On the Connectivity and Multihop Delay of Ad Hoc Cognitive Radio Networks

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***Abstract*—We analyze the multihop delay of ad hoc cognitive radio networks, where the transmission delay of each hop consists of the propagation delay and the waiting time for the availability of the communication channel (*i.e.,* the occurrence of a spectrum opportunity at this hop). Using theories and techniques from continuum percolation and ergodicity, we establish the scaling law of the minimum multihop delay with respect to the source-destination distance. We show the starkly different scaling behavior of the multihop delay in *instantaneously connected* networks as compared to networks that are only *intermittently connected* due to scarcity of spectrum opportunities.**

***Index Terms*—Cognitive radio network, multihop delay, connectivity, intermittent connectivity, continuum percolation.**

I. Introduction

The basic idea of opportunistic spectrum access is to achieve spectrum efficiency and interoperability through a hierarchical access structure with primary and secondary users [1]. Secondary users, equipped with cognitive radios [2] capable of sensing and learning the communication environment, identify and exploit instantaneous and local spectrum opportunities without causing unacceptable interference to primary users [1].

Using theories and techniques from continuum percolation and ergodicity, we analytically characterize the connectivity and multihop delay of the secondary network. Specifically, we consider a Poisson distributed secondary network overlaid with a Poisson distributed primary network in an infinite two-dimensional Euclidean space[1]. Due to the hierarchical structure of spectrum sharing, the transmission delay of each hop in the secondary network consists of two components: the propagation delay and the waiting time for the availability of the communication channel (*i.e.,* the occurrence of a spectrum opportunity at this hop).

A. Main Results

The contribution of this paper is twofold. First, We analytically characterize the connectivity of the secondary network, where the connectivity is defined by the finiteness of the minimum multihop delay (MMD) between two randomly chosen secondary users. Specifically, the network is disconnected if the MMD between two randomly chosen secondary users is infinite almost surely (a.s.). The network is connected if the MMD between two randomly chosen secondary users is finite with a positive probability (wpp.).

Due to the hierarchical structure of spectrum sharing, the connectivity of the secondary network depends on not only the topology of the secondary network but also the transmitting and receiving activities of the privileged primary network. It can thus be characterized by a partition of the $(\lambda_S, \lambda_{PT})$ plane as shown in Fig. 1, where λ_S is the density of the secondary users and λ_{PT} the density of the primary transmitters representing the traffic load of the primary network. Specifically, we show that when the temporal dynamics of the primary traffic is sufficiently rich (for example, independent realizations of active primary transmitters and receivers across slots), whether the secondary network is connected depends solely on its own density λ_S and is independent of the density λ_{PT} of the primary transmitters. When $\lambda_S > \lambda_c$, there is a.s. a unique infinite connected component in the secondary network formed by topological links connecting two secondary users within each other's transmission range. We further show that for any two secondary users in this infinite topologically connected component, the MDD is finite a.s.

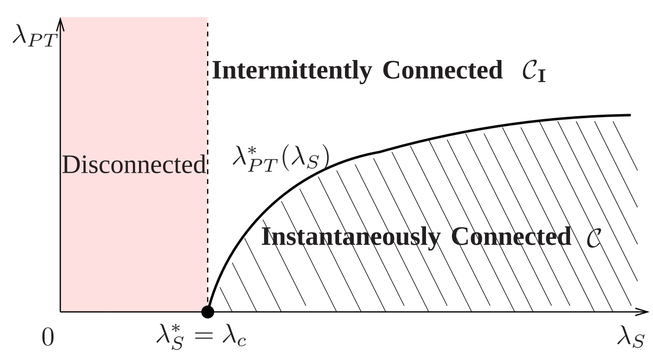

Fig. 1. Connectivity of ad hoc cognitive radio networks (the critical density λ^*_S of the secondary users is defined as the infimum density of the secondary users that ensures instantaneous connectivity under a *positive* density of the primary transmitters, and is equal to the critical density λ_c of a homogeneous ad hoc network; the upper boundary $\lambda^*_{PT}(\lambda_S)$ is defined as the supremum density of the primary transmitters that ensures instantaneous connectivity with a *fixed* density of the secondary users).

While the secondary network is connected and the MDD is finite wpp. whenever there are sufficient topological links (*i.e.,* $\lambda_S > \lambda_c$), there may not be sufficient *communication links* to make the network *instantaneously connected* at any given time. The latter is determined by the traffic load of the primary

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[1]This infinite network model is equivalent in distribution to the limit of a sequence of finite networks with a fixed density as the area of the network increases to infinity, *i.e.,* the so-called *extended network*.

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14. ABSTRACT
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15. SUBJECT TERMS
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network. As illustrated in Fig. 1, for any given density λ_S of the secondary users with $\lambda_S > \lambda_c$, there exists a maximum density $\lambda^*_{PT}(\lambda_S)$ of the primary transmitters beyond which the secondary network is only *intermittently connected*. When intermittently connected, the secondary network has no infinite connected component formed by communication links at any given time. Messages can only traverse the topological path connecting two secondary users by making stops in between to wait for spectrum opportunities.

It is thus natural to expect that the MDD will behave differently in an instantaneously connected secondary network as compared to an intermittently connected secondary network. Indeed, we show in this paper that the scaling behavior of the MDD with respect to the source-destination distance is starkly different depending on the type of the connectivity. To highlight the impact of the waiting time for spectrum opportunities on the MMD, we first ignore the propagation delay. Let μ be the source, ν the destination, $t(\mu,\nu)$ the MMD from μ to ν, and $d(\mu,\nu)$ the distance between μ and ν. We show that, a.s.

$$\lim_{d(\mu,\nu)\to\infty} \frac{t(\mu,\nu)}{d(\mu,\nu)} \begin{cases} = 0, & \text{if instantaneously connected;} \\ > 0, & \text{if intermittently connected.} \end{cases}$$

When the secondary network is instantaneously connected, a much stronger statement is actually shown, that is,

$$\lim_{d(\mu,\nu)\to\infty} \frac{t(\mu,\nu)}{g(d(\mu,\nu))} = 0 \text{ a.s.,}$$

where $g(d(\mu,\nu))$ is any monotonically increasing function of $d(\mu,\nu)$ satisfying $\lim\limits_{d(\mu,\nu)\to\infty} g(d(\mu,\nu)) = \infty$. It implies that the MMD $t(\mu,\nu)$ is asymptotically independent of the distance $d(\mu,\nu)$ as $d(\mu,\nu) \to \infty$. Thus when the propagation delay is negligible, an instantaneously connected cognitive radio (CR) network behaves almost the same as a homogeneous ad hoc network, in the sense that the waiting time for spectrum opportunities does not affect the scaling law of the MMD with respect to the source-destination distance.

The above scaling law may be illustrated with an analogy of traveling from a place μ to another place ν, where the waiting time for the spectrum opportunities is likened to the waiting time for traffic lights. Suppose that we can move fast enough such that the driving time on the road is negligible. When the secondary network is instantaneously connected, there exists a.s. an infinite connected component consisting of communication links which can be considered a highway without traffic lights between μ and ν. Given that both μ and ν are within a finite distance to the highway (independent of the distance between μ and ν), the travel time from μ to ν, which is the waiting time for traffic lights before entering the highway and after leaving the highway, is independent of the distance between μ and ν. When the secondary network is intermittently connected, such a highway between μ and ν can not be found. Then we have to use local paths and wait for traffic lights from time to time, leading to the linear scaling of the travel time with respect to the distance between μ and ν even when the driving time is ignored.

We also study the impact of the propagation delay on the MMD. When the propagation delay τ is nonnegligible, we show that the MMD scales linearly with the source-destination distance in both instantaneously connected and intermittently connected regimes, but with different rates. In particular, the limiting behavior of the rate as $\tau \to 0$ is distinct in the two regimes, *i.e.,* a.s.

$$\lim_{\tau\to 0}\lim_{d(\mu,\nu)\to\infty} \frac{t(\mu,\nu)}{d(\mu,\nu)} \begin{cases} = 0, & \text{if instantaneously connected;} \\ > 0, & \text{if intermittently connected.} \end{cases}$$

It indicates that when the propagation delay is sufficiently small, the scaling rate of the MMD for an instantaneously connected network is much smaller than the one for an intermittently connected network.

B. Related Work

As a fundamental indicator for the feasibility and efficiency of large-scale wireless networks, the scaling law has received increasing interest in the research community since the seminal work of Gupta and Kumar [3]. The capacity scaling law of CR networks has been analyzed in [4–6]. In [4], the authors also derive the capacity-delay tradeoff for a specific routing and scheduling algorithm which is shown to achieve the optimal one for homogeneous networks. A major difference of our work from theirs is that the scaling law of the MMD is not derived for any specific routing and scheduling algorithm. Instead, it provides a fundamental limit on the asymptotic delay performance of any routing and scheduling algorithm for CR networks. To our best knowledge, the scaling law of the MMD with respect to the source-destination distance in a CR network has not been characterized in the literature.

The scaling law of the multihop delay in homogeneous ad hoc networks has been well studied (see [7–15] and references therein). The multihop delay for a specific routing algorithm is analyzed in [7–9], and the capacity-delay tradeoff is revealed under a given network and mobility model in [10–12]. Based on continuum percolation theory, the scaling law of the multihop delay with respect to the source-destination distance is established in [13–15].

II. Network Model

We consider a Poisson distributed secondary network overlaid with a Poisson distributed primary network in an infinite two dimensional Euclidean space. The primary network adopts a synchronized slotted structure with a slot length T_S. The realizations of active primary transmitters vary from slot to slot and are assumed to be i.i.d. across slots[2]. Thus T_S can be considered as the time constant of the spectrum opportunities which are determined by the transmitting and receiving activities of the primary users. Without loss of generality, we assume that $T_S = 1$.

[2]The different realizations of active primary transmitters in different slots can be caused by the mobility of these users or changes in the traffic pattern or both. The i.i.d. model of the temporal dynamics of the primary traffic is assumed to simplify the analysis, and it is not necessary for our main results to hold (see Sec. III-B).

At the beginning of each slot, the primary transmitters are distributed according to a two-dimensional Poisson point process X_{PT} with density λ_{PT}. Primary receivers are uniformly distributed within the transmission range R_p of their corresponding transmitters. Here we have assumed that all the primary transmitters use the same transmission power and the transmitted signals undergo an isotropic path loss. It follows from the Displacement Theorem [16, Chapter 5] that the primary receivers form another Poisson point process X_{PR} with density λ_{PT}, which is correlated with X_{PT}.

The secondary users are distributed according to a two-dimensional Poisson point process X_S with density λ_S, which is independent of X_{PT} and X_{PR}. The locations of the secondary users are static over time, and they have a uniform transmission range r_p.

III. Connectivity

In this section, we examine the connectivity of the secondary network by analytically characterizing the partition of the $(\lambda_S, \lambda_{PT})$ plane illustrated in Fig. 1.

A. Topological Link vs. Communication Link

Topological links in the secondary network are independent of the primary network. A topological link exists between any two secondary users that are within each other's transmission range. In contrast, the existence of a communication link between two secondary users depends not only on the distance between them but also on the availability of the communication channel, *i.e.*, the presence of a spectrum opportunity offered by the primary network. As a result, even in a static secondary network, communication links are time-varying due to the temporal dynamics of spectrum opportunities. The presence of a spectrum opportunity is determined by the transmitting and receiving activities of the primary network as described below.

We consider the disk signal propagation and interference model as illustrated in Fig. 2. There exists an opportunity from μ, the secondary transmitter, to ν, the secondary receiver, if the transmission from μ does not interfere with *primary receivers* in the solid circle, and the reception at ν is not affected by *primary transmitters* in the dashed circle [17]. Referred to as the interference range of the secondary users, the radius r_I of the solid circle at μ depends on the transmission power of μ and the interference tolerance of the primary receivers, whereas the radius R_I of the dashed circle (the interference range of the primary users) depends on the transmission power of the primary users and the interference tolerance of ν.

It is clear from the above discussion that spectrum opportunities are *asymmetric*. Specifically, a channel that is an opportunity when μ is the transmitter and ν the receiver may not be an opportunity when ν is the transmitter and μ the receiver. Since unidirectional links are difficult to utilize, especially for applications with guaranteed delivery that require acknowledgements, we only consider bidirectional links in the secondary network when we define connectivity. As a result, the single-hop delay from μ to ν is the waiting time

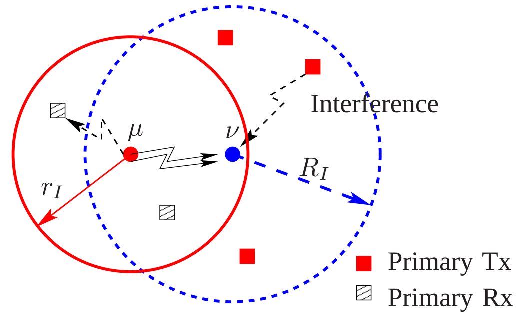


Fig. 2. Definition of spectrum opportunity.

for the presence of the first bidirectional opportunity plus the propagation delay τ.

B. Connectivity and the Finiteness of MMD

As stated in Sec. I, the connectivity of the secondary network is defined by the finiteness of the MDD between two randomly chosen secondary users. Consider an undirected random graph $\mathcal{G}_S(\lambda_S)$ consisting of all the secondary users and the topological links. Notice that $\mathcal{G}_S(\lambda_S)$ depends only on the Poisson point process X_S of the secondary network. Under the i.i.d. model of the temporal dynamics of the primary traffic, we show in Theorem 1 below that a necessary and sufficient condition for the a.s. finiteness of the MMD in the secondary network is the connectivity of $\mathcal{G}_S(\lambda_S)$ in the percolation sense.

Theorem 1: Let $t(\mu, \nu)$ denote the MMD between two randomly chosen secondary users μ and ν. Then wpp., $t(\mu, \nu) < \infty$ a.s. if and only if $\lambda_S > \lambda_c$ where λ_c is the critical density of homogeneous ad hoc networks.

Proof: This proof is based on the a.s. finiteness of the single-hop delay. The details are omitted due to the page limit, and are given in [18]. ■

Theorem 1 shows that under the i.i.d. model of the temporal dynamics of the primary traffic, the connectivity of the secondary network defined by the finiteness of the MMD is equivalent to the *topological* connectivity of $\mathcal{G}_S(\lambda_S)$ which is independent of the primary network. In other words, no matter how heavy the primary traffic is, the MMD between two secondary users in the infinite topologically connected component of $\mathcal{G}_S(\lambda_S)$ is finite a.s.

We point out that the i.i.d. model of the temporal dynamics of the primary traffic is not necessary for Theorem 1 to hold. This i.i.d. model can be considered as one end of the spectrum on the richness of the temporal dynamics of the primary traffic. The other end of the spectrum is given by a static set of primary transmitters and receivers. In this case, the finiteness of MMD can only be achieved through instantaneous connectivity using only communication links. It is an interesting future direction to obtain necessary conditions on the temporary dynamics of the primary traffic that ensures the equivalence between the finiteness of MMD and the topological connectivity of $\mathcal{G}_S(\lambda_S)$. From the proof of Theorem 1 we can see that this equivalence holds whenever the temporal dynamics of the primary traffic makes the single-hop delay have a proper distribution.

C. *Instantaneous Connectivity vs. Intermittent Connectivity*

In a primary slot t, we can obtain an undirected random graph $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$ consisting of all the secondary users and the communication links which represents the instantaneous connectivity of the secondary network in this slot. As illustrated in Fig. 3, this graph $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$ is determined by the three Poisson point processes in slot t: X_S, X_{PT}, and X_{PR}, where X_{PT} and X_{PR} are correlated.

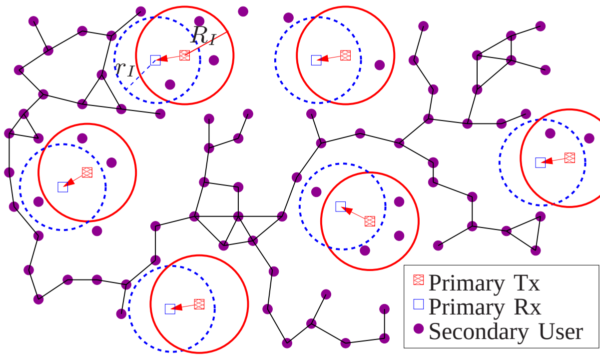


Fig. 3. A realization of the random graph $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$ which consists of all the secondary users and all the communication links in the primary slot t (denoted by solid lines). The solid circles denote the interference regions of the primary transmitters within which secondary users can not successfully receive, and the dashed circles denote the required protection regions for the primary receivers within which secondary users should refrain from transmitting.

We define the instantaneous connectivity of the secondary network as the a.s. existence of an infinite connected component in $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$ for all t. Given the transmission power and the interference tolerance of both the primary and the secondary users ($i.e.$, R_p, R_I, r_p, and r_I are fixed), the instantaneous connectivity region $\mathcal{C}$ is defined as

$$\mathcal{C} \triangleq \{(\lambda_S,\ \lambda_{PT}):\ \mathcal{G}_H(\lambda_S, \lambda_{PT}, t) \text{ is connected for all } t\}.$$

A detailed analytical characterization of $\mathcal{C}$ is given in [19].

Referred to as the critical density of the secondary users, λ_S^* is the infimum density of the secondary users that ensures instantaneous connectivity under a positive density of primary transmitters:

$$\begin{aligned}\lambda_S^* \quad &\triangleq \quad \inf\{\lambda_S : \exists \lambda_{PT} > 0 \text{ such that} \\ &\qquad\qquad \mathcal{G}_H(\lambda_S, \lambda_{PT}, t) \text{ is connected for all } t\}.\end{aligned}$$

It is shown in [19] that λ_S^* equals the critical density λ_c of a *homogeneous* ad hoc network.

$\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$ can also be obtained by removing topological links that do not see the opportunities in slot t from the random graph $\mathcal{G}_S(\lambda_S)$. Thus, even if the secondary network is connected (*i.e.*, $\mathcal{G}_S(\lambda_S)$ has an infinite connected component), it may not be instantaneously connected. Specifically, the infinite connected component in $\mathcal{G}_S(\lambda_S)$ may break into infinite number of finite connected components in $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$ due to scarcity of spectrum opportunities. In this case, we define the intermittent connectivity region $\mathcal{C}_I$ as

$$\begin{aligned}\mathcal{C}_I \quad \triangleq \quad &\{(\lambda_S,\ \lambda_{PT}):\ \lambda_S > \lambda_c \text{ and } \mathcal{G}_H(\lambda_S, \lambda_{PT}, t) \\ &\qquad\qquad \text{is disconnected for all } t\}.\end{aligned}$$

IV. Multihop Delay

In this section, we analytically characterize the scaling behavior of the MMD with respect to the source-destination distance. Let $C(\mathcal{G}_S(\lambda_S))$ be the infinite connected component in $\mathcal{G}_S(\lambda_S)$ when $\lambda_S > \lambda_c$, *i.e.*, the secondary network is either instantaneously connected or intermittently connected. We seek to establish the scaling law of the MMD between two arbitrary users in $C(\mathcal{G}_S(\lambda_S))$ with respect to the distance between them. As shown in the following two theorems which consider the two cases when the propagation delay $\tau = 0$ and $\tau > 0$, the type of the connectivity determines the scaling behavior of the MMD.

Theorem 2: Assume that $\tau = 0$. For any two secondary users μ, $\nu \in C(\mathcal{G}_S(\lambda_S))$, where $C(\mathcal{G}_S(\lambda_S))$ is the infinite connected component of $\mathcal{G}_S(\lambda_S)$, let $t(\mu, \nu)$ denote the MMD from μ to ν and $d(\mu, \nu)$ the distance between μ and ν, then

T2.1 if $(\lambda_S, \lambda_{PT}) \in \mathcal{C}$,

$$\lim_{d(\mu,\nu)\to\infty} \frac{t(\mu, \nu)}{g(d(\mu, \nu))} = 0 \text{ a.s.},$$

where $g(d)$ is any monotonically increasing function of d with $\lim_{d\to\infty} g(d) = \infty$;

T2.2 if $(\lambda_S, \lambda_{PT}) \in \mathcal{C}_I$, $\exists\ 0 < \beta < \infty$ such that

$$\lim_{d(\mu,\nu)\to\infty} \frac{t(\mu, \nu)}{d(\mu, \nu)} = \beta \text{ a.s.}, \tag{1}$$

where the value of β depends on $(\lambda_S, \lambda_{PT})$.

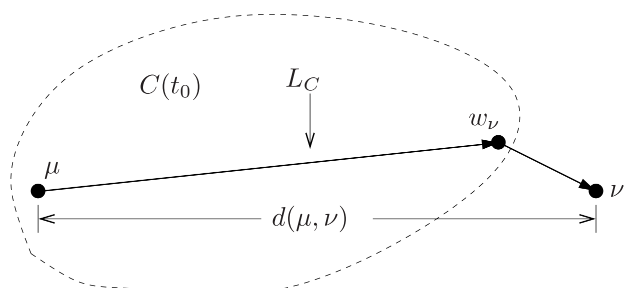


Fig. 4. An illustration of the constructed path L_C from μ to ν when $(\lambda_S, \lambda_{PT}) \in \mathcal{C}$. $C(t_0)$ is the infinite connected component of $\mathcal{G}(\lambda_S, \lambda_{PT}, t_0)$ which first contains μ, and w_ν is the user in $C(t_0)$ which is closest to ν.

Proof Sketch of T2.1: We use the infinite connected component[3] in $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t_0)$ during some primary slot t_0 to construct a path from μ to ν such that the multihop delay along this path is independent of the distance $d(\mu, \nu)$ (see Fig. 4 for an illustration). Let t_0 be the first primary slot such that μ belongs to the infinite connected component $C(t_0)$ of $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t_0)$, and w_ν the user in $C(t_0)$ which is closest to ν. Since the propagation delay $\tau = 0$, the multihop delay from μ to w_ν is zero. It follows that

$$t(\mu, \nu) = t_0 + t(w_\nu, \nu).$$

[3]It is shown in [19] that there exists either zero or one infinite connected component in $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$ a.s. for any given t.

Then it suffices to show that t_0 and $t(w_\nu, \nu)$ are independent of $d(\mu, \nu)$, which can be done by using continuum percolation theory and ergodic theory. ∎

Proof Sketch of T2.2: Based on the scaling argument [20, Chapter 2], we set the transmission range r_p of the secondary users to 1 without loss of generality. Take μ as the origin, and the line connecting μ and ν as the x-axis. Define an auxiliary node $\tilde{w}_i$ in $C(\mathcal{G}_S(\lambda_S))$ for every integer i:

$$\tilde{w}_i \triangleq \underset{w \in C(\mathcal{G}_S(\lambda_S))}{\arg\min} \; d(w, (i, 0)).$$

Obviously, $\tilde{w}_0 = \mu$. Let n be the closest integer to ν, then

$$\frac{t(\tilde{w}_0, \tilde{w}_n) - t(\tilde{w}_n, \nu)}{n+1} \leq \frac{t(\mu, \nu)}{d(\mu, \nu)} \leq \frac{t(\tilde{w}_0, \tilde{w}_n) + t(\tilde{w}_n, \nu)}{n-1}.$$

If $\tilde{w}_n = \nu$, then $t(\tilde{w}_n, \nu) = 0$; if $\tilde{w}_n \neq \nu$, then $t(\tilde{w}_n, \nu)$ is at most the single-hop delay because $d(\tilde{w}_n, \nu) \leq d(\tilde{w}_n, (n, 0)) + d(\nu, (n, 0)) \leq 2d(\nu, (n, 0)) \leq 1$.

Let $t_{m,n} = t(\tilde{w}_m, \tilde{w}_n)$ for any two integers m, n. Then to show T2.2, it suffices to prove that

$$\lim_{n \to \infty} \frac{t_{0,n}}{n} = \beta > 0 \text{ a.s.} \tag{2}$$

The proof of (2) is divided into two steps: first prove the existence of the limit based on the Subadditive Ergodic Theorem [21] and then derive a lower bound on $\frac{t(\mu,\nu)}{d(\mu,\nu)}$ by considering the fact that the message from μ can traverse only a finite distance towards ν during each primary slot. In the first step, we show that the five conditions of the Subadditive Ergodic Theorem hold for the sequence $\{t_{m,n}\}$ of MMD between auxiliary nodes, and then the existence of the limit follows immediately from the theorem. ∎

Theorem 3: Assume that $\tau > 0$. For any two secondary users μ, $\nu \in C(\mathcal{G}_S(\lambda_S))$, where $C(\mathcal{G}_S(\lambda_S))$ is the infinite connected component of $\mathcal{G}_S(\lambda_S)$, let $t^\tau(\mu, \nu)$ denote the MMD from μ to ν and $d(\mu, \nu)$ the distance between μ and ν, then $\exists\, \gamma = \gamma(\tau) > 0$ such that

$$\lim_{d(\mu,\nu) \to \infty} \frac{t^\tau(\mu, \nu)}{d(\mu, \nu)} = \gamma \geq \tau \text{ a.s.}. \tag{3}$$

Furthermore, if $(\lambda_S, \lambda_{PT}) \in \mathcal{C}$,

$$\lim_{\tau \to 0} \lim_{d(\mu,\nu) \to \infty} \frac{t^\tau(\mu, \nu)}{d(\mu, \nu)} = 0 \text{ a.s.}; \tag{4}$$

if $(\lambda_S, \lambda_{PT}) \in \mathcal{C}_I$,

$$\lim_{\tau \to 0} \lim_{d(\mu,\nu) \to \infty} \frac{t^\tau(\mu, \nu)}{d(\mu, \nu)} \geq \beta > 0 \text{ a.s.}, \tag{5}$$

where $\beta = \lim_{d(\mu,\nu) \to \infty} \frac{t(\mu,\nu)}{d(\mu,\nu)}$ is defined in (1).

Proof Sketch: The equality in (3) is based on the Subadditive Ergodic Theorem [21], while the inequality in (3) is established via a simple lower bound on $t^\tau(\mu, \nu)$. The basic idea behind establishing (4) is to consider the multihop delay along the path constructed in Fig. 4. Eqn. (5) follows immediately from the fact that $t^\tau(\mu, \nu) \geq t(\mu, \nu)$, where $t(\mu, \nu)$ is the MMD when $\tau = 0$. ∎

Due to the page limit, we omit the details of the above proofs, which can be found in [18].

V. Simulation Results

In this section, we present several simulation results. The density λ_S of the simulated secondary network is larger than the critical density λ_c. Thus, the secondary network is either instantaneously connected or intermittently connected, depending on the density λ_{PT} of the primary transmitters. Without loss of generality, we assume that the source is located at the origin. Each node in the network is a potential destination. This allows us to simulate different realizations of the source-destination pair using one Monte Carlo run.

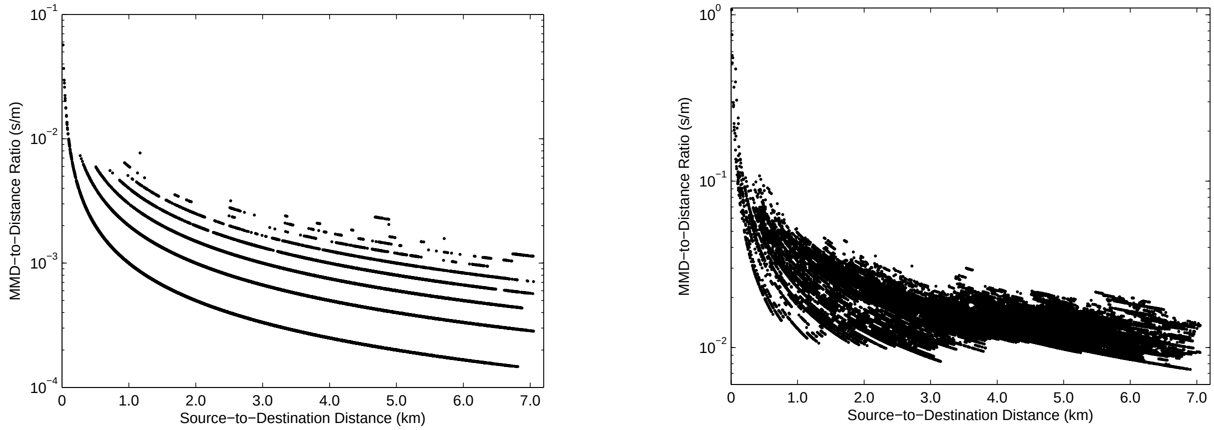
We obtain the MMD by considering the flooding scheme. Specifically, every user which has received the message (including the source) will transmit the message to its neighbors within its transmission range when it experiences a bidirectional spectrum opportunity to any of its neighbors. The transmission attempts will not stop until all its neighbors receive the message. The time that a user first receives the message during the flooding is considered as the MMD from the source to this user. It is easy to see that simulating this flooding scheme gives us the MMD when there is no contention between the secondary users' transmissions.

Fig. 5 shows the MMD-to-distance ratio as a function of the source-destination distance when the propagation delay τ is zero[4], where each dot represents a realization of the destination. We can see that if the secondary network is instantaneously connected (Fig. 5-(a)), the ratio decreases very fast as the distance increases, and it can be expected that the ratio will eventually tend to zero. On the other hand, if the secondary network is intermittently connected (Fig. 5-(b)), the decreasing rate of the ratio levels off as the distance increases, and the ratio will gradually approach a positive constant. Note that in Fig. 5-(a), the MMD-to-distance ratios of different realizations of the destination are grouped into several continuous curves, each associated with a fixed MMD. Specifically, since the message is mainly delivered via the infinite connected component consisting of communication links when the secondary network is instantaneously connected, the secondary users are actually grouped according to the first time that they are in an infinite connected component. From Fig. 5-(a) we can see that due to the temporal dynamics of spectrum opportunities, every node will be part of an infinite connected component within a few number of primary slots.

VI. Conclusion

We have studied the connectivity and multihop delay of ad hoc cognitive radio networks. The impact of connectivity on the multihop delay has been examined by establishing the scaling behavior of the minimum multihop delay with respect to the source-destination distance. Specifically, depending on whether the cognitive radio network is instantaneously connected or intermittently connected, the scaling of the minimum multihop delay behaves distinctly, in terms of either the scaling order when the propagation delay is negligible or the scaling

[4]The simulation results for nonzero propagation delay τ are omitted due to the page limit.



(a) instantaneously connected ($\lambda_{PT} = 10\text{km}^{-2}$, $\tau = 0$)

(b) intermittently connected ($\lambda_{PT} = 50\text{km}^{-2}$, $\tau = 0$)

Fig. 5. MMD-to-distance ratio (in logarithmic scale) vs. the source-to-destination distance. The secondary users are distributed within a square $[-5\text{km}, 5\text{km}] \times [-5\text{km}, 5\text{km}]$ according to a homogeneous Poisson point process with density $\lambda_S = 700\text{km}^{-2}$. Given the transmission range $r_p = 50m$ of the secondary users, we have that λ_S is larger than the critical density $\lambda_c(50) = 576\text{km}^{-2}$. Some other simulation parameters are given by $r_I = 80$m, $R_p = 50$m, $R_I = 80$m, and $T_S = 1$s.

rate when the propagation delay is nonnegligible. This result on scaling is independent of the random positions of the source and the destination, and it only depends on the network parameters (e.g., the density of the secondary users and the traffic load of the primary network). In establishing these results, we have used theories and techniques from continuum percolation and ergodicity including the concept of critical density and the Subadditive Ergodic Theorem.

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On the Connectivity and Multihop Delay of Ad Hoc Cognitive Radio Networks

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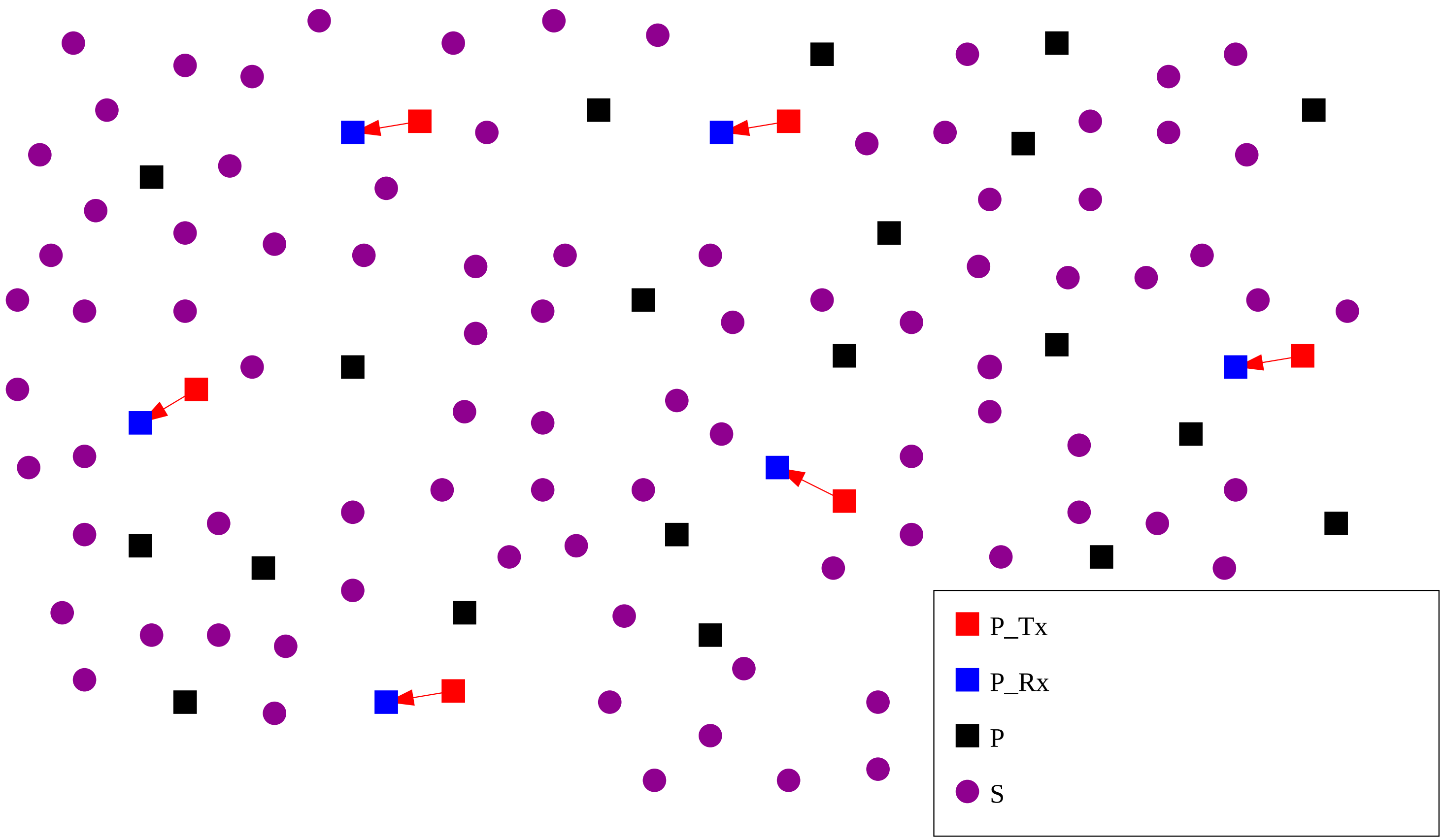
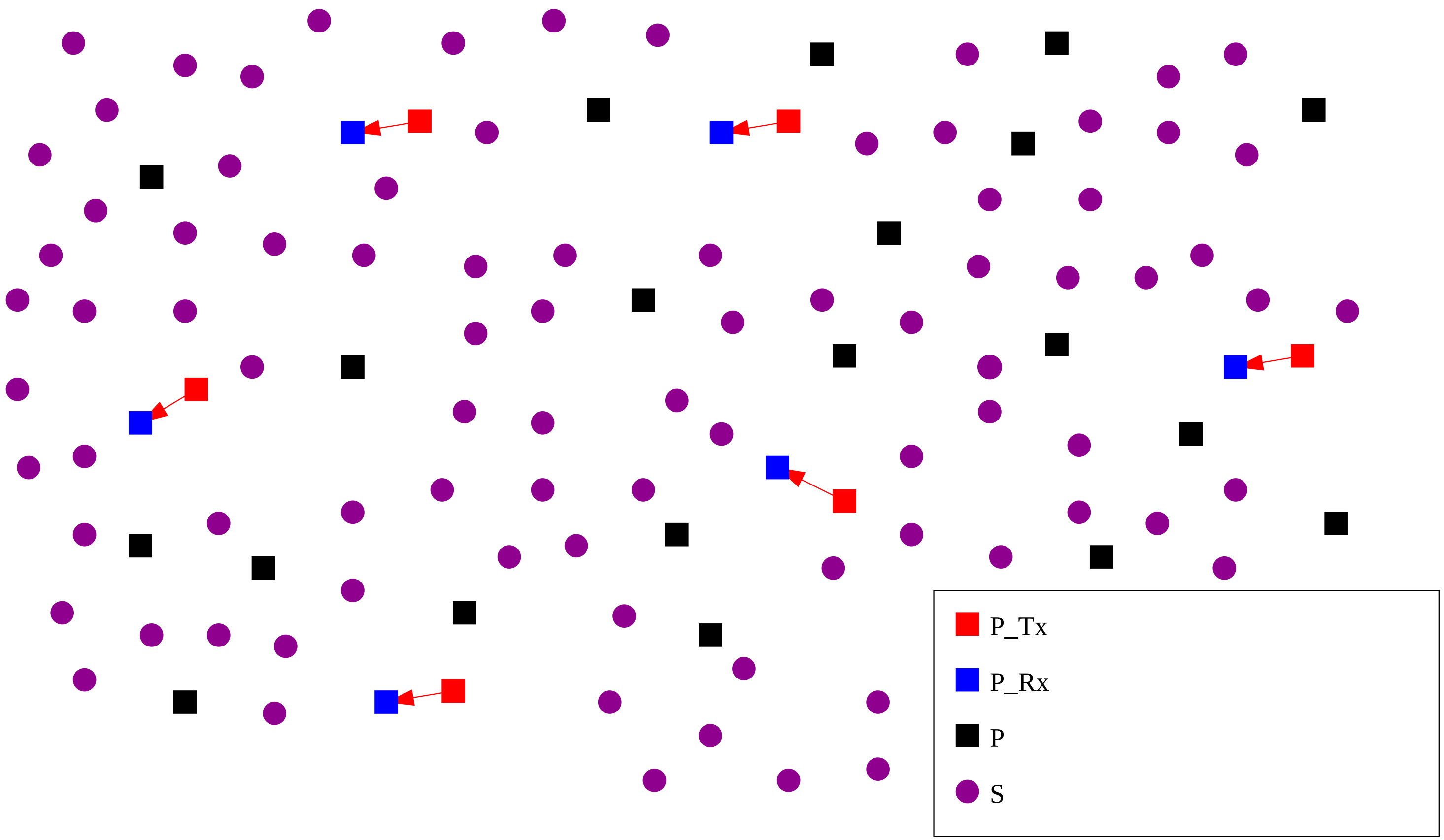
Poisson Primary + Poisson Secondary



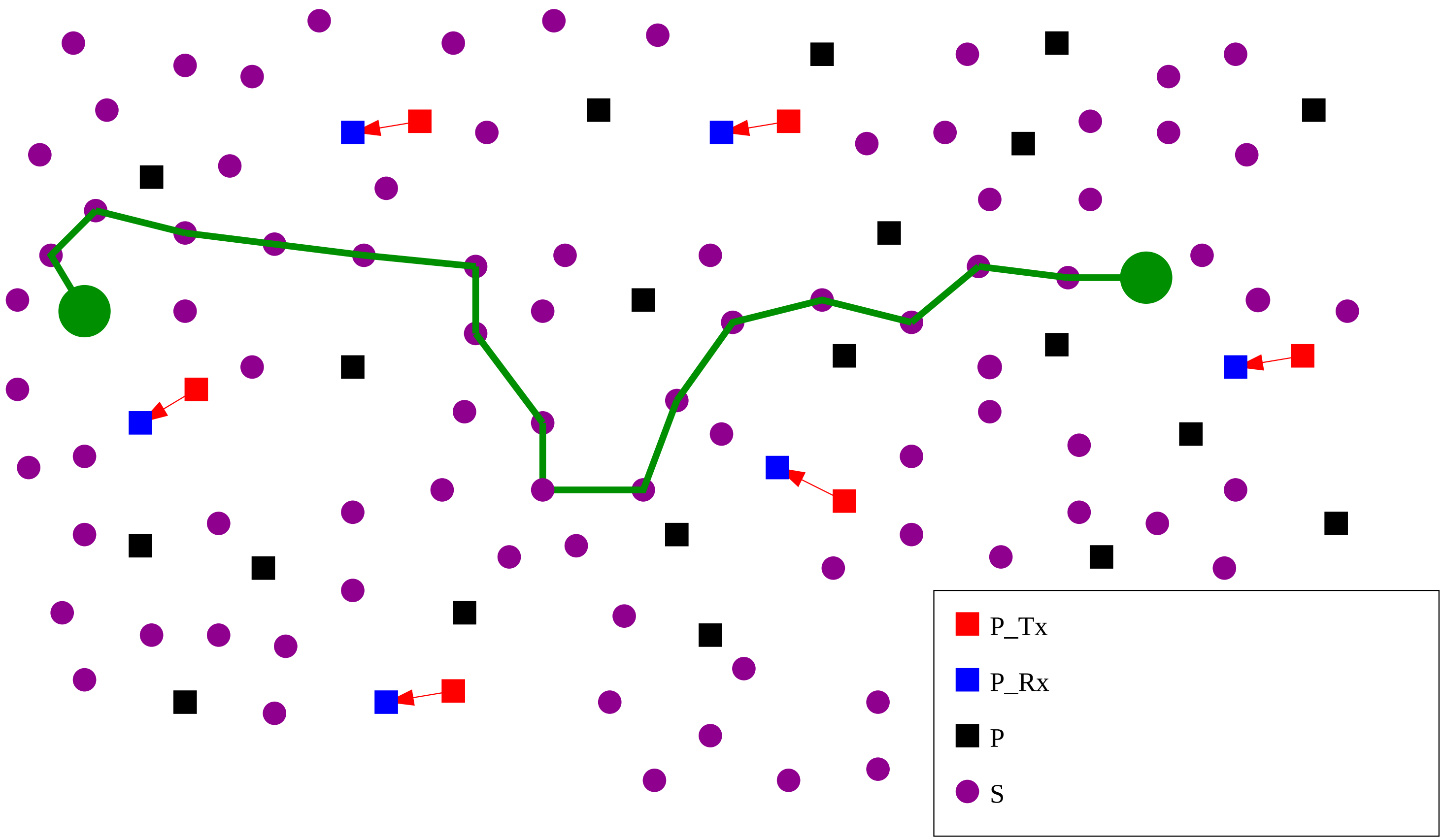
Poisson Primary + Poisson Secondary



Primary Network:

- Active primary Txs form a 2-D Poisson process.
- Primary Rxs are uniformly distributed within Tx range of their transmitters.
- Slotted transmission.

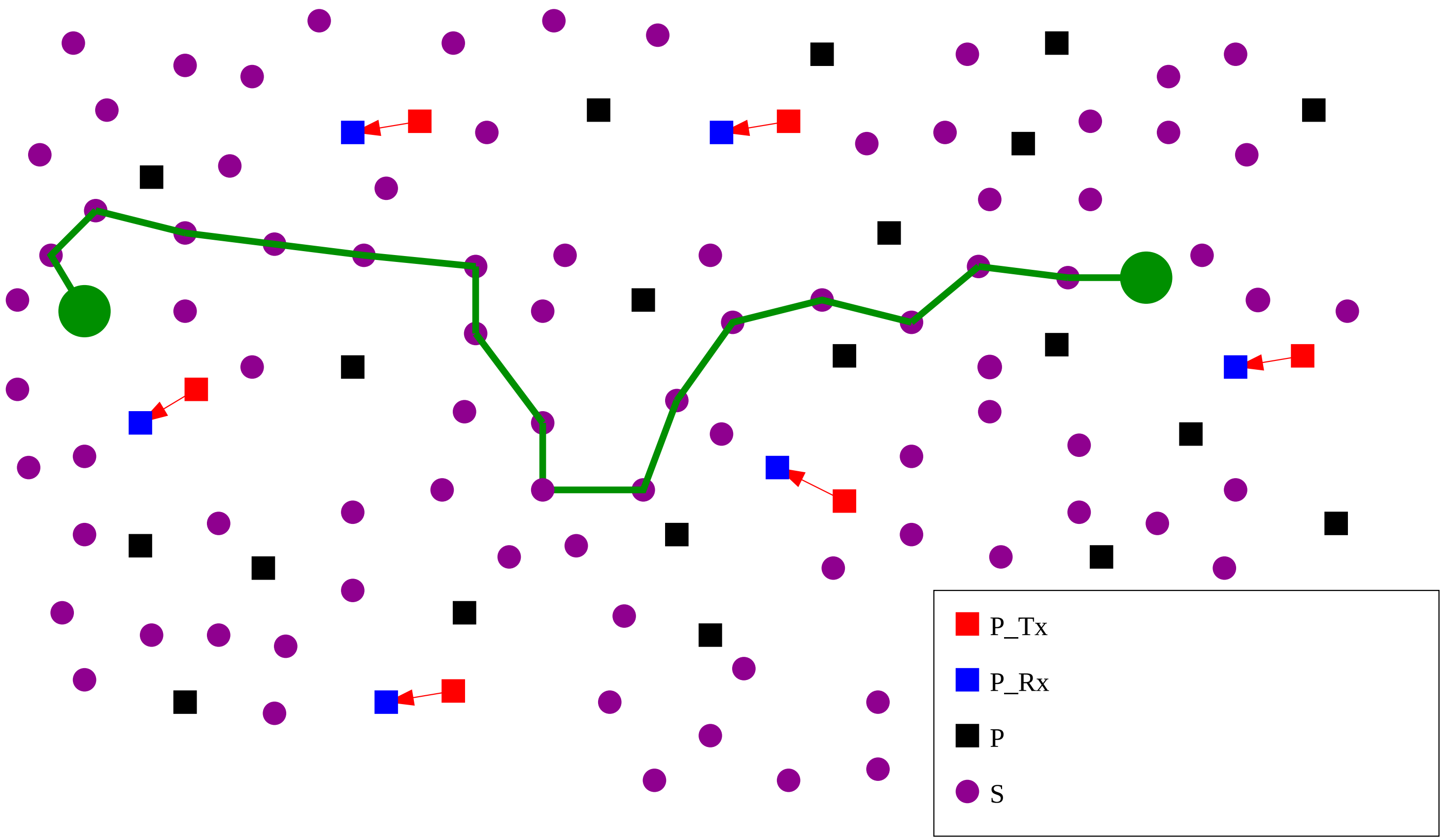
Multihop Transmission in Secondary Network



For two randomly chosen secondary users:

- can they communicate via multihop relay with finite delay (connectivity)?
- how does multihop delay scale with S-D distance?

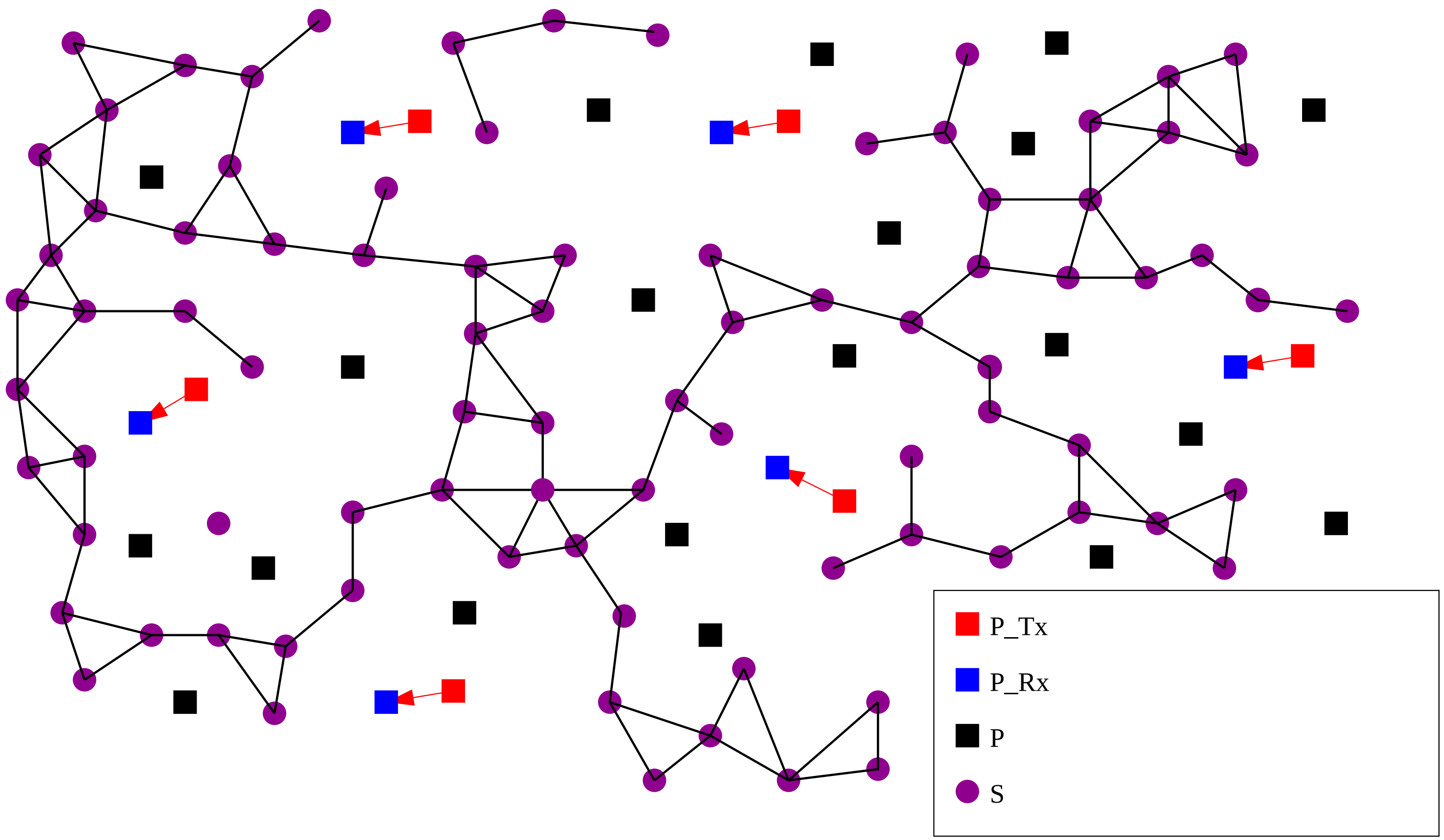
Multihop Transmission in Secondary Network



Unique Challenges: interaction between primary and secondary networks

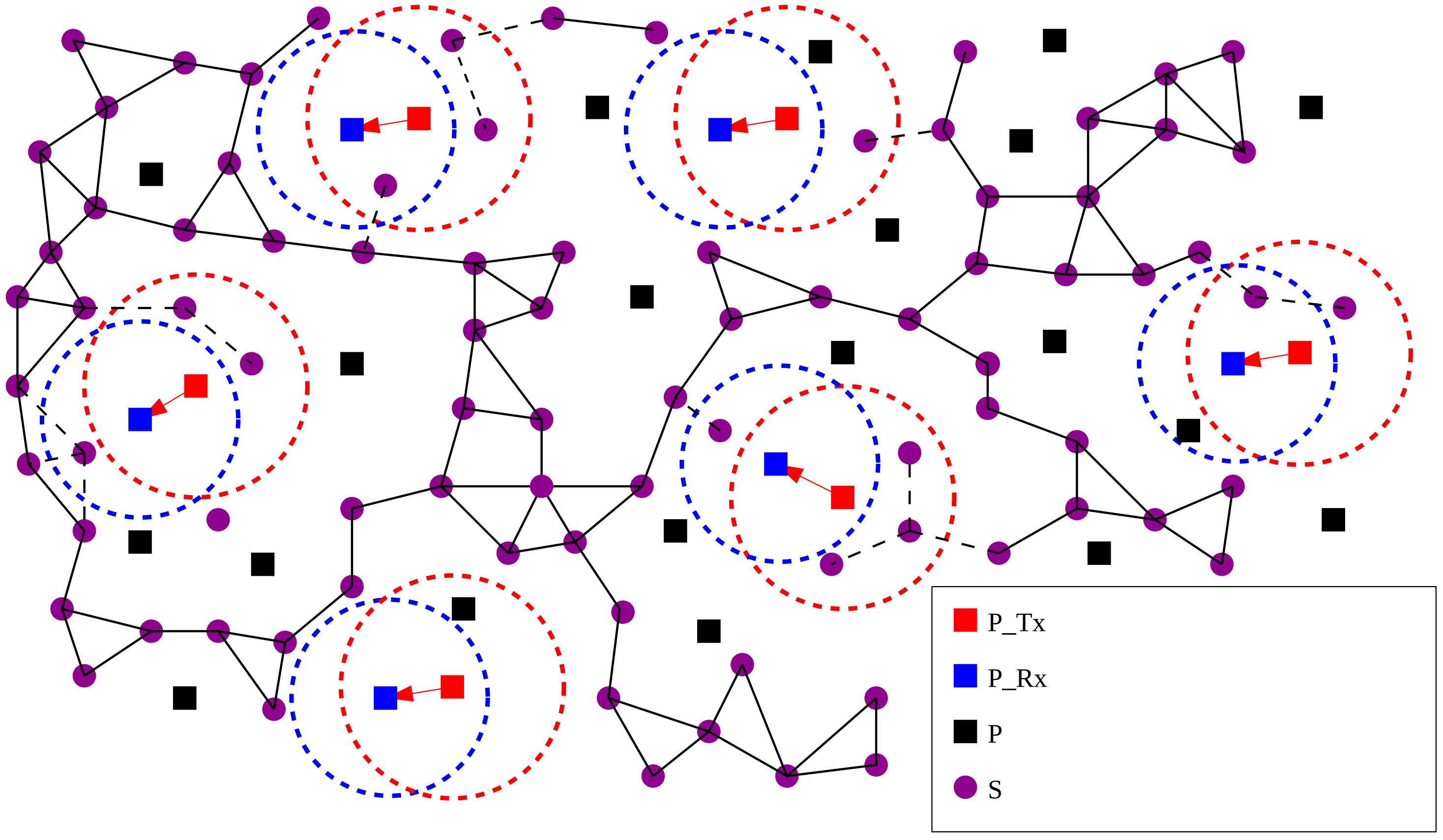
- Existence of a link depends on Tx/Rx activities of nearby PUs.
- Delay at each hop = propagation delay + waiting time for an opportunity.

Topological Links in Secondary Network



Topological Links: formed by secondary users within each other's Tx range.

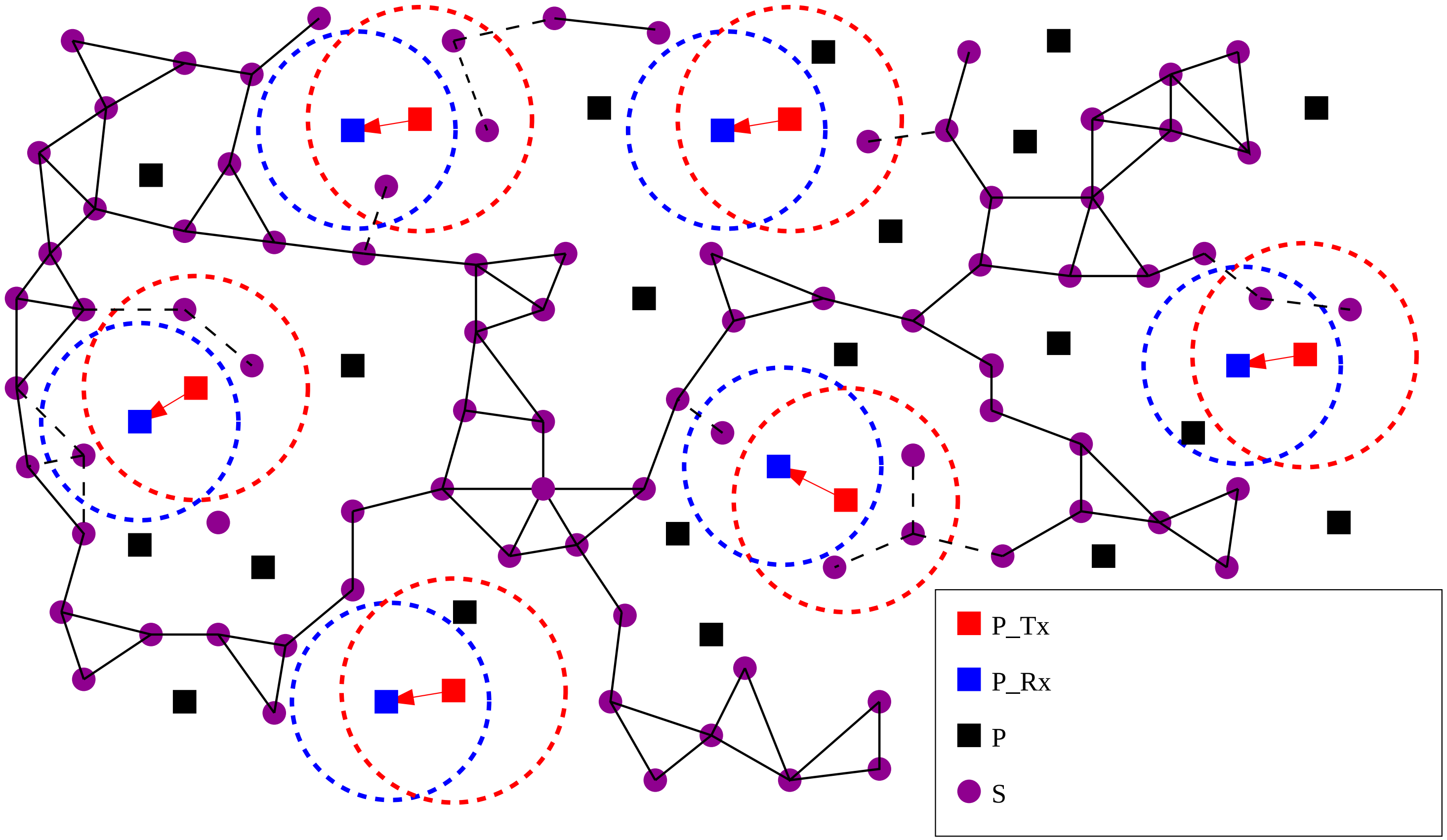
Communication Links in Secondary Network



Topological Links: formed by secondary users within each other's Tx range.

Communication Links: topological links that see bidirectional opportunities.

Connectivity of the Secondary Network

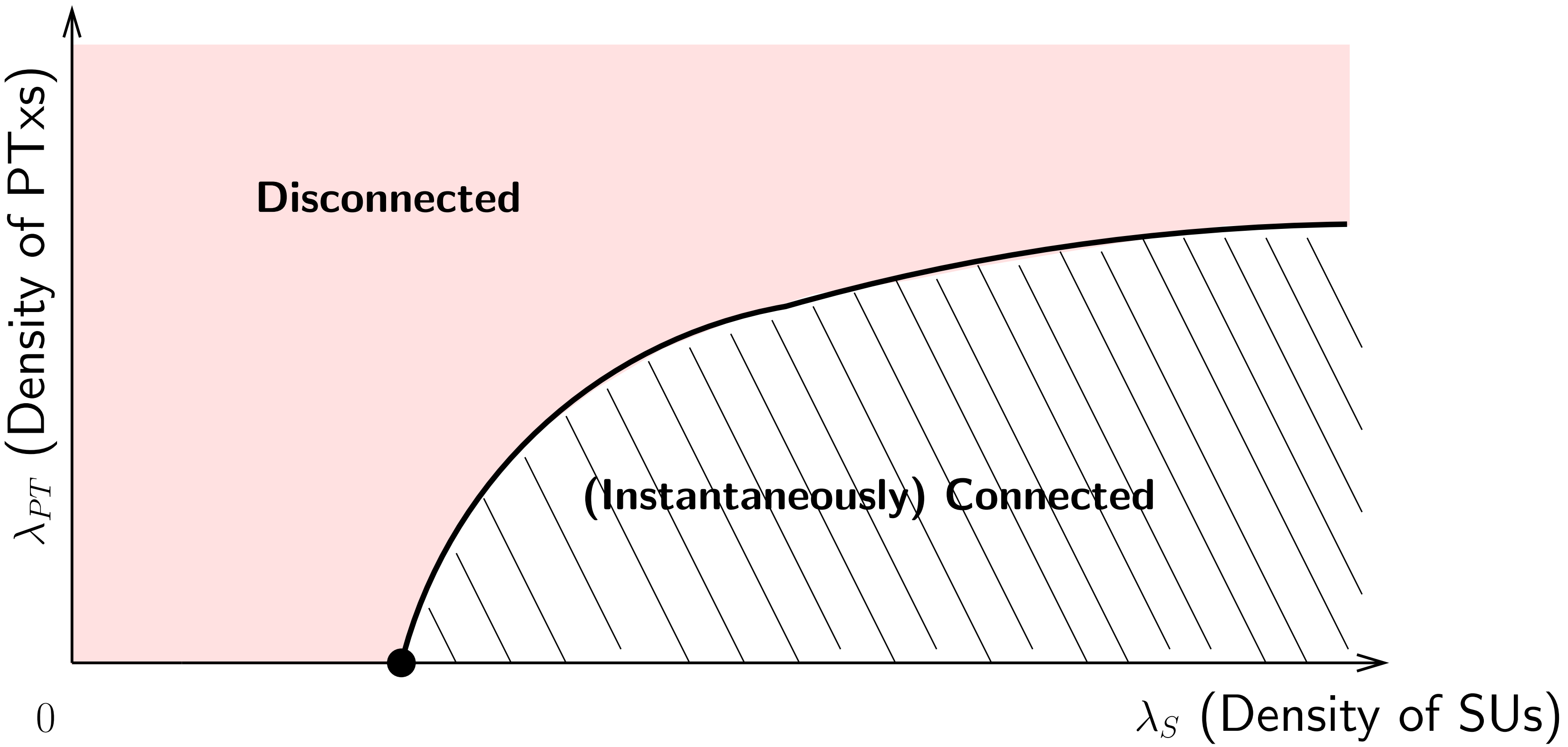


Connectivity:

- defined by finiteness of min multihop delay btw. two randomly chosen SUs.
- depends on the density of SUs and the traffic load of PUs.

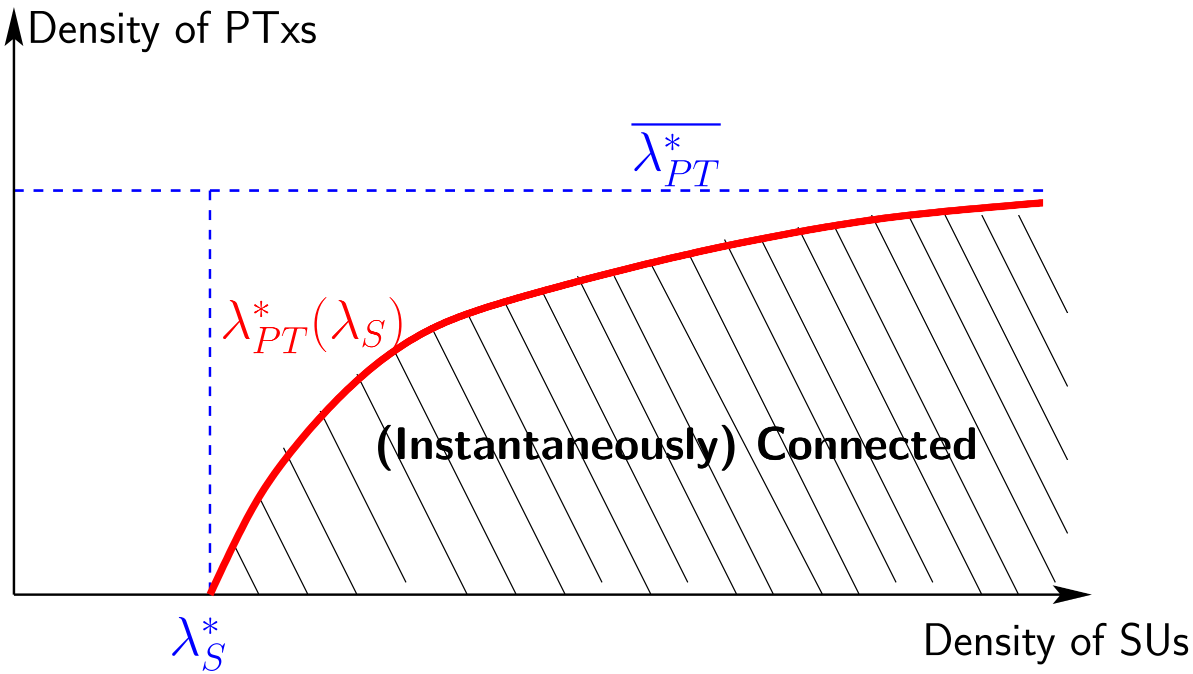
Connectivity: Static Case

When PTxs/PRxs are static over time:



Connectivity $\triangleq$ Finite MMD = Existence of inf. component connected by **comm. links**.

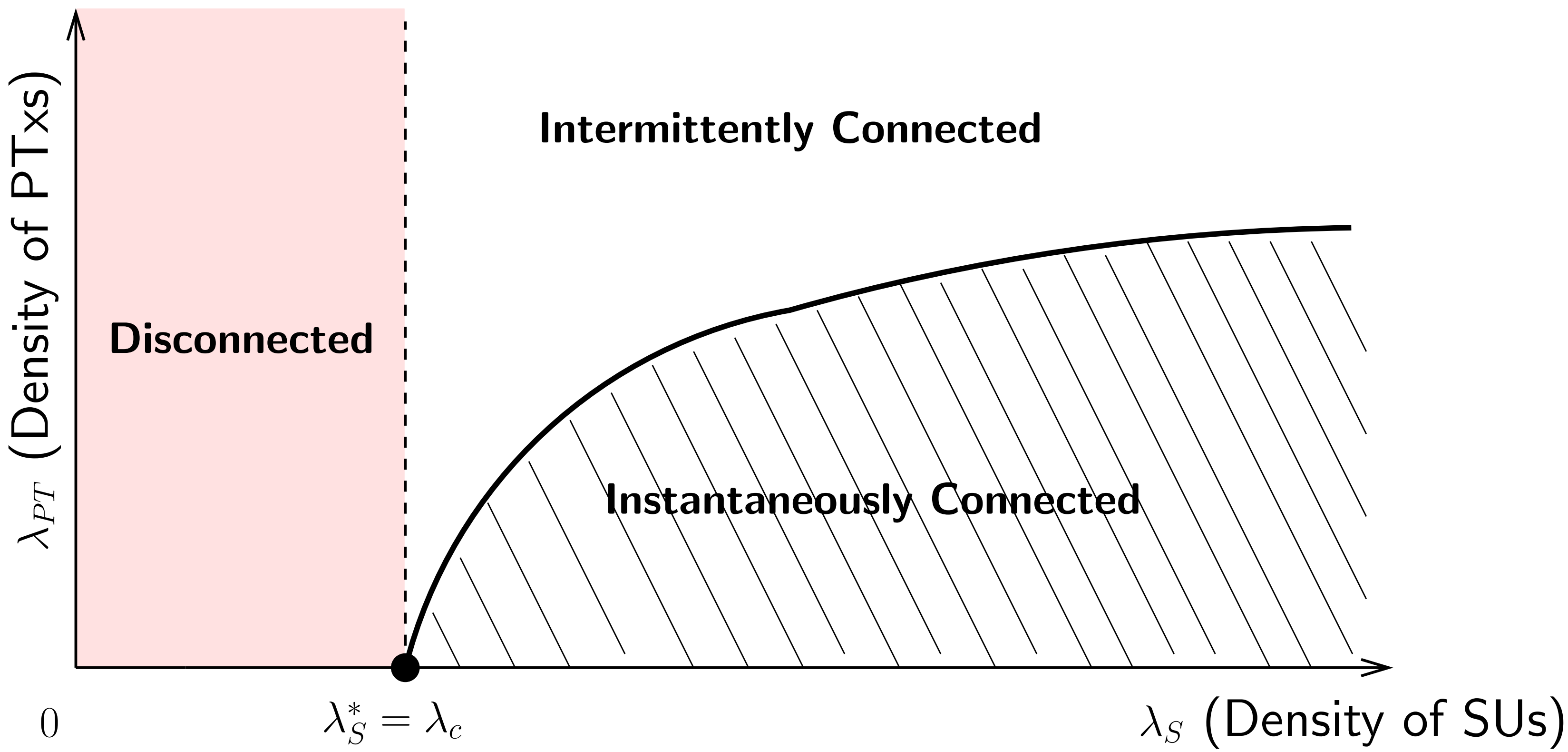
Connectivity Region



- $\forall(\lambda_S, \lambda_{PT}) \in \mathcal{C}$, there exists a *unique* infinite connected component.
- $\lambda_{PT}^*(\lambda_S)$ monotonically increases with λ_S.
- The critical density of secondary users: $\lambda_S^* = \lambda_c(r_{tx})$ (*CD of homogenous networks*).
- The critical density of primary Tx: $\overline{\lambda_{PT}^*} \leq \min\left\{\frac{1}{4\left(R_I^2 - r_p^2/4\right)}\lambda_c(1),\ \frac{1}{4\left(r_I^2 - r_p^2/4\right)}\lambda_c(1)\right\}$.

Connectivity: Dynamic Case

When temporal dynamics of PTxs/PRxs are sufficiently rich:



- Connectivity = Existence of inf. component connected by **topological links**.
- Connectivity of SUs is independent of the primary traffic load.

MMD under Negligible Propagation Delay

When instantaneously connected:

- MMD is **asymptotically independent** of the S-D distance:

$$\lim_{d(\mu,\nu)\to\infty} \frac{t(\mu,\nu)}{g(d(\mu,\nu))} = 0 \text{ a.s.},$$

where $g(d)$ is any monotonically increasing function of d with $\lim_{d\to\infty} g(d) = \infty$.

When intermittently connected:

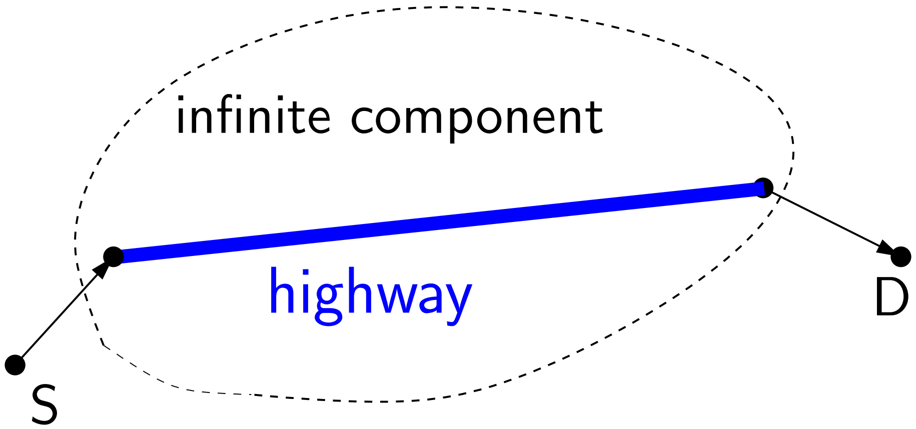
- MMD **grows linearly** with the S-D distance:

$$\lim_{d(\mu,\nu)\to\infty} \frac{t(\mu,\nu)}{d(\mu,\nu)} = \beta \text{ a.s.},$$

where the value of $\beta > 0$ depends on $(\lambda_S, \lambda_{PT})$ and the temporal dynamics of interference.

MMD under Negligible Propagation Delay

When instantaneously connected:

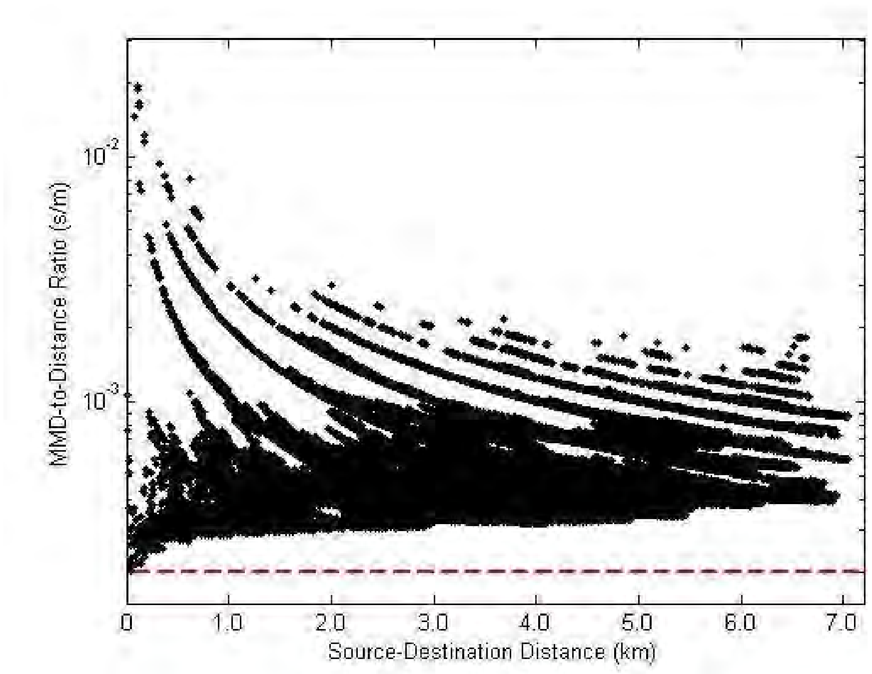


When intermittently connected:

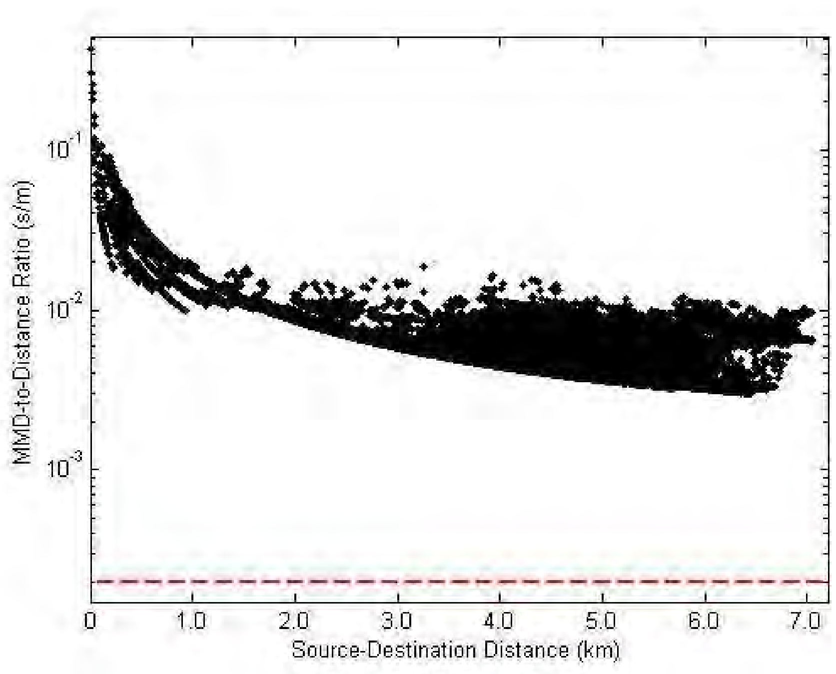
With Propagation Delay

With propagation delay:

- MMD scales linearly with S-D distance.
- Scaling rate for an instantaneously connected network can be orders of magnitude smaller than that for an intermittently connected network.



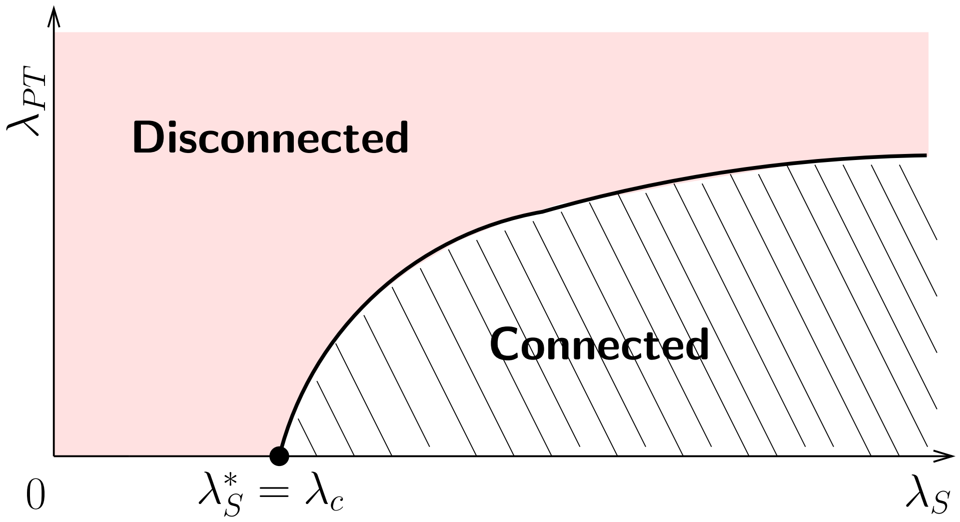
(a) instantaneously connected



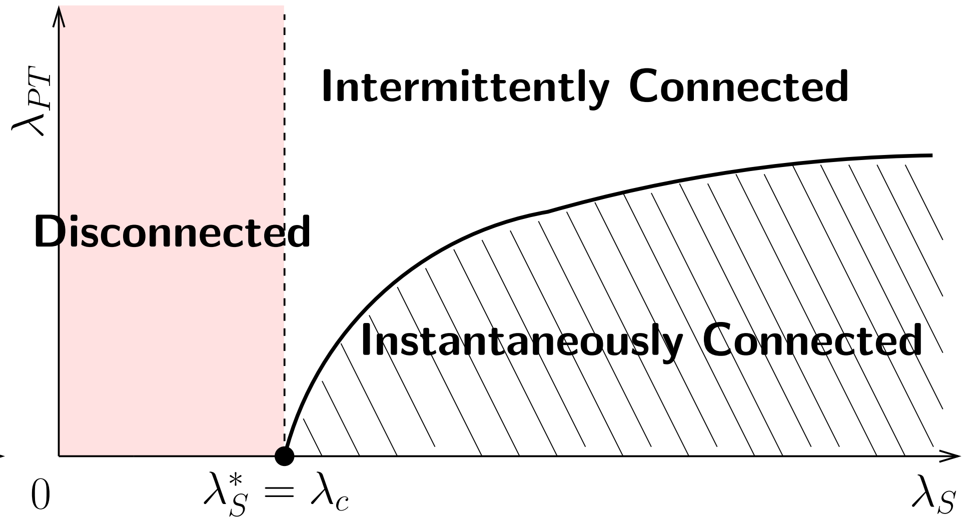
(b) intermittently connected

Conclusion and Future Directions

Static Case:



Dynamic Case:



Future Directions:

- Fading dealt with by a random connection model for continuum percolation.
- Contention among secondary users and interference aggregation.