Endogenous Split Awards as a Protest Management Tool: A Modeling & Computational Approach

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Endogenous Split Awards as a Protest Management Tool: A Modeling & Computational Approach

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Bid Protests & Split Awards: Agenda

- **Managing bid protests** in DoD procurement
- **Simple model** of bidding & protest process
- **Split awards** as a protest management tool
- Key question: What is the **right split**?
- Bids & prices with **fixed** split awards
- Bids & prices with **endogeneous** split awards
- **Conclusions**
- **Research agenda** moving forward
“Managing” Bid Protests

- **Objective is not to minimize** number of bid protests
- Protests intended to **correct procurement mistakes**
  - **Honest mistake**: Limited information & bounded rationality
  - **Dishonest mistake**: Bias or fraud by procurement officials
- **Objective is to “right size”** number of protests
  - Encourage protests that correct (significant) mistakes
  - Discourage protests that don’t make significant corrections
- **Modeling** the process could help identify, compare, & characterize **levers of control** for managing protests
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Modeling Bid Protests

- As noted, the intended role of bid protests is, in the most general terms, to correct procurement mistakes.

- Such mistakes – whether honest or dishonest – result from some form of imperfect decision-making.
  - How best to model such imperfection?

- Consider a model driven by imperfect information.
  - Imperfect info $\rightarrow$ small mistake more likely than big mistake.
  - Bias $\rightarrow$ small injustice more likely than big injustice.

- Imperfect information consistent with empirical results.
  - “Agency mis-evaluation” is by far the most commonly cited reason for sustaining a DoD bid protest (Gansler, et al.).
Simple Model of Bid & Protest Process

Vendor 1 with cost $C_1$ submits bid $P_1$

Vendor 1 decides whether or not to file protest

Vendor 2 with cost $C_2$ submits bid $P_2$

Buyer perceives $P_1 > P_2$

Buyer compares bids with imperfect information

Vendor 1 decides whether or not to file protest

Buyer perceives $P_1 < P_2$

Protestor incurs cost $K_p$

Vendor 2 decides whether or not to file protest

Vendor 1 profit = 0

Vendor 2 profit = $X(P_2 - C_2)$

Vendor 2 profit = 0

Buyer incurs cost $K_B$

Buyer cost = $XP_2$

Above excludes protest costs

Vendor 1 profit = $X(P_1 - C_1)$

Buyer cost = $XP_1$

Above excludes protest costs

No

No

Yes

Yes

$P_1 > P_2$

$P_1 < P_2$
Managing Vendor Protest Incentives

- Losing vendor 1 protests iff \( \text{Prob}(P_1 < P_2) \times X - K_P > 0 \)
- Recall the **two goals** of protest management:
  1. Encourage/allow “good” or efficient protests
  2. Discourage “bad” or inefficient protests
- Levers of control?
  - \( \text{Prob}(P_1 < P_2) \) ➔ Influence initial assessment accuracy
    ➔ Change or shift burden of proof
  - \( K_P \) ➔ Influence expected costs
    ➔ Different costs for successful vs. failed protests
  - \( X \) ➔ Influence gain from successful protest
    ➔ Split awards
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Bidding with Fixed Award Splits

Contract splits:

- $S_L$ = Share or split awarded low-price bidder
- $S_H$ = Share or split awarded high-price bidder
- $S_L + S_H = 1$
- $0 \leq S_H \leq \frac{1}{2}$ & $\frac{1}{2} \leq S_L \leq 1$

Award Determination:

- If final decision is that $P_1 < P_2$:
  - Vendor 1 awarded contract to produce $S_L X$ units
  - Vendor 2 awarded contract to produce $S_H X$ units
- If final decision is that $P_1 > P_2$:
  - Vendor 1 awarded contract to produce $S_H X$ units
  - Vendor 2 awarded contract to produce $S_L X$ units
Bid & Protest Process with Split Awards

Vendor 1 with cost $C_1$ submits bid $P_1$

Buyer compares bids with imperfect information

Vendor 1 decides whether or not to file protest

Yes

Vendor 2 decides whether or not to file protest

No

Vendor 1 profit = $S_H X \{P_1 - C_1\}$
Vendor 2 profit = $S_L X \{P_2 - C_2\}$
Buyer cost = $X(S_H P_1 + S_L P_2)$
Above excludes protest costs

Vendor 1 profit = $S_L X \{P_1 - C_1\}$
Vendor 2 profit = $S_H X \{P_2 - C_2\}$
Buyer cost = $X(S_L P_1 + S_H P_2)$
Above excludes protest costs

Buyer perceives $P_1 > P_2$

Yes

Protestor incurs cost $K_P$
Buyer incurs cost $K_B$

Vendor 2 with cost $C_2$ submits bid $P_2$

Vendor perceives $P_1 < P_2$

Vendor 2 decides whether or not to file protest

Yes

No
Revised Vendor Protest Incentives

- **Winner-take-all awards:** Losing vendor 1 protests iff $\text{Prob}(P_1<P_2) \times X - K_P > 0$

- **Split awards:** Losing vendor 1 protests iff $\text{Prob}(P_1<P_2) \times (S_L-S_H)X - K_P > 0$

- Split awards **raise the hurdle** for profitable protest
  – Is the hurdle high enough to limit “bad” protests?
  – Is the hurdle low enough to allow “good” protests?

- **Defacto split awards** already a *response* to protests
  – Alternative contracts, subcontracts, agency settlements, “Fed mail” buy-offs
  – Why not formalize this “under the table” process?
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Key Question: What is the Right Split?

- **Higher \( S_H \) \( \rightarrow \) lower protest incentive**
  - \( \Delta \Pi_1(\text{protest}) = \text{Prob}(P_1 < P_2) \times (1-2S_H)X - K \)
  - \( \frac{\Delta \Pi_1(\text{protest})}{\Delta S_H} = -2X \times \text{Prob}(P_1 < P_2) \)

- **Higher \( S_H \) \( \rightarrow \) higher total contract expense**
  - Winner-take-all cost = \( XP_L \)
  - Split-award cost = \( X(S_H P_H + (1-S_H)P_L) \)
  - Difference = \( XS_H(P_H - P_L) \)

- **Higher \( S_H \) \( \rightarrow \) incentive to submit higher bid**
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- Key question: What is the right split?
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Focus on bid-stage only (for now):

- Ignore “continuation value” of protest stage
  - Effect of protest on bidding strategy ambiguous
- Also ignore buyer’s imperfect information
  - Assume buyer perfectly informed regarding $P_1$ & $P_2$
  - Symmetric imperfect info $\Rightarrow$ neutral impact

**Expected profit function:**

\[
E\Pi_1(P_1) = X(P_1-C_1)[\text{Prob}(P_1>P_2)S_H + \text{Prob}(P_1<P_2)S_L] \\
= X(P_1-C_1)[S_L - \text{Prob}(P_1>P_2)(S_L - S_H)]
\]
Equilibrium Bidding with Fixed Splits

Expected profit function:

- Assume $C_1$, $C_2$ identically & independently distributed over interval $[0,M]$
- Symmetric bidding strategy $\lambda(C)$
  - $\lambda: [0,M] \sim [0,M]$
  - $\lambda(M) = M$

Equilibrium bidding strategy:

- $(C_1) = \frac{S_H M + (S_L S_H)(1 - F(C_1))E(C_2|C_2 > C_1)}{S_L \left(S_L S_H\right)F(C_1)}$
- Complete derivation included in appendix
Equilibrium Bidding with Fixed Splits

- Let $C_1, C_2 \sim U[0,100] \Rightarrow$

\[
(C_1) = \frac{S_H M + (S_L \ S_H)(1 - F(C_1)) E(C_2 \mid C_2 > C_1)}{S_L (S_L \ S_H) F(C_1)}
\]

\[
(C_1) = \frac{100S_H + (S_L \ S_H)(1 - \frac{1}{100}C_1) \frac{1}{2}(C_1 + 100)}{S_L \ 1/100(S_L \ S_H)C_1}
\]

\[
(C_1) = \frac{20,000S_H + (S_L \ S_H)(100-C_1)(C_1 + 100)}{200S_L \ 2(S_L \ S_H)C_1}
\]

\[
(C_1) = \frac{20,000S_H + (1 \ 2S_H)(10,000-C_1^2)}{200S_L \ 2C_1(S_L \ S_H)}
\]

\[
(C_1) = \frac{20,000S_H + 10,000-C_1^2}{200S_L \ 2C_1(S_L \ S_H)} = \frac{20,000S_H + 2S_HC_1^2}{200S_L \ 2C_1(S_L \ S_H)}
\]

\[
(C_1) = \frac{10,000+(2S_H \ 1)C_1^2}{200S_L \ 2C_1(S_L \ S_H)} = \frac{10,000 \ (S_L \ S_H)C_1^2}{200S_L \ 2C_1(S_L \ S_H)}
\]
Equilibrium Bidding with Fixed Splits

\[
S_H = 0 \quad (C_1) = \frac{10,000}{200} \frac{C_1^2}{2C_1} = \frac{(100 C_1)(100+C_1)}{2(100 C_1)} = 50 + \frac{1}{2} C_1
\]

\[
S_H = 0.1 \quad S_L = 0.9 \quad (C_1) = \frac{10,000}{180} \frac{0.8C_1^2}{1.6C_1}
\]

\[
S_H = 0.2 \quad S_L = 0.8 \quad (C_1) = \frac{10,000}{160} \frac{0.6C_1^2}{1.2C_1}
\]

\[
S_H = 0.3 \quad S_L = 0.7 \quad (C_1) = \frac{10,000}{140} \frac{0.4C_1^2}{0.8C_1}
\]

\[
S_H = 0.4 \quad S_L = 0.6 \quad (C_1) = \frac{10,000}{120} \frac{0.2C_1^2}{0.4C_1}
\]

\[
S_H = 0.5 \quad S_L = 0.5 \quad (C_1) = \frac{10,000}{100} \frac{0}{0} = 100
\]
Equilibrium Bidding with Fixed Splits

\[ S_H = 50\% \]
\[ S_H = 40\% \]
\[ S_H = 30\% \]
\[ S_H = 20\% \]
\[ S_H = 10\% \]
\[ S_H = 0\% \] (Winner-Take-All)

\[ C_1 \sim U[0,100] \]
\[ C_2 \sim U[0,100] \]
Average Price / Unit with Fixed Splits

Split Awarded to Higher-Priced Bidder

- 0%: 72
- 10%: 76
- 20%: 81
- 30%: 87
- 40%: 93
- 50%: 100

Average Price / Unit Paid
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Extension: Endogenous Split Awards

- Split awards **reduce frequency** of bid protest 😊
- **BUT 2 cost inflation effects** from split awards 😞
  - Direct additional cost = \( XS_H(P_H - P_L) \)
  - Indirect additional cost = bid inflation
- **Note:** Both inflation effects mitigated if size of \( S_H \) is inversely related to \( (P_H - P_L) \)
- Potential solution: **Endogenous split awards**
  - Let \( R_L = P_L / P_H \) (such that \( 0 \leq R_L \leq 1 \))
  - Let \( S_H = F(R_L) \)
  - \( 0 \leq F(R_L) \leq \frac{1}{2} \)
  - \( F(R_L) \) increasing in \( R_L \)
Example Split Award Function

- Let $S_H = \alpha R_L^\beta$
  - $\alpha = \text{maximum share}$ to high-price bidder $(0 \leq \alpha \leq \frac{1}{2})$
  - $\beta \geq 0$
  - $S_H$ is increasing in $\alpha$ & $R_L$
  - $S_H$ is decreasing in $\beta$

- Buyer decision: What are the best $\alpha$ & $\beta$?
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Better for High Bidder
Worse for Low Bidder

Worse for High Bidder
Better for Low Bidder
Split Award Scenarios with $S_H = \frac{1}{2} R_L^\beta$

![Graph showing the relationship between the share or split for high bidder ($S_H$) and the ratio of low bid to high bid ($R_L = P_L/P_H$). The graph includes lines for various values of $\beta$: $\beta = 0$, $\beta = 1/10$, $\beta = 1/2$, $\beta = 1$, $\beta = 2$, $\beta = 10$, and $\beta = 100$. Each line represents a different share of the award for the high bidder given the ratio of low bid to high bid.]
Split Award Scenarios with $S_H = \frac{2}{5} R_L^\beta$

- $\beta = 0$
- $\beta = 1/10$
- $\beta = 1/2$
- $\beta = 1$
- $\beta = 2$
- $\beta = 10$
- $\beta = \Box$

Ratio of Low Bid to High Bid: $R_L = \frac{P_L}{P_H}$
From Fixed Splits to Endogenous Splits

- Recall that the **equilibrium bidding strategy** under **fixed splits** of $S_H = 0.4$ & $S_L = 0.6$ with $C_1, C_2 \sim U[0,100]$ was given by:
  
  $$P_j = (C_j) = \frac{10,000 - 0.2C_j^2}{120 - 0.4C_j}$$

  - In equilibrium, this yielded an expected **price per unit** of 93

- Now, consider the following **endogenous split award function**:
  
  - $S_H = \alpha R_L^\beta$ with $\alpha = \frac{1}{2}$ & $\beta = 4$
  - $S_H = \frac{1}{2}R_L^4$

- If both vendors continue to bid according to the above fixed-split equilibrium bidding strategy, we have:
  
  - Average split (average value of $S_H = \frac{1}{2}R_L^4$) = 0.4
  - Median split (median value of $S_H = \frac{1}{2}R_L^4$) = 0.4

  - Thus, “**apples-to-apples” comparison** to compare bidding under these two award rules (one fixed, one endogenous)
Split Award Scenarios with $S_H = \alpha R_L^\beta$

\[ R_L = \frac{P_L}{P_H} \]

Note: If vendors bid according to fixed-price bidding strategy, average & median split will be the same under either split rule.
If vendors follow fixed-split bidding strategy for $S_H = 0.4$, expected & median values of the endogenous split should still be $S_H = 0.4$.

- But is this strategy still optimal when splits are endogenous?

So, when contracts splits are endogenous & given by $S_H = \frac{1}{2}R_L^4$:

- What is the equilibrium bidding strategy?
- What is the average price per unit paid by the buyer?

We answered these questions computationally

- Closed-form solution to equilibrium calculation is problematic
- Thus, solve via “iterative best-response”
  1. Start: Assume vendor 1 follows given fixed-price bid strategy
  2. Compute: What is vendor 2’s best-response bidding strategy?
  3. Iterate: What is vendor 1’s best-response to 2’s best-response?
  4. Repeat: Until you reach a “fixed-point” solution
Equilibrium Bidding with Endogenous Splits

\[ S_H = \frac{1}{2} R_L^4 \]

\( S_H = 40\% \)

\( S_H = 0\% \) (Winner-Take-All)

\( C_1 \sim U[0,100] \)

\( C_2 \sim U[0,100] \)
Average Price/Unit with Different Splits

Split Awarded to Higher-Priced Bidder

- 0%
- $\frac{1}{2}R_L^4$
- 40%

Average Price / Unit Paid:

- 72
- 81
- 93
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Bid Protests & Split Awards: Conclusions

- Objective is to **manage**, not minimize protests
  - Encourage protests that correct (significant) mistakes
  - Discourage protests that do not

- **Split awards** are lever for protest management
  - Raise the hurdle for profitable protest
  - Filters out unmerited protests more than merited

- Challenge is determining the **right split**
  - Higher split to 2\textsuperscript{nd}-vendor reduces protest incentive
  - BUT higher 2\textsuperscript{nd}-vendor split also increases costs
  - Higher **fixed** 2\textsuperscript{nd}-vendor split induces bid inflation

- **Endogenous** split awards offer potential solution
  - Retains protest “filtering” benefits
  - Reduces inflation of bids & average price paid
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Conclusions

- Research agenda moving forward
Research Agenda Moving Forward

- **Research questions:**
  - What is the *optimal split award function*?
    - Minimize expected and/or long-term buyer cost
    - Including cost of protests & corrective benefit of protests
    - Include impact of other benefits of split awards
  - What is the impact of *changes in key variables*?
    - Vendor & buyer information, costs of protest, etc.
  - What is the impact of *repeated procurements*?
    - Inter-temporal effects: Experience & innovation

- **Research methodology:**
  - Closed-form *game-theoretic solutions* & dynamics
  - Numerical *computation* & simulation
Endogenous Split Awards as a Protest Management Tool: A Modeling & Computational Approach

Appendix: Equilibrium Bid Strategy with Fixed-Splits

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Temporary Simplifying Assumptions

- For now, ignore “continuation value” of protest stage
  - Effect of protest on bidding strategy ambiguous
- For now, also ignore buyer’s imperfect information
  - Assume buyer perfectly informed regarding $P_1$ & $P_2$
  - Symmetric imperfect info $\implies$ neutral impact
Expected Profit Function (Bid Stage)

\[ E\Pi_1(P_1) = X(P_1-C_1)[\text{Prob}(P_1>P_2)S_H + \text{Prob}(P_1<P_2)S_L] \]
\[ = X(P_1-C_1)[\text{Prob}(P_1>P_2)S_H + [1-\text{Pr}(P_1>P_2)]S_L] \]
\[ = X(P_1-C_1)[S_L + \text{Prob}(P_1>P_2)(S_H-S_L)] \]
\[ = X(P_1-C_1)[S_L - \text{Prob}(P_1>P_2)(S_L-S_H)] \]
Cost Distribution & Bidding Strategy

- Assume $C_1$, $C_2$ identically & independently distributed over interval $[0,M]$
  - Distribution function $F$
  - Density function $f = F'$

- Symmetric bidding strategy $\lambda(C)$
  - $\lambda: [0,M] \sim [0,M]$
  - $\lambda(M) = M$
Equilibrium Bidding with Fixed Splits

- Calculate optimal bid $P_1$ for vendor 1 assuming:
  - Vendor 1 has cost $C_1$
  - Vendor 2 is bidding according to strategy $\lambda(C_2)$

- $\text{Prob}(P_2 < P_1) = \text{Prob}[\lambda(C_2) < P_1] = \text{Prob}[C_2 < \lambda^{-1}(P_1)]$
  \[ = F(\lambda^{-1}(P_1)) \]

- $\mathbb{E}\Pi_1(P_1) = X(P_1 - C_1)[S_L - \text{Prob}(P_1 > P_2)(S_L - S_H)]$
  \[ = X(P_1 - C_1)[S_L - F(\lambda^{-1}(P_1))(S_L - S_H)] \]

- Chain rule + inverse derivative theorem
  \[ \frac{\delta\mathbb{E}\Pi_1}{\delta P_1} = X[S_L - F(\lambda^{-1}(P_1))(S_L - S_H)] \]
  \[ - X(P_1 - C_1)(S_L - S_H)f(\lambda^{-1}(P_1))/\lambda'(\lambda^{-1}(P_1)) \]}
Equilibrium Bidding with Fixed Splits

- First-order condition

\[ S_L - F(\lambda^{-1}(P_1))(S_L - S_H) = (P_1 - C_1)(S_L - S_H)f(\lambda^{-1}(P_1))/\lambda'(\lambda^{-1}(P_1)) \]

\[ \lambda'(\lambda^{-1}(P_1))[S_L - F(\lambda^{-1}(P_1))(S_L - S_H)] = (P_1 - C_1)(S_L - S_H)f(\lambda^{-1}(P_1)) \]

- At symmetric equilibrium, \( P_1 = \lambda(C_1) \Rightarrow \lambda^{-1}(P_1) = C_1 \Rightarrow \)

\[ \lambda'(C_1)[S_L - F(C_1)(S_L - S_H)] = (\lambda(C_1) - C_1)(S_L - S_H)f(C_1) \]

\[ S_L \lambda'(C_1) = (S_L - S_H)[F(C_1)\lambda'(C_1) + \lambda(C_1)f(C_1) - C_1f(C_1)] \]

\[ (S_L - S_H)[F(C_1)\lambda'(C_1) + f(C_1)\lambda(C_1)] = S_L \lambda'(C_1) + C_1(S_L - S_H)f(C_1) \]

\[ \left(\frac{S_L}{S_H}\right)_{C_1} F(C_1) \left(\frac{C_1}{C_1}\right) = S_L \left(\frac{C_1}{C_1}\right) + C_1(S_L - S_H)f(C_1) \]
Equilibrium Bidding with Fixed Splits

\[
\begin{align*}
(S_L & \quad S_H)_{\frac{C_1}{C_1}} \quad F(C_1) \quad (C_1) = S_L \quad (C_1) + C_1(S_L \quad S_H)f(C_1) \\
(S_L & \quad S_H)_{C_1}^{M} \quad F(C_1) \quad (C_1) \quad dC_1 \\
& = S_L \quad (C_1) dC_1 + (S_L \quad S_H)_{C_1}^{M} \quad C_1 f(C_1) dC_1 \\
(S_L & \quad S_H) \quad F(M) \quad (M) \quad F(C_1) \quad (C_1) \\
& = S_L \quad (M) \quad (C_1) + (S_L \quad S_H)_{C_1}^{M} \quad C_1 f(C_1) dC_1 \\
(S_L & \quad S_H) \quad M \quad F(C_1) \quad (C_1) = S_L \quad M \quad (C_1) + (S_L \quad S_H)_{C_1}^{M} \quad C_1 f(C_1) dC_1 \\
S_H M & \quad (S_L \quad S_H)_{C_1}^{M} \quad C_1 f(C_1) dC_1 = (S_L \quad S_H)F(C_1) \quad (C_1) \quad S_L \quad (C_1) \\
S_H M + (S_L & \quad S_H)_{C_1}^{M} \quad C_1 f(C_1) dC_1 = (C_1) S_L \quad 1 \quad F(C_1) + S_H F(C_1) \quad (C_1)
\end{align*}
\]
Equilibrium Bidding with Fixed Splits

\[ S_{HM} + (S_L \quad S_H) \int_{c_1}^{M} C_1 f(C_1) \, dC_1 = (C_1)S_L \left(1 - F(C_1) \right) + (C_1)S_H F(C_1) \]

\[ S_{HM} + (S_L \quad S_H) \left(1 - F(C_1) \right) \int_{c_1}^{M} C_1 f(C_1) \, dC_1 \]

\[ = (C_1)S_L \left(1 - F(C_1) \right) + S_H F(C_1) \]

\[ S_{HM} + (S_L \quad S_H) \left(1 - F(C_1) \right) \mathbb{E}(C_2 \mid C_2 > C_1) \]

\[ = (C_1)S_L \quad S_L F(C_1) + S_H F(C_1) = (C_1)S_L \quad (S_L \quad S_H) F(C_1) \]

\[ (C_1) = \frac{S_{HM} + (S_L \quad S_H) \left(1 - F(C_1) \right) \mathbb{E}(C_2 \mid C_2 > C_1)}{S_L \quad (S_L \quad S_H) F(C_1)} \]
Equilibrium Bidding with Fixed Splits

Let $C_1, C_2 \sim U[0,100]$ →

$$(C_1) = \frac{SHM + (S_L \quad S_H) (1-F(C_1)) E(C_2|C_2 > C_1)}{S_L (S_L \quad S_H) F(C_1)}$$

$$(C_1) = \frac{100S_H + (S_L \quad S_H) (1- \frac{1}{100}C_1) \frac{1}{2}(C_1 + 100)}{S_L \frac{1}{100} (S_L \quad S_H) C_1}$$

$$(C_1) = \frac{20,000S_H + (S_L \quad S_H) (100-C_1) (C_1 + 100)}{200S_L \quad 2(S_L \quad S_H) C_1}$$

$$(C_1) = \frac{20,000S_H + (1 \quad 2S_H) (10,000-C_1^2)}{200S_L \quad 2C_1(S_L \quad S_H)}$$

$$(C_1) = \frac{20,000S_H + 10,000-C_1^2}{200S_L \quad 2C_1(S_L \quad S_H)} + \frac{20,000S_H + 2S_H C_1^2}{200S_L \quad 2C_1(S_L \quad S_H)}$$

$$(C_1) = \frac{10,000 + (2S_H \quad 1)C_1^2}{200S_L \quad 2C_1(S_L \quad S_H)} = \frac{10,000 \quad (S_L \quad S_H) C_1^2}{200S_L \quad 2C_1(S_L \quad S_H)}$$
Equilibrium Bidding with Fixed Splits

\[ \text{SH} = 0 \quad (C_1) = \frac{10,000}{200} \frac{C_1^2}{2C_1} = \frac{(100 - C_1)(100 + C_1)}{2(100 - C_1)} = 50 + \frac{1}{2} C_1 \]

\[ \text{SH} = 0.1 \quad S_L = 0.9 \quad (C_1) = \frac{10,000}{180} \frac{0.8C_1^2}{1.6C_1} \]

\[ \text{SH} = 0.2 \quad S_L = 0.8 \quad (C_1) = \frac{10,000}{160} \frac{0.6C_1^2}{1.2C_1} \]

\[ \text{SH} = 0.3 \quad S_L = 0.7 \quad (C_1) = \frac{10,000}{140} \frac{0.4C_1^2}{0.8C_1} \]

\[ \text{SH} = 0.4 \quad S_L = 0.6 \quad (C_1) = \frac{10,000}{120} \frac{0.2C_1^2}{0.4C_1} \]

\[ \text{SH} = 0.5 \quad S_L = 0.5 \quad (C_1) = \frac{10,000}{100} \frac{0}{0} = 100 \]
Endogenous Split Awards as a Protest Management Tool: A Modeling & Computational Approach

Unused Back-Up Slides

Peter J. Coughlan
William Gates
Graduate School of Business & Public Policy
Naval Postgraduate School
Bid Protests of Growing Concern

The rising problem of bid protests

Why large contracts are being sidelined

By ELISE CASTELLI
ecastelli@federaltimes.com

When the General Services Administration announced 29 winners of the $50 billion Alliant contract in 2007, agency leaders heralded it as the government's premier contract for information technology purchases.

With its expansive scope, access to the best in class in the private sector and ability to provide customized solutions tailored to agencies' unique IT needs, we can again prove that GSA is at the forefront of serving the acquisition needs of the federal government," GSA's Federal Acquisition Service Commissioner James Williams declared at the time. But 18 months later, a lot has changed. The contract was held up because of bid protests from several firms that didn't make the cut. A federal court ordered GSA to re-evaluate all bidders. And it wasn't until two weeks ago that GSA got the giant Alliant contract back on track by awarding it to 50 companies.

Alliant is one of a few high-profile, high-value procurements — another is the Air Force's tanker contract — that have been waylaid by protests in recent years. While these large procurements all get the attention, most bid protests concern smaller contracts.

Overall, the protests rose 44 percent since 2001 — in part because companies were recently allowed to protest not only final decisions. See PROTESTS, Page 19

ON THE RISE

The number of bid protests lodged each year has increased considerably since 2001:

<table>
<thead>
<tr>
<th>Year</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>1,146</td>
</tr>
<tr>
<td>2005</td>
<td>1,652</td>
</tr>
</tbody>
</table>

Hundreds of buildings across the country will be going green in the next year or two under the General Services Administration's plans for spending more than $5.5 billion in stimulus funds.

Many will benefit from features such as advanced meters to improve monitoring of electricity and water use, lighting controls and sensors that turn off lights when not needed, new or improved heating and air-conditioning systems, and renewable...
DoD Bid Protest Trends

- % Merit
- % Sustain
- # of Protests

2001: 603
2002: 667
2003: 704
2004: 731
2005: 706
2006: 739
2007: 775
2008: 838

- Merit Trends: 29% 24% 26% 29% 22% 20% 26% 16%
- Sustain Trends: 7% 3% 5% 5% 4% 5% 8% 4%
- Protests: 2001 - 2008
Vendor Protest Incentives

- Expected **Profit** from Protest
  - $\text{Expected Profit} = \text{Expected Benefits} - \text{Expected Costs}$

- Expected **Costs**
  - $\text{Expected Costs} = K$
  - $K = \text{Research} + \text{Legal} + \text{Reputation} + \text{Opportunity Costs}$

- Expected **Benefits**
  - $\text{Expected Benefits} = \text{Probability of Success} \times \text{Gain if Successful}$

- **Gain** if Successful
  - $\text{Gain if Successful} = \text{Contract Revenue} = X$

- **Probability** of Success
  - $\text{Probability of Success} = \text{Prob}(P_1 < P_2) \text{ given that buyer perceived } P_1 > P_2$

- Expected Profit from Protest
  - $\text{Expected Profit from Protest} = \text{Prob}(P_1 < P_2) \times X - K$
Modeling Buyer Imperfect Information

- Let $R_1 = \frac{P_1}{(P_1 + P_2)}$ & $R_2 = \frac{P_2}{(P_1 + P_2)}$
  - $0 \leq R_1 \leq 1$ & $0 \leq R_2 \leq 1$
  - $R_1 + R_2 = 1$

- Let $r_1 =$ buyer’s estimate of $R_1$
  - $r_1 = r / N$ where $r \sim \text{Bin}(N, R_1)$
  - Binominal with $N$ draws & success probability $= R_1$
  - Higher $N \Rightarrow$ more accurate estimate of $R_1$

- Let $r_2 =$ buyer’s estimate of $R_2$
  - $r_2 = 1 - r_1$
Perceived Probability of Protest Success

- Assume buyer discloses estimate $r_1$
  - $r_1 < \frac{1}{2} \implies$ vendor 1 wins
  - $r_1 > \frac{1}{2} \implies$ vendor 2 wins

- If vendor 1 loses, his estimate of the probability of a successful protest is:
  - $\text{Prob}(P_1 < P_2)$ given that buyer perceives $P_1 > P_2$
  - $\text{Prob}(R_1 < \frac{1}{2})$ given that buyer estimates $R_1$ at $r_1$
  - $\text{Prob}(R_1 < \frac{1}{2})$ given $N_{r_1}$ successes from $\text{Bin}(N, R_1)$
  - $\text{Prob}(R_1 < \frac{1}{2} \mid N_{r_1}$ out of $N)$
Perceived Probability of Protest Success

\[ \text{Prob}(R_1 < \frac{1}{2} | \text{Nr}_1 \text{ out of N}) \]

\[ = \frac{\text{Prob}(R_1 < \frac{1}{2}) \text{Prob}(\text{Nr}_1 \text{ out of N} | R_1 < \frac{1}{2})}{\text{Prob}(\text{Nr}_1 \text{ out of N})} \]

\[ = \frac{\int_0^{\frac{1}{2}} \text{Prob}(z) \text{Prob}(\text{Nr}_1 \text{ out of N} | R_1 = z) \, dz}{\int_0^1 \text{Prob}(z) \text{Prob}(\text{Nr}_1 \text{ out of N} | R_1 = z) \, dz} \]

\[ = \frac{\int_0^{\frac{1}{2}} \text{Prob}(z) \binom{N}{\text{Nr}_1} z^{\text{Nr}_1} (1 - z)^{N(1-\text{Nr}_1)} \, dz}{\int_0^1 \text{Prob}(z) \binom{N}{\text{Nr}_1} z^{\text{Nr}_1} (1 - z)^{N(1-\text{Nr}_1)} \, dz} \]

where \( \text{Prob}(z) \) reflects vendor 1's prior probability distribution of \( R_1 \).
Extension: Repeated Procurements

- What are the other benefits of split awards?
  - Why are split awards used currently?
- Split awards preserve competition for repeated or follow-on procurements

**Direct modeling implications:**
- Appropriate to model as repeated bidding game
- Implies presence of learning/experience effects

**Indirect modeling implications:**
- Incorporate innovation to avoid trivial outcomes
- Innovation driven by “shocks” or investment
Extension: Repeated Procurements

- Learning/Experience Effects
- Investment & Innovation
- Discounting Future Periods