TA5 Project 3:

Time-Dependent Reliability/Durability Methodologies for Acquisition, Maintenance, and Operation of Vehicle Systems

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**Time-Dependent Reliability/Durability Methodologies for Acquisition, Maintenance, and Operation of a Vehicle Systems**

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Excerpts from Memorandum dated 27 Mar 2004

...Published studies and audits have documented that reliability has a significant impact on mission effectiveness, logistics effectiveness, and life-cycle costs...."
Background

Vehicle

Input

Uncertainty (Quantified)

Propagation

Output

Uncertainty (Calculated)

Design

- Random Variable *(Time-Independent)*
- Random Process *(Time-Dependent)*

Challenges:

- **Quantification of a Random Process**
- **Estimation of time-dependent reliability**
Research Statement

➢ Develop methodologies to **assess and improve the reliability / durability** of vehicle systems using

  • Experimental (field) data
  • “Expert” opinion
  • Predictive tools (physics-of-failure data)

Previously and currently at TARDEC

➢ Use methodologies in **design for lifecycle cost and preventive maintenance**

Current research
Background

Random Process (Field)

Input Random Process

Vehicle

Output Random Process

Random Process leads to Time-Dependent Reliability
What is Reliability?
Cumulative Probability of Failure

Reliability at time $t$ is the probability that the system has not failed before time $t$.

$$F^c_T(t_L) = P(\exists t \in [0,t_L], \text{such that } g(X(t),t) \leq 0)$$

**Cumulative Prob. of Failure**

$$F^i_T(t_L) = P(g(X(t_L),t_L) \leq 0)$$

**Instantaneous Prob. of Failure**

**Time-Invariant Reliability**

$$F^i_T(t_L)$$

**Time-Variant Reliability**

$$F^c_T(t_L)$$
Maximum Response Approach

\[ y_{\text{max}}(X) = \max_{t_{\text{min}} \leq t \leq t_{\text{max}}} y(X,t) \]

\[ F_T^c(t_F) = P(y_{\text{max}}(X) \geq y^t) = P(y^t - y_{\text{max}}(X) \leq 0) \]

Time-independent composite limit state is defined as:

\[ g(X) = y^t - y_{\text{max}} \leq 0 \]
Observations:

- Niche center is an approximate MPP
- Niching GA finds ALL approximate MPPs

Local metamodels are driven by Niching GA exploration for multiple MPPs

- Error control using cross-validation
Definition of Lifecycle Cost

Lifecycle Cost = Production Cost

+ Inspection Cost

+ Expected Variable Cost

Quality

Time-Dependent System Reliability
Definition of Lifecycle Cost

\[ C_L(d, X, t_f, r) = C_P(d, X) + C_I(d, X, t_0) + C^E_V(d, X, t_f, r) \]

- Lifecycle Cost
- Production Cost
- Inspection Cost
- Expected Variable Cost

\[ C^E_V(d, X, t_f, r) = \int_0^{t_f} c_F(t)e^{-rt}f^c_T(t)dt \]

- Final time
- Interest rate
- Cost of failure at time \( t \)
- PDF of time to failure time

\[ F^c_T(t_L) = P(\exists t \in [0, t_L], such that \ g(X(t), t) \leq 0) \]
Design Using Lifecycle Cost

Using a Target System Reliability in Time

\[
\min_{d, \mu_X, \sigma_X} C_L(d, \mu_X, \sigma_X, t_f, r)
\]

s. t. \[F_T^i(d, X, t_0) \leq p^t_f(t_0)\]
\[F_T^c(d, X, t_1) \leq p^t_f(t_1)\]
\[F_T^c(d, X, t_f) \leq p^t_f(t_f)\]
\[d_L \leq d \leq d_U\]
\[\mu_{X_L} \leq \mu_X \leq \mu_{X_U}\]
\[\sigma_{X_L} \leq \sigma_X \leq \sigma_{X_U}\]
Estimation of Time for Preventive Maintenance

\[
\min_{d, \mu_X, \sigma_X} C_P(d, \mu_X, \sigma_X) + C_I(d, \mu_X, \sigma_X, t_0)
\]

s. t. \( F_T^c(d, X, t_M) \leq p_f^t \)

\( d_L \leq d \leq d_U \)

\( \mu_{X_L} \leq \mu_X \leq \mu_{X_U} \)

\( \sigma_{X_L} \leq \sigma_X \leq \sigma_{X_U} \)

Acceptable Reliability \((1 - p_f^t)\)
Design of a Roller Clutch

Constraints:

- **Contact angle** \( \alpha = 0.11 \pm 0.06 \) rad
- **Torque** \( \tau \geq 3000 \) Nm
- **Hoop stress** \( \sigma_h \leq 400 \) MPa

Random Variables: \( D, d, A \)

Due to degradation:

- \( D \rightarrow D(1-kt) \)
- \( d \rightarrow d(1-kt) \)
- \( A \rightarrow A(1+kt) \)

with: \( k = 2.5E-04 \) mm/ year

\[
g_1(D, d, A) = 0.05 - \cos^{-1}\left(\frac{D-d}{A-d}\right) \leq 0
\]

\[
g_2(D, d, A) = \cos^{-1}\left(\frac{D-d}{A-d}\right) - 0.17 \leq 0
\]

\[
g_3(D, d, A) = 3000 - NL\left(\frac{\sigma_c}{c_1}\right)^2 \frac{D^2d}{4(D+d)} \sqrt{1-S^2} \leq 0
\]

\[
g_4(D, d, A) = \frac{N}{2\pi} \left(\frac{\sigma_c}{c_1}\right)^2 \left(\frac{Dd}{(D+d)}\right) S \left(\frac{B^2 + A^2}{A\left(B^2 - A^2\right)}\right) - 400E06 \leq 0
\]
Roller Clutch: Problem Statement

Minimize Lifecycle Cost
\[ \min C_L(\mu_X, \sigma_X, t_f, r) \quad \sigma_{X_L} \leq \sigma_X \leq \sigma_{X_U} \]
\[ \mu_{X_L} \leq \mu_X \leq \mu_{X_U} \]

s. t.

Case 1
\[ F_T^i(\mu_X, \sigma_X, t_0 = 0) = P(\bigcup_i (g_i(D, d, A, t_0) < 0)) \leq p_f(t_0 = 0) = 0.0013 \]

Case 2
\[ F_T^i(\mu_X, \sigma_X, t_0 = 0) = P(\bigcup_i (g_i(D, d, A, t_0) < 0)) \leq p_f(t_0 = 0) = 0.0013 \]
\[ F_T^c(\mu_X, \sigma_X, t = 7.5) = P(\bigcup_i (g_i(D, d, A, t) < 0)) \leq p_f(t = 7.5) = 0.005 \]

Case 3
\[ F_T^c(\mu_X, \sigma_X, t = 10) = P(\bigcup_i (g_i(D, d, A, t) < 0)) \leq p_f(t = 10) = 0.0716 \]
Roller Clutch: Problem Statement

where:

Total Cost, \( C_L = C_P + C_I + C_V^E \)

\[
C_P = \left( 3.5 + \frac{0.75}{3\sigma_D} \right) + \left( 3.0 + \frac{0.65}{3\sigma_d} \right) + \left( 0.5 + \frac{0.88}{3\sigma_A} \right)
\]

\[
C_I = 20F_T^i(X, t_0)
\]

\[
C_V^E = \int_{0}^{t_f} 20e^{-rt} f_T^c(t) dt
\]

Scrap cost/unit

Failure cost/unit (warranty cost)

\( t_f = 10 \) years

\( r = 3\% \)
Roller Clutch: Results

Initial Design vs. Case 1

Case 1 vs. Case 2 and Case 3

<table>
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<th>Objective</th>
<th>Initial Design</th>
<th>Optimal Design</th>
<th>Case 1</th>
<th>Case 2</th>
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A Practical Issue

Vehicle speed: 20 mph; Mission distance: 100 miles

Simulation can be practically performed for a short-duration time
A novel MC-based method has been developed to calculate the time-dependent reliability (cumulative probability of failure) using short-duration data based on:

- Exponential extrapolation
- Poisson’s distribution
Can characterize a **stationary** or **non-stationary** input Random Process

\[ u_i - \bar{u} = \phi_1(u_{i-1} - \bar{u}) + \phi_2(u_{i-2} - \bar{u}) + \ldots + \phi_p(u_{i-p} - \bar{u}) + \varepsilon_i \]

Must estimate \( \phi_p \), \( \sigma_e^2 \)
Cumulative Probability of Failure

\[ R(t) = 1 - F_T^c(t) \]  \hspace{1cm} (1)

\[ \lambda(t) = \frac{P(t < T \leq t + dt/T > t)}{dt} = \frac{P(t < T \leq t + dt)}{dt * P(T > t)} = \frac{F(t + dt) - F(t)}{dt * R(t)} \Rightarrow \lambda(t) = \frac{f(t)}{1 - F_T^c(t)} \]  \hspace{1cm} (2)

From (1) and (2):

\[ F_T^c(t) = 1 - \exp \left[ -\int_0^t \lambda(t) dt \right] \]

All we need is the failure rate
Efficient MCS-based Approach

The Bathtub Curve
Hypothetical Failure Rate versus Time

\[ b = -\frac{1}{\lambda_0} \left( \frac{d\lambda}{dt} \right)_{t=0} \]

End of Life Wear-Out
Increasing Failure Rate

Normal Life (Useful Life)
Low "Constant" Failure Rate

Time

\[ t_{\text{int}} \]

Poisson's Formula

\[ F_T^c(t) = \begin{cases} 
-\int_0^t \hat{\lambda}(t)dt, & t \in [0, t_{\text{int}}] \\
1 - e^{-\int_0^t \hat{\lambda}(t)dt}, & t \in [t_{\text{int}}, t_f] 
\end{cases} \]

Exponential Extrapolation

\[ \hat{\lambda}(t) \approx \lambda_0 e^{-bt} \]
Quarter-Car Model on Stochastic Terrain

Constant design parameters:
\[ m_s = 1000 \text{ kg} \]
\[ m_u = 100 \text{ kg} \]
Vehicle speed = 20 mph

Random Input variables
Damping, \( b_s \sim N(7000,1400^2) \)
Stiffness, \( k_s \sim N(40 \times 10^3,(4 \times 10^3)^2) \)

Random Input Process: Experimental Stochastic Terrain from Yuma Proving Grounds.

Random Output Process (Vertical Acceleration, G’)
Threshold = 2G

Constant design parameters:
\[ m_s = 1000 \text{ kg} \]
\[ m_u = 100 \text{ kg} \]
Vehicle speed = 20 mph

Random Input variables
Damping, \( b_s \sim N(7000,1400^2) \)
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Random Input Process: Experimental Stochastic Terrain from Yuma Proving Grounds.

Random Output Process (Vertical Acceleration, G’)
Threshold = 2G
AR(3) model was identified based on:

Autocorrelation Function

Sample Autocorrelation Function (ACF)

Autocorrelation of Residual process

Sample Autocorrelation Function (ACF)

\[ u_i = 1.2456 \ u_{i-1} - 0.2976 \ u_{i-2} - 0.1954 \ u_{i-3} + \varepsilon_i (0, 0.5132^2) \]

Statistical tests were performed to verify the model
Quarter-Car Model: Results
(Failure Rate Estimation for Threshold = 2G)

Estimated parameters:
\[ \lambda_0 = 0.1708 \]
\[ b = 0.0818 \]

Exponential extrapolation
\[ \hat{\lambda}(t) \approx \lambda_0 e^{-bt} \]

Estimation requires short duration MCS

Poisson’s Formula
Quarter-Car Model: Results
Cumulative Probability of Failure for Threshold = 2G

Efficient MCS (blue) approach is close to true MCS results (red)
Quarter-Car Model: Results
(Failure Rate Estimation for Threshold = 2.65 G)

Estimated failure rate (black) is close to true failure rate (red)
Quarter-Car Model: Results
Cumulative Probability of Failure for Threshold = 2.65 G

Efficient MCS (blue) approach is close to true MCS results (red)
Summary

- Time-dependent reliability methodologies have been developed using math-based models.

- An approach to design for lifecycle cost and preventive maintenance has been developed.

- A novel MC-based approach was developed, using short-duration data, to compute time-dependent reliability in the presence of an input random process.

- Examples demonstrated the developed methods.
Future Work

- Develop an **importance sampling** method to improve the computational effort in estimating the time-dependent reliability of systems with a stationary and non-stationary input random process (June 2010).

- Demonstrate potential of developed methods in **preventive maintenance** (August 2010).

- Combine current research developments with existing or under development efforts at TARDEC in reliability area (December 2010).
Thanks for your attention!

Q & A

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