This experimental study investigated the response of dynamic flow structures of cryogenic coaxial nitrogen jets to pressure perturbations due to transverse acoustic forcing at a pressure antinode (PAN). The role of injector exit geometry on the flow response was examined using two shear coaxial injectors with different outer-to-inner jet area ratios. Flow conditions spanning subcritical (reduced pressure of 0.44) to supercritical (reduced pressure of 1.05) chamber pressures, varying outer-to-inner jet momentum flux ratios (0.5 – 20), and acoustic pressure antinode at the jet axis location were considered. A basic application of proper orthogonal decomposition on the intensity fluctuation of high-speed images enabled the extraction of the spatial and temporal characteristics of the dominant flow structures that existed in the flow field during exposure to acoustic forcing. Regardless of injector geometry or pressure regime, low outer-to-inner momentum flux ratio flows were found to be responsive to acoustic pressure antinode forcing. With increasing momentum flux ratio, however, the flow response to forcing depended on the injector geometry.
Proper Orthogonal Decomposition Analysis of Shear-Coaxial Injector Flows with and without Transverse Acoustic Forcing

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Motivation

• Feedback cycle between liquid rocket engine (LRE) combustion chamber pressure perturbations and unsteady combustion\textsuperscript{1,2}

• Large amplitude fluctuations in pressure and combustion heat release rates $\Rightarrow$ combustion instability


Objective

- Impose external acoustic perturbations, and examine the response and stability characteristic of shear-coaxial injector flow to pressure perturbation

![Acoustic/Pressure Perturbation](image1)

![Shear-Coaxial Injector Flow](image2)

- Investigate influence of injector geometry on flow response to external pressure perturbation

- Vary the outer-to-inner jet momentum flux ratio, $J$, under subcritical and nearcritical chamber pressure conditions, i.e., reduced pressures $Pr = 0.44, 1.05$

$$J = \frac{\rho_o u_o^2}{\rho_i u_i^2} \quad Pr = \frac{P_{chamber}}{P_{critical,N_2}} \quad P_{critical,N_2} = 493 \text{ psi (3.4 MPa)}$$

- Apply proper orthogonal decomposition of high-speed image pixel intensity fluctuations to extract spatial and temporal characteristics of prevalent coherent flow structures
Previous Works on Jet Instability

- Michalke and Hermann (1982) did linear, inviscid instability analysis of a circular jet with coflow
  - Showed that with increasing coflow velocity, $U_\infty$
    - Helical disturbances more unstable than axisymmetric ones farther downstream of exit
    - Jet flow becomes less unstable, but spectrum of spatial growth rate becomes broader and the peak shifts to higher frequencies

- Dahm et al. (1992), Wicker and Eaton (1994) conducted experimental investigation of large-scale vortex structures in the near field of coaxial jets
  - For outer-to-inner jet velocity ratios greater than one, found that coherent structures in the outer shear layer dominate those in the inner shear layer
  - At large axial distances, shear-layer vortices exhibit helical structures
Schematic of Experimental Facility

- Main Pressure Chamber
- Inner Chamber (Test Section)
- GN2 (Ambient T)
- Inner Jet
- Outer Jet
- GN2 (Ambient T)
- Chamber Pressurization GN2
- Acoustic Waveguide
- Xenon Arc-Lamp
- Exhaust
- High-Speed Camera

Distribution A: Approved for Public Release; Distribution Unlimited
Image of Experimental Facility

- Piezo-Siren
- Waveguide
- High-Speed Camera
- Coaxial Injector
- Differential Pressure Transducers
- Thermocouple Probe
- Inner Chamber

Distribution A: Approved for Public Release; Distribution Unlimited
Injector Configuration

- Two types of outer-to-inner jet cross-sectional area ratios
  - Large Area Ratio (LAR)
  - Small Area Ratio (SAR)

<table>
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<th>Injector</th>
<th>$t$</th>
<th>$D_1$</th>
<th>$t/D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
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<td>0.13</td>
<td>0.89</td>
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<tr>
<td>SAR</td>
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<td>1.65</td>
<td>2.44</td>
<td>3.94</td>
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</table>
Acoustic Field Set-Up: Pressure Antinode

- Pressure antinode (PAN) – condition of maximum pressure perturbation in the acoustic field
- Piezo-sirens forced in-phase
- Superposition of quasi-1D acoustic waves traveling in opposite directions ⇒ PAN at the jet location (geometric center of test section)
Proper Orthogonal Decomposition

• Proper Orthogonal Decomposition (POD) or Principal Component Analysis (PCA) was used for extracting dominant dynamical processes embedded in high-speed images.

• A time-resolved set of images $A(x,t)$ can be represented as a linear combination of orthonormal basis functions $\phi_k$ (aka proper orthogonal modes)$^{1,2}$:

$$A(x,t) = \sum_{k=1}^{M} a_k(t)\phi_k(x)$$

where $a_k(t)$ are time dependent orthonormal amplitude coefficients and $M$ is the number of modes.

• Main idea: POD modal amplitudes capture the maximum possible “energy” in an average sense$^3$, i.e.,

$$\sum \langle a_k(t)a_k(t) \rangle \geq \sum \langle b_k(t)b_k(t) \rangle$$

where $b_k(t)$ are the temporal coefficients of a decomposition with respect to an arbitrary orthonormal basis $\psi_k$.

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Construction of Data Set

• First, form a row vector consisting of all pixel intensity values of each snapshot image (with resolution of $n$ rows by $m$ columns) in order of increasing columns, then increasing rows.

![Image Frame]

- Pixel Intensity
- $m$ columns
- $n$ rows
- $N$ frames

• Then, combine all such row vectors for $N$ sequences of image frames resulting in a matrix $A$ consisting of $N$ rows by $P = n \times m$ columns of intensity values.

$$A = \begin{bmatrix}
\end{bmatrix}$$

$P = n \times m$ pixel intensities

$N$ time steps
Orthogonal Decomposition Technique

- Eigenvalue decomposition or singular value decomposition (SVD) can be used
- SVD preferred since
  1. Applicable to non-square matrices (most likely the case)
  2. Decomposition matrices are orthogonal
  3. Subroutine readily available in MATLAB®
- Subtracted temporal mean of $\mathbf{A} \Rightarrow$ matrix of intensity fluctuations $\tilde{\mathbf{A}}$
- Applied SVD

$$\Rightarrow \tilde{\mathbf{A}} = \mathbf{U}\mathbf{S}\mathbf{V}^T = \mathbf{Q}\mathbf{V}^T$$

- Columns of $\mathbf{Q} \sim a_k(t)$ contain temporal information
- Columns of $\mathbf{V} \sim \phi_k(x)$ contain spatial information
- Orthogonal Matrix of Left Singular Column Vectors of $\tilde{\mathbf{A}}$
- Orthogonal Matrix of Right Singular Column Vectors of $\tilde{\mathbf{A}} \Leftrightarrow$ proper orthogonal modes (POM)
- Diagonal Matrix of Singular Values
Results – Subcritical Baseline at Low $J$

- LAR, $Pr = 0.44$, $J = 0.5$

Antisymmetric Structures
Identified with Characteristic Frequencies

Amplitude information contained in singular values

Power Spectral Densities (PSD) of Temporal Coefficients of POMs 1 and 2
Results – Subcritical PAN at Low $J$

- LAR, $Pr = 0.44$, $J = 0.5$, forcing Frequency, $f_F = 3.14$ kHz
Cross-Power Spectral Density (CPSD)

- CPSD yields the FFT of the cross-correlation of the temporal coefficients
- Magnitude and phase plots used to determine existence of propagating structures

LAR, $Pr = 0.44$, $J = 0.5$
Sample Animation – PAN ($f_F = 3.14 \text{ kHz}$)

- $\text{LAR } Pr = 0.44, J = 0.5$

Superposition of POMs 1 and 2 Resulted in Downstream Propagating Structures
Results – LAR, $Pr = 0.44$, Baseline

- Antisymmetric flow structures indicated helical type flow instabilities for all $J$

Characteristic peaks broadened and shifted to higher frequencies with increasing outer jet velocity
Results – LAR, $Pr = 0.44$, PAN

- Gradual shift from symmetric to antisymmetric flow structures with increasing $J$
- Response at forcing frequency, $f_F$, dominant at lower $J$

$J = 2.1$
($f_F = 3.12$ kHz)

$J = 5.2$
($f_F = 3.12$ kHz)

$J = 11$
($f_F = 3.10$ kHz)

$J = 20$
($f_F = 3.11$ kHz)
Results – LAR, $Pr = 1.05$, Baseline

- Antisymmetric flow structures indicated helical type flow instabilities for all $J$

Similar to $Pr = 1.05$, peaks broadened and shifted to higher frequencies with increasing outer jet velocity
Results – LAR, $Pr = 1.05$, PAN

- Trend in response with varying $J$ similar to $Pr = 0.44$
- Gradual shift from symmetric to antisymmetric flow structures with increasing $J$
- Response at $f_F$ still took over natural (baseline) frequency at lower $J$

$J = 0.5$  
(f$_F$ = 3.10 kHz) 

$J = 1.9$  
(f$_F$ = 3.10 kHz) 

$J = 5.0$  
(f$_F$ = 3.41 kHz) 

$J = 12$  
(f$_F$ = 3.10 kHz)
Results – SAR, $Pr = 0.44$, Baseline

- Helical type flow instabilities became more well-defined with increasing $J$

Unlike LAR flows, characteristic peaks showed minimal variation in frequency with outer jet velocity.
Results – SAR, $Pr = 0.44$, PAN

- Symmetric structures persist despite increasing $J$
- Response at $f_F$ strong at highest $J$

$J = 2.0$
($f_F = 2.97$ kHz)

$J = 5.2$
($f_F = 3.02$ kHz)

$J = 12$
($f_F = 2.92$ kHz)

$J = 17$
($f_F = 2.90$ kHz)
Results – SAR, $Pr = 1.05$, Baseline

- Antisymmetric flow structures indicated helical type flow instabilities

Similar to $Pr = 0.44$, characteristic peaks showed minimal variation in frequency with increasing outer jet velocity
Results – SAR, $Pr = 1.05$, PAN

- Similar to $Pr = 0.44$, symmetric structures persist even at high $J$.
- Vortex-pairing interactions were most dominant response at $0.5f_F$.
- Response at $f_F$ strong at highest $J$.

![Graphs showing CPDF magnitude vs. frequency for Baseline and PAN, indicating differences in response.]
Conclusion

- **Proper orthogonal decomposition** of high-speed image intensity fluctuation data revealed key **spatial** and **temporal characteristics** of flow structures
- In both pressure regimes, **LAR** injector:
  - Peak frequencies of baseline flow instabilities became broader and shifted to higher frequencies with increasing $J$
  - PAN forcing at low $J$ produced symmetric flow structures, while at higher $J$, influence of forcing subsided
  - Spectral magnitude plots showed **decreasing influence of PAN** forcing with increasing $J$
- In both pressure regimes, **SAR** injector:
  - Increasing $J$ had minimal influence on peak frequencies of baseline flow instabilities
  - PAN forcing produced symmetric flow structures regardless of $J$
  - Spectral plots showed **strong response** to PAN forcing at low and high $J$
- Operated at **high** enough $J$, LAR injector flows **less vulnerable to external pressure disturbances**
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  – Randy Harvey, David Hill, Earl Thomas (ERC)
  – Todd Newkirk (Jacobs Engineering)

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• Discussion and SAR Data
  – Dr. Juan Rodriguez

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Back-Up Slides
### Data Summary Tables - LAR

**Pr = 0.44, LAR**

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<th>$\rho_{\text{chamber}}$ (kg/m$^3$)</th>
<th>$P_{\text{chamber}}$ (MPa)</th>
<th>$T_{\text{outer}}$ (K)</th>
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**Pr = 1.05, LAR**

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</table>
### Data Summary Tables - SAR

**Pr = 0.44, SAR**

| $J$ | $R$ | $T_{chamber}$ (K) | $\rho_{chamber}$ (kg/m$^3$) | $P_{chamber}$ (MPa) | $T_{outer}$ (K) | $\dot{m}_{outer}$ (mg/s) | $\rho_{outer}$ (kg/m$^3$) | $u_{outer}$ (m/s) | $Re_{outer} (10^4)$ | $T_{inner}$ (K) | $\dot{m}_{inner}$ (mg/s) | $\rho_{inner}$ (kg/m$^3$) | $u_{inner}$ (m/s) | $Re_{inner} (10^4)$ |
|-----|-----|-------------------|----------------------------|---------------------|----------------|--------------------------|--------------------------|----------------|----------------|----------------|----------------|--------------------------|----------------|----------------|----------------|
| 2.0 | 6.9 | 246               | 21                        | 1.49                | 195           | 450                      | 27                       | 6.6            | 1.1            | 109            | 925            | 630                      | 0.96           | 1.5            |
| 5.2 | 11  | 217               | 24                        | 1.49                | 184           | 750                      | 29                       | 10             | 1.9            | 110            | 925            | 620                      | 0.97           | 1.5            |
| 12  | 17  | 222               | 23                        | 1.49                | 194           | 1100                     | 27                       | 16             | 2.6            | 108            | 925            | 640                      | 0.94           | 1.4            |
| 17  | 20  | 217               | 24                        | 1.48                | 194           | 1300                     | 27                       | 19.3           | 3.1            | 108            | 925            | 638                      | 0.95           | 1.4            |

**Pr = 1.05, SAR**

| $J$ | $R$ | $T_{chamber}$ (K) | $\rho_{chamber}$ (kg/m$^3$) | $P_{chamber}$ (MPa) | $T_{outer}$ (K) | $\dot{m}_{outer}$ (mg/s) | $\rho_{outer}$ (kg/m$^3$) | $u_{outer}$ (m/s) | $Re_{outer} (10^4)$ | $T_{inner}$ (K) | $\dot{m}_{inner}$ (mg/s) | $\rho_{inner}$ (kg/m$^3$) | $u_{inner}$ (m/s) | $Re_{inner} (10^4)$ |
|-----|-----|-------------------|----------------------------|---------------------|----------------|--------------------------|--------------------------|----------------|----------------|----------------|----------------|--------------------------|----------------|----------------|----------------|
| 9.4 | 9.9 | 214               | 59                        | 3.58                | 203           | 1460                     | 63                       | 9.2            | 3.2            | 109            | 925            | 650                      | 0.93           | 1.3            |
| 19  | 14  | 215               | 59                        | 3.56                | 207           | 2060                     | 62                       | 13             | 4.5            | 111            | 925            | 635                      | 0.95           | 1.4            |