On the Performance Evaluation of Query-Based Wireless Sensor Networks

Guvenc Degirmenci[1] and Jeffrey P. Kharoufeh[2]
Department of Industrial Engineering
University of Pittsburgh
1048 Benedum Hall
3700 O'Hara Street
Pittsburgh, PA 15261 USA

and

Rusty O. Baldwin[3]
Department of Electrical and Computer Engineering
Air Force Institute of Technology
2950 Hobson Way (AFIT/ENG)
Wright Patterson AFB, OH 45433 USA

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Abstract

We present a queueing-theoretic framework to evaluate the performance of large-scale, query-based wireless sensor networks whose nodes detect and advertise significant events that are useful for only a limited time; queries generated by the nodes of the network are also time-limited. The main performance parameter is the steady state proportion of generated queries that fail to be answered on time. Using an infinite transmission range model, we first provide an approximation for this parameter that is insensitive to the size of the network. Subsequently, we approximate the proportion of failed queries when the transmission range is limited and show that this proportion converges to its infinite range counterpart as the sensor transmission range tends to infinity. The analytical approximations are shown to be remarkably accurate when compared with benchmark values obtained using a commercial network simulator.

1 Introduction

This paper evaluates the performance of large-scale, query-based wireless sensor networks (WSNs) whose sensors detect and advertise significant events that are useful for only a limited time before they expire (e.g., detecting hazardous biological agents, military surveillance, environmental monitoring, etc.). Event expiration times are established to ensure that sensor nodes have the most up-to-date information to share with other nodes in the network. Query-based WSNs derive their name from the fact that communication between nodes is either event- or query-driven. That is, either the witnessing of an event (e.g., a sudden increase in temperature), or the generation of a query (e.g., a request for the temperature reading at a distant region of the network) triggers communication between nodes which must act as routers for other nodes' packets due to a limited

[1]Email: gdegirmenci@gmail.com.
[2]Corresponding author. Email: jkharouf@pitt.edu; Ph: (412) 624-9832.
[3]Email: rusty.baldwin@afit.edu; Ph: (937) 255-3636 ext. 4445.

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14. ABSTRACT
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sensor transmission range. A query, which itself has a limited lifetime, traverses the network according to a two-dimensional random walk until it either locates the desired information or expires. For this type of network, the total proportion of generated queries that are not answered prior to their expiration time is a critical performance parameter. We devise simple analytical approximations for this quantity, along with other network quality-of-service measures, within a queueing framework. Our analytical approach is unique in that it explicitly accounts for the realism of limited event and query lifetimes which are generally distributed.

Large-scale wireless sensor networks are emerging in such diverse applications as ecological and environmental monitoring, structural health monitoring of aging infrastructure, industrial process control and military surveillance. The ever-increasing interest in WSNs stems from their ability to sense and convey critical information about objects, their surroundings, and the interaction between the two autonomously. Large-scale WSNs are composed of thousands, or hundreds of thousands, of low-cost sensing devices, typically linked via a wireless channel, that cooperate to perform specific network tasks in a distributed manner. Due to their small physical dimensions, the sensing nodes have very limited energy reserves, local memory, and computational capabilities. Moreover, to conserve power and alleviate contention for access to the transmission medium, each node's transmission range may be limited to that required to ensure a connected network.

Wireless sensor networks have been analyzed from a variety of perspectives including design considerations, routing protocols, and resource management strategies, to name only a few. Some useful survey papers related to WSN sensing tasks, applications, design issues, and communications architectures include Akyildiz et al. [5], Yick et al. [37], and Dietrich and Dressler [17]. Owing to the fact that sensor nodes are energy-constrained, defining WSN lifetime and operating policies has emerged as a critical issue. Dietrich and Dressler [17] surveyed many definitions of WSN lifetime including the number of "alive" nodes, network coverage, network connectivity, and quality-of-service considerations (e.g., event detection rates). Other authors (cf. Anastasi et al. [8]) have classified energy conservation approaches (e.g., sensor sleep/wake protocols, data acquisition schemes, mobile sink-based approaches, etc.). The lifetime of WSNs has been further discussed in [7, 28, 29, 38].

Routing protocols constitute the largest area of research related to the performance of wireless sensor networks. A variety of techniques are reviewed in a cogent survey by Al-Karaki and Kamal [6]. Most protocols aim to minimize the energy expended by the network while satisfying quality-of-service guarantees. Routing protocols are broadly labeled as flat-based, hierarchical (see [30]), and location-based. Flat-based (or data-centric) protocols assume all sensor nodes have equal capabilities and similar roles whereas hierarchical protocols assign different roles to the nodes. Location-based protocols use sensor node position information to make routing decisions. The model we analyze here falls into the category of flat routing and, more specifically, query-based flat routing. Classical data-centric approaches include flooding and gossiping (see [20]) which are known to be energy- and bandwidth-inefficient. Alternatively, rumor-routing protocols (see [4, 14, 10, 18, 31, 34, 36]) can be used. Rumor routing uses packets with relatively long lifetimes called *agents*. When a node detects an event, it adds information pertaining to the event in a local *event table* and immediately creates a time-limited agent that "advertises" the local information to distant nodes via subsequent packet transmissions. Consequently, when any node of the network generates a query, a node with the information stored in its local event table can respond, if it receives the query. This approach obviates the need for flooding, thereby reducing energy expenditure. Rumor routing is effective (relative to flooding) when the arrival rate of events is relatively small but generally requires significant overhead. Specifically, witnessed events are assigned a time-to-live (TTL) counter, or *resource replication level*, that is tracked while query lifetimes must also be tracked.

The resource replication level is the number of times a witnessed event is replicated in the network, and studies related to this parameter are relatively sparse. Bellavista et al. [11] developed a simulation model (REDMAN) to explore resource replication levels and related network settings. Krishnamachari and Ahn [22] derived cost expressions as a function of the resource replication level for unstructured networks in which the source node is unknown. They used expanding ring queries to search for the information and formulated a nonlinear programming (NLP) model to determine the optimal number of resource replicates, subject to a network storage capacity constraint. Ahn and Krishnamachari [3] extended the results of [22] to structured networks in $d-$dimensional space, and in [1], studied structured and unstructured two-dimensional grid and random topology networks. Ahn and Krishnamachari [2] presented a model to obtain the optimal resource replication level that minimizes the total expected cost of replication and searching, subject to a storage capacity constraint. An algorithm for dissemination and retrieval of information that ensures an even geographical distribution of the informed nodes is proposed for unstructured wireless ad-hoc networks by Miranda et al. [26]. Most relevant to our work here, Mann et al. [25] used a queueing framework to obtain the optimal replication level that minimizes a proxy for energy expenditure, subject to a performance guarantee on the steady state proportion of failed queries. Their approach is unique in that it considers time-limited event agents and queries but is limited to memoryless lifetimes. Bisnik and Abouzeid [13] used a queueing network model to analyze random access, multi-hop wireless networks and derived the average end-to-end delay. Niyato and Hossain [27] developed a queueing model to investigate the performance of different sleep and wake-up strategies. Chiasserini et al. [15] proposed a fluid queueing model that accounts for energy consumption, active/sleep dynamics, and traffic routing. Ata [9] considered the problem of dynamically choosing the transmission rate in a general wireless communications network such that the average energy consumption per time unit is minimized, subject to a quality-of-service constraint. In that work, the transmission queue was modeled as a finite-buffer, $M/M/1$ system. With the exception of Mann et al. [25], none of the analytical models described herein account explicitly for limited packet lifetimes.

This paper provides a queueing-theoretic framework for evaluating the steady state proportion of query failures (i.e., the proportion of generated queries that fail to be answered on time) in a large-scale WSN with time-critical data. While the network model itself is similar to the one described in [25], it has several important distinguishing attributes. Moreover, the analysis approach we present here is quite different. In fact, the model of Mann et al. [25] is strictly limited to exponentially distributed inter-event times and query lifetimes, and it essentially ignores the network's topology by assuming that sensors have an infinite transmission range. We generalize their model by considering general event and query lifetimes and a finite transmission range. Derived are valid approximations that explicitly account for (1) time-limited events and queries, (2) the limited transmission range of sensor nodes, and (3) generally-distributed resource and query lifetimes. The first approximation (derived from an infinite transmission range model) is shown to be insensitive to the network's size. The finite transmission range approximation is shown to be asymptotically valid, and extensive numerical comparisons with simulated networks verify the exceptional accuracy of the approximations, even when exponential assumptions are violated.

The remainder of the paper is organized as follows. Section 2 provides a description of the network model, the queueing models of sensor node elements, and the most relevant attributes. In Section 3, we derive the steady state proportion of query failures assuming an unlimited sensor transmission range, while Section 4 presents an approximation that explicitly accounts for the limited transmission range of sensors. Section 5 presents extensive numerical results that validate our analytical approximations.

2 Model Description

Consider a multi-hop wireless sensor network (WSN) represented by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ where $\mathcal{N} = \{1, 2, \ldots, N\}$ is the node set (or set of vertices), N is the number of sensor nodes in the network, and $\mathcal{A}$ is the arc set of the sensor network. An arc (i, j) is an element of $\mathcal{A}$ if and only if nodes i and j are within transmission range of each other. Once deployed, the sensor nodes are spatially stationary (i.e., they are not mobile). In this research, we consider only networks with sensor nodes deployed in a sensor field R, a subset of Euclidean 2-space. The nodes are assumed to be spatially uniformly distributed in R. The node density of the network, ψ, is the average number of nodes per unit area (in nodes/m^2) given by $\psi = N/A$ where A is the area of sensor field R. For each $i \in \mathcal{N}$, denote by $\boldsymbol{x}_i$ the position of sensor node i in Euclidean 2-space. Then for $j \in \mathcal{N}$, $j \neq i$, the Euclidean distance between $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ is $\rho(i, j) \equiv \|\boldsymbol{x}_i - \boldsymbol{x}_j\|$ where $\|\cdot\|$ denotes the Euclidean norm. Assuming each sensor node has a transmission range r (in meters), the *degree* of node $i \in \mathcal{N}$ is the number of nodes within transmission range of i given by

$$d_i(r) \equiv \sum_{j \in \mathcal{N} \setminus \{i\}} \mathbf{1}(\rho(i, j) \leq r),$$

where $\mathbf{1}(x)$ is an indicator function that assumes the value 1 if condition x holds and 0 otherwise. Obviously, $d_i(r)$ depends on the deployment of nodes in R, the network topology, and the transmission range of individual sensor nodes. Finally, the average degree of the network is

$$\bar{d}(r) \equiv \frac{1}{N} \sum_{i=1}^{N} d_i(r).$$

A node $i \in \mathcal{N}$ for which $d_i(r) = 0$ is said to be *isolated*. Isolated nodes are essentially useless to the WSN since they cannot exchange information with other nodes. The WSN is said to be *disconnected* if there exists at least one isolated node in $\mathcal{N}$ but is *completely connected* if there exists at least one path between nodes i and j for every $i, j \in \mathcal{N}$. Obviously, it is undesirable for the network to be disconnected, particularly when the information relayed by nodes is time sensitive. When the nodes are uniformly distributed in R with homogeneous node density ψ, the minimum transmission range needed to ensure the network is connected with probability p is (see Theorem 1 of [12])

$$\widehat{r} \geq \sqrt{\frac{-\ln\left(1 - p^{1/N}\right)}{\pi\,\psi}}. \tag{1}$$

The lower bound in (1) can be used, for example, to create discrete-event simulation models of wireless sensor networks that ensure connectivity with high probability.

Next, we describe individual sensor nodes in greater detail. (This discussion is similar to that of Mann [25].) It is assumed that sensor nodes are identical, i.e., they have identical resource requirements, physical limitations, and performance limitations. They are also similar with respect to their information requirements and the rates at which they observe and report relevant phenomena. Each sensor node is equipped with processing, transmitting, and sensing capabilities as well as a limited power supply (in the form of an on-board battery) that is generally difficult, if not impossible, to replace.

In hybrid push-pull wireless sensor networks, sensor nodes serve as both producers and consumers of network resources. A node produces a *resource* when (1) it monitors the environment and gathers data on the occurrence of pertinent events; or (2) it offers a particular service to the network. In addition to data gathering, nodes are also required to execute specific applications in support of the network's goals. When a node requires access to a resource that is not available locally, the node is forced to traverse the network to locate the necessary information and/or services.

When a node witnesses a relevant phenomenon, or offers a particular service to the network, it broadcasts this information to a subset of the network by means of an *event agent* – a packet that describes the resource available, the location of the resource (or, alternatively, the data itself), and the duration of time the resource is available or valid. In this research, we assume that agents are transmitted from node-to-node via a random walk until either the witnessed event expires (i.e., it reaches its deadline), or it exhausts its *time-to-live* (TTL) counter – a positive integer representing the maximum number of times the resource may be replicated in the network. It is worth mentioning that a variety of routing protocols can be assumed (cf. [24]), but the results herein assume transmission to a randomly selected neighbor. Each sensor node is equipped with an on-board *event table*. Whenever an event agent is received, or an event is witnessed by the node, the contents of the event agent are added to the event table, and the node is labeled as *informed* as long as the event agent has not expired.

In addition to witnessing and forwarding events, nodes generate *queries* to request data or resources from the network. A query contains at least three pieces of information: the identifier and/or location of the node originating the request, the resource sought, and the maximum amount of time the query is permitted to traverse the network in search of an informed node. Only informed nodes are capable of answering the queries of uninformed nodes. Similar to event agents, queries are forwarded from node-to-node via a random walk. If a query is received by an informed node, the query is terminated and the informed node generates a *response* that is returned to the query origin node via shortest-path routing. The response packet contains the information stored in the informed node's event table and, if available, the desired data. If a query cannot locate an informed node before its designated expiration time, the query fails.

Our main objective is to assess a critical quality-of-service measure for query-based WSNs, namely the long-run proportion of queries that fail to be answered on time. To this end, we create a queueing network model that leads to simple analytical expressions and accommodates easy computational implementation. Next we describe infinite- and single-server queueing models of the sensor node's event table and transmitter, respectively.

2.1 Queueing Models of Node Elements

For each $i \in \mathcal{N}$, events are assumed to arrive according to a Poisson process with rate λ. Each witnessed event is time sensitive, i.e., it is useful for only a limited time before it expires. Therefore, once an event is witnessed by a node, it is added to the node's event table and assigned an expiration time or *lifetime*, Z, a non-negative, non-defective random variable. Event expiration times (across all nodes) are independent and identically distributed (i.i.d.) random variables with common cumulative distribution function (c.d.f.) $G(w) \equiv \mathbb{P}(Z \leq w)$, $w \geq 0$, and mean $\mathbb{E}(Z) = 1/\delta < \infty$. As long as the event agent has not expired in the event table, the node is informed and can answer queries arriving from other nodes. Because event agents are mutually independent and do not necessarily expire in their order of arrival, the event table can be modeled as an $M/G/\infty$ queueing system whose input is a Poisson process with aggregate arrival rate Λ and whose service time is generally distributed with c.d.f. G (see Figure 1). The event arrival rate Λ depends on many

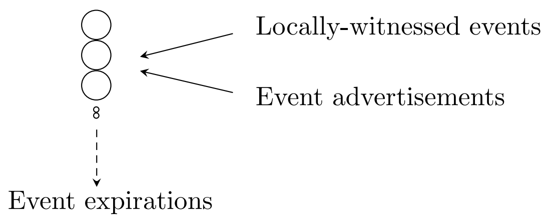

Figure 1: Graphical depiction of a sensor node's event table as an $M/G/\infty$ queue.

factors, not the least of which is the transmission range r. We pause here to remark that, in general, the superposition of locally-witnessed events and externally-generated event advertisements does not necessarily form a Poisson process since the latter do not (in general) originate from a Poisson stream. Furthermore, the event table may not realistically have infinite capacity. Therefore, the evolution of the number of busy servers in the $M/G/\infty$ model must be viewed as an approximation of the evolution of event table content. However, it will be shown in Section 5 that this assumption is not overly restrictive and that the proportion of query failures is surprisingly insensitive to the Poisson assumption. We choose the $M/G/\infty$ model for its tractability and generality with respect to event lifetimes. Specifically, it provides a simple expression for the steady state proportion of time an arbitrary node in the WSN is uninformed given by

$$\pi_0 \equiv \mathbb{P}(E = 0) = \exp\left(-\Lambda/\delta\right),$$

where E is the steady state number of events in the event table. Once a node witnesses an event, the information is forwarded until its TTL counter is exhausted. Henceforth, we denote this integer value by $\ell \in \mathbb{N}$.

Each sensor node contains a transmitter along with an (assumed) infinite buffer for storing data packets (queries, event agents, or responses). When an event agent arrives to a node, and its time-to-live counter has not been exhausted, the agent joins the transmission queue after a copy has been added to the node's event table. Moreover, when a node receives a query, either the query or the response is sent to the transmission queue, depending on whether the node is informed or uninformed. In either case, the query fails if the time elapsed from the moment of its inception until it locates an informed node exceeds its expiration time. The node's transmission queue is modeled as a single-server queueing system as depicted in Figure 2.

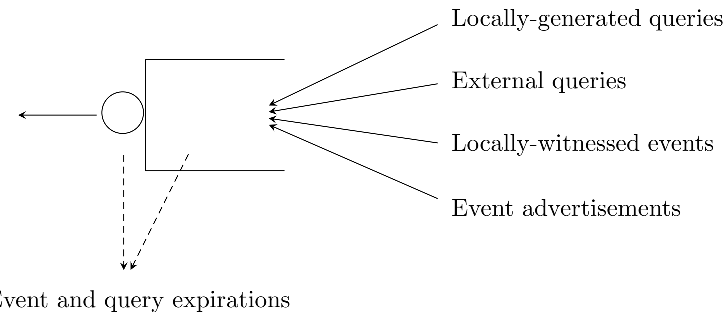

Figure 2: Graphical depiction of the sensor node's transmission queue.

Specifically, we assume that each node's transmission queue operates as a non-prioritized, multi-class $M/M/1$ queueing system with a first-come-first-served (FCFS) queueing discipline. Arrivals are assumed to originate from a Poisson process with rate λ_q. As depicted in Figure 2, the aggregate arrival process is comprised of locally-witnessed events, advertisements from other nodes, locally-generated queries, and queries arriving from other nodes. When a query is generated or received by a node, it joins the transmission queue only if the node is uninformed. The service time is the time needed to transmit a query packet or an event packet (either a locally-witnessed event or an advertisement from other nodes). Irrespective of the packet type, we assume the transmission time τ is an exponential random variable with parameter μ, c.d.f. $F(x) \equiv \mathbb{P}(\tau \leq x) = 1 - \exp(-\mu x)$, and finite mean $\mathbb{E}(\tau) = 1/\mu$. The transmission queue is stable if and only if $\mu > \lambda_q$. This condition is usually met in practice since transmission rates are generally very high. It is important to note that the total arrival rate of traffic to the transmission queue serves as a proxy for energy expenditure at a node since transmitting is the primary energy consuming activity (cf. [32]).

2.2 Network Performance Parameters

The primary concern of this research is the assessment of the steady state probability that a generated query fails to be answered on time. We refer to this performance parameter as the proportion of query failures. A query is said to fail if it expires awaiting transmission or while being transmitted. Our main aim is to provide easy-to-use analytical expressions for this parameter that allow us to circumvent costly, time-consuming simulation experiments for large-scale networks. To this end, let T be a non-negative random variable denoting the total time needed for a query to locate an informed node as measured from the time the query is generated at a node $n \in \mathcal{N}$. This random time depends on the status of node n at the time of creation. Define the indicator variable

$$I_n = \begin{cases} 1, & \text{if node } n \text{ is informed,} \\ 0, & \text{if node } n \text{ is uninformed.} \end{cases}$$

The c.d.f. of $[T|I_n = 0]$ is denoted by $B(t) \equiv \mathbb{P}(T \leq t|I_n = 0)$, $t \geq 0$ for any $n \in \mathcal{N}$. At the time of generation, the query is assigned a lifetime, X, so that if T exceeds X, the query does not locate the desired information before expiring, and it fails. The c.d.f. of X is $H(x) \equiv \mathbb{P}(X \leq x)$, $x \geq 0$, and its mean is $\mathbb{E}(X) = 1/\beta < \infty$. Proposition 1 characterizes the primary performance parameter.

Proposition 1 *The unconditional proportion of query failures is*

$$\Delta \equiv \mathbb{P}(T > X) = \pi_0 \int_0^\infty [1 - B(x)] \, \mathrm{d}H(x). \tag{2}$$

Proposition 1 can be proved using a simple conditioning argument. The expression for the proportion of query failures is straightforward except that the distribution function B is difficult to characterize in all but a few cases. The next section considers one such case when the sensors all use an infinite transmission range.

3 Unlimited Sensor Transmission Range

In this section, we provide an approximation for Δ when $r = \infty$, i.e., when each node's transmission range is large enough to ensure that any other node in the network can be reached with a single hop. This simplified model provides an approximation for the proportion of failed queries because realistic sensor nodes have limited transmission capabilities and rely on their neighbors to propagate witnessed events, queries, and responses to queries. Although several assumptions are employed, the primary purpose of this model is to provide a framework for a more realistic limited transmission range model.

3.1 Approximating Network Traffic

Here we establish approximations for event agent and query arrival rates at an arbitrary node of the network, assuming nodes are identical. The first result bounds the aggregate event arrival rate to the sensor node's event table. This bound sets the stage for an approximation of the steady state proportion of time that any node is uninformed.

Proposition 2 *Assume events arrive locally to each $n \in \mathcal{N}$ according to a Poisson process with rate λ. Then the total event arrival rate Λ to an arbitrary $n \in \mathcal{N}$ is bounded above by $\lambda\,(1 + \ell)$ where ℓ is the time-to-live counter.*

Proof. The aggregate event arrival rate Λ consists of the Poisson rate of locally-witnessed events, and the aggregate rate of witnessed events arriving from the other $N - 1$ nodes in the network. Therefore, $\Lambda = \lambda + \Lambda_x$ where Λ_x denotes the rate of external event arrivals. An event agent can be forwarded to, at most, ℓ nodes. Since $r = \infty$, each node can be reached with a single hop; hence, each event advertisement is equally likely to be received by one of the other $N - 1$ nodes. That is, a particular node receives one of the (potential) ℓ advertisements with probability $\ell/(N - 1)$, and since $N - 1$ other nodes transmit event agents,

$$\Lambda_x \leq \lambda\,(N - 1) \left(\frac{\ell}{N - 1} \right) = \lambda\,\ell.$$

Therefore, $\Lambda \leq \lambda + \lambda\,\ell = \lambda\,(1 + \ell)$. ∎

Next, we provide a lower bound for the steady state proportion of time a node is uninformed. An event agent is assigned a lifetime Z once it enters the event table. The random variable Z has c.d.f. G and finite mean $\mathbb{E}(Z) = 1/\delta$. As noted in Section 2, the event table is approximated by an $M/G/\infty$ queue with (Poisson) arrival rate Λ due to the insensitivity of this model to the event lifetime distribution. Using well-known results for the steady state behavior of the $M/G/\infty$ system, the limiting proportion of time a node is uninformed (i.e., the limiting probability that the event table is empty), is

$$\pi_0 = \exp(-\Lambda/\delta), \quad 0 < \delta < \infty.$$

By Proposition 2, the event arrival rate to a node is bounded above by $\lambda\,(1+\ell)$. Therefore,

$$\pi_0 \geq \exp\left[\frac{-\lambda(1+\ell)}{\delta}\right]. \tag{3}$$

Similarly, the event arrival rate to the node's *transmission queue*, λ_e, is also bounded above. Specifically, $\lambda_e \leq \lambda\ell$ since a locally-witnessed event is certainly added to the transmission queue. On the other hand, when a witnessed event is advertised through the network, since each of the other $N-1$ nodes is equally likely to receive the advertisement, a single node receives the first transmission and adds the event agent to its transmission queue with probability $1/(N-1)$, the second transmission with the same probability, and so forth. The event agent is forwarded at most ℓ times, but the node which receives it at the ℓth transmission does not add the agent to its transmission queue since the agent's time-to-live counter will have expired. Therefore,

$$\lambda_e \leq \lambda + \lambda(N-1)\sum_{i=1}^{\ell-1}\frac{1}{N-1} = \lambda\ell. \tag{4}$$

While the bounds of (3) and (4) are legitimate, they may not be tight since they do not explicitly account for the expiration of event agents waiting in the transmission queue, or those that expire during transmission. Proposition 3 provides an improved bound for λ_e (by considering the effect of event expirations) that leads to an improved approximation for π_0. In what follows, let α_j denote the probability that an event agent expires at the jth visited node, $j = 1, 2, \ldots$, where the first visited node is the event witnessing node. For simplicity, define the expiration probability at the first node by $\alpha \equiv \alpha_1$. Assuming the event lifetime distribution function G has an increasing failure rate (IFR), then $0 < \alpha \leq \alpha_2 \leq \alpha_3 \leq \cdots$.

Proposition 3 *Suppose G is an IFR distribution function so that $0 < \alpha \leq \alpha_2 \leq \alpha_3 \leq \cdots$. Then for a fixed time-to-live counter ℓ,*

$$\lambda_e \leq \lambda\left[\frac{1-(1-\alpha)^{\ell}}{\alpha}\right] \leq \lambda\ell.$$

Proof. Since each of the $N-1$ nodes is equally likely to receive an advertised event agent, an individual node receives the kth transmission with probability

$$\frac{1}{N-1}\prod_{j=1}^{k}(1-\alpha_j), \quad k = 1, 2, \ldots, \ell-1.$$

That is, the event agent is forwarded at most ℓ times; however, the last node that receives the agent does not add it to its transmission queue since the agent's time-to-live counter will have expired. Therefore, the approximate total rate of event arrivals to a node's transmission queue is given by

$$\begin{aligned}
\lambda_e &= \lambda + \lambda(N-1) \cdot \sum_{k=1}^{\ell-1} \frac{1}{N-1} \prod_{j=1}^{k} (1-\alpha_j) \\
&\leq \lambda + \lambda \cdot \sum_{k=1}^{\ell-1} (1-\alpha)^k \\
&= \lambda \sum_{k=0}^{\ell-1} (1-\alpha)^k .
\end{aligned}$$

Since $\alpha \in (0,1)$, we apply the well-known identity for geometric summations

$$\sum_{k=0}^{m} x^k = \frac{1-x^{m+1}}{1-x}, \quad x \in [0,1),$$

to obtain the final result,

$$\lambda_e \leq \lambda \left[\frac{1-(1-\alpha)^{\ell}}{\alpha} \right] \leq \lambda \ell, \quad \alpha \in (0,1),$$

where the second inequality follows from $\alpha \in (0,1)$. ■

For the results that follow, we use the approximation,

$$\lambda_e \approx \lambda \left[\frac{1-(1-\alpha)^{\ell}}{\alpha} \right],$$

to improve the approximation of π_0. Similarly, it can be shown that the event arrival rate to the event table is approximated by

$$\Lambda \approx \widehat{\Lambda} = \lambda \left[\frac{1-(1-\alpha)^{\ell+1}}{\alpha} \right]. \tag{5}$$

Therefore, when $r = \infty$, the approximate steady state proportion of time a node is uninformed is given by

$$\pi_0 \approx \exp\left[-\frac{\lambda}{\delta} \left(\frac{1-(1-\alpha)^{\ell+1}}{\alpha} \right) \right]. \tag{6}$$

Next, we examine the total traffic experienced at the transmission queue. Let λ_q be the total arrival rate of events and queries to a sensor node's transmission queue. Each node generates local queries according to a Poisson process with rate γ. When a query is generated locally, or received from another node, it is added to the transmission queue only if the subject node is uninformed. The arrival rate of locally-generated queries to the transmission queue, q_l, is $q_l = \pi_0 \gamma$. Let q_x denote the rate at which external queries arrive at a node. In steady state, the query visits an informed node with probability $1-\pi_0$ and an uninformed node with probability π_0, independent of the number of hops prior to the current visit. Consequently, the number of hops before a query locates an informed node is a geometric random variable with success probability $1-\pi_0$, and mean $1/(1-\pi_0)$. Since any node is equally likely to receive a query, the probability of receiving a query generated by any other node is $1/((1-\pi_0)(N-1))$. Because all other $N-1$ nodes generate queries identically, q_x is approximately

$$q_x = \gamma\pi_0(N-1)\left[\frac{1}{(1-\pi_0)(N-1)}\right] = \frac{\gamma\pi_0}{1-\pi_0}. \tag{7}$$

Consequently, we approximate the total arrival rate of traffic to a node's transmission queue as

$$\lambda_q \approx \widehat{\lambda}_e + q_l + q_x = \lambda\left[\frac{1-(1-\alpha)^\ell}{\alpha}\right] + \pi_0\gamma\left(\frac{2-\pi_0}{1-\pi_0}\right). \tag{8}$$

By (6) and (8), we see that π_0 and, consequently, λ_q are explicit functions of α. Therefore, the approximation of λ_q is written as

$$\lambda_q \approx c(\alpha) \equiv \lambda\left[\frac{1-(1-\alpha)^\ell}{\alpha}\right] + \gamma e^{-g(\alpha)}\left[\frac{2-e^{-g(\alpha)}}{1-e^{-g(\alpha)}}\right], \tag{9}$$

where

$$g(\alpha) = \frac{\lambda}{\delta}\left[\frac{1-(1-\alpha)^{\ell+1}}{\alpha}\right].$$

We are now prepared to provide an expression for α, the probability that an event agent expires in the first transmission queue. The result is approximate since the input to the transmission queue is assumed to be the superposition of independent Poisson arrival streams. The equilibrium random variable Z_e associated to the lifetime Z with c.d.f. G and mean $\mathbb{E}(Z)$ has c.d.f.

$$G_e(z) \equiv \mathbb{P}(Z_e \le z) = \frac{1}{\mathbb{E}(Z)}\int_0^z [1-G(u)]\mathrm{d}u.$$

We make use of the equilibrium distribution of the event agent lifetime in the following proposition that characterizes α.

Proposition 4 *Assume $\mu > c(\alpha)$ for each λ and δ such that $0 < \lambda < \infty$ and $0 < \delta < \infty$. Let W be the total time spent at a node's transmission queue (delay plus transmission time) by an arbitrary arrival in steady state. Then*

$$\alpha = \mathbb{P}(W > Z_e) = \widetilde{G}_e(\mu - c(\alpha)) = \mathbb{E}\left[e^{-[\mu-c(\alpha)]Z_e}\right], \tag{10}$$

where $\widetilde{G}_e(\mu - c(\alpha))$ denotes the Laplace-Stieltjes transform (LST) of G_e evaluated at $\mu - c(\alpha)$.

Proof. The transmission queue is modeled as an $M/M/1$ queueing system with mean transmission time $1/\mu$ and aggregate arrival rate $c(\alpha)$. Let W_n be the total time spent in the transmission queue (i.e., the delay time plus the transmission time) by the nth arrival to the queue, either an event agent or a query. It is well known that if $\mu > c(\alpha)$, $W_n \Rightarrow W$ as $n \to \infty$ where W is exponentially distributed with mean $1/(\mu - c(\alpha))$ and ($\Rightarrow$) is convergence in distribution (or weak convergence). Suppose an event agent arrives at time t so that $Z - t$ is the residual lifetime of the event. Using basic results from renewal theory, $Z - t \Rightarrow Z_e$ as $t \to \infty$. Therefore, by conditioning on Z_e, we obtain

$$\alpha = \mathbb{P}(W > Z_e) = \int_0^\infty e^{-(\mu - c(\alpha))z} \mathrm{d}G_e(z) = \widetilde{G}_e(\mu - c(\alpha)) = \mathbb{E}\left[e^{-(\mu - c(\alpha))Z_e}\right].$$

■

As an illustration, if the event lifetime is exponentially distributed with mean $1/\delta$, then $G_e(z) = G(z)$ for all $z \geq 0$, and the unique probability α is the solution to the fixed point problem

$$\alpha = \frac{\delta}{\mu - c(\alpha) + \delta}.$$

Recall that our aim is to approximate Δ of equation (2) by assuming $r = \infty$. To this end, let $\widetilde{T}$ denote the time to locate an informed node when $r = \infty$ and let

$$\Delta_\infty \equiv \mathbb{P}(\widetilde{T} > X) = \pi_0 \int_0^\infty [1 - B(x)] \mathrm{d}H(x).$$

The c.d.f. of $\widetilde{T}$ is a function of both λ_q ($c(\alpha)$) and π_0, both of which are determined by α. The next section shows how to obtain Δ_∞.

3.2 Approximate Query Failure Rate

Queries, which can be generated at any node $n \in \mathcal{N}$, are forwarded via a random walk to one-hop neighbors until either an informed node is located or the query expires while awaiting transmission in some node's transmission queue (or while being transmitted). Once generated, a query is assigned a lifetime X having c.d.f. $H(x) \equiv \mathbb{P}(X \leq x)$, $x \geq 0$. Let us assume for the moment that a query generated at an uninformed node can be forwarded indefinitely (i.e., $X = \infty$ w.p. 1), and let M be the (integer) number of hops needed to first locate an informed node.

Let T_k denote the time spent by a query at its kth location. That is, T_1 denotes the time spent at the original query node (which is uninformed), T_2 is the time spent at the second node, which might be informed or uninformed, and so forth. To simplify notation, let $\widetilde{T}_u \equiv [\widetilde{T} | I_n = 0]$ be the elapsed time between creation of the query at an *uninformed* node and the time it first locates an informed node. It is easy to see that

$$\widetilde{T}_u = \sum_{k=1}^{M} T_k.$$

Because we assume $r = \infty$ and identical nodes, a query visits an informed node with probability $1 - \pi_0$ and an uninformed node with probability π_0, independent of any prior visits, in steady state. Thus, M is a geometric random variable with success probability $1 - \pi_0$ and mean $1/(1 - \pi_0)$, i.e., $\widetilde{T}_u$ is a geometric sum of i.i.d. exponential random variables.

Lemma 1 *Given that a query is generated at an uninformed node, the time to locate an informed node is exponentially distributed with parameter* $(1 - \pi_0)(\mu - \lambda_q)$*, i.e.,*

$$B(x) \equiv \mathbb{P}(\widetilde{T}_u \leq x) = 1 - \exp\left[-(1 - \pi_0)(\mu - \lambda_q)x\right], \quad x \geq 0, \tag{11}$$

where $\lambda_q \equiv c(\alpha)$ *is obtained using the value of* α *that solves the fixed point equation* (10).

Using Lemma 1, we next provide our approximate expression for the steady state proportion of query failures when $r = \infty$.

Proposition 5 *Assuming Poisson event arrivals and query generation, the proportion of query failures in an infinite-range WSN is*

$$\Delta_\infty = \mathbb{P}(\widetilde{T} > X) = \pi_0\, \widetilde{H}[(1-\pi_0)(\mu - \lambda_q)], \tag{12}$$

where $\widetilde{H}(s) = \mathbb{E}\left(e^{-sX}\right)$ *is the LST of the query lifetime distribution function* H.

Proof. The proof follows directly by conditioning on the lifetime X and utilizing Lemma 1. Specifically,

$$\begin{aligned}
\Delta_\infty = \mathbb{P}(\widetilde{T} > X) &= \int_0^\infty \mathbb{P}(\widetilde{T} > X | I_n = 0, X = x)\mathbb{P}(I_n = 0)\mathrm{d}H(x) \\
&= \pi_0 \int_0^\infty e^{-(1-\pi_0)(\mu-\lambda_q)x}\mathrm{d}H(x) \\
&= \pi_0\, \widetilde{H}\left[(1-\pi_0)(\mu-\lambda_q)\right].
\end{aligned}$$

■

Proposition 5 holds for all query lifetime distributions that possess a LST. However, if the distribution function H is heavy-tailed and does not possess an LST, the transform approximation method (TAM) developed by [19], or its modification by [35], can be used to approximate $\widetilde{H}$.

3.3 Computing the Proportion of Query Failures

Here, we describe a simple fixed point algorithm to compute the probability α to obtain Δ_∞ using equation (12). Let $\pi_0^{(k)}$, $\lambda_q^{(k)}$ and $\alpha^{(k)}$ be the approximated values of π_0, λ_q and α at the kth iteration of the algorithm, respectively.

Algorithm to Compute α:

Step 0: *Initialization via the bounds of* (3) *and* (4).

$$\begin{aligned}
&k := 0; \\
&\pi_0^{(k)} := \exp\left[-\lambda(1+\ell)/\delta\right]; \\
&\lambda_q^{(k)} := \lambda\,\ell + \gamma\pi_0^{(k)}\left(\frac{2-\pi_0^{(k)}}{1-\pi_0^{(k)}}\right); \\
&\alpha^{(k)} := \widetilde{G}_e\left(\mu - \lambda_q^{(k)}\right).
\end{aligned}$$

Step 1: *Update the approximations.*

$$\begin{aligned}
&k := k+1; \\
&\pi_0^{(k)} := \exp\left[-\frac{\lambda}{\delta}\left(\frac{1-\left(1-\alpha^{(k-1)}\right)^{\ell+1}}{\alpha^{(k-1)}}\right)\right]; \\
&\lambda_q^{(k)} := \lambda\left[\frac{1-\left(1-\alpha^{(k-1)}\right)^{\ell}}{\alpha^{(k-1)}}\right] + \gamma\pi_0^{(k)}\left[\frac{2-\pi_0^{(k)}}{1-\pi_0^{(k)}}\right];
\end{aligned}$$

$$\alpha^{(k)} := \widetilde{G}_e\left(\mu - \lambda_q^{(k)}\right).$$

Step 2: *Check convergence criterion.*

If $\left|\alpha^{(k)} - \alpha^{(k-1)}\right| > \epsilon$, return to Step 1;
Else $\alpha := \alpha^{(k)}$;
Stop.

Scalability of the WSN is an important issue as realistic networks are envisioned to have thousands or even hundreds of thousands of sensor nodes. The infinite range approximations of this section are appealing due to their insensitivity to N. For large N, the probability that a given node is visited more than once by a specific event agent or query is negligible. That is, when a node witnesses an event, it forwards an event packet to at most ℓ *distinct* nodes. Similarly, when a query is generated, the present model assumes it visits a distinct node at each hop, independently of all prior hops. However, to conserve power, realistic WSN sensors use a limited transmission range, so the probability of revisiting neighbors can be significant as highlighted by Rodero-Merino et al. [33]. This revisiting effect increases the proportion of time nodes are uninformed, the time to locate an informed node, and consequently, the proportion of failed queries. In the next section, we account for a limited sensor transmission range.

4 Accounting for Transmission Range

In this section, we present an approximation for the steady state proportion of query failures that explicitly accounts for the limited transmission range of wireless sensors. Specifically, we approximate Δ_r, the steady state proportion of query failures when the sensor nodes have transmission range of r ($r < \infty$). Additionally, we show that for large N, the approximation converges appropriately to Δ_∞ as $r \to \infty$.

4.1 Modeling Query Dynamics

Here we consider the status (and movement) of an individual query from its inception until it either locates an informed node or fails due to expiration. If a query is generated at an informed node, then it is answered immediately and never forwarded; therefore, we focus on the case when a query is generated at an uninformed node $n \in \mathcal{N}$. At its inception the query is instantaneously assigned a lifetime (or expiration time) X with c.d.f. $H(x)$ and mean $\mathbb{E}(X) = 1/\beta$ ($0 < \beta < \infty$). It is forwarded to a randomly selected node within the r-radius of the current node until either an informed node is located, or the query expires, in which case it is destroyed. In what follows, all random quantities are conditioned on the event $\{X = x\}$; therefore, we make the dependence on x explicit.

For each integer k ($k \geq 0$), let Q_k be the status of the query just before it joins the transmission queue of the kth visited node. (Note that Q_0 is the query's status at the moment it is generated at the query origin node.) The query can be in one of three mutually exclusive and exhaustive states: *active* (state 0), *answered* (state 1), or *expired* (state 2). The query is added to the kth visited node's transmission queue if and only if it is active. For each $k \geq 0$, $Q_k \in S \equiv \{0, 1, 2\}$

where $Q_k = 0$ means the query, having been successfully transmitted k times, has not expired but has not been answered; $Q_k = 1$ means the query, having been successfully transmitted k times is answered at the kth visited node (i.e., the kth visited node is informed); and $Q_k = 2$ means the query was successfully transmitted $k-1$ times but expired awaiting its kth transmission (or during its kth transmission) at the $(k-1)$st visited node. (Note that $\mathbb{P}(Q_0 = 2) = 0$.) The process $Q \equiv \{Q_k : k \geq 0\}$ is an S-valued discrete-time Markov chain (DTMC) with *temporally-nonhomogeneous* one-step transition probability matrix, $\mathbf{P}(k,x)$, given by

$$\mathbf{P}(k,x) = \begin{bmatrix} p_{00}(k,x) & p_{01}(k,x) & p_{02}(k,x) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad k \geq 0, \;\; x \geq 0 \tag{13}$$

where for each $i, j \in S$,

$$p_{ij}(k,x) \equiv \mathbb{P}_x(Q_{k+1} = j | Q_k = i), \quad k \geq 0,$$

denotes the probability that the status of the query transitions from i to j at the $(k+1)$st step and $\mathbb{P}_x(A) \equiv \mathbb{P}(A|X = x)$ for any measurable event A. Once a query locates an informed node, it is no longer forwarded to a neighbor node, and if the query expires awaiting transmission (or during its transmission), it is destroyed; therefore, states 1 and 2 are absorbing states of the DTMC. Row 0 of $\mathbf{P}(k,x)$ contains the critical transition probabilities. In particular, $p_{02}(k,x)$ is the probability that the query fails at the kth visited node, given it was active just before being added to the kth visited node's transmission queue. Likewise, $p_{00}(k,x)$ is the probability the query remains active just before being added to the $(k+1)$st visited node's transmission queue, given it was active just before being added to the kth node's transmission queue. Finally, $p_{01}(k,x)$ is the probability that a query is answered at the $(k+1)$st visited node, given it was active just before being added to the kth visited node's transmission queue.

Obviously, the DTMC $\{Q_k : k \geq 0\}$ is reducible with one transient state (state 0) and two closed communicating classes, namely $C_1 = \{1\}$ and $C_2 = \{2\}$; therefore, its limiting behavior is fairly easy to characterize. Before examining the limiting behavior, we characterize the distribution of $\{Q_k : k \geq 0\}$ at a particular step k. Let $v_j^k(x) = \mathbb{P}_x(Q_k = j)$ be the (unconditional) probability that the query is in state $j \in S$ just before joining the transmission queue of the kth node, and let $\boldsymbol{v}^k(x) = [v_j^k(x)]_{j \in S}$ be a (1×3) row vector comprised of these values. Because $\{Q_k : k \geq 0\}$ possesses a time-nonhomogeneous transition probability matrix, the vector $\boldsymbol{v}^k(x)$ can be obtained recursively (cf. [21]) by

$$\boldsymbol{v}^{k+1}(x) = \boldsymbol{v}^k(x)\,\mathbf{P}(k,x), \quad k \geq 0,$$

whose solution is given by

$$\boldsymbol{v}^{k+1}(x) = \boldsymbol{v}^0(x) \prod_{n=0}^{k} \mathbf{P}(n,x), \quad k \geq 0. \tag{14}$$

The square matrix on the right-hand side of (14) is the $(k+1)$-step transition probability matrix of $\{Q_k : k \geq 0\}$. The transient analysis of $\{Q_k : k \geq 0\}$ will facilitate an analysis of its limiting behavior which, in turn, is used to derive an expression for the steady state probability that a query fails to locate an informed node before expiring.

To this end, let us define the limiting probability vector

$$\boldsymbol{v}(x) \equiv \lim_{k\to\infty} \boldsymbol{v}^{k+1}(x) = \lim_{k\to\infty} \boldsymbol{v}^0(x) \prod_{n=0}^{k} \mathbf{P}(n,x) = \boldsymbol{v}^0(x) \lim_{k\to\infty} \prod_{n=0}^{k} \mathbf{P}(n,x). \tag{15}$$

Before approximating this vector, we first establish the existence and form of the limit in the right-most term of (15) via Theorem 1.

Theorem 1 *For a fixed expiration time x ($x > 0$), there exist real numbers $\alpha_1(x)$ and $\alpha_2(x)$ such that*

$$\mathbf{A}(x) \equiv \lim_{k\to\infty} \prod_{n=0}^{k} \mathbf{P}(n,x) = \begin{bmatrix} 0 & \alpha_1(x) & \alpha_2(x) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $\alpha_1(x), \alpha_2(x) \in (0,1)$ and $\sum_{i=1}^{2} \alpha_i(x) = 1$.

Proof. Using induction, it can be shown that the $(k+1)$-step transition probability matrix is given by

$$\prod_{n=0}^{k} \mathbf{P}(n,x) = \begin{bmatrix} \prod_{n=0}^{k} a_n & \sum_{n=0}^{k} b_n \left(\prod_{j=0}^{n-1} a_j\right) & \sum_{n=0}^{k} c_n \left(\prod_{j=0}^{n-1} a_j\right) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{16}$$

where $a_n \equiv p_{00}(n,x)$, $b_n \equiv p_{01}(n,x)$, $c_n \equiv p_{02}(n,x)$, $n \geq 0$, and $a_{-1} \equiv 1$. First, note that rows 1 and 2 of $\prod_{n=0}^{k} \mathbf{P}(n,x)$ are as given in (16) for any $k \in \mathbb{N}$; hence, we need only concern ourselves with row 0. Allowing $k \to \infty$ on both sides of (16), and noting that $0 < a_n < 1$, we see immediately that

$$\lim_{k\to\infty} \prod_{n=0}^{k} a_n = 0,$$

$$\alpha_1(x) \equiv \lim_{k\to\infty} \sum_{n=0}^{k} b_n \prod_{j=0}^{n-1} a_j = b_0 + \sum_{n=1}^{\infty} b_n \prod_{j=0}^{n-1} a_j \geq b_0 > 0, \tag{17}$$

and

$$\alpha_2(x) \equiv \lim_{k\to\infty} \sum_{n=0}^{k} c_n \prod_{j=0}^{n-1} a_j = c_0 + \sum_{n=1}^{\infty} c_n \prod_{j=0}^{n-1} a_j \geq c_0 > 0. \tag{18}$$

Since each row of $\mathbf{A}(x)$ is comprised of nonnegative real numbers, and the row sums must be unity (cf. [16]), we conclude that $\alpha_1(x) + \alpha_2(x) = 1$ which, in light of (17) and (18), implies that $0 < \alpha_1(x) < 1$ and $0 < \alpha_2(x) < 1$. ∎

For computational purposes, we approximate $\boldsymbol{v}(x)$ by truncating the infinite product of (15) at an appropriate integer q. Specifically, for a sufficiently large $q \in \mathbb{N}$, the approximation for $\boldsymbol{v}(x)$ is given by

$$\boldsymbol{v}(x) \approx \boldsymbol{v}^{q+1}(x) = \boldsymbol{v}^0(x) \prod_{n=0}^{q} \mathbf{P}(n,x), \tag{19}$$

where q is chosen such that $\|\boldsymbol{v}^{q+1}(x) - \boldsymbol{v}^q(x)\|_\infty < \epsilon$ with $\|\cdot\|_\infty$ the usual ∞-norm and ϵ a convergence threshold. Let Δ_r be the limiting probability of query failure provided each sensor's transmission range is r ($r < \infty$) and let

$$v_2(x) \equiv \lim_{k\to\infty} v_2^k(x) = \lim_{k\to\infty} \mathbb{P}_x(Q_k = 2).$$

The unconditional proportion of query failures is approximately

$$\Delta_r = \int_0^\infty v_2(x)\mathrm{d}H(x). \tag{20}$$

Let $\pi_0(r)$ be the steady state proportion of time an arbitrary node is uninformed when the transmission range is r. Note that $v_2(x)$ depends implicitly on r through $\pi_0(r)$ since $\boldsymbol{v}^0(x) = (\pi_0(r), 1-\pi_0(r), 0)$, and $\mathbf{A}(x)$ depends on $\pi_0(r)$. However, we suppress this dependence on r for ease of notation. To compute $\boldsymbol{v}(x)$ (or its approximation $\boldsymbol{v}^q(x)$ via (19)), we now provide an expression for $p_{02}(k,x)$ and, subsequently, expressions for $p_{00}(k,x)$ and $p_{01}(k,x)$.

Lemma 2 *For a fixed expiration time x ($x > 0$), the transition probability $p_{02}(k,x)$ is*

$$p_{02}(k,x) = \frac{e^{-(\mu-\lambda_q)x}}{G(k,x)} \cdot \frac{[(\mu-\lambda_q)x]^k}{k!}, \quad k \geq 0, \tag{21}$$

where for each $k \geq 1$, $G(k,x)$ is the c.d.f. of a k-phase Erlang random variable with parameter $\mu - \lambda_q$ and $G(0,x) \equiv 1$.

Proof. If the query is transmitted to the $(k+1)$st node, then it had k successful prior transmissions without expiring. Let T_i denote the sojourn time at the ith visited node, $i \geq 1$. Because each node's transmission queue is modeled as a stable $M/M/1$ queue, $\{T_i : i \geq 1\}$ is an i.i.d. sequence of random variables with parameter $\mu - \lambda_q$. Denote by Y_k the total time elapsed from the moment a query is generated at an uninformed node up to and including its kth transmission, i.e.,

$$Y_k = \sum_{i=1}^k T_i,$$

where Y_k is a k-phase Erlang random variable with parameter $\mu - \lambda_q$. It is well-known (cf. [23]) that, for $k \geq 1$, the c.d.f. of Y_k is

$$G(k,x) \equiv \mathbb{P}(Y_k \leq x) = 1 - \sum_{n=0}^{k-1} e^{-(\mu-\lambda_q)x} \frac{[(\mu-\lambda_q)x]^n}{n!}.$$

We can express the conditional probability $p_{02}(k,x)$ in terms of the random variables Y_k and Y_{k+1} by noting that

$$p_{02}(k,x) = \mathbb{P}(Y_{k+1} > x | Y_k \leq x)$$

is the probability the query expires at the kth visited node while awaiting its $(k+1)$st transmission, given it had successfully made k prior transmissions and was active just before joining the kth node's transmission queue. When $k = 0$, $p_{02}(0,x)$ is the probability the query expires in the transmission queue of the query origin node given by

$$p_{02}(0,x) = \mathbb{P}(Y_1 > X | X = x) = \mathbb{P}(T_1 > x) = e^{-(\mu-\lambda_q)x}.$$

For $k \geq 1$, using basic conditional probability,

$$p_{02}(k,x) = \mathbb{P}(Y_{k+1} > x | Y_k \leq x) = \frac{G(k,x) - G(k+1,x)}{G(k,x)} = \frac{e^{-(\mu-\lambda_q)x}}{G(k,x)} \frac{[(\mu-\lambda_q)x]^k}{k!}.$$

■

The remaining probabilities in row 0 of $\mathbf{P}(k,x)$, $p_{00}(k,x)$ and $p_{01}(k,x)$, depend on whether or not the query revisits uninformed nodes during its lifetime when $r < \infty$. For this reason, it is necessary to first compute the probability that the query visits a particular node $n \in \mathcal{N}$ for the first time at its kth visit.

To this end, let U_k be the location of the query just after its kth hop and note that $\{U_k : k \geq 0\}$ is a time-homogeneous DTMC with state space $\mathcal{N} = \{1, \ldots, N\}$. Define its one-step transition probability matrix by $\boldsymbol{\theta}(r) = [\theta_{ij}(r)]_{i,j \in \mathcal{N}}$. As in section 2, for $j \neq i$, let $\rho(i,j) = \|\boldsymbol{x}_i - \boldsymbol{x}_j\|$ and let $d_i(r)$ be the degree of node $i \in \mathcal{N}$. Assuming any neighbor of the current node is equally likely to receive a query transmission, for $i, j \in \mathcal{N}$ such that $j \neq i$, the transition probability $\theta_{ij}(r)$ is

$$\theta_{ij}(r) = \begin{cases} 1/d_i(r), & \text{if } \rho(i,j) \leq r, \\ 0, & \text{if } \rho(i,j) > r. \end{cases}$$

(Note that $\theta_{ii}(r) = 0$ for all $i \in \mathcal{N}$ as a query cannot be transmitted to the current node.)

Now let $q(k,r)$ be the probability that a query (or event agent) visits any one of the N nodes for the first time at the kth visit, and let $u_r(i,j,k)$ be the probability of visiting node j *at least once* before the $(k+1)$st visit, given that the query (or agent) originates at node i. Let $w_r(i,j,k)$ denote the probability the query visits state j for the first time on the kth visit, given it originated at node i. We have the following important lemma.

Lemma 3 *For each $k \in \mathbb{N}$ and $r \in (0, \infty)$,*

$$q(k,r) \approx \widehat{q}(k,r) = \frac{1}{N} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \{i\}} [u_r(i,j,k) - u_r(i,j,k-1)] \tag{22}$$

where

$$u_r(i,j,k) = \begin{cases} \theta_{ij}(r) + \displaystyle\sum_{m \in \mathcal{N} \setminus \{j\}} \theta_{im}(r)\, u_r(m,j,k-1), & k \geq 1, \\ 0, & k = 0. \end{cases}$$

Proof. The lemma is proved using standard results for DTMCs. Specifically, define

$$T_{ij}^r = \inf\{k \geq 1 : U_k = j | U_0 = i\}$$

as the first passage time to node $j \in \mathcal{N}$, given that the query (or event agent) was generated at node $i \in \mathcal{N}$. Then,

$$u_r(i,j,k) = \mathbb{P}(T_{ij}^r \leq k),$$

and these probabilities can be obtained recursively by conditioning on the location of the query after its first transmission. The derivation is similar to that outlined in Theorem 4.1 of [23] and shows that for $k \geq 1$,

$$u_r(i,j,k) = \theta_{ij}(r) + \sum_{m \in \mathcal{N} \setminus \{j\}} \theta_{im}(r)\, u_r(m,j,k-1), \quad i,j \in \mathcal{N},$$

where $u_r(i,j,0) \equiv 0$ for each $i,j \in \mathcal{N}$. Using $u_r(i,j,k)$, the probability the query's first visit to node j is the kth visit, given the query originated at node i, is

$$w_r(i,j,k) \equiv \mathbb{P}(T_{ij}^r = k) = u_r(i,j,k) - u_r(i,j,k-1), \quad k \geq 1.$$

Assuming a query is generated at any $i \in \mathcal{N}$ with equal probability (i.e., $\mathbb{P}(U_0 = i) = 1/N$ for all $i \in \mathcal{N}$), via unconditioning, the approximate probability a query visits a distinct node at the kth visit is

$$q(k,r) \approx \widehat{q}(k,r) = \frac{1}{N} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N} \setminus \{i\}} w_r(i,j,k), \quad k \geq 1.$$

■

Lemma 3 facilitates simple approximations for the transition probabilities $p_{00}(k,x)$ and $p_{01}(k,x)$, $k \geq 0$, which are provided in the next proposition.

Proposition 6 *The transition probabilities $p_{00}(k,x)$ and $p_{01}(k,x)$, $k \geq 0$, are respectively approximated by*

$$p_{00}(k,x) \approx [1 - \widehat{q}(k+1,r)(1-\pi_0(r))]\,[1 - p_{02}(k,x)], \tag{23}$$

$$p_{01}(k,x) \approx \widehat{q}(k+1,r)[1-\pi_0(r)]\,[1 - p_{02}(k,x)]. \tag{24}$$

Proof. This approximation assumes that if node i is uninformed when a query first visits the node, it remains uninformed during any subsequent visits to node i by the same query. We justify this assumption by noting that the mean recurrence time to node i is proportional to r. To approximate $p_{00}(k,x)$, condition on whether or not the $(k+1)$st visited node is distinct. First, given the query does not expire at the kth visited node, the $(k+1)$st visited node is not distinct with probability $1 - \widehat{q}(k+1,r)$. In the second case, given the query does not expire at the kth visited node, the $(k+1)$st node is distinct with probability $\widehat{q}(k+1,r)$, and it is uninformed with probability $\pi_0(r)$. Therefore, the probability of locating an uninformed node at the $(k+1)$st visit, given the query was active just before joining the transmission queue of kth node is, for $k \geq 0$,

$$\begin{aligned} p_{00}(k,x) &\approx [1-\widehat{q}(k+1,r)][1-p_{02}(k,x)] + \widehat{q}(k+1,r)\pi_0(r)[1-p_{02}(k,x)] \\ &= [1 - \widehat{q}(k+1,r)(1-\pi_0(r))]\,[1-p_{02}(k,x)]. \end{aligned}$$

To approximate $p_{01}(k,x)$, note that the query moves from state 0 (active) to state 1 (answered) if it was successfully transmitted from the kth visited node to a distinct node that is informed. Therefore, for $k \geq 0$,

$$p_{01}(k,x) \approx \widehat{q}(k+1,r)[1-\pi_0(r)][1-p_{02}(k,x)].$$

■

4.2 Range-Dependent Query Failure Rate

Using the approximation of $\mathbf{P}(k,x)$, we now provide improved approximations for the WSN traffic rates, the steady state proportion of time nodes are uninformed, and the steady state proportion of failed queries. It was shown in Section 3 that, if each sensor's range is such that all $N-1$

other nodes belong to its neighborhood, the total arrival rate of witnessed events (both local and external) to the node's event table is

$$\Lambda \approx \widehat{\Lambda} = \lambda \left[\frac{1-(1-\alpha)^{\ell+1}}{\alpha} \right].$$

The approximation $\widehat{\Lambda}$ does not account for the revisiting effects noted in this section. The following result uses $\widehat{q}(k,r)$ to correct for revisits and improve the approximate total arrival rate to the event table. To distinguish these values, let $\Lambda(r)$ be the total arrival rate of local and external events as a function of r. Then we can write

$$\begin{aligned} \Lambda(r) \approx \widehat{\Lambda}(r) &= \lambda + \lambda\, \bar{d}(r) \left(\frac{\widehat{q}(1,r)(1-\alpha)}{\bar{d}(r)} + \frac{\widehat{q}(2,r)(1-\alpha)^2}{\bar{d}(r)} + \cdots + \frac{\widehat{q}(\ell,r)(1-\alpha)^\ell}{\bar{d}(r)} \right) \\ &= \lambda \left[1 + \sum_{i=1}^{\ell} \widehat{q}(i,r)(1-\alpha)^i \right], \end{aligned}$$

where $\bar{d}(r)$ is the network's average node degree. Using $\widehat{\Lambda}(r)$, the steady state proportion of time nodes are uninformed, $\pi_0(r)$, is

$$\pi_0(r) \approx \exp\left[-\frac{\lambda}{\delta} \left(1 + \sum_{i=1}^{\ell} \widehat{q}(i,r)(1-\alpha)^i \right) \right]. \tag{25}$$

Equation (25) is used to compute the elements of $\mathbf{P}(k,x)$, the limiting matrix $\mathbf{A}(x)$, and ultimately the limiting probability $v_2(x)$ to obtain Δ_r via (20). The asymptotic validity of this approximation is discussed in the next subsection.

4.3 Asymptotic Validity of Approximation

In this subsection, we show that the finite transmission range approximation is asymptotically valid by proving that, for large N, the proportion of query failures converges to Δ_∞ as $r \to \infty$. To this end, we have the following important lemma.

Lemma 4 *For large N, as $r \to \infty$, $\widehat{q}(k,r) \to 1$ for each $k \in \mathbb{N}$.*

Proof. First note that

$$\lim_{r\to\infty} d_i(r) = \lim_{r\to\infty} \sum_{j \in \mathcal{N}\setminus\{i\}} \mathbf{1}(\rho(i,j) \le r) = N-1.$$

Therefore, for $i, j \in \mathcal{N}$ with $j \neq i$,

$$\theta_{ij}(r) = \frac{1}{d_i(r)} \to \frac{1}{N-1}$$

as $r \to \infty$. By induction on $k \in \mathbb{N}$, we now characterize the limiting behavior of $u_r(i,j,k)$ as $r \to \infty$. For $k = 1$, note that $u_r(i,j,1) = \theta_{ij}(r) \to 1/(N-1)$. For $k = 2$, it is easy to show that

$$\begin{aligned}
\lim_{r\to\infty} u_r(i,j,2) &= \lim_{r\to\infty}\left(\theta_{ij}(r) + \sum_{m\in\mathcal{N}\setminus\{j\}} \theta_{im}(r)\, u_r(m,j,1)\right) \\
&= \frac{1}{N-1} + \sum_{m\in\mathcal{N}\setminus\{i,j\}} \left(\frac{1}{N-1}\right)^2 \\
&= \frac{2}{N-1} + O(N^{-2}),
\end{aligned}$$

where $O(N^{-2}) \to 0$ as $N \to \infty$. For the inductive step, assume $u_r(i,j,n) \to n/(N-1) + O(N^{-2})$ for any $n \in \mathbb{N}$. With some simplification we obtain

$$\begin{aligned}
\lim_{r\to\infty} u_r(i,j,n+1) &= \lim_{r\to\infty}\left(\theta_{ij}(r) + \sum_{m\in\mathcal{N}\setminus\{j\}} \theta_{im}(r)\, u_r(m,j,n)\right) \\
&= \frac{1}{N-1} + \sum_{m\in\mathcal{N}\setminus\{i,j\}} \frac{1}{N-1}\left[\frac{n}{N-1} + O(N^{-2})\right] = \frac{n+1}{N-1} + O(N^{-2}),
\end{aligned}$$

which completes the induction proof. Therefore, for each $k \in \mathbb{N}$ and $i, j \in \mathcal{N}$ with $j \neq i$,

$$\lim_{r\to\infty} w_r(i,j,k) \equiv \lim_{r\to\infty} [u_r(i,j,k) - u_r(i,j,k-1)] = \frac{1}{N-1},$$

and consequently,

$$\lim_{r\to\infty} \widehat{q}(k,r) = \lim_{r\to\infty} \frac{1}{N} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}\setminus\{i\}} w_r(i,j,k) = \frac{1}{N} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}\setminus\{i\}} \frac{1}{N-1} = 1.$$

■

Lemma 4 will now be used to prove Theorem 2 which asserts that, as $r \to \infty$, the approximate event arrival rate, proportion of time uninformed, and the proportion of query failures all converge appropriately to their respective infinite-range counterparts for large networks.

Theorem 2 *For large N, as $r \to \infty$, $\widehat{\Lambda}(r) \to \Lambda$, $\pi_0(r) \to \pi_0$, and $\Delta_r \to \Delta_\infty$.*

Proof. By Lemma 4, $\widehat{q}(k,r) \to 1$ for each $k \in \mathbb{N}$ as $r \to \infty$. Therefore,

$$\begin{aligned}
\lim_{r\to\infty} \widehat{\Lambda}(r) = \lim_{r\to\infty}\left[\lambda + \lambda \sum_{i=1}^{\ell} \widehat{q}(i,r)(1-\alpha)^i\right] &= \lambda \lim_{r\to\infty} \sum_{i=0}^{\ell} \widehat{q}(i,r)(1-\alpha)^i \\
&= \lambda\left[\frac{1-(1-\alpha)^{\ell+1}}{\alpha}\right] = \Lambda.
\end{aligned}$$

Consequently, by (25) we see that $\pi_0(r) \to \pi_0$ as $r \to \infty$. Next, recall that for $r < \infty$,

$$\Delta \approx \Delta_r = \int_0^\infty v_2(x)\mathrm{d}H(x)$$

where $v_2(x) = \lim_{k\to\infty} v_2^k(x)$. So as $r \to \infty$, we substitute $\boldsymbol{v}^0(x) = (\pi_0,\, 1-\pi_0,\, 0)$ in the expression

$$\boldsymbol{v}^k(x) = \boldsymbol{v}^0(x) \prod_{n=0}^{k-1} \mathbf{P}(n,x).$$

Using (13), (21), (23), and (24), we now show by induction on k that the elements of $\boldsymbol{v}^k(x)$ are

$$\begin{aligned}
v_0^k(x) &= \pi_0^{k+1} G(k,x), && (26)\\
v_1^k(x) &= \sum_{n=1}^{k} \left[\pi_0^n (1-\pi_0) G(n,x)\right] + 1 - \pi_0, && (27)\\
v_2^k(x) &= \pi_0 \left[1 - \pi_0^{k-1} G(k,x) - (1-\pi_0) \sum_{n=1}^{k-1} \pi_0^{n-1} G(n,x)\right]. && (28)
\end{aligned}$$

For $k=1$, applying (14) with $\boldsymbol{v}^0(x) = (\pi_0,\, 1-\pi_0,\, 0)$, it is easy to see that

$$\begin{aligned}
v_0^1(x) &= \pi_0^2 G(1,x),\\
v_1^1(x) &= \pi_0(1-\pi_0) G(1,x) + 1 - \pi_0,\\
v_2^1(x) &= \pi_0[1 - G(1,x)],
\end{aligned}$$

where the summation in (28) is 0 when $k=1$. Similarly, for $k=2$,

$$\begin{aligned}
v_0^2(x) &= \pi_0^3 G(2,x),\\
v_1^2(x) &= \pi_0^2(1-\pi_0) G(2,x) + \pi_0(1-\pi_0) G(1,x) + 1 - \pi_0,\\
v_2^2(x) &= \pi_0\left[1 - (1-\pi_0) G(1,x) - \pi_0 G(2,x)\right];
\end{aligned}$$

therefore, the result holds for $k=1,2$. For the inductive step, assume that (26)–(28) hold for an arbitrary $m \in \mathbb{N}$. Then, after some matrix algebra, we obtain

$$\begin{aligned}
v_0^{m+1}(x) &= \pi_0^{m+2} G(m+1,x),\\
v_1^{m+1}(x) &= \sum_{n=1}^{m+1} \left[\pi_0^n (1-\pi_0) G(n,x)\right] + 1 - \pi_0,\\
v_2^{m+1}(x) &= \pi_0 \left[1 - \pi_0^{m} G(m+1,x) - (1-\pi_0) \sum_{n=1}^{m} \pi_0^{n-1} G(n,x)\right],
\end{aligned}$$

and the induction proof is complete. Now, as $r \to \infty$,

$$\begin{aligned}
v_2(x) = \lim_{k\to\infty} v_2^k(x) &= \lim_{k\to\infty} \left[\pi_0 \left(1 - \pi_0^{k-1} G(k,x) - (1-\pi_0) \sum_{n=1}^{k-1} \pi_0^{n-1} G(n,x)\right)\right]\\
&= \pi_0 \left[1 - (1-\pi_0) \sum_{n=1}^{\infty} \pi_0^{n-1} G(n,x)\right].
\end{aligned}$$

We obtain a closed-form expression for $v_2(x)$ via its Laplace-Stieltjes transform, $\tilde{v}_2(s)$, given by

$$\begin{aligned}\tilde{v}_2(s) \equiv \int_0^\infty e^{-sx} \mathrm{d}v_2(x) &= \pi_0 \left[1 - \frac{1-\pi_0}{\pi_0} \sum_{n=1}^{\infty} \left(\frac{\pi_0(\mu-\lambda_q)}{\mu-\lambda_q+s}\right)^n\right] \\ &= \pi_0 \left[1 - \frac{1-\pi_0}{\pi_0} \left(\sum_{n=0}^{\infty} \left(\frac{\pi_0(\mu-\lambda_q)}{\mu-\lambda_q+s}\right)^n - 1\right)\right] \\ &= \pi_0 \left[1 - \frac{(1-\pi_0)(\mu-\lambda_q)}{(1-\pi_0)(\mu-\lambda_q)+s}\right],\end{aligned}$$

Now, $\tilde{v}_2(s)$ can be inverted analytically to obtain

$$v_2(x) = \mathcal{L}^{-1}\left\{\frac{\tilde{v}_2(s)}{s}\right\} = \pi_0\, e^{-(1-\pi_0)(\mu-\lambda_q)x},$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform operator. Finally, we obtain

$$\begin{aligned}\lim_{r\to\infty} \Delta_r &= \int_0^\infty \pi_0\, e^{-(1-\pi_0)(\mu-\lambda_q)x} \mathrm{d}H(x) \\ &= \int_0^\infty \mathbb{P}(\widetilde{T} > X | I_n = 0, X = x)\pi_0 \mathrm{d}H(x) \\ &= \mathbb{P}(\widetilde{T} > X) \\ &= \Delta_\infty.\end{aligned}$$

■

By considering a limited transmission range, the model presented herein explicitly accounts for query revisiting *and* boundary effects (i.e., the effect of nodes adjacent to the boundary of the sensor field). In Section 5, we illustrate and assess the quality of the finite and infinite transmission range approximations by comparing the steady state proportion of time uninformed and proportion of query failures with results obtained by a commercial network simulator.

5 Numerical Examples and Validation

The analytical approximations of Sections 3 and 4 provide a relatively easy way to evaluate the behavior of query-based wireless sensor networks. In this section, we assess the quality and validity of these approximations by comparing them with simulated values obtained using the OPNET commercial network simulator. Presented herein are summary tables and figures for uniform-topology networks with a variety of distributional assumptions and sensor transmission ranges. For each experiment, the minimum transmission range was chosen to ensure a connected network with probability $p = 0.9999$ using (1). Initially, results for 1000- and 5000-node networks are provided before presenting an extensive validation study that examines impact of our model assumptions.

For each scenario, the performance parameter is the maximum absolute deviation (MAD) between the approximated value and its simulated counterpart over a finite set of time-to-live values, $L \equiv \{1, 2, \ldots, 30\}$. We choose this set because, for many typical wireless applications, a TTL counter between 3 and 25 is suitable. For each $\ell \in L$, let π_0^ℓ be the approximate steady state proportion of time nodes are uninformed, assuming $r = \infty$, which is obtained via (6), i.e.,

$$\pi_0^\ell = \exp\left[-\frac{\lambda}{\delta}\left(\frac{1-(1-\alpha)^{\ell+1}}{\alpha}\right)\right].$$

Similarly, let $\pi_0^\ell(r)$ be the same value, assuming $r < \infty$, obtained by (25). That is,

$$\pi_0^\ell(r) = \exp\left[-\frac{\lambda}{\delta}\left(1+\sum_{i=1}^{\ell}\widehat{q}(i,r)(1-\alpha)^i\right)\right].$$

For both cases, the probability α is approximated using the fixed point algorithm described in Section 3. To express the dependence of Δ on the TTL value ℓ, let Δ_∞^ℓ and Δ_r^ℓ denote the steady proportion of query failures when $r = \infty$ and $r < \infty$, respectively. Using (12) and (20), respectively, we compute

$$\Delta_\infty^\ell \equiv \pi_0^\ell \int_0^\infty \exp\left[-(1-\pi_0^\ell)(\mu-\lambda_q)x\right] \mathrm{d}H(x)$$

and

$$\Delta_r^\ell \equiv \int_0^\infty v_2(x)\mathrm{d}H(x),$$

where $v_2(x)$ is obtained via (19). In cases where the integrals cannot be evaluated in closed form, we perform numerical integration via the trapezoidal rule. Finally, we define $\pi_0^s(\ell)$ as the simulated steady state proportion of time nodes are uninformed, and $\Delta^s(\ell)$ as the simulated steady state proportion of query failures when the TTL counter is $\ell \in L$.

The MAD between the true (simulated) values and their corresponding analytical approximations are therefore

$$D_\pi \equiv \max_{\ell \in L} \left|\pi_0^s(\ell) - \widehat{\pi}_0(\ell)\right|, \tag{29}$$

where $\widehat{\pi}_0(\ell) = \pi_0^\ell$ if $r = \infty$, and $\widehat{\pi}_0(\ell) = \pi_0^\ell(r)$ if $r < \infty$. Similarly, let

$$D_\Delta \equiv \max_{\ell \in L} \left|\Delta^s(\ell) - \widehat{\Delta}_0(\ell)\right|, \tag{30}$$

where $\widehat{\Delta}(\ell) = \Delta_\infty^\ell$ if $r = \infty$, and $\widehat{\Delta}(\ell) = \Delta_r^\ell$ if $r < \infty$. For Examples 1 and 2 that follow, a few parameter values were held constant; these values are summarized in Table 1. Moreover, we assumed event lifetimes are exponentially distributed with parameter δ in these two cases, but this assumption is relaxed in Example 3.

Table 1: Summary of parameter values for OPNET simulation: Examples 1 and 2.

Parameter	Parameter description	Value
μ	Transmitter's exponential transmission rate	5.000
λ	Poisson rate of locally-witnessed events (for all $n \in \mathcal{N}$)	0.005
γ	Poisson rate of locally-generated queries (for all $n \in \mathcal{N}$)	0.050
$1/\delta$	Mean event lifetime	10.000
$1/\beta$	Mean query lifetime (for all distributions)	5.000

The analytical approximations were coded in the C programming language and executed in Microsoft® Visual Studio® 2008 on a personal computer equipped with an Intel® Core™ 2 Duo CPU operating at 3.00GHz with 2.00 GB of RAM. The simulated values were obtained via a discrete-event simulation model created in the OPNET Modeler® Wireless Suite v. 15. Ten (10) independent replications were performed for each $\ell \in L$ to ensure a standard error less than 5×10^{-4}. The reported simulated values represent the average of the 10 replications. The run length was 3720s, including a 120s warm-up period for each replication. The simulation experiments were conducted on a personal computer equipped with an Intel® Core™ i7 CPU operating at 2.67GHz with 2.00 GB of RAM.

Example 1: 1000-Node Network: Here, we present results for a 1000-node wireless sensor network with nodes distributed randomly in a 3335m $\times$ 3335m sensor field. The node density is $\psi \approx 9.00 \times 10^{-5}$ nodes per square meter. To ensure a connected network with probability 0.9999, the minimum required sensor transmission range is $r = 239$m. Therefore, we considered the following transmission ranges: 350m, 500m, 1000m, 5000m. By doing so, we are able to assess the quality of both the infinite and finite range approximations. Table 2 summarizes the MAD in the proportion of time uninformed using each transmission range. The column labeled "$r = \infty$" corresponds to the infinite transmission range approximation, and the column labeled "$r < \infty$" is the finite range approximation.

Table 2: MAD in the proportion of time uninformed (D_π) when $N = 1000$.

Query lifetime	350m		500m		1000m		5000m	
	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$
Exponential(0.2)	0.0523	0.0045	0.0289	0.0065	0.0202	0.0116	0.0024	0.0060
Triangular(0.1, 5.0, 9.9)	0.0529	0.0059	0.0296	0.0055	0.0173	0.0088	0.0173	0.0131
Uniform(0.1, 9.9)	0.0507	0.0068	0.0285	0.0041	0.0171	0.0093	0.0039	0.0070
Rayleigh(5.645)	0.0511	0.0050	0.0298	0.0055	0.0176	0.0088	0.0043	0.0069
Weibull(3.0, 5.6)	0.0511	0.0061	0.0302	0.0061	0.0173	0.0088	0.0035	0.0072

Table 2 indicates an order of magnitude improvement by including the revisiting effect, especially when the actual transmission range is small (350m). Because queries are more likely to revisit neighbors when the transmission range is small, the difference between the two approximations is quite pronounced. Table 2 also illustrates the performance of the approximations when the query lifetime distribution is not memoryless. The results are consistent with the case of exponential query lifetimes. Figure 3 shows the performance of the approximations and reveals that the finite range approximation is superior to the infinite range approximation for all TTL values when r is small. Indeed, the gap between the latter approximation and OPNET simulation values increases with ℓ since the revisiting effect is more pronounced when the time-to-live counter is large. For larger ranges, the approximations nearly coincide and both closely track the simulated values.

Results for the steady state proportion of query failures are summarized in Table 3. Both approximation schemes perform extremely well (the maximum absolute deviation over all cases is less than 0.049). It is also worth noting that the finite range approximation outperforms the

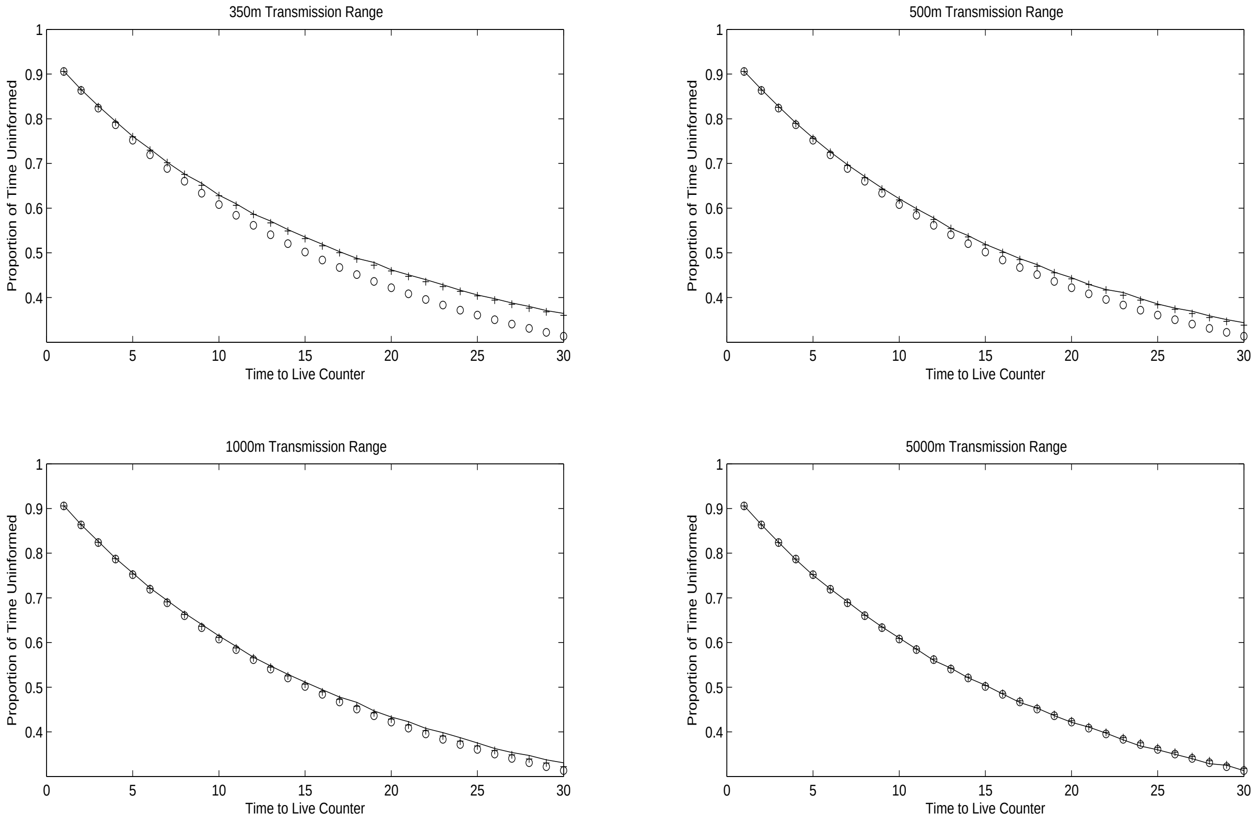

Figure 3: Comparison of π_0 values with Weibull query lifetimes ($N = 1000$): (-) OPNET; (o) $r = \infty$; (+) $r < \infty$.

infinite range approximation, particularly when r is relatively small. The results are consistent for non-memoryless query lifetimes. Figure 4 graphically depicts the four cases.

Table 3: MAD in the proportion of failed queries (D_Δ) when $N = 1000$.

Query lifetime	350m		500m		1000m		5000m	
	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$
Exponential(0.2)	0.0371	0.0246	0.0168	0.0103	0.0047	0.0055	0.0060	0.0052
Triangular(0.1, 5.0, 9.9)	0.0459	0.0301	0.0170	0.0128	0.0030	0.0008	0.0082	0.0024
Uniform(0.1, 9.9)	0.0383	0.0258	0.0178	0.0121	0.0035	0.0015	0.0051	0.0016
Rayleigh(5.645)	0.0237	0.0127	0.0065	0.0158	0.0256	0.0270	0.0255	0.0247
Weibull(3.0, 5.6)	0.0485	0.0306	0.0230	0.0149	0.0031	0.0014	0.0022	0.0014

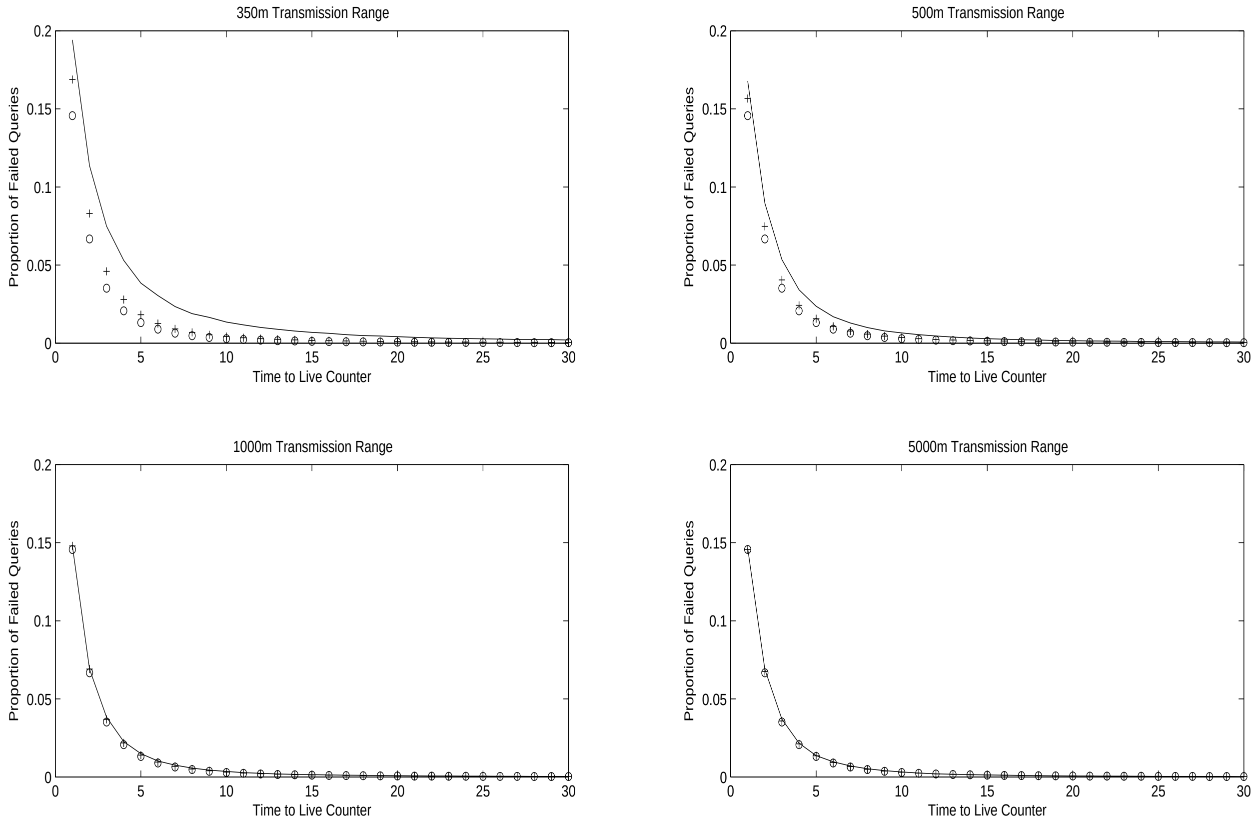

Figure 4: Comparison of Δ values with Weibull query lifetimes ($N = 1000$): (-) OPNET; (o) $r = \infty$; (+) $r < \infty$.

Example 2: 5000-Node Network: Here, we consider a 5000-node wireless sensor network with nodes deployed in the same region as the 1000-node case but with node density $\psi \approx 4.50 \times 10^{-4}$ nodes per square meter. To ensure a connected network with probability 0.9999, the minimum required sensor transmission range is $r = 112$m. Therefore, we considered the following transmission ranges: 115m, 350m, 500m, and 5000m. Table 4 illustrates the quality of both approximations for the 5000-node network. The maximum absolute deviation for the proportion of time uninformed is less than 0.082 for $r = \infty$, and it is reduced to, at most, 0.0174 when the revisiting effect is included. As before, the superiority of the finite range approximation is generally more pronounced for smaller transmission ranges. Figure 5 depicts the simulated and approximated values of π_0 when the query

Table 4: MAD in the proportion of time uninformed (D_π) when $N = 5000$.

Query lifetime	115m		350m		500m		5000m	
	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$
Exponential(0.2)	0.0605	0.0172	0.0107	0.0019	0.0082	0.0031	0.0040	0.0049
Triangular(0.1, 5.0, 9.9)	0.0612	0.0174	0.0100	0.0015	0.0068	0.0026	0.0053	0.0062
Uniform(0.1, 9.9)	0.0819	0.0054	0.0105	0.0013	0.0074	0.0025	0.0048	0.0058
Rayleigh(5.645)	0.0595	0.0172	0.0101	0.0016	0.0074	0.0026	0.0051	0.0060
Weibull(3.0, 5.6)	0.0611	0.0174	0.0099	0.0014	0.0068	0.0028	0.0073	0.0083

lifetime follows a triangular distribution. When the transmission range is small (115m), we see some discrepancy between the two approximation schemes. However, for the other three cases, the approximations nearly coincide and are very similar to the simulated results ($D_\pi < 0.011$).

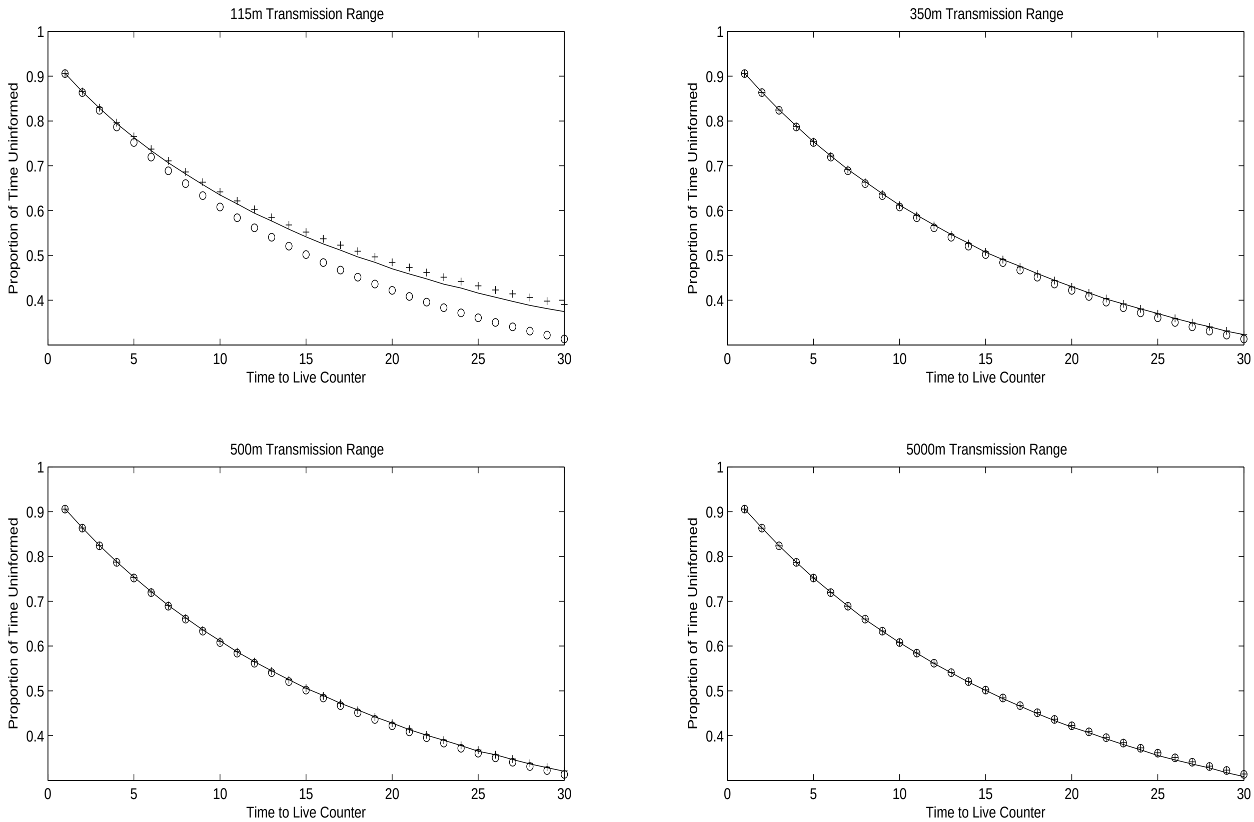

Figure 5: Comparison of π_0 values with triangular query lifetimes ($N = 5000$): (-) OPNET; (o) $r = \infty$; (+) $r < \infty$.

Next, we compare the maximum absolute deviation of the proportion of query failures. Table 5 shows that the maximum deviation values are bounded above by 0.0725. Again, the finite range approximation outperforms the infinite range version when the transmission range is small. However, for larger ranges, the results nearly coincide and closely track the simulated values.

Table 5: MAD in the proportion of query failures (D_Δ) when $N = 5000$.

Query lifetime	115m		350m		500m		5000m	
	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$
Exponential(0.2)	0.0493	0.0283	0.0046	0.0022	0.0045	0.0043	0.0061	0.0044
Triangular(0.1, 5.0, 9.9)	0.0588	0.0333	0.0052	0.0026	0.0013	0.0024	0.0044	0.0045
Uniform(0.1, 9.9)	0.0724	0.0493	0.0044	0.0047	0.0051	0.0021	0.0078	0.0023
Rayleigh(5.645)	0.0371	0.0170	0.0214	0.0192	0.0265	0.0225	0.0286	0.0228
Weibull(3.0, 5.6)	0.0619	0.0351	0.0071	0.0064	0.0025	0.0041	0.0039	0.0021

Figure 6 graphically depicts the simulated and approximated values of Δ and illustrates the high quality of the approximations. In the worst case (115m), the MAD is less than 0.0725 and 0.05 for $r = \infty$ and $r < \infty$, respectively.

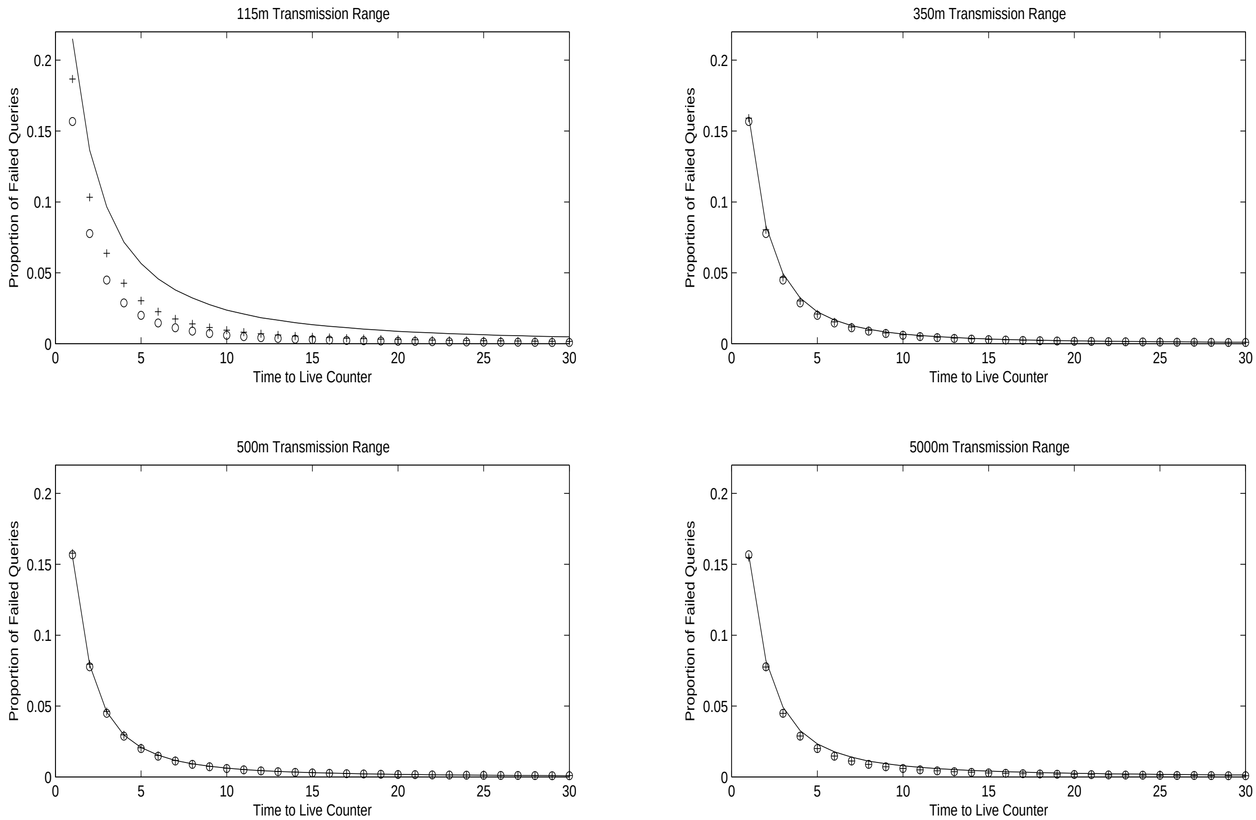

Figure 6: Comparison of Δ values with triangular query lifetimes ($N = 5000$): (-) OPNET; (o) $r = \infty$; (+) $r < \infty$.

Example 3: Model Validation: Finally, we conducted an experiment to validate the approximations when some of the model assumptions are violated. For the benchmark simulation experiments presented here, events arrive according to a renewal process with a specified (non-exponential) interarrival time distribution (i.e., the event arrival process is not Poisson). This experiment also employs non-exponential event agent and query lifetimes, both of which are incorporated in the approximations of Sections 3 and 4.

Table 6 provides a summary of the numerical results for 45 distinct test cases using a 1000-node wireless sensor network with nodes distributed randomly in a 3335m $\times$ 3335m sensor field. The node density is $\psi \approx 9.00 \times 10^{-5}$ nodes per square meter. To ensure a connected network with probability 0.9999, the minimum required sensor transmission range is $r = 239$m; therefore, we set $r = 350$m.

Table 6: MAD in the proportion of time uninformed (D_π) and proportion of failed queries (D_Δ).

Trial	Event interarrival time	Event lifetime	Query lifetime	D_π		D_Δ	
				$r = \infty$	$r < \infty$	$r = \infty$	$r < \infty$
1	Erlang(5, 40.0)	Erlang(4, 2.5)	Erlang(5, 1.0)	0.0711	0.0233	0.0421	0.0265
2	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Erlang(5, 1.0)	0.0693	0.0215	0.0428	0.0273
3	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Erlang(5, 1.0)	0.0596	0.0124	0.0424	0.0273
4	Erlang(5, 40.0)	Erlang(4, 2.5)	Rayleigh(5.645)	0.0696	0.0213	0.0201	0.0106
5	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Rayleigh(5.645)	0.0699	0.0220	0.0203	0.0106
6	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Rayleigh(5.645)	0.0594	0.0130	0.0213	0.0106
7	Erlang(5, 40.0)	Erlang(4, 2.5)	Triangular(0.1, 5.0, 9.9)	0.0687	0.0213	0.0399	0.0262
8	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0682	0.0206	0.0407	0.0267
9	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0584	0.0100	0.0400	0.0259
10	Erlang(5, 40.0)	Erlang(4, 2.5)	Uniform(0.1, 9.9)	0.0691	0.0218	0.0340	0.0209
11	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Uniform(0.1, 9.9)	0.0690	0.0211	0.0342	0.0205
12	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Uniform(0.1, 9.9)	0.0589	0.0118	0.0344	0.0211
13	Erlang(5, 40.0)	Erlang(4, 2.5)	Weibull(3.0, 5.6)	0.0699	0.0229	0.0424	0.0278
14	Erlang(5, 40.0)	Triangular(0.1, 10.0, 19.9)	Weibull(3.0, 5.6)	0.0677	0.0201	0.0427	0.0282
15	Erlang(5, 40.0)	Uniform(0.1, 19.9)	Weibull(3.0, 5.6)	0.0584	0.0096	0.0428	0.0278
16	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Erlang(5, 1.0)	0.0739	0.0261	0.0438	0.0282
17	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Erlang(5, 1.0)	0.0713	0.0235	0.0429	0.0274
18	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Erlang(5, 1.0)	0.0615	0.0145	0.0433	0.0282
19	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Rayleigh(5.645)	0.0718	0.0235	0.0213	0.0106
20	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Rayleigh(5.645)	0.0728	0.0250	0.0215	0.0106
21	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Rayleigh(5.645)	0.0614	0.0150	0.0217	0.0114
22	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Triangular(0.1, 5.0, 9.9)	0.0713	0.0239	0.0410	0.0273
23	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0725	0.0249	0.0405	0.0265
24	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0523	0.0052	0.0418	0.0277
25	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Uniform(0.1, 9.9)	0.0709	0.0237	0.0343	0.0212
26	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Uniform(0.1, 9.9)	0.0717	0.0238	0.0356	0.0219
27	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Uniform(0.1, 9.9)	0.0618	0.0146	0.0296	0.0163
28	Triangular(1.0, 200.0, 399.0)	Erlang(4, 2.5)	Weibull(3.0, 5.6)	0.0718	0.0247	0.0437	0.0290
29	Triangular(1.0, 200.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Weibull(3.0, 5.6)	0.0721	0.0246	0.0431	0.0286
30	Triangular(1.0, 200.0, 399.0)	Uniform(0.1, 19.9)	Weibull(3.0, 5.6)	0.0519	0.0037	0.0438	0.0288
31	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Erlang(5, 1.0)	0.0737	0.0259	0.0453	0.0297
32	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Erlang(5, 1.0)	0.0675	0.0201	0.0459	0.0304
33	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Erlang(5, 1.0)	0.0630	0.0158	0.0457	0.0306
34	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Rayleigh(5.645)	0.0742	0.0260	0.0243	0.0127
35	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Rayleigh(5.645)	0.0737	0.0258	0.0234	0.0123
36	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Rayleigh(5.645)	0.0637	0.0167	0.0233	0.0133
37	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Triangular(0.1, 5.0, 9.9)	0.0751	0.0276	0.0435	0.0298
38	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0745	0.0269	0.0440	0.0300
39	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Triangular(0.1, 5.0, 9.9)	0.0634	0.0155	0.0444	0.0300
40	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Uniform(0.1, 9.9)	0.0749	0.0277	0.0369	0.0255
41	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Uniform(0.1, 9.9)	0.0749	0.0270	0.0356	0.0219
42	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Uniform(0.1, 9.9)	0.0634	0.0165	0.0379	0.0252
43	Uniform(1.0, 399.0)	Erlang(4, 2.5)	Weibull(3.0, 5.6)	0.0728	0.0257	0.0458	0.0311
44	Uniform(1.0, 399.0)	Triangular(0.1, 10.0, 19.9)	Weibull(3.0, 5.6)	0.0746	0.0270	0.0464	0.0319
45	Uniform(1.0, 399.0)	Uniform(0.1, 19.9)	Weibull(3.0, 5.6)	0.0648	0.0161	0.0469	0.0319

Table 6 reveals some very interesting results. First, we note that the performance of the finite-range approximation is similar to that reported in Example 1 which assumed Poisson-generated events. Despite the fact that the event arrival process is distinctly non-Poisson, and the query and event lifetimes are non-memoryless, the proportion of failed queries is approximated very closely using the finite-range approximation (as compared to the OPNET simulated values). Over all 45 test cases, the maximum absolute deviation is on the order of 0.03. These findings are significant because they provide empirical evidence that the approximations are not heavily influenced by the Poisson arrival assumption at the event tables or the transmission queues. Moreover, for each instance, the approximated proportion of time uninformed and proportion of query failures ions were computed in less than 20 minutes as opposed to the OPNET simulation results, which required approximately 2 hours.

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