PARAMETERIZATION AND VALIDATION OF AN INTEGRATED ELECTRO-THERMAL LFP BATTERY MODEL

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ABSTRACT

In this paper the parameterization of an integrated electro-thermal model for an A123 26650 LiFePO4 battery is presented. The electrical dynamics of the cell are described by an equivalent circuit model. The resistances and capacitances of the equivalent circuit model are identified at different temperatures and SOC’s, for charging and discharging. Functions are chosen to characterize the fitted parameters. A two-state thermal model is used to capture the core and surface temperatures of the battery. The electrical model is coupled with the thermal model through heat generation. Parameters of the thermal model are identified using a least squares algorithm. The electro-thermal model is then validated against voltage and surface temperature measurements from a realistic drive cycle experiment.

1 INTRODUCTION

Lithium Ion Batteries are attractive energy storage devices for Hybrid Electric (HEV), Plug In Hybrid Electric (PHEV), and Electric Vehicles (EV) due to their reasonable power and energy density. The ability to accurately predict the electrical and temperature dynamics of a battery is critical for designing onboard battery management systems (BMS), and thermal management systems.

Electrical models vary in complexity. For some applications, a simple model capturing the basic electrical behavior can be sufficient (eg. an OCV-R model). There are more complex electrochemical models [1–3] that are highly accurate [4–6], but hard to be fully parameterized [6], and require large computational capacity. Therefore, they are not suitable for control oriented modeling. Equivalent circuit models are commonly used, which offer a tradeoff between accuracy and simplicity, and are suitable for control oriented applications [7–11].

The equivalent circuit model can capture the terminal voltage of the battery and has been widely adopted since the work in [12]. The voltage supply in the equivalent circuit, shown in Fig. 1, represents the open circuit voltage ($V_{OCV}$) which is a function of state of charge. The series resistance ($R_s$) represents internal resistance of the battery. The voltage drop across the two resistor-capacitor (RC) pairs ($V_1$ and $V_2$) are used to model the dynamic voltage losses due to lithium diffusion in the solid phase and in the electrolyte [13]. These circuit elements depend on state of charge (SOC), temperature, and current direction as shown in [10]. These parameter dependencies are important for accurately capturing the dynamics of battery terminal voltage throughout a usable range of temperature and state of charge.

In addition to predicting the terminal voltage, an accurate model of the battery temperature is needed for control and thermal management to constrain the operating temperature range. In applications with high power demands, such as automotive traction batteries, the internal temperature of the battery may rise quickly, due to joule heating, and can be higher than the surface temperature. The internal temperature of the battery should be limited to extend battery life [14]; however, in practice only the surface temperature of the battery may be measured. Therefore, a thermal model is needed to predict the missing information for use in the equivalent circuit electrical model and for thermal
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management.

Coupled electro-thermal models have been investigated using PDE based electrical models in [2, 4, 15], and equivalent circuit based electrical models in [14, 16–18]. The thermal models used in these studies have either been complex, or very simple only capturing the lumped temperature. Complex thermal models that capture the detailed temperature distribution in a cell have been used [15, 19, 20], but require a large amount of computational resources, making them unsuitable for control oriented modeling. A simple thermal model that predicts the critical temperature of a cylindrical cell is desired, such as the two state thermal model. A simple thermal model that predicts the critical temperature distribution in a cell can be expanded to a battery pack configuration to estimate unmeasured temperatures as presented in [22].

In this paper an OCV-R-RC-RC equivalent circuit electrical model is integrated with a two state thermal model. In Section 2, the coupling between the heat generation and temperature in the integrated electro-thermal model is highlighted by the temperature dependence of the equivalent circuit parameters. In Section 3, we first show how the electric model can be parameterized using a low current rate so that isothermal conditions could be assumed. The identified parameters and their dependencies on SOC, current direction, and temperature are examined. Basis functions are chosen to represent the temperature and SOC dependence of the circuit elements. Next the parameters of the thermal model are identified using the heat generation calculated by the modeled open circuit voltage for a high C-rate drive cycle. Finally in Section 4, the coupled electro-thermal model is validated against the measured terminal voltage and surface temperature data from a drive cycle experiment.

2 BATTERY MODEL

In this section the electrical and thermal battery models are presented. The model parameter dependencies are introduced, and an electro-thermal model is formed through a heat generation term.

2.1 Electrical Model

The battery state of charge (SOC) is defined by current integration as,

$$\dot{SOC} = -\frac{1}{\Delta SOC} I.$$  \hspace{1cm} (1)

The nominal capacity of the cell $C_n (Ah)$ is found by cycling the battery cell per manufacturer recommendation [24]. The charging profile consists of a Constant Current - Constant Voltage (CC-CV) charging cycle that is terminated when the current tapers below 50 mA, and the voltage at the end of discharge is 2.0 V.

The heat generation $Q$ in the battery cell is defined by the polarization heat from joule heating and energy dissipated in the electrode over-potentials [19]. The effect of the entropic heat generation is excluded for simplicity, as it is relatively small compared to the total heat generation for an LiFePO4 cell as shown by [23]. The entropic heat would contribute less than 1% of mean $Q$ for the drive cycle used in this paper.
2.2 Thermal Model

Figure 2. SINGLE CELL LUMPED THERMAL MODEL.

The two state thermal model is defined by,

\[
\begin{align*}
C_c \dot{T}_c &= Q + \frac{T_s - T_c}{R_c} \\
C_s \dot{T}_s &= \frac{T_f - T_c}{R_u} - \frac{T_s - T_c}{R_c}.
\end{align*}
\]

(5)

where \(T_c(\circ C)\) and \(T_s(\circ C)\) represent the core and surface temperature states respectively. The temperature used by the equivalent circuit model is the mean of the core and surface temperatures defined as \(T_m(\circ C)\). The inputs are the inlet air coolant temperature \(T_f(\circ C)\) and the heat generation \(Q\) calculated by the electrical model shown by Eq. 4. The parameters \(C_c(J/K)\) and \(C_s(J/K)\) are the lumped heat capacities of the core and surface respectively, \(R_c(K/W)\) is the equivalent conduction resistance between the core and surface respectively, \(R_u(K/W)\) is the equivalent convection resistance around the cell. The convective resistance \(R_u\) depends on the flow condition, and can be modeled for different types of coolants as described in [30, 31].

2.3 Model Coupling

The electro-thermal model is formed by taking the calculated heat generation from the electrical model as an input to the thermal model. The thermal model then generates the battery surface and core temperatures, used to find the mean battery temperature for the parameters of the electrical model, as shown in Fig. 3. The inputs of the electro-thermal model are the current \(I\) for the electrical model, and the air inlet temperature \(T_f\) for the thermal model. The electro-thermal model outputs are SOC, voltage, and the battery temperatures.

The coupling results in a stable system because of the negative feedback for the temperature states. The negative feedback coupling between the thermal and electrical models can be explained by the temperature dependence of battery internal resistance. To understand this coupling, consider a constant current. Under this condition the heat generation \(Q\) will decrease with increasing cell temperature, because the reaction kinetics become more favorable, which further reduces the internal resistance.

3 MODEL PARAMETER IDENTIFICATION

In this section the electrical and thermal model parameterization methods are described. First the parameters of the equivalent circuit model are identified from pulse current discharge/charge and relaxation experiments at different SOC’s and temperatures with the battery placed inside a thermal chamber. Then using the calculated heat generation of the cell, the parameterization of the thermal model is presented using a least-squared fitting algorithm originally developed in [32].

There are different methods of identifying equivalent circuit model parameters such as electro impedance spectroscopy (EIS) [11], genetic algorithm (GA) optimization [25], and nonlinear least squares curve fitting techniques [10]. Most of these involve identifying parameters with respect to SOC as in [7–9, 11, 33], in addition the parameters are shown to depend on temperature and current direction [10, 14, 25]. The method selected here is to identify parameters from experimental pulse current data using nonlinear least squares curve fitting. Assuming isothermal conditions the identification is performed at each temperature and SOC grid point (5 parameters per pulse) in order to avoid simultaneous identification of the full parameter set (ie. 360 parameters in this model). This reduces the computational burden and allows us to investigate the equivalent circuit’s parameter dependence on temperature and SOC.

3.1 Electrical Model Parameterization

Experiments to parameterize the electrical model for a 2.3Ah A123 26650 LiFePO₄ cell were conducted using a Yokogawa GS-610 Source Measure Unit to control the current, and a Cincinnati Sub-Zero ZPHS16-3.5-SCT/AC environmental chamber to regulate the air coolant temperature. The tests were
conducted in the environmental chamber. The battery temperature is assumed to be isothermal and \( T_m \) equal to the ambient temperature in the chamber due to the low C-rate experiments. This assumption is consistent with the small measured rise in surface temperature of the battery cell, less than 0.7°C, during the pulsed discharge.

First the capacity of the cell is measured by cycling the battery at low rate (C/20). The \( V_{OCV} \) curve is assumed to be the average of the charge/discharge curves corresponding to the same C/20 cycle test at 25°C. The effect of hysteresis in this cell chemistry results in a voltage gap between the charge and discharge curves as explained in [25–28]. Since hysteresis is not being modeled in this paper, the average curve is used for \( V_{OCV} \). It is shown in [10], that there is a minimal effect on \( V_{OCV} \) with respect to the temperature range of study here for an LiFePO\(_4\) cell. Therefore, \( V_{OCV} \) is modeled with an SOC dependence.

After \( V_{OCV} \) and capacity are determined, the experiments to generate data for parameterization of the RC elements are conducted. First the cell sits at a constant temperature set point for 2h to ensure thermal equilibrium. The battery is then charged up to 100% SOC using a 1C CCCV charge protocol at the 3.6V maximum until a 50mA CV cutoff current is reached. It is then discharged by 10%SOC at 1C rate, and relaxed for 2h. This process is repeated until the 2V minimum is reached. The pulse current followed by a 2h relaxation profile is repeated for the charge direction up to the 3.6V maximum. The pulse discharging and charging is conducted at different temperatures, resulting in 15°C, 25°C, 35°C, 45°C datasets. The voltage and current profile of one of the pulse discharge tests at 15°C is shown in Fig. 4.

The equivalent circuit parameter \( R_s(\Omega) \) is found using Ohm’s law and the measured initial voltage jump \( \Delta V_s \) (shown in the inset of the top subplot of Fig. 4) defined as,

\[
R_s = \frac{\Delta V_s}{I}.
\]

Where \( I \) is the current applied during the pulse discharge/charge before the relaxation period (eg. 2.3A as shown in the inset of the bottom subplot of Fig. 4). The remaining equivalent circuit model parameters are identified by minimizing the error in voltage between the model and data during the relaxation period,

\[
J_{Electrical} = \min \sum_{i=1}^{n} (V_{data}(i) - V_{model}(i))^2,
\]

using the lsqcurvefit function in MATLAB. Each instance is represented by \( i \), starting from the first voltage relaxation datapoint \( i = 1 \), up to the last datapoint \( i = n \).

The voltage recovery during relaxation, \( V_{relax}(t_r) \), is derived by solving Eq. (2), assuming the capacitor voltages \( V_1, V_2 \) at the end of the previous rest period are zero

\[
V_{relax}(t_r) = IR_1(1 - \exp(-\frac{t_{pulse}}{RC_1})) + IN_2(1 - \exp(-\frac{t_r}{RC_2})).
\]

Where \( t_{pulse} \) is duration of the constant current pulse prior to the relaxation period, and \( t_r \) is the time since the start of relaxation, as shown in Fig. 4.

The inclusion of two or more RC pairs in the equivalent circuit model increases the accuracy of the cell voltage dynamic prediction as seen in [7, 9, 11, 25]. A comparison of the performance for best fit single RC, double RC, and triple RC models is shown Fig. 5. One can see that the single RC pair model yields large error especially during the first 500 seconds of relaxation, whereas the double RC and triple RC pair models yield less error across the entire dataset time period. It is evident that the higher order RC models can achieve a better fit to the relaxation voltage data than that of the single RC pair model. The double RC pair model has sufficient accuracy with the lowest number of states and therefore is an appropriate choice.

### 3.2 Equivalent Circuit Parameters

The equivalent circuit parameters can then be characterized as functions of SOC, and temperature for the discharge and charge direction as shown in [10]. The calculated internal resistance \( R_s \), from Eq. (6), is shown in Fig. 6 with respect to SOC and temperature for discharge and charge. The internal resistance \( R_s \) has a minimal dependence on SOC over the range of 10 to 90 %, but depends strongly on temperature and current direction. Therefore, the \( R_s \) parameters can be represented by an exponential function of the mean temperature \( T_m \), for the discharging and
charging cases, as shown by,

\[
R_s = \begin{cases} 
R_{sd}, & I > 0 \quad \text{(discharge)} \\
R_{sc}, & I < 0 \quad \text{(charge)} 
\end{cases}
\]

\[
R_{sd} = R_{sd0} \exp \left( \frac{T_{refRd}}{T_m - T_{shiftRd}} \right)
\]

Where \( * = d, c \) represents the value during discharging and charging respectively. The characterized \( R_s \) functions in Eq. (9) are plotted along with the \( R_s \) values fit from the relaxation data using Eq. (7), in Fig. 6. The values for Eq. (9) are shown in Tab. 1.

The parameters \( R_1, R_2 \) are characterized by including an SOC dependency to the function in Eq. (9) used for the parameter \( R_s \). The corresponding \( R_1, R_2 \) functions including SOC and temperature dependence for discharge and charge are shown in Fig. 7.

### 3.3 Thermal Model Parameterization

The experiment procedure used to identify the thermal model parameters is the Urban Assault Cycle (UAC), scaled for the A123 26650 cell, as explained in [32]. This cycle has been presented in [34], for a 13.4 ton armored military vehicle. The cell is first charged to 100% SOC using a 1C CC-CV protocol until the 50mA CV cutoff current is reached. It is then discharged at 1C to about 50% SOC. The UAC current profile is then applied to the cell under constant coolant flow, with the measured inlet temperature \( T_f \) as shown in Fig. 7.

<table>
<thead>
<tr>
<th>( R_{fd} )</th>
<th>( R_{soc} )</th>
<th>( T_{refRd} )</th>
<th>( T_{refRc} )</th>
<th>( T_{shiftRd} )</th>
<th>( T_{shiftRc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0048</td>
<td>0.0055</td>
<td>31.0494</td>
<td>22.2477</td>
<td>-15.3253</td>
<td>-11.5943</td>
</tr>
</tbody>
</table>

Eq. (19) and Eq. (20). The characterized functions are plotted along with the \( R_1, R_2 \) parameter values in Fig. 11 and Fig. 12.

The parameters \( C_1, C_2 \) are represented by polynomial SOC functions including temperature dependence. The \( C_1, C_2 \) functions including SOC and temperature dependence for discharge and charge are shown in Eq. (21) and Eq. (22). They are plotted along with the \( C_1, C_2 \) parameter values in Fig. 13 and Fig. 14.

The experiment is done by using a Bitrode FTV1-200/50/2-60. The battery cell is placed in a designed flow chamber as shown in Fig. 8, where a Pulse Width Modulated (PWM) fan is mounted at the end to regulate the air flow rate around the cell. This flow chamber is used to emulate cooling conditions of a cell in a pack, where the flowrate is adjustable. Two T-type thermocouples are used for temperature measurement, one attached to
the aluminum casing of the cell to measure the surface temperature $T_s$ and the other near the battery inside the flow chamber to measure the air flow temperature $T_f$. This thermal identification experiment setup is also presented in [32].

The non-recursive least squares thermal model identification method described in [22, 32] is implemented here by using the heat generation from Eq. (4) as the input for the thermal model. Where $V_T$ is the measured voltage, $V_{OCV}$ is the modeled open circuit voltage, and $I$ is the measured current. The objective is to minimize the sum of the squared errors between the the modeled surface temperature $T_s$, and the measured surface temperature $T_{s, data}$ as shown by the cost function,

$$J_{thermal} = \min \sum_{i=1}^{n} (T_s(i) - T_{s, data}(i))^2.$$  \hspace{1cm} (10)

Where each instance is represented by $i$, starting from the first surface temperature datapoint $i = 1$, up to the last datapoint $i = n$.

A parametric model in the form of [35],

$$z = \theta^T \phi,$$  \hspace{1cm} (11)

is used for the thermal model parameter least squares identification [22]. Where the observation $z$ and the independent regressors $\phi$ should be measured. The parameters in $\theta$ are calculated by the non-recursive least squares after the experimental data is taken over a period of time $t_1, t_2, \ldots, t$ by [35],

$$\theta(t) = (\Phi^T(t)\Phi(t))^{-1} \Phi(t)Z(t),$$  \hspace{1cm} (12)

where

$$Z(t) = \begin{bmatrix} z(t_1) \\ m(t_1) \\ z(t_2) \\ m(t_2) \\ \vdots \\ \vdots \\ m(t) \end{bmatrix}^T$$

$$\Phi(t) = \begin{bmatrix} \phi^T(t_1) \\ \Phi(t_1) \\ \phi^T(t_2) \\ \Phi(t_2) \\ \vdots \\ \vdots \\ \phi^T(t) \end{bmatrix}^T,$$  \hspace{1cm} (13)

$m(t)$ is the normalization factor to enhance the robustness of parameter identification as explained in [22]. For this purpose, the parametric model for the linear model identification with initial battery surface temperature condition $T_{s,0}$ is first derived. The thermal model in Eq. (5) becomes [22],

$$s^2 T_s - sT_{s,0} = \frac{1}{C_s C_c R_C} Q + \frac{1}{C_s C_c R_C R_u} (T_f - T_s)$$  

$$- \left( \frac{C_s + C_t}{C_s C_c R_C} + \frac{1}{C_s R_u} \right) (sT_s - T_{s,0})$$  \hspace{1cm} (14)

after a Laplace transformation and substitution of the unmeasurable $T_c$ by the measurable $T_f$, $T_s$. In order for this algorithm to be implemented, a proper parametric model must be used. A filter is designed and applied to the parametric model in Eq. (11),

$$\frac{z}{\Lambda} = \theta^T \frac{\phi}{\Lambda},$$  \hspace{1cm} (15)

where the observation $z$ and the independent regressors $\phi$ are measured. The parameter vector $\theta$ is defined as [22],

$$z = s^2 T_s - sT_{s,0}$$

$$\phi = [Q \ T_f - T_s \ sT_s - T_{s,0}]^T$$

$$\theta = [\alpha \ \beta \ \gamma]^T$$  \hspace{1cm} (16)

where the parameters $\alpha, \beta, \gamma$ are [22],

$$\alpha = \frac{1}{C_s C_c R_C}$$

$$\beta = \frac{1}{C_s C_c R_C R_u}$$

$$\gamma = -\left( \frac{C_s + C_t}{C_s C_c R_C} + \frac{1}{C_s R_u} \right)$$  \hspace{1cm} (17)

Assuming $C_c$, the parameters $\alpha, \beta, \gamma$ are identified to find $C_s, R_u$, and $R_C$. This is achieved based on the measured observation and
The parameter \( C_c \) is assumed to be close to that in [23, 32] and the resulting identified parameters \( C_c, R_c, \) and \( R_u \) from the thermal identification scheme are shown in Tab. 2. The parameters \( C_c, R_c, \) and \( R_u \) should not change significantly within the lifetime of the battery cell due to their physical properties. The parameter \( R_u \) can change with respect to the flow around the cell as previously mentioned. In this case it is identified as a constant for a steady flow condition.

4 MODEL VALIDATION AND RESULTS

The electro-thermal model is implemented in Simulink to validate its performance under the UAC experiment. The SOC, temperature, and current direction dependencies of the equivalent circuit model parameters are included using lookup tables. The current \( I \) and air inlet temperature \( T_f \) inputs are shown in Fig. 7. The voltage and temperature responses of the electro-thermal model are compared to the experimental measurements. SOC is shown in Fig. 9 for this experiment.

![Figure 9. UAC SIMULATION SOC RESULTS.](image)

The SOC varies between 50% and 43% under these conditions. The measured surface temperature \( T_s, data \) and terminal voltage \( V_T, data \) are compared to the predicted surface temperature \( T_s \) and voltage \( V_T \) as shown in Fig. 10. The root mean square error (RMSE) in predicted surface temperature is 0.33°C and voltage is 18.4mV. The voltage RMSE is comparable with published results in [29], using a similar type of drive cycle profile for this type of cell. The predicted core temperature \( T_c \) is also shown in Fig. 10, which is 1.77°C higher than the predicted surface temperature \( T_s \) under this cycle. A higher \( T_c \) prediction is presented in [32], using a different heat generation under the same experimental conditions. The heat generation for our case is smaller than [32], causing slightly different identified parameters and a lower \( T_c \) prediction. Further investigation is required to determine if the calculated heat generation \( Q \) and core temperature \( T_c \) prediction are correct. Including a hysteresis model in the electro-thermal model will also need to be investigated to determine if better results can be achieved.

![Figure 10. UAC TEMPERATURE AND VOLTAGE RESULTS.](image)

5 CONCLUSION AND FUTURE WORK

In this study an equivalent circuit electrical model along with a two state thermal model for an A123 26650 LiFePO4 cell were parameterized. The models were integrated into an electro-thermal model in MATLAB/Simulink through a coupling heat generation and temperature feedback. The resulting electro-thermal model matches experimental measurements with minimal error. This shows that the parameterization schemes used are adequate for battery modeling.

Future work will involve modeling of hysteresis as in [25, 27, 28], which will then cause the heat generation to change due to the new \( V_{OCV} \) term. Measurement of the core temperature \( T_c \) is also planned to validate the electro-thermal model.
REFERENCES
[25] Hu, Y., Yurkovich, B. J., Yurkovich, S., and Guezenec,


[34] Lee, T. K., Kim, Y., Stefanopoulou, A., and Filipi, Z. S., 2011. “Hybrid electric vehicle supervisory control design reflecting estimated lithium-ion battery electrochemical dynamics”. In 2011 American Control Conference (ACC).


**Appendix: Calculated Parameters and Functions**

\[
R_1 = \begin{cases} 
R_{1d}, & I > 0 \ (\text{discharge}) \\
R_{1c}, & I < 0 \ (\text{charge}) 
\end{cases}
\]

\[
R_{1s} = R_{10} + R_{11s}(SOC) + R_{12s}(SOC)^2 \exp\left(\frac{T_{ref/R1s}}{T_m-T_{shift/R1s}} \right) 
\]  

(19)

\[
R_2 = \begin{cases} 
R_{2d}, & I > 0 \ (\text{discharge}) \\
R_{2c}, & I < 0 \ (\text{charge}) 
\end{cases}
\]

\[
R_{2s} = (R_{20} + R_{21s}(SOC) + R_{22s}(SOC)^2) \exp\left(\frac{T_{ref/R2s}}{T_m} \right) 
\]  

(20)

![Figure 11. Calculated R1 versus parametric functions describing their dependence on temperature and SOC.](image1)

![Figure 12. Calculated R2 versus parametric functions describing their dependence on temperature and SOC.](image2)

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**Table 3. Parametric R1 function parameters.**

<table>
<thead>
<tr>
<th>(R_{1d} )</th>
<th>(R_{10} )</th>
<th>(R_{11d} )</th>
<th>(R_{11c} )</th>
<th>(R_{12d} )</th>
<th>(R_{12c} )</th>
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<th>(T_{shift/R1d} )</th>
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Table 4. PARAMETRIC \( R_2 \) FUNCTION PARAMETERS.

<table>
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<th>( R_{21_d} )</th>
<th>( R_{21_c} )</th>
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<table>
<thead>
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<th>( R_{22_d} )</th>
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<td>0.0605</td>
<td>0.0339</td>
<td>16.6712</td>
<td>17.0224</td>
</tr>
</tbody>
</table>

\[ C_1 = \begin{cases} C_{10d}, & I > 0 \ (\text{discharge}) \\ C_{11c}, & I < 0 \ (\text{charge}) \end{cases} \]

\[ C_{1s} = C_{10s} + C_{11s}(SOC) + C_{12s}(SOC)^2 + (C_{13s} + C_{14s}(SOC) + C_{15s}(SOC)^2)T_m \]  \hspace{1cm} (21)

---

\[ C_2 = \begin{cases} C_{20d}, & I > 0 \ (\text{discharge}) \\ C_{21c}, & I < 0 \ (\text{charge}) \end{cases} \]

\[ C_{2s} = C_{20s} + C_{21s}(SOC) + C_{22s}(SOC)^2 + (C_{23s} + C_{24s}(SOC) + C_{25s}(SOC)^2)T_m \]  \hspace{1cm} (22)

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![Figure 13. CALCULATED \( C_1 \) VERSUS PARAMETRIC FUNCTIONS DESCRIBING THEIR DEPENDENCE ON TEMPERATURE AND SOC.](image)

![Figure 14. CALCULATED \( C_2 \) VERSUS PARAMETRIC FUNCTIONS DESCRIBING THEIR DEPENDENCE ON TEMPERATURE AND SOC.](image)

Table 5. PARAMETRIC \( C_1 \) FUNCTION PARAMETERS.

<table>
<thead>
<tr>
<th>( C_{10_d} )</th>
<th>( C_{10_c} )</th>
<th>( C_{11_d} )</th>
<th>( C_{11_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>335.4518</td>
<td>523.215</td>
<td>3.1712e+3</td>
<td>6.4171e+3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( C_{12_d} )</th>
<th>( C_{12_c} )</th>
<th>( C_{13_d} )</th>
<th>( C_{13_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.3214e+3</td>
<td>-7.5555e+3</td>
<td>53.2138</td>
<td>50.7107</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( C_{14_d} )</th>
<th>( C_{14_c} )</th>
<th>( C_{15_d} )</th>
<th>( C_{15_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-65.4786</td>
<td>-131.2298</td>
<td>44.3761</td>
<td>162.4688</td>
</tr>
</tbody>
</table>

Table 6. PARAMETRIC \( C_2 \) FUNCTION PARAMETERS.

<table>
<thead>
<tr>
<th>( C_{20_d} )</th>
<th>( C_{20_c} )</th>
<th>( C_{21_d} )</th>
<th>( C_{21_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1887e+4</td>
<td>6.2449e+4</td>
<td>-1.1593e+5</td>
<td>-1.055e+5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( C_{22_d} )</th>
<th>( C_{22_c} )</th>
<th>( C_{23_d} )</th>
<th>( C_{23_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0493e+5</td>
<td>4.4432e+4</td>
<td>60.3114</td>
<td>198.9753</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( C_{24_d} )</th>
<th>( C_{24_c} )</th>
<th>( C_{25_d} )</th>
<th>( C_{25_c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0175e+4</td>
<td>7.5621e+3</td>
<td>-9.5924e+3</td>
<td>-6.9365e+3</td>
</tr>
</tbody>
</table>