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Complexity, Robustness, and Network Thermodynamics in Large-Scale and Multiagent Systems: A Hybrid Control Approach

Final Report
FA9550-09-1-0429

by

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Contents

1. Introduction

1.1. Research Objectives

As part of this research program we proposed the development of a network thermodynamic stabilization framework for hybrid control design of large-scale and multiagent aerospace systems. In particular, we concentrated on hybrid control, hierarchical control, impulsive dynamical systems, nonnegative dynamical systems, compartmental systems, large-scale systems, nonlinear switching control, cooperative control, and adaptive control. Application areas include large flexible interconnected space structures, spacecraft stabilization, cooperative control of unmanned air vehicles, network systems, swarms of air and space vehicle formations, and pharmacological systems.

1.2. Overview of Research

Controls research by the Principal Investigator [1–45] has concentrated on network thermodynamics for large-scale and multiagent systems. In our research we used and are using these results to develop a coordination control framework for finite-time consensus and parallel formation for multiagent systems with switching information topologies involving state-dependent communication links for addressing communication link failures and communication dropouts. In addition, a new neuroadaptive control architecture for nonlinear uncertain dynamical systems is developed. The proposed framework involves a novel controller architecture involving additional terms in the update laws that are constructed using a moving time window of the integrated system uncertainty. These terms can be used to identify the ideal system weights of the neural network as well as effectively suppress and cancel system uncertainty without the need for persistency of excitation. A nonlinear parametrization of the system uncertainty is considered and state and output feedback neuroadaptive controllers are developed.

Semistable and finite-time semistable protocols for dynamical networks with switching topologies are also developed. Specifically, we developed distributed static and dynamic output feedback controller architectures for coordination control for finite-time consensus, rendezvous, and parallel formation for multiagent systems with switching information topologies involving state-dependent communication links for addressing communication link failures and communication dropouts. In addition, using set-valued supply rate maps and set-valued storage maps consisting of piecewise continuous storage functions, dissipativity properties for discontinuous dynamical systems are presented. These results are used to develop feedback interconnection stability results for discontinuous systems by appropriately combining the set-valued storage maps for the forward and feedback systems. Finally, we developed a new and novel output feedback control framework for nonminimum phase multivariable systems for output stabilization, command following, and disturbance rejection.

Modern complex large-scale dynamical systems exist in virtually every aspect of science and engineering, and are associated with a wide variety of physical, technological, environmental, and social phenomena, including aerospace, power, communications, and network systems, to name just a few examples. As part of our research, we developed a general stability analysis and control design framework for nonlinear large-scale interconnected dynamical systems using vector Lyapunov function methods, vector dissipativity theory, and decentralized control architectures. Specifically, we addressed continuous-time and hybrid large-scale systems using finite-time decentralized control architectures, thermodynamic modeling, maximum entropy control, and energy-based decentralized control. In addition, we developed a stability analysis and control design framework for time-varying sets of nonlinear time-varying dynamical systems using vector Lyapunov functions. Using this framework, we designed distributed control algorithms for multivehicle coordination. Finally, we developed dissipativity notions for dynamical systems with discontinuous vector fields. In particular, we introduce a generalized definition of dissipativity for discontinuous dynamical systems with Lebesgue measurable and locally essentially bounded vector fields characterized by differential inclusions involving Filippov set-valued maps specifying a set of directions for the system velocity and admitting Filippov solutions with absolutely continuous curves. In future research this framework will be used as a design tool for developing group coordination algorithms for multiagent systems possessing a dynamic (i.e., switching) topology.

1.3. Goals of this Report

The main goal of this report is to summarize the progress achieved under the program during the past three years. Since most of the technical results appeared or will soon appear in over 45 archival journal and conference publications, we shall only summarize these results and remark on their significance and interrelationship.

2. Description of Work Accomplished

The following partial research accomplishments have been completed over the past three years.

2.1. Semistability, Finite-Time Stability, Differential Inclusions, and Discontinuous Dynamical Systems Having a Continuum of Equilibria

Numerous engineering applications give rise to discontinuous dynamical systems. Specifically, in impact mechanics the motion of a dynamical system is subject to velocity jumps and force discontinuities leading to nonsmooth dynamical systems. In mechanical systems subject to unilateral constraints on system positions, discontinuities occur naturally through system-environment interaction. Alternatively, control of networks and control over networks with dynamic topologies also give rise to discontinuous systems [12, 26]. In particular, link failures or creations in network multiagent systems result in switchings of the communication topology. In this case, the vector field defining the dynamical system is a discontinuous function of the state, and hence, system stability can be analyzed using nonsmooth Lyapunov theory involving concepts such as weak and strong stability notions, differential inclusions, and generalized gradients of locally Lipschitz continuous functions and proximal subdifferentials of lower semicontinuous functions.

In many applications of discontinuous dynamical systems such as mechanical systems having rigid-body modes, isospectral matrix dynamical systems, and consensus protocols for dynamical networks, the system dynamics give rise to a continuum of equilibria. Under such dynamics, the limiting system state achieved is not determined completely by the dynamics, but depends on the initial system state as well. For such systems possessing a continuum of equilibria, *semistability* [46], and not asymptotic stability, is the relevant notion of stability. Semistability is the property whereby every trajectory that starts in a neighborhood of a Lyapunov stable equilibrium converges to a (possibly different) Lyapunov stable equilibrium.

To address the stability analysis of discontinuous dynamical systems having a continuum of equilibria, in this research [9, 12, 26] we extend the theory of semistability to discontinuous time-invariant dynamical systems. In particular, we develop sufficient conditions to guarantee weak and strong invariance of Fillipov solutions. Moreover, we present Lyapunov-based tests for semistability of autonomous differential inclusions. In addition, we develop sufficient conditions for finite-time semistability of autonomous discontinuous dynamical systems. Future extensions will focus on using these results to develop a coordination control framework for finite-time information consensus and parallel formation in dynamical networks with switching topologies involving state-dependent communication links for addressing communication link failures and communication dropouts.

2.2. Semistability of Switched Dynamical Systems

An essential feature of multiagent network systems is that these systems possess a continuum of equilibria [12]. Since every neighborhood of a nonisolated equilibrium contains another equilibrium, a non-isolated equilibrium cannot be asymptotically stable. Hence, asymptotic stability is not the appropriate notion of stability for systems having a continuum of equilibria. As discussed in Section 2.1, such systems possess a continuum of equilibria and hence semistability [46] is the relevant notion of stability. It is important to note that semistability is not equivalent to set stability of the equilibrium set. Indeed, it is possible for trajectories to approach the equilibrium set without any trajectory approaching any single equilibrium [46].

Since communication links among multiagent systems are often unreliable due to multipath effects and exogenous disturbances, the information exchange topologies in network systems are often dynamic. In particular, link failures or creations in network multiagent systems result in switchings of the communication topology. This is the case, for example, if information between agents is exchanged by means of line-of-sight sensors that experience periodic communication dropouts due to agent motion. Variation in network topology introduces system discontinuities, which in turn give rise to switched dynamical systems. In this case, the vector field defining the dynamical system is a discontinuous function of the state and/or time, and hence, system stability should involve analysis of semistability of switched systems having a continuum of equilibria.

In this research [5, 6], we develop semistability and uniform semistability analysis results for switched linear and nonlinear systems. Since solutions to switched systems are a function of both the system initial conditions and the admissible switching signals, uniformity here refers to the convergence rate to a Lyapunov stable equilibrium as the switching signal ranges over a given switching set. Our main results involve sufficient conditions for semistability and uniform semistability using multiple Lyapunov functions and sufficient regularity assumptions on the class of switching signals considered. Specifically, using multiple Lyapunov functions whose derivatives are negative semidefinite, semistability for switched linear and nonlinear systems are established. If, in addition, the admissible switching signals have infinitely many disjoint intervals of length bounded from below and above, uniform semistability can be concluded.

2.3. A New Neuroadaptive Control Architecture for Nonlinear Uncertain Dynamical Systems: Beyond σ- and e-Modifications

Neural networks have been extensively used for adaptive system identification as well as adaptive and neuroadaptive control of highly uncertain systems. The goal of adaptive and neuroadaptive control is to achieve system performance without excessive reliance on system models. The fundamental difference between adaptive control and neuroadaptive control can be traced back to the modeling and treatment of the system uncertainties as well as the structure of the basis functions used in constructing the regressor vector. In particular, adaptive control is based on *constant*, *linearly parameterized* system uncertainty models of a known structure but unknown parameters. This uncertainty characterization allows for the system nonlinearities to be parameterized by a *finite* linear combination of basis functions within a class of function approximators such as rational functions, spline functions, radial basis functions, sigmoidal functions, and wavelets. However, this linear parametrization with a given basis function cannot, in general, exactly capture the system uncertainty.

To approximate a larger class of nonlinear system uncertainty, the uncertainty can be expressed in terms of a neural network involving a parameterized nonlinearity. Hence, in contrast to adaptive control, neuroadaptive control is based on the universal function approximation property, wherein any continuous nonlinear system uncertainty can be *approximated* arbitrarily closely on a compact set using a neural network with appropriate size, structure, and weights, all of which are not necessarily known a priori. Hence, while neuroadaptive control has advantages over standard adaptive control in the ability to capture a much larger class of uncertainties, further complexities arise when the basis functions are not known. In particular, the choice and the structure of the basis functions as well as the size of the neural network and the approximation error over a compact domain become important issues to address in neuroadaptive control. This difference in the modeling and treatment of the system uncertainties results in the ability of adaptive controllers to guarantee asymptotic closed-loop system stability versus ultimate boundness as is the case with neuroadaptive controllers.

To improve robustness and the speed of adaptation of adaptive and neuroadaptive controllers several controller architectures have been proposed in the literature. These include the σ- and e-modification architectures used to keep the system parameter estimates from growing without bound in the face of system uncertainty. In this research [10, 13, 30], a new neuroadaptive control architecture for nonlinear uncertain dynamical systems is developed. Specifically, the proposed framework involves a new and novel controller architecture involving additional terms, or *Q-modification terms*, in the update laws that are constructed using a moving time window of the integrated system uncertainty. The Q-modification terms can be used to identify the ideal neural network system weights which can be used in the adaptive law. In addition, these terms effectively suppress system uncertainty.

Even though the proposed approach is reminiscent to the composite adaptive control framework, the Q-modification framework does not involve filtered versions of the control input and system state in the update laws nor does it involve a least-squares exponential forgetting factor. Rather, the update laws involve auxiliary terms predicated on an estimate of the unknown neural network weights which in turn are characterized by an auxiliary equation involving the integrated error dynamics over a moving time interval. Our results address vector uncertainty structures with nonlinear parameterizations. In addition, state and output feedback controllers are developed. Finally, to illustrate the efficacy of the proposed approach we apply our results to a spacecraft model involving an unknown moment of inertia matrix as well as a Boeing unmanned combat aerial vehicle model with an actuator system failure and compare our results with standard neuroadaptive control methods.

2.4. Finite-Time Semistability, Filippov Systems, and Consensus Protocols for Nonlinear Dynamical Networks with Switching Topologies

Modern complex dynamical systems are highly interconnected and mutually interdependent, both physically and through a multitude of information and communication networks. Distributed decision-making for coordination of networks of dynamic agents involving information flow can be naturally captured by graph-theoretic notions. These dynamical network systems cover a very broad spectrum of applications including cooperative control of unmanned air vehicles (UAV's), autonomous underwater vehicles (AUV's), distributed sensor networks, air and ground transportation systems, swarms of air and space vehicle formations, and congestion control in communication networks, to cite but a few examples. Hence, it is not surprising that a considerable research effort has been devoted to control of networks and control over networks in recent years.

Since communication links among multiagent systems are often unreliable due to multipath effects and exogenous disturbances, the information exchange topologies in network systems are often dynamic. In particular, link failures or creations in network multiagent systems result in switchings of the communication topology. This is the case, for example, if information between agents is exchanged by means of line-of-sight sensors that experience periodic communication dropouts due to agent motion. Variation in network topology is introduced through control input discontinuities, which in turn give rise to discontinuous dynamical systems. In this case, the vector field defining the dynamical system is a discontinuous function of the state, and hence, system stability can be analyzed using nonsmooth Lyapunov theory involving concepts such as weak and strong stability notions, differential inclusions, and generalized gradients of locally Lipschitz continuous functions and proximal subdifferentials of lower semicontinuous functions.

In many applications involving multiagent systems, groups of agents are required to agree on certain quantities of interest. In particular, it is important to develop information consensus protocols for networks of dynamic agents wherein a unique feature of the closed-loop dynamics under any control algorithm that achieves consensus is the existence of a continuum of equilibria representing a state of equipartitioning or *consensus*. Under such dynamics, the limiting consensus state achieved is not determined completely by the dynamics, but depends on the initial system state as well. Information consensus protocols are key in addressing rendezvous problems, formation control, flocking, and attitude alignment in multiagent systems. For such systems possessing a continuum of equilibria, *semistability*, and not asymptotic stability, is the relevant notion of stability.

Semistability is the property whereby every trajectory that starts in a neighborhood of a Lyapunov stable equilibrium converges to a (possibly different) Lyapunov stable equilibrium. From a practical viewpoint, it is not sufficient to only guarantee that a network converges to a state of consensus since steady state convergence is not sufficient to guarantee that small perturbations from the limiting state will lead to only small transient excursions from a state of consensus. It is also necessary to guarantee that the equilibrium states representing consensus are Lyapunov stable, and consequently, semistable. It is important to note that semistability is not merely equivalent to asymptotic stability of the set of equilibria, and hence, is a more natural stability notion than considering stability of the consensus subspace.

To address agreement problems in switching networks with state-dependent topologies, in this research [9, 12] we extend the theory of semistability to discontinuous time-invariant dynamical systems [9]. In particular, we develop sufficient conditions to guarantee weak and strong invariance of Fillipov solutions. Moreover, we present Lyapunov-based tests for strong and weak semistability for autonomous differential inclusions. In addition, we develop sufficient conditions for finite-time semistability of autonomous discontinuous dynamical systems [12]. Achieving agreement in finite time allows the dynamical network to use exact information in addressing other system tasks. Furthermore, using our consensus algorithms we develop a coordination framework for multivehicle rendezvous.

2.5. Dissipativity Theory for Discontinuous Dynamical Systems: Basic Input, State, and Output Properties, and Finite-Time Stability of Feedback Interconnections

In control engineering, dissipativity theory provides a fundamental framework for the analysis and control design of dynamical systems using an input, state, and output system description based on system-energy-related considerations. The notion of energy here refers to abstract energy notions for which a physical system energy interpretation is not necessary. The dissipation hypothesis on dynamical systems results in a fundamental constraint on their dynamic behavior, wherein a dissipative dynamical system can deliver only a fraction of its energy to its surroundings and can store only a fraction of the work done to it.

Dissipativity theory along with Lyapunov stability theory for feedback interconnections of dissipative systems has been extensively developed for dynamical systems possessing continuous flows [46]. However, numerous engineering applications give rise to discontinuous dynamical systems. Specifically, in impact mechanics the motion of a dynamical system is subject to velocity jumps and force discontinuities leading to nonsmooth dynamical systems [9]. In mechanical systems subject to unilateral constraints on system positions, discontinuities occur naturally through system-environment interaction. Alternatively, open-loop and feedback controllers also give rise to discontinuous dynamical systems. In particular, bang-bang controllers discontinuously switch between maximum and minimum control input values to generate minimum-time system trajectories, whereas sliding mode controllers use discontinuous feedback control for system stabilization. In switched systems [5, 6] switching algorithms are used to select an appropriate plant (or controller) from a given finite parameterized family of plants (or controllers) giving rise to discontinuous systems. As for dynamical systems with continuous flows [46], dissipativity theory can play a fundamental role in addressing robustness, disturbance rejection, stability of feedback interconnections, and optimality for discontinuous dynamical systems.

In light of the fact that energy notions involving conservation, dissipation, and transport also arise naturally for discontinuous systems, it seems natural that dissipativity theory can play a key role in the analysis and control design of discontinuous dynamical systems. Specifically, as in the analysis of dynamical systems with continuous flows, dissipativity theory for discontinuous dynamical systems can involve conditions on system parameters that render an input, state, and output system dissipative. In addition, robust stability for discontinuous dynamical systems can be analyzed by viewing a discontinuous dynamical system as an interconnection of discontinuous dissipative dynamical subsystems. Alternatively, discontinuous dissipativity theory can be used to design discontinuous feedback controllers that add dissipation and guarantee stability robustness allowing discontinuous stabilization to be understood in physical terms.

In this research [8], we develop dissipativity notions for discontinuous dynamical systems. In particular, we introduce a generalized definition of dissipativity for discontinuous dynamical systems in terms of set-valued supply rate maps and set-valued storage maps consisting of locally Lebesgue integrable supply rates and piecewise continuous storage functions, respectively. The collection of storage functions and supply rates satisfy a set of dissipation inequalities reflecting the fact that the dissipated generalized energies of a discontinuous dissipative system is nonnegative and is given by the difference of what is supplied and what is stored. Our dissipativity definition includes set-valued connective supply maps consisting of locally Lebesgue integrable connective supply rates to reflect the fact that an inactive storage function, corresponding to a Filippov set-valued map, can still change since multiple storage functions within a set-valued storage map have common state variables.

Next, we develop extended Kalman-Yakubovich-Popov set-valued conditions in terms of the discontinuous system dynamics for characterizing passivity and nonexpansivity via generalized Clarke gradients of locally Lipschitz continuous storage functions for discontinuous systems. In addition, using the concepts of dissipativity for discontinuous dynamical systems with appropriate set-valued storage maps and set-valued supply rate maps, we construct set-valued Lyapunov functions for discontinuous feedback systems by appropriately combining the set-valued storage maps for the forward and feedback subsystems. General stability criteria are given for Lyapunov, asymptotic, and finite-time stability for feedback interconnections of discontinuous dynamical systems. In the case where the set-valued supply rate map consists of supply rates involving net system power or weighted input-output energy, these results provide extensions of the positivity and small gain theorems to discontinuous dynamical systems.

2.6. Output Feedback Adaptive Command Following for Nonminimum Phase Uncertain Dynamical Systems

Mathematical models are critical in capturing and studying physical phenomena that undergo spatial and temporal evolution arising in most applications of science and engineering. These models are often based on first-principles of physics and are derived using fundamental physical laws. However, due to system complexity, nonlinearities, uncertainty, and disturbances, first-principle models are often based on simplifying approximations resulting in system modeling errors. For systems where the system model does not adequately capture the physical system due to idealized assumptions, model simplification, and model parameter uncertainty, adaptive control methods can be used to achieve system performance without excessive reliance on system models.

Direct adaptive controllers require less system modeling information than robust controllers and can address system uncertainties and system failures. These controllers adapt feedback gains in response to system variations without requiring a parameter estimation algorithm. This property distinguishes them from indirect adaptive controllers that employ an estimation algorithm to estimate the unknown system parameters and adapt the controller gains. Direct adaptive controllers can be classified as either full state feedback or output feedback designs.

Full state feedback designs assume knowledge of the state variables, and this assumption leads to computationally simpler adaptive controller algorithms as compared to output feedback algorithms. Output feedback direct adaptive controllers, however, are required for most applications that involve high-dimensional models such as active noise suppression, active control of flexible structures, fluid flow control systems, and combustion control processes. Models for these applications vary from (reasonably) accurate low frequency models in the case of structural control problems, to less accurate low-order models in the case of active control of noise, vibrations, flows, and combustion processes.

There has been a number of results in recent decades focused on output feedback direct adaptive controllers. These results require an observer for unknown state variables, an observer for output tracking errors, an output predictor, and/or estimation of Markov parameters that lead to adaptive control algorithms with varying sets of assumptions. These assumptions include knowledge of the relative degree of the regulated system output and the dimension of the system, as well as the requirement that the system be minimum phase or passive.

Virtually all output feedback adaptive controllers are developed under a minimum-phase assumption. In this research [32], we develop an output feedback adaptive control framework for nonminimum phase multivariable systems for output stabilization, command following, and disturbance rejection. The approach is based on a nonminimal state space realization that generates an expanded set of states using the filtered inputs and filtered outputs of the original system. Specifically, a direct adaptive controller for the nonminimal state space model is constructed using the expanded states of the nonminimal realization and is shown to be effective for multi-input, multi-output nonminimum phase systems with unstable dynamics. The adaptive controller does not require any model information except for an expanded compatibility condition involving the nonminimal model, which is far less restrictive than standard matching conditions for model reference output feedback adaptive control involving the actual system dynamics. In addition, the proposed adaptive controller does not require knowledge of the nonminimum phase system zeros.

2.7. A Neuroadaptive Control Architecture for Nonlinear Uncertain Dynamical Systems with Amplitude, Rate, and Time-Delay Constraints

Any electromechanical control actuation device is subject to amplitude and/or rate constraints leading to saturation nonlinearities enforcing limitations on control amplitudes and control rates. Actuator nonlinearities can severely degrade closed-loop system performance, and in some cases drive the system to instability, if not accounted for in the control design process. These effects are even more pronounced for adaptive controllers which continue to adapt when the feedback loop has been severed due to the presence of actuator saturation causing unstable controller modes to drift, which in turn leads to severe windup effects and unacceptable transients after saturation.

Many practical applications involve nonlinear dynamical systems with simultaneous control amplitude and rate saturation. The presence of control rate saturation may further exacerbate the problem of control amplitude saturation. For example, in advanced tactical fighter aircraft with high maneuverability requirements, pilot-induced oscillations can cause actuator amplitude and rate saturation in the control surfaces, leading to catastrophic failures.

In addition to actuator amplitude and rate saturation, actuator time-delay constraints as well as actuator dynamics need to be accounted for in the control design process. Accounting for input time delays is critical in adaptive control design since they can quantify time delay margin which can translate in increasing system gain margins. Finally, accounting for actuator dynamics can capture the effects of slow actuator effects which can degrade system performance, and potentially lead to instability.

In this research [36], we develop a new model reference adaptive control architecture for nonlinear uncertain dynamical systems with input actuator and time delay constraints. In particular, we consider both linear and nonlinear in the parameters neural network approximations to design neuroadaptive controllers for stabilization and command following in the presence of actuator dynamics that can effectively account for actuator amplitude and rate saturation constraints. In addition, we extend our framework to additionally account for actuator time-delay constraints.

2.8. Stability and Control of Large-Scale Dynamical Systems

Modern complex large-scale dynamical systems arise in virtually every aspect of science and engineering, and are associated with a wide variety of physical, technological, environmental, and social phenomena. Such systems include large-scale aerospace systems, power systems, communications systems, network systems, transportation systems, large-scale manufacturing systems, integrative biological systems, economic systems, ecological systems, and process control systems. These systems are strongly interconnected and consist of interacting subsystems exchanging matter, energy, or information with the environment. In addition, the subsystem interactions often exhibit remarkably complex system behaviors. Complexity here refers to the quality of a system wherein interacting subsystems form multiechelon hierarchical evolving structures exhibiting emergent system properties.

The sheer size, or dimensionality, of large-scale dynamical systems necessitates decentralized analysis and control system synthesis methods for their analysis and control design. Specifically, in analyzing complex large-scale interconnected dynamical systems it is often desirable to treat the overall system as a collection of interacting subsystems. The behavior and properties of the aggregate large-scale system can then be deduced from the behaviors of the individual subsystems and their interconnections. Often the need for such an analysis framework arises from computational complexity and computer throughput constraints. In addition, for controller design the physical size and complexity of large-scale systems imposes severe constraints on the communication links between system sensors, processors, and actuators, which can render centralized control architectures impractical. This leads to consideration of decentralized controller architectures involving multiple sensor-processor-actuator subcontrollers without real-time intercommunication. The design and implementation of decentralized controllers is a nontrivial task involving control-system architecture determination and actuator-sensor assignments for a particular subsystem, as well as processor software design for each subcontroller of a given architecture.

In this research [2], we develop a unified stability analysis and control design framework for nonlinear large-scale interconnected dynamical systems based on vector Lyapunov function methods, vector dissipativity theory, and decentralized control architectures. The use of vector Lyapunov functions in dynamical system theory offers a very flexible framework for stability analysis since each component of the vector Lyapunov function can satisfy less rigid requirements as compared to a single scalar Lyapunov function. Moreover, in the analysis of large-scale interconnected nonlinear dynamical systems, several Lyapunov functions arise naturally from the stability properties of each individual subsystem. In addition, since large-scale dynamical systems have numerous input, state, and output properties related to conservation, dissipation, and transport of energy, matter, or information, extending classical dissipativity theory to capture conservation and dissipation notions on the subsystem level provides a natural energy flow model for large-scale dynamical systems. Aggregating the dissipativity properties of each of the subsystems by appropriate storage functions and supply rates, allows us to study the dissipativity properties of the composite large-scale system using the newly developed notions of vector storage functions and vector supply rates.

Finally, a novel class of fixed-order, energy-based hybrid decentralized controllers is proposed as a means for achieving enhanced energy dissipation in large-scale vector lossless and vector dissipative dynamical systems. These dynamic decentralized controllers combine a logical switching architecture with continuous dynamics to guarantee that the system plant energy is strictly decreasing across switchings. The general framework leads to hybrid closed-loop systems described by impulsive differential equations [2]. In addition, we construct hybrid dynamic controllers that guarantee that each subsystem-subcontroller pair of the hybrid closed-loop system is consistent with basic thermodynamic principles. Special cases of energy-based hybrid controllers involving state-dependent switching are described, and several illustrative examples as well as an experimental testbed is designed to demonstrate the efficacy of the proposed approach.

2.9. Coordination Control for Multiagent Interconnected Systems

Modern complex multiagent dynamical systems are highly interconnected and mutually interdependent, both physically and through a multitude of information and communication networks. Distributed decision-making for coordination of networks of dynamic agents involving information flow can be naturally captured by graph-theoretic notions. These dynamical network systems cover a very broad spectrum of applications including cooperative control of unmanned air vehicles (UAV's), autonomous underwater vehicles (AUV's), distributed sensor networks, air and ground transportation systems, swarms of air and space vehicle formations, and congestion control in communication networks, to cite but a few examples. Hence, it is not surprising that a considerable research effort has been devoted to control of networks and control over networks in recent years.

A key application area of multiagent network coordination within aerospace systems is cooperative control of vehicle formations using distributed and decentralized controller architectures. Distributed control refers to a control architecture wherein the control is distributed via multiple computational units that are interconnected through information and communication networks, whereas decentralized control refers to a control architecture wherein local decisions are based only on local information. Vehicle formations are typically dynamically decoupled, that is, the motion of a given agent or vehicle does not directly affect the motion of the other agents or vehicles. The multiagent system is coupled via the task which the agents or vehicles are required to perform.

The complexity of cooperative manoeuvres that multiagent systems need to perform as well as environmental constraints often necessitate the design of control algorithms that use information on current position and velocity of each vehicle to steer them while maintaining a specified formation. In particular, for mobile agents operating in a foggy environment or located far from each other, open-loop visual control for coordinated motion becomes impractical. In this case, feedback control algorithms are required for individual vehicle steering which determine how a given vehicle maneuvers based on positions and velocities of nearby vehicles and/or on those of a formation leader. The leader could be real, that is, one of the vehicles in a formation leads the others, or the leader could be virtual, that is, vehicles synthesize a leader and the motions of the vehicles in a formation are defined with respect to a virtual agent whose positions and velocities are known at each instant of time.

Analysis and control design for networks of mobile agents has received considerable attention in the literature. Common formations of multiagent systems include flocking, cyclic pursuit, (virtual) leader following, and rendezvous. Graph-theoretic notions are essential in the analysis and control design for a system of mobile agents performing a common task. Several researchers have proposed different techniques for analyzing network systems. Specifically, graph theory has been used to model interconnected systems and analyze the stability of formations of large number of agents. In addition, potential functions have been used to analyze flocking.

In this research [2], we develop a stability analysis and control design framework for multiagent coordination predicated on vector Lyapunov functions. In multiagent systems, several Lyapunov functions arise naturally where each agent can be associated with a generalized energy function corresponding to a component of a vector Lyapunov function. Furthermore, since a specified formation of multiple vehicles can be characterized by a time-varying set in the state space, the problem of control design for multiagent coordinated motion is equivalent to design of stabilizing controllers for time-varying sets of nonlinear dynamical systems. Thus, using a stability and control design framework for time-varying sets, we design distributed control algorithms for stabilization of multi-vehicle formations. These distributed control algorithms use only local information of the individual vehicle relative position and velocity with respect to the leader in order to maintain a specified formation for a system of multiple vehicles. Finally, we specialize the results obtained for time-varying sets to address stabilization of time-invariant sets and develop stabilizing control algorithms for static formations (rendezvous) of multiple vehicles. The developed cooperative control algorithms are shown to globally exponentially stabilize both moving and static formations.

2.10. Dissipative Differential Inclusions, Set-Valued Energy Storage and Supply Rate Maps, and Discontinuous Dynamical Systems

The key foundation in developing dissipativity theory for nonlinear dynamical systems with continuous flows was presented by Willems in his seminal two-part paper on dissipative dynamical systems. In particular, Willems introduced the definition of dissipativity for general nonlinear dynamical systems in terms of a *dissipation inequality* involving a generalized system power input, or *supply rate*, and a generalized energy function, or *storage function.* The dissipation inequality implies that the increase in generalized system energy over a given time interval cannot exceed the generalized energy supply delivered to the system during this time interval. The set of all possible system storage functions is convex and every system storage function is bounded from below by the *available system storage* and bounded from above by the *required energy supply.*

In this research [23], we develop dissipativity notions for dynamical systems with discontinuous vector fields. Specifically, we consider dynamical systems with Lebesgue measurable and locally essentially bounded vector fields characterized by differential inclusions involving Filippov set-valued maps specifying a set of directions for the system velocity and admitting Filippov solutions with absolutely continuous curves. In particular, we introduce a generalized definition of dissipativity for discontinuous dynamical systems in terms of set-valued supply rate maps and set-valued storage maps consisting of locally Lebesgue integrable supply rates and piecewise continuous storage functions, respectively. The collection of storage functions and supply rates satisfy a set of dissipation inequalities reflecting the fact that the dissipated generalized energies of a discontinuous dissipative system is nonnegative and is given by the difference of what is supplied and what is stored.

In addition, we introduce the notion of a *set-valued available storage map* and a *set-valued required supply map*, and show that if these maps have closed convex images they involve single-valued maps corresponding to the *smallest available storage* and the *largest required supply* of the differential inclusion, respectively. Furthermore, we show that all system storage functions are bounded from above by the largest required supply and bounded from below by the smallest available storage, and hence, a dissipative differential inclusion can deliver to its surroundings only a fraction of its generalized stored energy and can store only a fraction of the generalized work done to it. Finally, we develop analogous results for lossless differential inclusions as well as specialize our results to switched differential inclusions to address the notion of dissipativity for switched dynamical systems. Future extensions will focus on using discontinuous dissipativity theory as a design tool for developing group coordination algorithms for multiagent systems possessing a dynamic (i.e., switching) topology.

2.11. Heat Flow, Work Energy, Chemical Reactions, and Thermodynamics: A Dynamical Systems Perspective

There is no doubt that thermodynamics is a theory of universal proportions whose laws reign supreme among the laws of nature and are capable of addressing some of science's most intriguing questions about the origins and fabric of our universe. The laws of thermodynamics are among the most firmly established laws of nature and play a critical role in the understanding of our expanding universe. In addition, thermodynamics forms the underpinning of several fundamental life science and engineering disciplines, including biological systems, physiological systems, chemical reaction systems, ecological systems, information systems, and network systems, to cite but a few examples. While from its inception its speculations about the universe have been grandiose, its mathematical foundation has been amazingly obscure and imprecise [47]. This is largely due to the fact that classical thermodynamics is a physical theory concerned mainly with equilibrium states and does not possess equations of motion. The absence of a state space formalism in classical thermodynamics, and physics in general, is quite disturbing and in our view largely responsible for the monomeric state of classical thermodynamics.

In recent research [47], we combined the two universalisms of thermodynamics and dynamical systems theory under a single umbrella to develop a dynamical system formalism for classical thermodynamics so as to harmonize it with classical mechanics. While it seems impossible to reduce thermodynamics to a mechanistic world picture due to microscopic reversibility and Poincaré recurrence, the system thermodynamic formulation of [47] provides a harmonization of classical thermodynamics with classical mechanics. In particular, our dynamical system formalism captures all of the key aspects of thermodynamics, including its fundamental laws, while providing a mathematically rigorous formulation for thermodynamical systems out of equilibrium by unifying the theory of heat transfer with that of classical thermodynamics. In addition, the concept of entropy for a nonequilibrium state of a dynamical process is defined, and its global existence and uniqueness is established. This state space formalism of thermodynamics shows that the behavior of heat, as described by the conservation equations of thermal transport and as described by classical thermodynamics, can be derived from the same basic principles and is part of the same scientific discipline.

Connections between irreversibility, the second law of thermodynamics, and the entropic arrow of time are also established in [47]. Specifically, we show a state irrecoverability and, hence, a state irreversibility nature of thermodynamics. State irreversibility reflects time-reversal non-invariance, wherein time-reversal is not meant literally; that is, we consider dynamical systems whose trajectory reversal is or is not allowed and not a reversal of time itself. In addition, we show that for every nonequilibrium system state and corresponding system trajectory of our thermodynamically consistent dynamical system, there does not exist a state such that the corresponding system trajectory completely recovers the initial system state of the dynamical system and at the same time restores the energy supplied by the environment back to its original condition. This, along with the existence of a global strictly increasing entropy function on every nontrivial system trajectory, establishes the existence of a completely ordered time set having a topological structure involving a closed set homeomorphic to the real line giving a clear time-reversal asymmetry characterization of thermodynamics and establishing an emergence of the direction of time flow.

In this research [14], we reformulate and extend some of the results of [47]. In particular, unlike the framework in [47] wherein we establish the existence and uniqueness of a global entropy function of a specific form for our thermodynamically consistent system model, in this research we assume the existence of a continuously differentiable, strictly concave function that leads to an entropy inequality that can be identified with the second law of thermodynamics as a statement about entropy increase. We then turn our attention to stability and convergence. Specifically, using Lyapunov stability theory and the Krasovskii-LaSalle invariance principle, we show that for an adiabatically isolated system the proposed interconnected dynamical system model is Lyapunov stable with convergent trajectories to equilibrium states where the temperatures of all subsystems are equal. Finally, we present a state-space dynamical system model for chemical thermodynamics. In particular, we use the law of mass-action to obtain the dynamics of chemical reaction networks. Furthermore, using the notion of the chemical potential [14], we unify our state space mass-action kinetics model developed in [1, 7] with our thermodynamic dynamical system model involving energy exchange. In addition, we show that entropy production during chemical reactions is nonnegative and the dynamical system states of our chemical thermodynamic state space model converge to a state of temperature equipartition and zero affinity (i.e., the difference between the chemical potential of the reactants and the chemical potential of the products in a chemical reaction).

2.12. A Variational Approach to the Fuel Optimal Control Problem for UAV Formations

The pivotal role of unmanned aerial vehicles (UAVs) in modern aircraft technology is evidenced by the large number of civil and military applications they are employed in. For example, UAVs successfully serve as platforms carrying payloads aimed at land monitoring, wildfire detection and management, law enforcement, pollution monitoring, and communication broadcast relay, to name just a few.

A formation of UAVs, defined by a set of vehicles whose states are coupled through a common control law, is often more valuable than a single aircraft because it can accomplish several tasks concurrently. In particular, UAV formations can guarantee higher flexibility and redundancy, as well as increased capability of distributed payloads. For example, an aircraft formation can successfully intercept a vehicle which is faster than its chasers. Alternatively, a UAV formation equipped with interferometic synthetic aperture radar (In-SAR) antennas can pursue both along-track and cross-track interferometry, which allow harvesting information that a single radar cannot detect otherwise.

Path planning is one of the main problems when designing missions involving multiple vehicles; a UAV formation typically needs to accomplish diverse tasks while meeting some assigned constraints. For example, a UAV formation may need to intercept given targets while its members maintain an assigned relative attitude. Trajectories should also be optimized with respect to some performance measure capturing minimum time or minimum fuel expenditure. In particular, trajectory optimization is critical for mini and micro UAVs (μUAVs) because they often operate independently from remote human controllers for extended periods of time and also because of limited amount of available energy sources.

In this research [18], we provide a rigorous and sufficiently broad formulation of the optimal path planning problem for UAV formations, modeled as a system of n 6-degrees of freedom (DoF) rigid bodies subject to a constant gravitational acceleration and aerodynamic forces and moments. Specifically, system trajectories are optimized in terms of control effort, that is, we design a control law that minimizes the forces and moments needed to operate a UAV formation, while meeting all the mission objectives. Minimizing the control effort is equivalent to minimizing the formation's fuel consumption in the case of vehicles equipped with conventional fuel-based propulsion systems and is a suitable indicator of the energy consumption for vehicles powered by batteries or other power sources.

In this research [18], we also derive an optimal control law which is independent of the size of the formation, the system constraints, and the environmental model adopted, and hence, our framework applies to aircraft, spacecraft, autonomous marine vehicles, and robot formations. The direction and magnitude of the optimal control forces and moments is a function of the dynamics of two vectors, namely the translational and rotational primer vectors. In general, finding the dynamics of these two vectors over a given time interval is a demanding task that does not allow for an analytical closed-form solution, and hence, a numerical approach is required. Our main result involves necessary conditions for optimality of the formations' trajectories.

2.13. Output Feedback Adaptive Stabilization and Command Following for Minimum Phase Uncertain Dynamical Systems

As discussed in Section 2.6, there has been a number of results in recent decades focused on output feedback direct adaptive controllers. These results require an observer for unknown state variables, an observer for output tracking errors, an output predictor, and/or estimation of Markov parameters that lead to adaptive control algorithms with varying sets of assumptions. These assumptions include knowledge of the relative degree of the regulated system output and the dimension of the system, as well as the requirement that the system be minimum phase or passive. The main reason for the minimum phase assumption is because direct adaptive controllers employ high gain feedback that can drive nonminimum phase systems to instability.

In this research [22], we extend the disturbance free adaptive control framework presented in [37] to develop an output feedback adaptive control framework for continuous-time minimum phase multivariable uncertain dynamical systems with exogenous disturbances for output stabilization and command following. The approach is based on a nonminimal state space realization that generates an expanded set of states using the filtered inputs and filtered outputs and their derivatives of the original system. Specifically, a direct adaptive controller for the nonminimal state space model is constructed using the expanded states of the nonminimal realization and is shown to be effective for multi-input, multi-output linear dynamical systems with unmatched disturbances, unmatched uncertainties, and unstable dynamics. The proposed adaptive control architecture requires knowledge of the open-loop system's relative degree as well as a bound on the system's order. Several illustrative numerical examples are provided to demonstrate the efficacy of the proposed approach.

2.14. Robust Adaptive Control Architecture for Disturbance Rejection and Uncertainty Suppression with $\mathcal{L}_\infty$ Transient and Steady-State Performance Guarantees

One of the fundamental problems in feedback control design is the ability of the control system to guarantee robust stability and robust performance with respect to system uncertainties in the design model. To this end, adaptive control along with robust control theory have been developed to address the problem of system uncertainty in control-system design. The fundamental differences between adaptive control design and robust control design can be traced to the modeling and treatment of system uncertainties as well as the controller architecture structures.

In particular, adaptive control is based on constant linearly parameterized system uncertainty models of a known structure but unknown variation, whereas robust control is predicated on structured and/or unstructured linear or nonlinear (possibly time-varying) operator uncertainty models consisting of bounded variation. Hence, for systems with constant real parametric uncertainties with large unknown variations, adaptive control is clearly appropriate, whereas for systems with time-varying parametric uncertainties and nonparametric uncertainties with norm bounded variations, robust control may be more suitable.

In contrast to fixed-gain robust controllers, which are predicated on a mathematical model of the system uncertainty and which maintain specified constants within the feedback control law to *sustain* robust stability and performance over the range of system uncertainty, adaptive controllers directly or indirectly adjust feedback gains to maintain closed-loop stability and *improve* performance in the face of system uncertainties. Specifically, indirect adaptive controllers utilize parameter update laws to identify unknown system parameters and adjust feedback gains to account for system variation, whereas direct adaptive controllers directly adjust the controller gains in response to plant variation. In either case, the overall process of parameter identification and controller adjustment constitutes a nonlinear control law architecture, which makes validation and verification of guaranteed transient and steady-state performance, as well as robustness margins of adaptive controllers extremely challenging.

While adaptive control has been used in numerous applications to achieve system performance without excessive reliance on system models, the necessity of high-gain feedback for achieving fast adaptation can be a serious limitation of adaptive controllers. Specifically, in certain applications fast adaptation is required to achieve stringent tracking performance specifications in the face of large system uncertainties and abrupt changes in system dynamics. This, for example, is the case for high performance aircraft systems that can be subjected to system faults or structural damage which can result in major changes in aerodynamic system parameters. In such situations, high-gain adaptive control is necessary in order to rapidly reduce and maintain system tracking errors. However, fast adaptation using high-gain feedback can result in high-frequency oscillations which can excite unmodeled system dynamics resulting in system instability. Hence, there exists a critical trade-off between system stability and control adaptation rate.

Virtually all adaptive control methods developed in the literature have averted the problem of high-gain control. Notable exceptions include the use of a low-pass filter that effectively subverts high frequency oscillations that can occur due to fast adaptation while using a predictor model to reconstruct the reference system model. In particular, this method involves a robust adaptive control architecture that provides sufficient conditions for stability and performance in terms of $\mathcal{L}_1$-norms of the underlying system transfer functions despite fast adaptation, leading to uniform bounds on the $\mathcal{L}_\infty$-norms of the system input-output signals.

In this research [19], a new adaptive control architecture for linear and nonlinear uncertain dynamical systems is developed to address the problem of high-gain adaptive control. Specifically, the proposed framework involves a new and novel controller architecture involving a modification term in the update law that minimizes an error criterion involving the distance between the weighted regressor vector and the weighted system error states. This modification term allows for fast adaptation without hindering system robustness. In particular, we show that the governing tracking closed-loop system error equation approximates a Hurwitz linear time-invariant dynamical system with $\mathcal{L}_\infty$ input-output signals. This key feature of our framework allows for robust stability analysis of the proposed adaptive control law using $\mathcal{L}_1$ system theory. Specifically, in the face of fast adaptation, uniform transient and steady-state system performance bounds are derived in terms of $\mathcal{L}_1$-norms of the closed-loop system error dynamics. We further show that by properly choosing the design parameters in the modification term we can adjust the bandwidth of the adaptive controller, the transient and steady-state closed-loop performance, and the size of the ultimate bound of the closed-loop system trajectories independently of the system adaptation rate. Several illustrative numerical examples are provided to demonstrate the efficacy of the proposed approach.

2.15. Multistability, Bifurcations, and Biological Neural Networks: A Synaptic Drive Firing Model for Cerebral Cortex Transition in the Induction of General Anesthesia

Advances in neuroscience have been closely linked to mathematical modeling beginning with the integrate-and-fire model of Lapicque and proceeding through the modeling of the action potential by Hodgkin and Huxley to the current era of mathematical neuroscience. Neuroscience has always had models to interpret experimental results from a high-level complex systems perspective; however, expressing these models with dynamic equations rather than words fosters precision, completeness, and self-consistency. Nonlinear dynamical system theory, in particular, can provide a framework for a rigorous description of the behavior of large-scale networks of neurons. A particularly interesting application of nonlinear dynamical systems theory to the neurosciences is to study phenomena of the central nervous system that exhibit nearly discontinuous transitions between macroscopic states. One such example exhibiting this phenomenon is the induction of general anesthesia.

The rational, safe, and effective utilization of any drug in the practice of medicine is grounded in an understanding of the pharmacodynamics of the drug, loosely defined as what the drug does to the body. A very important measure of the pharmacodynamics of any drug is the drug concentration parameter EC_{50}, which reflects the drug dose at which the therapeutic effect is achieved in 50% of the cases. This concept is certainly applicable for the administration of general inhalational anesthetics, where the potency of the drug is defined by the minimum alveolar concentration (MAC) of the drug needed to prevent a response to noxious stimuli in 50% of administrations.

The MAC concept is intrinsically embedded in a probabilistic framework. It is the concentration at which the probability of a response to a noxious stimulus is 0.5. Typically the MAC of a particular anesthetic is determined by administering various doses of the agent to a population of patients and determining the dose at which there is a 0.5 chance of responding to a noxious stimulus. (Technically, we identify the concentration in the alveoli, the fundamental functional gas exchange units of the lung, at which the chance of response is 0.5.) It has been possible, however, to conduct studies of single subjects, varying the anesthetic concentration and determining responsiveness. When this has been done, it has been noted that the transition from responsiveness to non-responsiveness in the individual patient is very sharp, almost an all-or-none transition. This simply confirms the observations of generations of clinicians. And this raises the question of how to account for such a transition in terms of the known molecular properties of the anesthetic agent.

Although general anesthesia has been used in the clinical practice of medicine for over 150 years, the mechanism of action is still not fully understood and is still under considerable investigation. Theories range from a nonspecific perturbation of the lipid bilayer membrane of neurons, the cells responsible for the "information" function of the central nervous system, to the interaction of the anesthetic agent with specific protein receptors. Early theories postulated that anesthesia is produced by disturbance of the physical properties of cell membranes. The work of Meyer and Overton demonstrated that for some anesthetics there was a correlation between anesthetic potency and solubility in fat-like solvents. This led to a theory that anesthesia resulted from a nonspecific perturbation of the lipid bilayer membrane of neurons. Subsequent research then found that membrane proteins performed functions of excitability and this led to a focus on anesthetic binding and perturbation of hydrophobic regions of membrane proteins. Further research also revealed that some anesthetic gases follow the Meyer-Overton correlation but do not produce anesthesia and some Meyer-Overton gases are excitatory and can cause seizures. These results led to the more common modern focus on the interaction of the anesthetic agent with specific protein receptors.

In particular, there has been extensive investigation of the influence of anesthetic agents on the binding of neurotransmitters to their postsynaptic receptors. A plethora of receptors have been investigated, including receptors for glycine, serotonin type 2 and 3, N-methyl-d-aspartate (NMDA), α-2 adrenoreceptors, α-amino-3-hydroxy-5-methyl-4- isoxazolepropionic acid (AMPA), histamine, acetycholine, and γ-aminobutyric acid (GABA). One attractive aspect of this focus on postsynaptic receptors is it facilitates mathematical analysis on the basis of the effect of receptor binding on the postsynaptic potential. This is in marked contrast to the Meyer-Overton hypothesis, which failed to explicitly detail how a nonspecific perturbation of the lipid membrane would result in the anesthetic state.

In parallel with the investigation of the molecular interactions of general anesthetic agents, there has also been active investigation of the anatomic pathways involved in the transition from consciousness to anesthesia. There is compelling evidence that the immobility created by some anesthetics is mediated at the level of the spinal cord. In contrast, functional imaging and electroencephalograph analysis has suggested that the site of suppression of consciousness is the thalamus, and thalamocortical tracts may play a critical role in the suppression of consciousness.

Despite these advances in our understanding of the molecular interactions of anesthetic agents and of specific anatomic loci for the action of anesthetic agents, there has been less development of a mathematical framework to understand this fascinating and clinically important phenomenon. It is certainly possible that if the mechanism of general anesthesia is the binding of the anesthetic agent to a specific receptor protein, then the nearly all-or-none transition from the awake state to the anesthetized state could be explained by a highly cooperative binding of the anesthetic to the receptor. In fact, it has been common to mathematically model the probability of responsiveness to drug concentration using the Hill equation, a simplified equation originally derived in 1909 to describe the cooperative binding of oxygen to the hemoglobin molecule. However, to date, no single unifying receptor mediating general anesthesia has been identified.

Rather, the most likely explanation for the mechanisms of action of anesthetics lies in the network properties of the brain. It is well established that there are two general types of neurons in the central nervous system—excitatory and inhibitory—interconnected in a complex network. The action potential of a spiking neuron is propagated along the axon to synapses where chemical neurotransmitters are released that generate a postsynaptic potential on the dendrites of connected neurons. Excitatory neurons generate a depolarizing postsynaptic potential on the dendrite of the connected neuron and if the depolarization is of sufficient magnitude, then a spike will be induced in the connected neuron. In contrast, inhibitory neurons generate a hyperpolarizing postsynaptic potential; an effect that acts to maintain a quiescent state.

The human central nervous system involves a complex large-scale interconnected neural network involving feedforward and feedback (or recurrent) networks, with the brain serving as the central element of this network system. The brain is interconnected to receptors that transmit sensory information to the brain, and in turn the brain delivers action commands to effectors. The neural network of the brain consists of approximately 10^{11} neurons (nerve cells) with each having 10^4 to 10^5 connections interconnected through subnetworks or nuclei. The nuclei in turn consist of clusters of neurons each of which performs a specific and defined function.

The most basic characteristic of the neurons that comprise the central nervous system is the electrochemical potential gradient across the cell membrane. All cells of the human body maintain an electrochemical potential gradient between the inside of the cell and the surrounding milieu. Neurons have the capacity of excitability. If stimulated beyond a threshold the neuron will "fire" and produce a large voltage spike (the action potential) before returning to the resting potential. The neurons of the brain are connected in a complex network in which the firing of one neuron can be the stimulus for the firing of another neuron. A major focus of theoretical neuroscience has been describing neuronal behavior in terms of this electrochemical potential, both at the single neuron level but more ambitiously, at the level of multi-neuron networks. In this type of analysis the specific properties of the single neuron that are most relevant are how the spike of a one neuron alters the electrochemical potential of another neuron, and how this change in the potential results in a neuronal spike. The physical connection between neurons occurs in the synapse, a small gap between the axon, the extension of the cell body of the transmitting neuron, and the dendrite, the extension of the receiving neuron. The signal is transmitted by the release of a neurotransmitter from the axon into the synapse. This neurotransmitter diffuses across the synapse, binds to a postsynaptic receptor membrane protein on the dendrite, and alters the electrochemical potential of the receiving neuron.

There is considerable evidence that general anesthetics alter postsynaptic potentials. For example, it is possible that the anesthetic bifurcation to unconsciousness or the nearly all-or-none characteristic induction of anesthesia is a type of phase transition of the neural network. This possibility was first considered by Steyn-Ross *et al.* Their focus was on the mean voltage of the soma, or cell body, of neurons. Specifically, they show that the biological change of state to anesthetic unconsciousness is analogous to a thermodynamic phase change involving a liquid to solid phase transition. For certain ranges of anesthetic concentrations, their first-order model predicts the existence of multiple steady states for brain activity leading to a transition from normal levels of cerebral cortical activity to a quiescent, low-firing state.

In this research [16, 39], we present an alternative approach to the possibility of neuronal network phase transition in terms of neuronal firing rates, using the concept of multistability for dynamical systems. Multistability is the property whereby the solutions of a dynamical system can alternate between two or more mutually exclusive Lyapunov stable and convergent states under asymptotically slowly changing inputs or system parameters. In particular, multistable systems give rise to the existence of multiple (isolated and/or a continuum of) stable equilibria involving a quasistatic-like behavior between these multiple semistable steady states [16]. Semistability is the property whereby the solutions to a dynamical system converge to Lyapunov stable equilibrium points determined by the system initial conditions [46]. Multistability is ubiquitous in biological systems ranging from biochemical networks to ecosystems to gene regulation and cell replication. Since molecular studies suggest that one possible mechanism of action of anesthetics is the inhibition of synaptic transmission in cortical neurons, this suggests that general anesthesia is a phenomenon in which different equilibria can be attained with changing anesthetic agent concentrations. Hence, multistability theory can potentially provide a theoretical foundation for describing general anesthesia.

In this research [16, 39], we present a thorough discussion on biological neural networks. In particular, the fundamental building block of the central nervous system, the neuron, is represented as a dynamic element that is "excitable," and can generate a pulse or spike whenever the electrochemical potential across the cell membrane of the neuron exceeds a threshold value. More specifically, a nonlinear discontinuous system is derived for describing the relationship between the synaptic current and firing rates of excitatory and inhibitory neural networks. Then, we develop some basic results on differential inclusions which provide the mathematical foundation for discontinuous dynamical systems generating Filippov and Carathéodory solutions. To establish convergence and semistability for discontinuous dynamical systems we introduce the notion of nontangency between a discontinuous vector field and a weakly invariant or weakly negatively invariant subset of the level or sublevel sets of a Lyapunov function. Specifically, to capture the notion of nontangency we introduce the direction cone of a discontinuous vector field [9].

Next, we use positive limit sets, restricted prolongations, and nontangency to develop Lyapunov analysis for convergence and semistability to establish multistability for discontinuous dynamical systems. Here, the restricted prolongation of a point is a subset of its positive prolongation. In particular, we establish connectedness and invariance properties of restricted prolongations, and give inclusion results for restricted prolongations in terms of invariant and negatively invariant subsets of the level sets of a Lyapunov function and its derivative. Then, using nontangency, we obtain Lyapunov results for convergence and semistability to develop sufficient conditions for multistability for discontinuous dynamical systems.

Finally, we apply our results to excitatory-inhibitory firing neural models. While there is ongoing debate as to whether information is encoded by the firing rates (i.e., rate-coding) of spiking neurons or by precise timing of single neuron spikes (i.e., temporal coding), it is evident that firing rates do characterize central nervous system activity. Firing rates are nonnegative entities and the nonnegativity constraint for neural network activity can be easily incorporated within nonlinear dynamical system theories using solutions of differential equations evolving in cones [1]. Next, we develop a two-class model for characterizing mean excitatory and inhibitory synaptic drives to explain the underling mechanism of action for anesthesia and consciousness. In particular, we demonstrate how the full synaptic drive firing model can be reduced to a model involving mean excitatory and inhibitory synaptic drives, and then use our results to show multistability.

3. Research Personnel Supported

Faculty

Wassim M. Haddad, Principal Investigator

Graduate Students

K. Volyanskyy, Ph. D, and T. Sadikhov, Ph. D.

Several other students (T. Yucelen and A. L'Afflitto) were involved in research projects that were closely related to this program. Although none of these students were financially supported by this program, their research did directly contribute to the overall research effort. Furthermore, one Ph. D. dissertation was completed under partial support of this program; namely

K. Volyanskyy, *Adaptive and Neuroadaptive Control for Nonnegative and Compartmental Dynamical Systems*, August 2010.

Dr. Volyanskyy is presently a postdoctoral fellow of the School of Civil Engineering at Georgia Tech.

4. Interactions and Transitions

4.1. Participation and Presentations

The following conferences were attended over the past three years.

American Control Conference, St. Louis, MO, June 2009.
IEEE Conference on Decision and Control, Shanghai, P. R. China, December 2009.
American Control Conference, Baltimore, MD, July 2010.
IEEE Conference on Decision and Control, Atlanta, GA, December 2010.
American Control Conference, San Francisco, CA, June 2011.
IEEE Conference on Decision and Control, Orlando, FL, December 2011.

Furthermore, conference articles [25–45] were presented.

4.2. Transitions

Our work on adaptive and neuroadaptive control of drug delivery [1] partially supported under this program continues to transition to clinical studies at the Northeast Georgia Medical Center in Gainsville, Georgia, under the direction of Dr. James M. Bailey (770-534-1312), director of cardiac anesthesia and consultant in critical care medicine. This work has recently transitioned from operating room (OR) hypnosis to intensive care unit (ICU) sedation. In addition, this work was communicated to Colonel Leopoldo C. Cancio (210-916-3301) of the US Army Institute of Surgical Research in Fort Sam Houston, San Antonio, in order to provide improvements for combat casualty care in current and future battlefields. Collaboration with the US Army Institute of Surgical Research is underway.

5. Research Publications

5.1. Journal Articles and Books

[1] W. M. Haddad, V. Chellaboina, and Q. Hui, *Nonnegative and Compartmental Dynamical Systems.* Princeton, NJ: Princeton University Press, 2010

[2] W. M. Haddad and S. G. Nersesov, *Stability and Control of Large-Scale Dynamical Systems: A Vector Dissipative Systems Approach.* Princeton, NJ: Princeton University Press, 2011

[3] W. M. Haddad, V. Chellaboina, and B. Gholami, "$\mathcal{H}_2$ Suboptimal Estimation and Control for Nonnegative Dynamical Systems," *Opt. Contr. Appl. Meth.*, vol. 30, pp. 27-47, 2009.

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[11] Q. Hui, W. M. Haddad, and S. P. Bhat, "On Robust Control Algorithms for Network Consensus Protocols," *Int. J. Robust and Nonlinear Control*, vol 20, pp. 268-284, 2010.

[12] Q. Hui, W. M. Haddad, and S. P. Bhat, "Finite-Time Semistability, Filippov Systems, and Consensus Protocols for Nonlinear Dynamical Networks with Switching Topologies," *Nonlinear Analysis: Hybrid Systems*, vol. 4, pp. 557-573, 2010.

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