Design for Lifecycle Cost using Time-Dependent Reliability

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# Report Documentation Page

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Standard Form 298 (Rev. 8-98)  
Prescribed by ANSI Std Z39-18
Response(t) = f [ E(t), Degradation/Wear(t), Load(t) ]

Random Process approach to reliability-based design is needed → time-dependent reliability
Problem Definition

**Input variables**

Variable #1

Frequency

Time

Degradation

Variable #n

Frequency

Time

Degradation

**System Responses**

Response #1

Frequency

Time

**System Reliability**

System Reliability

Quality

**Quality = Reliability (t = 0)**
What can we get from Time-Dependent Reliability?

- Design for:
  - Lifecycle cost
  - Quality
  - Warranty
  - Maintenance schedule for CBM
Design for Lifecycle Cost

\[
\text{Lifecycle Cost} = \text{Production Cost} + \text{Inspection Cost} + \text{Expected Variable Cost}
\]

Accurate and efficient predictive tools are, therefore, needed to estimate **Time-dependent System Reliability.**
Design for Lifecycle Cost

\[ C_L(d, X, t_f, r) = C_P(d, X) + C_I(d, X, t_0) + C_V^E(d, X, t_f, r) \]

- Lifecycle Cost
- Production Cost
- Inspection Cost
- Expected Variable Cost

\[ C_V^E(d, X, t_f, r) = \int_{0}^{t_f} c_F(t) e^{-rt} f_T^c(t) dt \]

- Final time
- Interest rate
- Cost of failure at time t
- PDF of time to failure time

\[ F_T^c(t_L) = P(\exists t \in [0, t_L], such that g(X(t), t) \leq 0) \]
Problem Statement: 
Design for Lifecycle Cost

\[ \min_{d, \mu_x, \sigma_x} C_L (d, \mu_x, t_f, r) \]

s. t. 
\[ F^i_{Q_i} (d, X, t_0) \leq p_f (t_0) \]
\[ F^c_R (d, X, t_k) \leq p_f (t_k) \]

\[ d_L \leq d \leq d_U \]
\[ \mu_{x_L} \leq \mu_x \leq \mu_{x_U} \]

Quantification of time-dependent reliability is a major challenge in this research.
Definitions / Observations

**Reliability:** Ability of a system to carry out a function in a time period \([0, t_L]\)

\[
p_f^c = P(t \leq t_L) = F_T^c(t_L)
\]

**Prob. of Time to Failure**

\[
F_T^c(t_L) = P(\exists t \in [0, t_L], \text{such that } g(X(t), t) \leq 0)
\]

**Cumulative Prob. of Failure**

\[
F_T^i(t_L) = P(g(X(t_L), t_L) \leq 0)
\]

**Instantaneous Prob. of Failure**

**Time-Invariant Reliability**

\[
0 \quad \frac{t_L}{\text{time}} \quad F_T^i(t_L)
\]

**Time-Variant Reliability**

\[
0 \quad \frac{t_L}{\text{time}} \quad F_T^c(t_L)
\]
Calculation of Cumulative Probability of Failure

• State-of-the-art Approaches


➢ Set-Based approach (Son and Savage, *Quality & Rel. Engin.*, 2007)

\[ t_F = K \Delta t \]

• State-of-the-art approaches are in general, inaccurate due to:

➢ Choice of \( \Delta t \)

➢ Not including contribution of all discrete times
Cumulative Probability of Failure

Composite Limit State

Example 1: Linear Limit State

\[ g(X_1, X_2, t) = X_1 + tX_2 \]

\[ X_1 \sim N(-5, 1^2) \quad X_2 \sim N(0, 1^2) \]

\[ t_0 < t \leq t_K \]

“Composite” limit state

\[ F_T^c(t_F) = P\left( \bigcup_{k=0}^{K} g(X(t_k), t_k) \right) \]

Single MPP of instantaneous limit states evolves into multiple MPPs of composite limit state.
Cumulative Probability of Failure

Composite Limit State

Example 2:

\[ g(X_1, X_2) = 1 - \frac{X_1 - 1000X_2 \sin(4\pi t + X_2)\alpha}{12000\alpha}, \quad \alpha = 52966 \]

\[ X_1 \sim N\left(4.58E08, \ (5E07)^2\right) \]

\[ X_2 \sim N\left(0, \ 0.7^2\right) \]
Composite Limit State: Example 2

$U_2$

$U_1$

$t = 0$
Composite Limit State: Example 2

t = 0  t = 0.125
Composite Limit State: Example 2
Composite Limit State: Example 2

- $t = 0$
- $t = 0.125$
- $t = 0.25$
- $t = 0.375$
Composite Limit State: Example 2
Composite Limit State: Example 2
Reliability Index Approach

– Limit State is kept Time-dependent i.e. \( g(d, X, t) = 0 \)

Maximum Response Approach

– Limit State is converted into Time-Independent i.e \[ g(d, X) = 0 \]
Calculation of Probability of Failure

- **Reliability Index Approach:**

\[ \beta = \min_{U,t} \|U\|_2 \]

\[ \text{s.t.} \quad g(U, t) = 0 \]

\[ t_0 \leq t \leq t_{\text{max}} \]

Time is treated as an **additional** design variable in RIA optimization.
Cumulative Probability of Failure

- Maximum Response Approach:

\[ y_{\text{max}}(d, X) = \max_{t_{\text{min}} \leq t \leq t_{\text{max}}} y(d, X, t) \]

\[ F_{T}^{C}(t_F) = P(y_{\text{max}}(X) > y^t) = P(y^t - y_{\text{max}}(X) < 0) \]

Composite Limit-State as time-independent is defined as:

\[ g(d, X) = y^t - y_{\text{max}} \leq 0 \]
Calculation of $p_f$:
Two-DOF System
Two-DOF System

\[ m_c \sim N(\mu_m, \sigma_m^2), \quad \mu_m = 55 \text{ Kg}, \quad \sigma_m = 5 \text{ Kg} \]

\[ k_s \sim N(\mu_k, \sigma_k^2), \quad \mu_k = 33E04 \text{ N/m} \]

\[ \sigma_k = 3E04 \text{ N/m} \]

\[ u(t) \text{: unit impulse; } \quad 0 \leq t \leq 5s \]
Two-DOF System - Multiple MPPs

- Maximum Response method
- Niching GA optimization to search for multiple MPPs

T=0.2 sec

T=1 sec

Local Metamodel based MCS: 100000 samples
Two-DOF System - Comparison of Pf

![Graph showing comparison of cumulative Pf for different models.](image-url)
Design of a Roller Clutch using Lifecycle Cost
Roller Clutch

**Random Design Variables:**
- **D:** Hub diameter, mm
- **d:** Roller diameter, mm
- **A:** Cage inner diameter, mm

**D, d, and A are normally distributed**

**Due to degradation:**
- \( D \rightarrow D(1 - kt) \)
- \( d \rightarrow d(1 - kt) \)
- \( A \rightarrow A(1 + kt) \)

with: \( k = 2.5E-04 \text{ mm/ year} \)
Roller Clutch

**Constraints:**

- **Contact angle** \( \alpha = 0.11 \pm 0.06 \text{ rad} \)
- **Torque** \( \tau \geq 3000 \text{ Nm} \)
- **Hoop stress** \( \sigma_h \leq 400 \text{ MPa} \)

\[
0.05 \leq \cos^{-1}\left(\frac{D-d}{A-d}\right) \leq 0.17
\]

\[
g_1(D, d, A) = 0.05 - \cos^{-1}\left(\frac{D-d}{A-d}\right) \leq 0
\]

\[
g_2(D, d, A) = \cos^{-1}\left(\frac{D-d}{A-d}\right) - 0.17 \leq 0
\]

\[
g_3(D, d, A) = 3000 - NL\left(\frac{\sigma_c}{c_1}\right)^2 \frac{D^2d}{4(D+d)} \sqrt{1-S^2} \leq 0
\]

\[
g_4(D, d, A) = \frac{N}{2\pi}\left(\frac{\sigma_c}{c_1}\right)^2 \left(\frac{Dd}{(D+d)}\right) \frac{S}{A}\left(\frac{B^2 + A^2}{B^2 - A^2}\right) - 400E06 \leq 0
\]
Roller Clutch: Problem Statement

Minimize Lifecycle Cost

\[ \min \ C_L (\mu_X, \sigma_X, t_f, r) \]

s. t.

\[ \sigma_{XL} \leq \sigma_X \leq \sigma_{XU} \]

\[ \mu_{XL} \leq \mu_X \leq \mu_{XU} \]

Case 1

\[ F^i(\mu_X, \sigma_X, t_0 = 0) = \ P(\bigcup_i (g_i(D,d,A,t_0) < 0)) \leq p_f(t_0 = 0) = 0.0013 \]

Case 2

\[ F^i(\mu_X, \sigma_X, t_0 = 0) = \ P(\bigcup_i (g_i(D,d,A,t_0) < 0)) \leq p_f(t_0 = 0) = 0.0013 \]

\[ F^c(\mu_X, \sigma_X, t = 7.5) = \ P(\bigcup_i (g_i(D,d,A,t) < 0)) \leq p_f(t = 7.5) = 0.005 \]

Case 3

\[ F^c(\mu_X, \sigma_X, t = 10) = \ P(\bigcup_i (g_i(D,d,A,t) < 0)) \leq p_f(t = 10) = 0.0716 \]
Roller Clutch: Problem Statement

where:

Total Cost, \( C_L = C_P + C_I + C_V^E \)

\[
C_P = \left( 3.5 + \frac{0.75}{3\sigma_D} \right) + \left( 3.0 + \frac{0.65}{3\sigma_d} \right) + \left( 0.5 + \frac{0.88}{3\sigma_A} \right)
\]

\( C_I = 20F_Q(X,t_0) \)

\( C_V^E = \int_0^{t_f} 20e^{-rt} f_R^c(t) \, dt \)

\( t_f = 10 \text{ years} \)

\( r = 3\% \)

Scrap cost/unit

Failure cost/unit (warranty cost)
Roller Clutch: Results

Initial Design vs. Case 1

<table>
<thead>
<tr>
<th>Objective</th>
<th>Initial Design</th>
<th>Optimal Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production Cost</td>
<td>17.3900</td>
<td>21.3340</td>
</tr>
<tr>
<td>Inspection Cost</td>
<td>0.7677</td>
<td>0.0260</td>
</tr>
<tr>
<td>Expected Variable Cost</td>
<td>10.0697</td>
<td>2.5161</td>
</tr>
</tbody>
</table>
Summary/Conclusions

- A new method to calculate the Cumulative Probability of failure is presented for linear and non-linear problems.

- The design study of the roller clutch showed that:
  - Lifecycle cost can be reduced by controlling the probability of failure though time.
  - Higher lifecycle cost due to higher initial quality does not guarantee acceptable reliability.
Challenges/Future Work

➢ Improve further efficiency by:
  ▪ Random process characterization using time-series modeling techniques.
  ▪ Solving RBDO problem using Probabilistic Re-Analysis which uses a single MCS

➢ Apply presented ideas/approaches to the Army related problems
Q & A

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