### REPORT DOCUMENTATION PAGE

#### Nonlinear dynamics and quantum transport in small systems

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**Sponsoring/Monitoring Agency:** Air Force Office of Scientific Research, 801 N Randolph St., Arlington, VA 22203

**Approval Statement:** Approved for public release; distribution is unlimited.

**Abstract:** MEMS/NEMS are characterized by their small size, low power consumption, and ultra-fast speed. Recent years have witnessed a growing interest in the fundamental dynamics of MEMS/NEMS and their potential applications. It is conceivable that future military systems may contain MEMS/NEMS components. To understand and generate various nonlinear dynamical phenomena including chaos in MEMS/NEMS can thus be of great interest from the standpoint of defense. We investigated a number of fundamental issues in this area. In particular, we developed an understanding of the dynamical mechanism of the intrinsic localized modes (ILMs) in MEMS oscillator arrays, articulated a global control scheme to induce intrinsic localized modes at an arbitrary site in MEMS cantilever arrays, and performed a detailed bifurcation analysis for a common class of electrostatically driven nanowires using a multiphysics model. We further explored potential applications of extensive chaos in nanoscale systems: ultra-fast random number generators. All these were collaborative works with Dr. David Dietz from AFRL at Kirtland AFB.

There is tremendous interest in graphene recently due to its potential applications in nano-scale electronic devices and circuits. It is possible that future military systems may contain some graphene components. To understand various fundamental aspects of quantum transport dynamics is key to developing graphene-based devices. During the project period, the following problems were investigated: (1) relativistic quantum scars, (2) energy-level statistics in graphene systems, (3) electronic transport in graphene nanoribbons and stochastic resonance, (4) geometry-dependent conductance fluctuations in graphene quantum dots, (5) Quantum chaotic scattering in graphene systems, (6) Klein tunneling and fractal-like conductance fluctuations in graphene quantum point contacts, and (7) Modulating quantum transport by transient chaos.

In addition, we investigated a number of basic issues in nonlinear dynamics, such as synchronization in chaotic systems and the effect of noise, robust chaos and experimental investigation in electronic circuits, nonlinear-dynamics based characterization of two-point flows, chaos and branched wave structure in optical metamaterials, and compressive-sensing based prediction of catastrophe. The last problem was a result of collaboration with Dr. Vassilis Kovanis in the Sensors Directorate at Wright Patterson AFB.

**Subject Terms:** Microelectromechanical systems, nanoelectromechanical systems, intrinsic localized modes, extensive chaos, chaotic scattering, graphene systems, conductance fluctuations, quantum dots, quantum point contacts, Klein tunneling, metamaterials, compressive sensing

**Security Classification:** U

**Limitation of Abstract:** UU

**Number of Pages:** 15

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**Supplementary Notes:**

**Date of Report:** 22-02-2012

**Report Type:** Final Report

**Period Covered:** 12/1/2008 - 11/30/2011

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**Sponsor/monitor's ACRONYM:** AFOSR

**Sponsor/monitor's report number:** AFRL-OSR-VA-TR-2012-0522

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**Name of Responsible Person:** Ying-Cheng Lai
Final Report

This report summarizes activities under Air Force Office of Scientific Research (AFOSR) Grant No. FA9550-09-1-0260 entitled “Nonlinear Dynamics and Quantum Transport in Small Systems.” The PI is Ying-Cheng Lai from Arizona State University. The duration of the project was 12/1/2008-11/30/2011. The report is divided into the following Sections:

1. Objectives
2. Accomplishments and New Findings
3. Personnel Supported and Theses Supervised by PI
4. List of Publications
5. Interactions/Transitions
6. Past Honors
1 Objectives

- Nonlinear dynamics and chaos in microelectromechanical (MEM) and nanoelectromechanical (NEM) systems;
- Electronic transport in graphene systems.

2 Accomplishments and New Findings

2.1 Nonlinear dynamics and chaos in microelectromechanical (MEM) systems

MEMS/NEMS are characterized by their small size, low power consumption, and ultra fast speed. Recent years have witnessed a growing interest in the fundamental dynamics of MEMS/NEMS and their potential applications. It is conceivable that future military systems may contain MEMS/NEMS components. To understand and generate various nonlinear dynamical phenomena including chaos in MEMS/NEMS can thus be of great interest from the standpoint of defense.

During the project period, we first focused on understanding the dynamical mechanism of intrinsic localized modes (ILMs) in MEMS oscillator arrays, a phenomenon that had been observed in a number of experiments. We found that spatiotemporal chaos is ubiquitous and it provides a natural platform for actual realization of various ILMs through frequency control. In particular, unstable periodic orbits associated with ILMs are pivotal for chaos to arise and these orbits are key to stabilizing ILMs from spatiotemporal chaos by frequency modulation. We then articulated a global control scheme to induce intrinsic localized modes at an arbitrary site in MEM cantilever arrays. The idea is to locate the particular cantilever beam in the array that one wishes to drive to an oscillating state with significantly higher amplitude than the average, and then apply small adjustments to the electrical signal that drives the whole array system. We developed detailed theoretical and computational analyses to validate the method. The control scheme may be useful in applications where the goal is to defeat certain MEM based electronic devices. Finally, we turned our attention to NEMS and carried out a detailed bifurcation analysis for a common class of electrostatically driven nanowires using a multi-physics model. We found that the nano-scale system can exhibit distinct chaotic states: chaos with symmetry-breaking and extensive chaos possessing the full symmetry of the system. We further explored potential applications of extensive chaos in nanowire systems: ultra-fast random number generators. All these were collaborative works with Dr. David Dietz from AFRL at Kirtland AFB.

2.2 Electronic transport in graphene systems

There is tremendous interest in graphene recently due to its potential applications in nano-scale electronic devices and circuits. It is possible that future military systems may contain some graphene components. To understand various fundamental aspects of quantum transport dynamics is key to developing graphene-based devices.

2.2.1 Relativistic quantum scars

A remarkable phenomenon in quantum systems whose dynamics in the classical limit are chaotic is scarring. In particular, in the semiclassical regime a wavefunction can be regarded locally as a superposition of many plane waves. Due to classical chaos, the directions of the plane waves are uniformly distributed. Intuitively, one may expect the wave functions to have uniform concentration in the position space. Signatures of non-uniform distribution of the wave function were, however, discovered by McDonald and Kaufman in 1979. Heller discovered in 1984 the striking “scarring” phenomenon that the wavefunctions often tend to concentrate on paths corresponding to unstable periodic orbits in the classical limit, and the eigenfunctions associated with different
eigenvalues can focus on different periodic orbits. In solid-state electronic devices the quantum scarring states, also referred to as quantum “pointer states,” can have a significant effect on electronic transport and conductance.

Most existing works on scarring were concerned exclusively with non-relativistic quantum mechanical systems described by the Schrödinger equation, where the dependence of the particle energy on momentum is quadratic. A question is whether scarring can occur in relativistic quantum systems described by the massless Dirac equation, where the energy-momentum relation is linear. This question, besides being fundamental to physics, is also important for device applications. In particular, graphene, a single, one-atom-thick sheet of carbon atoms arranged in a honeycomb lattice, was realized in experiments in 2004. Due to its peculiar hexagonal lattice structure, the band structure exhibits a linear dependence of the energy on the wave vector about the Dirac points, signifying relativistic motion. Devices made of graphene are potentially capable of operating at much higher speed than those based on conventional silicon electronics.

The ASU work reported the first evidence of relativistic quantum scars in closed graphene confinements exhibiting chaos in the classical limit. Signatures of such scars were also found in open graphene quantum dots. This result is expected to be fundamental to relativistic quantum solid-state electronics, a new field in applied physics and engineering.

2.2.2 Energy-level statistics in graphene systems

We then addressed one of the outstanding problems in quantum chaos: energy-level statistics in relativistic quantum systems. We demonstrated, using chaotic graphene confinements where electronic motions are governed by the Dirac equation in the low-energy regime, that the level-spacing statistics are those given by Gaussian orthogonal ensemble (GOE) random matrices. Weak magnetic field can change the level-spacing statistics to those of Gaussian unitary ensemble (GUE) for electrons in graphene. For sufficiently strong magnetic field, the GOE statistics are restored due to the appearance of Landau levels. Our results indicate that graphene systems can have properties not shared by either non-relativistic quantum or purely relativistic quantum systems, and the distribution of energy levels may have implications to graphene-based devices that use quantum dots, a kind of "open" billiard structure.

2.2.3 Electronic transport in graphene nanojunctions and stochastic resonance

We also investigated electronic transport in graphene nanojunctions and found that the transmission (or the conductance) can exhibit a non-monotonic behavior with respect to variation in the disorder strength, mimicking a stochastic resonance. The general setting for this remarkable phenomenon is where the graphene device possesses localized states in the absence of disorder, i.e., localized edge states specific to graphene. A small amount of disorder can then break the localization and lead to an enhancement in the transmission. For strong disorder, Anderson localization sets in, causing the transmission to decrease. The phenomenon is robust and can occur with or without magnetic field.

2.2.4 Geometry-dependent conductance fluctuations in graphene quantum dots

Quantum point contacts (QPCs) are common in the metal-graphene interface for various device applications. Utilizing graphene quantum dots with zigzag horizontal boundaries as a paradigm, we found that the conductance of the dots can exhibit significant oscillations with the positions of the QPCs. The oscillation patterns are a result of quantum interference determined by the band structure of the underlying graphene nanoribbon. In particular, the power spectrum of the conductance variation scars on a selective set of bands of the ribbon. The computational results were substantiated by a heuristic theory that provides selection rules for quantum scarring.
2.2.5 Quantum chaotic scattering in graphene systems

We investigated transport fluctuations in both non-relativistic quantum dots and graphene quantum dots with both hyperbolic and nonhyperbolic chaotic scattering dynamics in the classical limit. We found that nonhyperbolic dots generate sharper resonances than those in the hyperbolic case. Strikingly, for the graphene dots, the resonances tend to be much sharper. This means that transmission or conductance fluctuations are characteristically greatly enhanced in relativistic as compared to non-relativistic quantum systems.

2.2.6 Klein tunneling and fractal-like conductance fluctuations in graphene quantum point contacts

The ASU work addressed the quantum transport problem in a general graphene system subject to external potential, a situation that can be expected in all kinds of future graphene based electronic devices with quantum dots and quantum point contacts. The main finding was that electrons often tend to take on propagating paths that have absolutely no counterpart in non-relativistic quantum systems. Strikingly, such uniquely relativistic quantum paths can lead to an extreme form of conductance fluctuations, not seen previously in any quantum transport systems. This phenomenon has profound implications to the development of graphene based devices that require stable electronic properties.

From a general viewpoint, the answer to the question, “When a free particle enters a region where there is an external potential, which paths will the particle follow?” is perhaps trivial, because conventional wisdom holds that the particle will travel through regions where the potential energy is smaller than the particle energy. Indeed this answer is correct in classical physics and even in non-relativistic quantum mechanics. What was uncovered is that, for graphene systems, the favorable paths for particle can actually be in regions where the potential energy is much larger than the particle energy. This can be understood only by relativistic quantum mechanics. The most significant manifestation of this phenomenon is that the quantum transport properties can depend extremely sensitively on the external potential, posing an obstacle that must be overcome if graphene is to be used in future electronic devices.

2.2.7 Modulating quantum transport by transient chaos

We proposed a scheme to modulate quantum transport in nanostructures based on classical chaos. By applying external gate voltage to generate a classically forbidden region, transient chaos can be generated and the escape rate associated with the underlying non-attracting chaotic set can be varied continuously by adjusting the gate voltage. We demonstrated that this can effectively modulate the quantum conductance-fluctuation patterns. A theory based on self-energies and the spectrum of the generalized non-Hermitian Hamiltonian of the open quantum system was developed to understand the modulation mechanism.

2.3 Nonlinear wave and chaos in optical metamaterials

2.3.1 Transient chaos in optical metamaterials

We investigated the dynamics of light rays in two classes of optical metamaterial systems: (1) time-dependent system with a volcano-shaped, inhomogeneous and isotropic refractive-index distribution, subject to external electromagnetic perturbations, and (2) time-independent system consisting of three overlapping or non-overlapping refractive-index distributions. Utilizing a mechanical-optical analogy and coordinate transformation, the wave-propagation problem governed by the Maxwell’s equations can be modeled by a set of ordinary differential equations for light rays. We found that transient chaotic dynamics, hyperbolic or nonhyperbolic, are common in optical metamaterial systems. Due to the analogy between light-ray dynamics in metamaterials and the motion of light and matter as described by general relativity, our results reinforce the idea that chaos in gravitational systems can be observed and studied in laboratory experiments.
2.3.2 Branched wave structure and scaling in random optical medium

Experiments had revealed that branched, fractal-like wave patterns can arise in a variety of physical situations ranging from microwave and optical systems to solid-state devices, and that the wave-intensity statistics are non-Gaussian and typically exhibit a long-tail distribution. The origin of branched wave patterns has been an issue of active debate. We proposed and investigated a “minimal” model of optical wave propagation and scattering with two generic ingredients: (1) a finite-size medium for linear wave propagation and (2) random scatterers characterized by a continuous refractive-index profile. We found that branched waves can emerge as a general phenomenon in a wide parameter regime in between the weak-scattering limit and Anderson localization, and the distribution of high intensities follows an algebraic scaling law. The minimal model can provide insights into the physical origin of branched waves in other physical systems as well.

2.4 General research on nonlinear dynamics, chaos, and complex systems

In addition, we investigated a number of basic issues in nonlinear dynamics, such as synchronization in chaotic systems and the effect of noise, robust chaos and experimental investigation in electronic circuits, nonlinear-dynamics based characterization of two-phase flows, and compressive-sensing based prediction of catastrophe. Particularly worth mentioning is the last problem, where we developed a general approach to predicting catastrophes in nonlinear dynamical systems under the assumption that the system equations are completely unknown and only time series reflecting the evolution of the dynamical variables of the system are available. Our idea was to expand the vector field or map of the underlying system into a suitable function series and then to use the compressive-sensing technique to accurately estimate the various terms in the expansion. This work was a result of collaboration with Dr. Vassilios Kovanis in the Sensors Directorate at Wright Patterson AFB.

3 Personnel Supported and Theses Supervised by PI

3.1 Personnel Supported

The following people received salaries from the AFOSR Project in various time periods.

- **Faculty (partial summer salary):**
  Ying-Cheng Lai (PI), Professor of Electrical Engineering, Professor of Physics

- **Post-Doctoral Fellows (full-time or part-time appointments):**
  Liang Huang (12/1/2008-11/30/2011)

- **Graduate Students (part-time appointments in various time periods):**
  1. Qingfei Chen, Ph.D. student in Electrical Engineering (graduated in May 2010).
  2. Rui Yang, Ph.D. student in Electrical Engineering (to graduate in May 2012).
  3. Xuan Ni, Ph.D. student in Electrical Engineering (to graduate in August 2012).
  4. Amogh S. Deshpande, Master student in Electrical Engineering (graduated in May 2010).

3.2 Theses supervised by PI with AFOSR support


4. Xuan Ni, Ph.D. in Electrical Engineering expected, August 2012. Dissertation: *Branched waves in optical media and relativistic quantum tunneling dynamics in Dirac fermion and graphene systems.*

4 List of Publications

- **Nonlinear dynamics and chaos in MEM and NEM systems**


- **Electronic transport in graphene systems**


- **Nonlinear wave and chaos in optical metamaterials**


- **General research on nonlinear dynamics, chaos, and complex systems**


19. Z.-K. Gao, N.-D. Jin, W.-X. Wang, and Y.-C. Lai, “Motif distributions in phase-space networks for characterizing experimental two-phase flow patterns with chaotic features,” *Physical Review E* **82**, 016210, 1-8 (2010). Figure 1(b) from this paper was selected for “Kaleidoscope” of PRE. This work was also selected by the Virtual Journal of Biological Physics Research for the August 1, 2010 issue (http://www.vjbio.org).


5 **Interactions/Transitions**

Collaboration with AFRL scientists:

- Dr. David Dietz, AFRL at Kirtland AFB, on nonlinear dynamics and chaos in MEM and NEM systems.
- Dr. Vassilios Kovanis, AFRL at Wright Patterson AFB, on compressive sensing and complex dynamical systems.

Invited talks on topics derived from the project


5. “Phase synchronization and applications to stochastic resonance and epileptic signal analysis,” TIMS (Taida Institute for Mathematical Science) lecture, National Taiwan University, Taipei, Taiwan; January 15, 2010.

6. “Extensive chaos in silicon nanowires and applications,” Plenary talk, Workshop on Chaos in Nonlinear PDEs, Department of Mathematics, Kyungpook National University, Daegu, Korea; March 16, 2010.

7. “Predicting complex networks based on time series,” Seminar, Department of Mathematics, Kyungpook National University, Daegu, Korea; March 17, 2010.

8. “Nonlinear dynamics and chaos in microelectromechanical and nanoelectromechanical systems,” Joint Seminar, National University of Singapore Graduate School, Centre for Computational Science and Engineering, Department of Physics, National University of Singapore; June 30, 2010.


12. “Relativistic quantum chaos in graphene systems,” Colloquium, Department of Physics, Wesleyan University; October 28, 2010.


15. “Conductance fluctuations in graphene quantum point contacts and control of quantum transport by chaos,” Invited talk, AFOSR PI meeting on Effect of High-Power Microwave on Circuits, AFRL, Albuquerque, NM; April 19, 2011.


17. “Reverse engineering of nonlinear dynamical systems and complex networks - a compressive-sensing based approach,” Plenary talk, International Conference on Modeling Life Sciences, Fudan University, Shanghai, China; September 26, 2011.

18. “Reverse engineering of nonlinear dynamical systems and complex networks,” Colloquium, Department of Physics, Eastern China Normal University, Shanghai, China; September 27, 2011.


20. “Quantum transport and conductance fluctuations in graphene systems,” Colloquium, Department of Physics, Lanzhou University, Lanzhou, China; October 24, 2011.

21. “Introduction to transient chaos,” Undergraduate colloquium, Department of Physics, Lanzhou University, Lanzhou, China; October 25, 2011.
6 Past Honors


2. Election as a Fellow of the American Physical Society, 1999. Citation: *For his many contributions to the fundamentals of nonlinear dynamics and chaos.*