**Title and Subtitle**

**Inverse Problems and Optical Imaging with Nanoscale Resolution**

**Abstract**

The objective of this basic research project is to describe an imaging protocol which will allow optical imaging to resolve nanoscale features (Near Field Tomography), both at the surface and below, of various components. The AF anticipates exploiting ever smaller devices and the manufacturing/inspection of these will benefit from the basic research done here. The PI will continue his seminal research into near-field EM. In this not-well understood area the classical rules of thumb regarding wave-length limited resolution do not apply since recourse is now made to the evanescent waves always present in the near field. The PI proposes a mathematical approach involving principles from Inverse Scattering theory requiring the inversion of a suitable Laplace transform.
Final Report:
Inverse Problems and Optical Imaging with Nanoscale Resolution

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The goal of this project is to develop new mathematical tools for optical imaging at the nanoscale. A second goal is the prediction of novel physical effects at subwavelength scales. The results of this study were presented in the following publications. Note that the source of funding was acknowledged in each case.


   We develop a classical theory of electron confinement in conducting nanoparticles. The theory is used to compute the nonlinear optical response of the nanoparticle to a harmonic external field.


   A model for the interaction of the scanning probe in near-field scanning optical microscopy is presented. Multiple scattering of the illuminating field with the probe is taken into account. The implications of this so-called strong tip model for the solution of the associated inverse scattering problem are studied through simulations.


   A theoretical and numerical analysis of spectral self-interference microscopy (SSM) is presented with the goal of expanding the realm of SSM applications. In particular, this work is intended to enable SSM imaging in low-signal applications such as single-molecule studies. A comprehensive electromagnetic model for SSM is presented, allowing arbitrary forms of the excitation field, detection optics, and tensor sample response. An evanescently excited SSM system, analogous to total internal reflection microscopy, is proposed and investigated through Monte Carlo simulations. Nanometer-scale axial localization for single-emitter objects is demonstrated, even in low-signal environments.

We consider the photoacoustic effect for multiply scattered light in a random medium. Within the accuracy of the diffusion approximation to the radiative transport equation, we present a general analysis of the sensitivity of a photoacoustic wave to the presence of one or more small absorbing objects.


The inverse scattering problem (ISP) for diffuse waves consists of recovering the spatially varying optical properties of the interior of a bounded domain from measurements taken on its boundary. The problem has been widely studied in the context of optical tomography and is closely related to the inverse scattering problem for wave-fields with evanescent components, as arise in near-field optics. The ISP for diffuse waves (in the near-field) is usually formulated as a nonlinear optimization problem. At present, the iterative methods which are used to solve this problem are not well understood mathematically, since error estimates and convergence results are not known. We have shown that, to some extent, it is possible to fill this gap. In particular, we will study the solution to the ISP which arises from inversion of the Born series. In previous work we have utilized such series expansions as tools to develop fast, direct image reconstruction algorithms. In this work we have characterized their convergence, stability and reconstruction error.


We have obtained a short-distance expansion for the half-space, frequency domain electromagnetic Green’s tensor. The small parameter of the theory is $\omega \epsilon_1 L/c$ where $\omega$ is the frequency, $\epsilon_1$ is the permittivity of the upper half-space, in which both the source and the point of observation are located, and which is assumed to be transparent, $c$ is the speed of light in vacuum, and $L$ is a characteristic length, defined as the distance from the point of observation to the reflected (with respect to the planar interface) position of the source. In the case when the lower half-space (the substrate) is characterized by a generally complex permittivity $\epsilon_2$, we compute the expansion to third order. For the case when the substrate is a transparent dielectric, we compute the imaginary part of the Green’s tensor to seventh order. The analytical calculations are verified numerically. The results of this study are relevant to the theory of radiative processes near interfaces, nano-optics and near-field imaging.


Near-field scanning optical microscopy (NSOM) is a method for nano-scale optical imaging that achieves subwavelength resolution by detecting the evanescent wave fields which are accessible in the near-zone of a scattering object. In conventional NSOM, the intensity of the optical field in a plane above a three-dimensional sample is interpreted as a two-dimensional image of the sample. This interpretation is known to problematic, since the three-dimensional structure of the medium is not uniquely determined by the two-dimensional intensity map which constitutes the NSOM image. Means to interpret NSOM images have been developed,
including the solution of a three-dimensional near-field inverse scattering problem (ISP); the resulting methods are known as near-field optical tomography (NFOT). NFOT experiments require the acquisition of multiple complete NSOM data sets for varying directions of illumination or observation of the eld. This requirement presents serious obstacles to experimental implementation, except in special cases such as the imaging of two-dimensional samples. The results of NSOM experiments in which the probe is scanned over a three-dimensional volume outside the sample suggest that the three-dimensional structure of the sample may be unambiguously determined. That is, the so-called approach curves, or measured intensity as a function of height above the sample, are seen in to vary depending on the depth of subsurface features of the sample. Such an experiment is attractive because, though multiple NSOM data sets must be acquired (at a variety of scan heights above the sample), the directions of the far-field illumination and observation may be held xed. The independence of data acquired at multiple heights is somewhat unexpected. In illumination or collection mode NSOM, data from different planes are related by propagation or back-propagation of the scattered eld. In backscattered mode NSOM, however, data sets collected at different distances are independent. Thus three-dimensional reconstruction of the sample may be obtained by a new method, to which we refer as volume-scanning near-field optical tomography (vNFOT).


In this paper we extend our previous results (item 1 above) by studying numerically the convergence of the inverse Born series for a medium with radial symmetry. We make use of exact solutions to the forward problem to calculate the scattering data. Results in both two and three dimensions are presented and reconstructions are computed up to fifth order in the inverse series. We find that the series appears to converge quite rapidly for low contrast objects. As the contrast is increased, the higher order terms systematically improve the reconstructions until, at sufficiently large contrast, the series diverges.


We propose a method for optical nanoimaging in which the structure of a three-dimensional inhomogeneous medium may be recovered from far-field power measurements. Neither phase control of the illuminating field nor phase measurements of the scattered field are necessary. The method is based on the solution to the inverse scattering problem for a system consisting of a weakly-scattering dielectric sample and a strongly-scattering nanoparticle tip. Numerical simulations are used to illustrate the results.


We show that refraction of a narrow collimated beam at a negative angle into strongly anisotropic non-magnetic crystals—a phenomenon which was observed in a number of experiments—should not be confused with negative refraction and lacks important physical characteristics of the latter. In particular, it is shown that there is no contradiction between the theory previously developed by Markel (Opt. Express 16 19152 (2009)), where it is claimed that negative refraction is impossible, and the above mentioned experiments.


We study the linear inverse problem for scanning near-field optical microscopy with broadband illumination. Inversion formulae are derived and illustrated with numerical simulations.


We consider the inverse scattering problem that arises in two-photon quantum imaging with interferometric measurements. We show that the quantum two-point correlation function of the field contains information about the scattering medium at a spatial frequency of twice the Rayleigh bandwidth. The linearized inverse problem, however, yields reconstructions with a resolution of $\lambda/2$, where $\lambda$ is the wavelength of light, suggesting that resolution beyond the Rayleigh limit may possibly be obtained by solving the corresponding nonlinear inverse problem.