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Predicting the Shadow in an Oblique View of a Rectangular Target

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ABSTRACT

This note describes a method for predicting the sun shadow produced by a rectangular ground or sea target and visible in an arbitrary oblique view.

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Predicting the Shadow in an Oblique View of a Rectangular Target

Executive Summary

When land or sea targets are sought in an oblique view of a sunlit scene, their shadows may be their most conspicuous features. It is therefore useful to predict how the shadow of a target will appear in the image, given its shape and orientation and the positions of the sun and the camera.

This note considers the prediction of the visible shadows of a simple but representative target shape, the rectangular block, including those cast on the ground or water and those formed on the walls of the block. It gives a complete analysis of the method, and includes a simple method for predicting the sun's position.

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Robert Whatmough was employed as Cadet Defence Science at the Weapons Research Establishment, Department of Supply, later DSTO. He was awarded the degree of B Sc with honours in 1969.

Until 1985 he was Experimental Officer, Scientific Officer then Research Scientist in Computing Services Group. His work included mathematical data handling techniques, random number generation, printed circuit board design, simulation of batch computer operations, curve fitting and smoothing, time-critical computing, thermal modelling, computer graphics and display, remote sensing and image processing, and assistance of computer users with complex computing problems.

Since 1986 he has worked in various Divisions in fields related to image processing. These have included restoration, enhancement and classification of visual, multispectral and synthetic aperture radar images, geographic information systems, prediction of oblique aerial and navigation radar images, matching models to objects in images, shape inference from perspective distortion and video sequences, registration and mosaicking of aerial images, and enhancement of video sequences.

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1. Introduction

When land or sea targets are sought in an oblique view of a sunlit scene, their shadows may be their most conspicuous features. It is therefore useful to predict how the shadow of a target will appear in the image, given its shape and orientation and the positions of the sun and the camera and assuming an orthographic projection appropriate to a distant camera.

This note considers a simple but representative target shape, the rectangular block. The shadow areas include those cast on the ground or water and those on the sides of the block that face away from the sun. Some or all of the shadows can be hidden from the camera by the block itself.

In the rest of this note, Section 2 describes a method for predicting the sun's position if this is not given. Section 3 describes the shadow prediction for the ground or sea areas, and Section 4 describes the method for the block walls.

2. Sun position

For prediction of sun shadows, the azimuth and elevation of the sun are required. The most accurate predictions would take account of interacting planetary orbits and precession of the Earth's rotation, but it will be sufficient here to consider a simpler second-harmonic approximation as used in ground temperature predictions (eg Carlson & Roland, 1978). The main assumptions are:

1. The Earth's orbital period is 365.2425 days, so the movements repeat after 400 years including the 97 leap years of the Gregorian calendar system (in use since October 1582).
2. The sun's position for an earth that rotates once per orbit is adequately predicted by a second-harmonic approximation, which allows for the eccentricity of the orbit. The result may be expressed as the declination (angle north or south of the equatorial plane) and the time of noon.
3. The true position is then found by a further rotation at a steady rate of once every 24 hours.
4. The bending of the rays by the atmosphere when the sun is low is ignored. (It raises the position slightly and lengthens the day.)

The position in the orbit, expressed as a number of days, is the sum of

1. 365 times the year number (assumed positive)
2. 1 for each leap year before the present year within the 400 year cycle
3. The number of days in earlier months of the current year, allowing for a possible leap year
4. The day of the month
5. $1/24$ times the time of day in hours, reduced by the time zone number (hours ahead of UTC)

6. A constant, approximately 1/24, determined by adjustment to predict the correct times of equinoxes

The position is scaled by 360/365.2425 to express it as an angle θ in degrees, modulo 360.

The declination D and the time of noon expressed as an angle ϕ_n are now approximately

$$D = \sin^{-1} \left\{ \sin 23.4417 \sin \left(\theta + 279.9348 + 1.914827 \sin \theta - 0.079525 \cos \theta \right) \right. \\ \left. + 0.019938 \sin 2\theta - 0.00162 \cos 2\theta \right\}$$

$$\phi_n = 15 \left(12 + 0.12357 \sin \theta - 0.004289 \cos \theta \right. \\ \left. + 0.153809 \sin 2\theta + 0.060783 \cos 2\theta + t_z \right) - \phi.$$

Here t_z is the time zone number and ϕ is the longitude. All angles are in degrees and all times are in hours.

The sun time expressed as an angle in degrees is $\phi_r = 15t - \phi_n$, where t is the time of day in hours. The direction cosines for the sun eastwards, northwards and upwards are

$$x_E = -\cos D \sin \phi_r$$

$$x_N = \sin D \cos \lambda - \cos D \cos \phi_r \sin \lambda$$

$$x_U = \sin D \sin \lambda + \cos D \cos \phi_r \cos \lambda.$$

The sun azimuth and elevation are then

$$A = \tan^{-1} \frac{x_E}{x_N}$$

$$E = \sin^{-1} x_U.$$

More elaborate methods that take into account planetary interactions, refraction and parallax due to observation from the earth's surface instead of its centre are described in Montenbruck & Pflieger (1991).

3. Shadows on the ground or sea

3.1 Problem definition and solution approach

Figure 1 illustrates the difficulties of predicting the shadow cast by a rectangular block. There are two views of the same target at the same time, differing only by a 180 degree change in camera azimuth. The visibility of the shadow is much less in the second view because it is partly occluded by the target. Evidently the shadow area is polygonal, but the number of sides varies erratically as the sun and camera move.

The problem to be solved can now be stated exactly:

“Given the direction of a line parallel to the length of a rectangular block target on a horizontal supporting plane, its length, breadth and height, the azimuth and elevation of the sun and the azimuth and elevation of the camera, describe the camera's orthographic view of the visible shadow of the target on the supporting plane as the vertices of a polygon.”

Some useful simplifying assumptions are now made:

1. The sun is a point source at infinity.
2. If the sun is in the plane of a target wall, it casts no shadow at the foot of the wall.
3. If the sun is vertically overhead it is in the planes of all walls and there is no shadow.
4. If the sun elevation is zero or negative there is no shadow.
5. The camera has no swing (ie, upward vertical lines in the scene are upward vertical in the image). (This assumption can be removed by adding a final step to the prediction process, before any matters of camera resolution are dealt with.)
6. The length of the target has been rotated to the east-west direction and the corresponding changes have been made to the sun and camera azimuths, with no effect on the resulting view. (No assumption need be made about the relative sizes of "length" and "breadth"; these are a matter of convenience.)

The problem can be decomposed into three simpler problems, two of them analogous, as follows:

1. The shadow areas visible to the camera (obliquely) are those that are occluded by the target when viewed from the sun, and not occluded by the target when viewed from the camera. More precisely, the set of visible shadow points in the supporting plane is $S_p = O_s \setminus O_c$, where O_s contains the points occluded from the sun and O_c those occluded from the camera.
2. If points under the target are included in both sets, then each of O_s and O_c is a closed convex polygon, as will be confirmed below, and each contains every interior point of the set under the target, and in particular the point C under its centre, as an interior point.
3. The two sets can be found by a single method, described in the next subsection, using the sun position in one case and the camera position in the other.
4. S_p can be then found from O_s and O_c by using C as the origin, as described in subsection 3.3.
5. C can also be assumed projected to the origin of the image plane.

3.2 Finding the shadow and occlusion polygons

Figure 2 shows the 11 usefully distinguishable cases of sun shadow in the set O_s , viewed vertically. The sun can be vertical, shine parallel to two walls in four ways, or illuminate two walls in four ways. In all cases the set is a closed convex polygon. Each vertex on the ground is at a target corner or is the shadow of a corner of the top of the target, displaced horizontally away from the sun azimuth by $(\text{target height})/\tan(\text{sun elevation})$.

Two of the nine cases can be further subdivided according to where the vertices fall in relation to the north direction. In each of the resulting 11 cases, the vertex positions relative to the target centre can be listed in order clockwise from north at C without any need for sorting, following the numbering in the figure.

The points occluded from the camera, in O_C , can be found in the same way and again listed as vertex positions in order clockwise from north at C .

3.3 Finding the visible shadow set

Representing O_S and O_C by lists of vertices in order from a common starting direction (with zero azimuth) makes the construction of S_p efficient. The first step in this construction is to make the azimuths of the vertices of the two polygons coincide, by adding redundant vertices on the sides wherever necessary.

The azimuths of the vertices from C are computed. Since C is an interior point of each polygon, each list of azimuths is circular and implies connections right around C . The two lists are merged. When an azimuth appears for one polygon and not the other, a dummy vertex is inserted at the same azimuth on the other polygon.

The calculation for insertion is a special case of the following. Given two non-parallel line segments (x_1, y_1) to (x_2, y_2) and (x_3, y_3) to (x_4, y_4) , there is a point of intersection (possibly requiring extension) at

$$(x_1, y_1) + \lambda(x_2 - x_1, y_2 - y_1) = (x_3, y_3) + \mu(x_4 - x_3, y_4 - y_3)$$

where λ, μ are found from

$$\begin{bmatrix} x_2 - x_1 & x_3 - x_4 \\ y_2 - y_1 & y_3 - y_4 \end{bmatrix} \begin{bmatrix} \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} x_3 - x_1 \\ y_3 - y_1 \end{bmatrix}. \quad (1)$$

For insertion, (x_3, y_3) is a vertex point and (x_4, y_4) is the origin, but (1) still applies.

Once the merge has been completed, there is a combined list which gives, for each azimuth in a common sequence, one original or inserted vertex at that azimuth for each polygon. From this list, S_p can be determined as follows:

1. Search the combined list for an azimuth at which the O_S vertex is further from the origin than the O_C vertex. (If there is none, then no shadow is visible.)
2. Search anticlockwise for the first azimuth at which the O_S vertex is not further from the origin than the O_C vertex. If the vertices are at the same distance they are the same point; start a list of output vertices with this point. Otherwise, the range of exposed shadow as a function of azimuth became non-zero at an intermediate point on at least one polygon; this intermediate point can be found by applying (1) to find the intersection of two sides, one from each polygon; start the list of output vertices with the intersection.
3. Search clockwise for the next azimuth at which the O_S vertex is again not further from the origin than the O_C vertex. Add to the list of output vertices any original (not inserted) vertex of O_S passed on the way. Find either a vertex common to the polygons, or an intersection of sides, and add it to the list as well.

4. Step back through the azimuths again to the one found in step 2. Add to the list of output vertices any original vertex of O_C passed on the way.

The above procedure lists the vertices of S_p in a clockwise sequence. Figure 3 illustrates a representative case of its use, in vertical view. The target (thick line) occludes points as far as the solid line from the sun and points as far as the dotted line from the camera. Each of the lines radiating from the target centre C represents an azimuth at which there is an original vertex common to both polygons (closed square) or an original vertex in one polygon (closed circle) and an inserted vertex in the other (open circle). Step 1 finds the original vertex S of O_S beyond an inserted vertex of O_C . Step 2 finds the common vertex P and puts it in the output list. Step 3 adds vertices Q, R and S to the output list, locates the intersection T and puts that in the list. Step 4 finds the vertex P again and puts U and V in the list on the way. The list now holds the sequence PQRSTUV, the vertices of the required polygon for S_p .

The final step of shadow prediction for the supporting surface is simply to project the visible shadow areas to the image plane, suitably oriented for the given camera position. The projection is a rotation followed by vertical scaling by the sine of camera elevation, and full scaling according to the focal length.

4. Shadows on the target walls

Predicting visible shadows on the target walls is far simpler than predicting them on the ground or sea. Each wall is either sunlit or not (with edge-on sunlight counted as shadow) and either visible or not (with edge-on viewing counted as no visibility). The camera can see either one wall only, or two adjacent walls, so the visible shaded wall area may be no wall, one wall or two adjacent walls.

The prediction method is then as follows:

1. Rotate the length of the target to the east-west direction and make the corresponding change to the sun and camera azimuths, as in the ground or sea prediction.
2. Check the visible-shadow condition for each wall.
3. Find a pair of adjacent walls with visible shadow, or failing that, a single such wall. List their corner positions in three dimensions in a circuit around one or two walls. If no visible shaded wall is found, return an indication of no visible shadow.
4. Project the corner positions to the image plane and scale them according to focal length.

5. Discussion and Conclusions

This note has described how to predict the shadows on the ground or sea and on the walls around a rectangular target in the form of polygons in the image plane in the absence of camera swing. Further analysis of the visibility of shadows requires knowledge of the

swing (if any) and of the horizontal and vertical resolution of image formation. The decision must be made which of the two shadow categories will affect the detection - it may depend on the target and substrate colours.

6. References

Carlson, T.N. & Roland, F.E. (1978). "Analysis of Urban-Rural Canopy Using a Surface Heat-flux/Temperature Model", *Journal of Applied Meteorology*, 17, 998-.

Montenbruck O. & Pflieger T. (1991). "Astronomy on the Personal Computer". (Dunlop S. translator.) Springer-Verlag, Berlin.

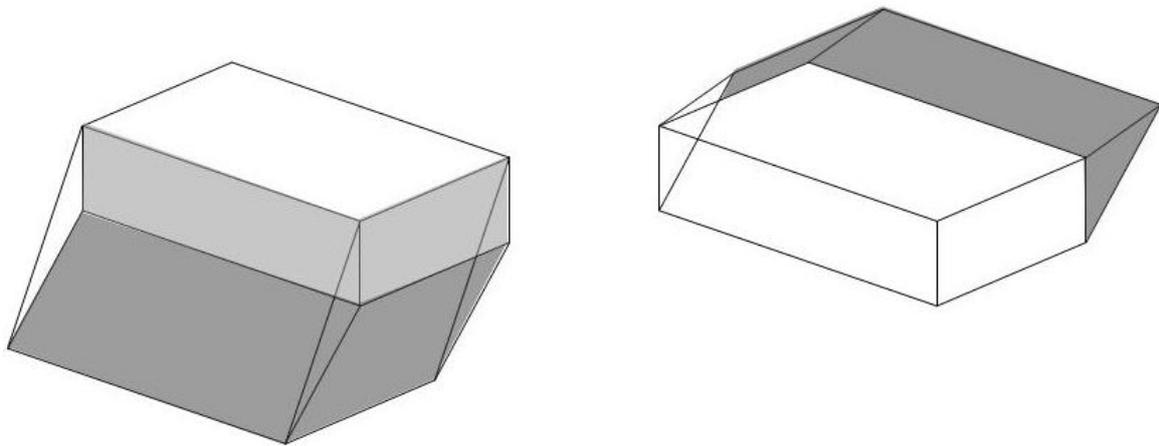


Figure 1. Two orthographic views of a sunlit rectangular block target. Only the azimuths of the camera positions are different, by 180 degrees, but occlusion by the target greatly affects the amount of visible shadow on the ground and walls.

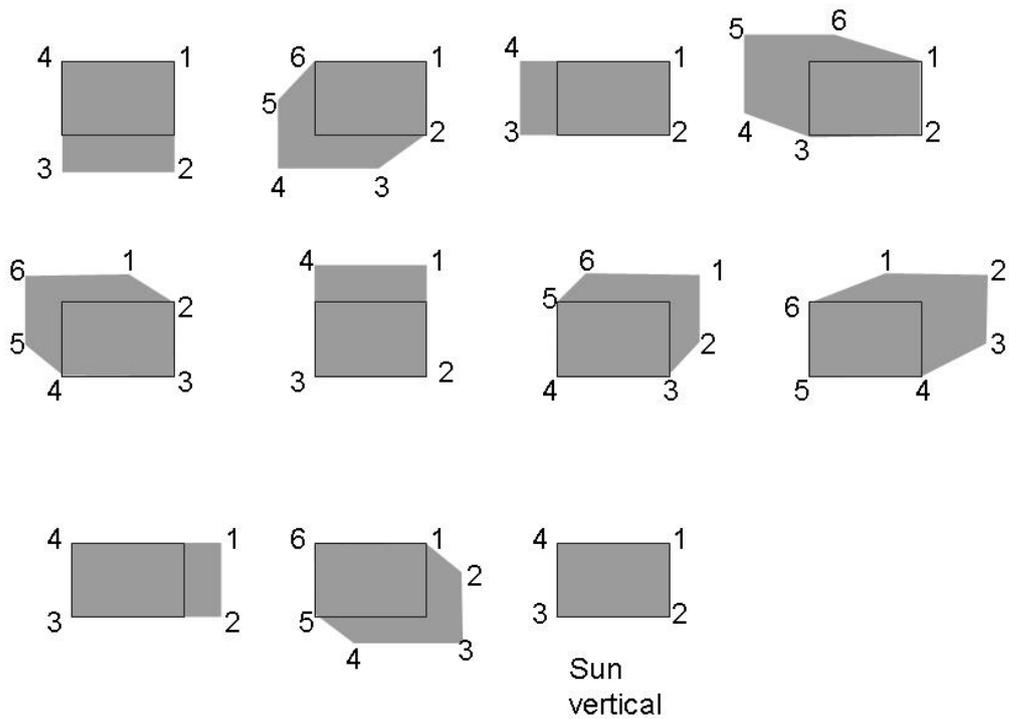


Figure 2. The 11 cases of the area occluded from the sun, with polygon vertices numbered clockwise from north measured from the target centre.

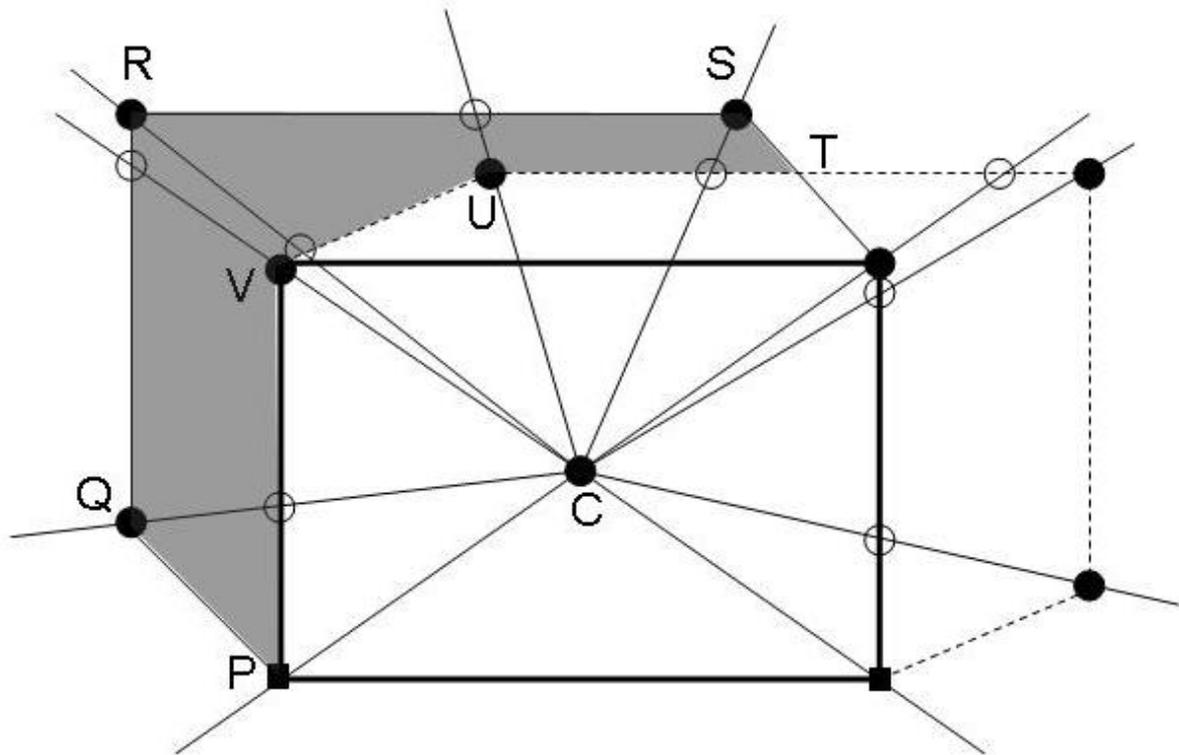


Figure 3. Finding the visible shadow from the polygonal areas occluded from the sun and from the camera by the method in Section 3.3. (See description in text.)

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