation of various inequalities on the material properties of the scattering object from a knowledge of the far field pattern of the scattered wave. Also covered are the approximation properties of Herglotz wave functions, the behavior of solutions to the interior transmission problem, a novel interior boundary value problem, and numerical examples of the inversion scheme. The authors give various numerical examples of the linear sampling method throughout the text. This book is intended for mathematicians and engineers performing research in inverse electromagnetic scattering theory. It is also appropriate for an advanced graduate course on inverse problems.

28. D. Colton and R. Kress Inverse Scattering, in *Handbook of Mathematical Methods in Imaging*, Omar Scherzer, editor, Springer, 2011, 552-598.

    Abstract: We give a survey of the mathematical basis of inverse scattering theory, concentrating on the case of time-harmonic acoustic waves. After an introduction and historical remarks we give an outline of the direct scattering problem. This is then followed by sections on uniqueness results in inverse scattering theory and iterative and decomposition methods to reconstruct the shape and material properties of the scattering object.We conclude by discussing qualitative methods in inverse scattering theory, in particular the linear sampling method and its use in obtaining lower bounds on the constitutive parameters of the scattering object.

29. H-H Qin and F. Cakoni, Nonlinear integral equations for shape reconstruction in the inverse interior scattering problem, *Inverse Problems* **27** (2011).

    Abstract: In this paper, we consider the inverse scattering problem of recovering the shape of a perfectly conducting cavity from one source and several measurements placed on a curve inside the cavity. Under restrictive assumptions on the size of the cavity, a uniqueness theorem for infinitely many excitations is given. Based on a system of nonlinear and ill-posed integral equations for the unknown boundary, we apply a regularized Newton iterative approach to find the boundary. We present the mathematical foundation of the method and give several numerical examples to show the viability of the method.

30. R. Falk, P. Gatto, and P. Monk, Hexahedral $H(\mathrm{div})$ and $H(\mathrm{curl})$ finite elements, *ESAIM: Mathematical Modeling and Numerical Analysis*, **45** (2011), 115-143.

Abstract: We study the approximation properties of some finite element subspaces of $H(\text{div};\Omega)$ and $H(\text{curl};\Omega)$ defined on hexahedral meshes in three dimensions. This work extends results previously obtained for quadrilateral $H(\text{div};\Omega)$ finite elements and for quadrilateral scalar finite element spaces. The finite element spaces we consider are constructed starting from a given finite dimensional space of vector fields on the reference cube, which is then transformed to a. space of vector fields on a hexahedron using the appropriate transform (e.g., the Piola transform) associated to a trilinear isomorphism of the cube onto the hexahedron. After determining what vector fields are needed on the reference element to insure $O(h)$ approximation in $L^2(\Omega)$ and in $H(\text{div};\Omega)$ and $H(\text{curl};\Omega)$ on the physical element, we study the properties of the resulting finite element spaces.