ION-CYCLOTRON INSTABILITY IN CURRENT-CARRYING LORENTZIAN (KAPPA) AND MAXWELLIAN PLASMAS WITH ANISOTROPIC TEMPERATURES: A COMPARATIVE STUDY

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**Ion-Cyclotron Instability in Current-Carrying Lorentzian (Kappa) and Maxwellian Plasmas with Anisotropic Temperatures: A Comparative Study**

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Current-driven electrostatic ion-cyclotron instability has so far been studied for Maxwellian plasma with isotropic and anisotropic temperatures. Since satellite-measured particle velocity distributions in space are often better modeled by the generalized Lorentzian (kappa) distributions and since temperature anisotropy is quite common in space plasmas, theoretical analysis of the current-driven, electrostatic ion-cyclotron instability is carried out in this paper for electron-proton plasma with anisotropic temperatures, where the particle parallel velocity distributions are modeled by kappa distributions and the perpendicular velocity distributions are modeled by Maxwellian distributions. Stability properties of the excited ion cyclotron modes and, in particular, their dependence on electron to ion temperature ratio and ion temperature anisotropy are presented in more detail. For comparison, the corresponding results for bi-Maxwellian plasma are also presented. Although the stability properties of the ion cyclotron modes in the two types of plasmas are qualitatively similar, significant quantitative differences can arise depending on the values of je and ji. The comparative study is based on the numerical solutions of the respective linear dispersion relations. Quasilinear estimates of the resonant ion heating rates due to ion-cyclotron turbulence in the two types of plasma are also presented for comparison.

Current-driven, electrostatic, ion-cyclotron, instability, Maxwellian, turbulence, anisotropic, Kappa
Ion-cyclotron instability in current-carrying Lorentzian (kappa) and Maxwellian plasmas with anisotropic temperatures: A comparative study

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I. INTRODUCTION

The satellite-measured particle velocity distributions in the solar wind and in many space plasmas often exhibit non-Maxwellian suprathermal tails that decrease as a power-law of the velocity.\(^1\) Such deviations from the Maxwellian distributions are expected in plasmas with sufficiently low degree of collisionality. The distribution function that can better model such particle velocity distributions is the so-called generalized Lorentzian or kappa ($\kappa$) distribution.\(^2\) The kappa distribution with a finite value of the spectral index $\kappa$ has a power-law tail at velocities larger than the thermal velocity and, consequently, has a substantially larger number of suprathermal particles in comparison with the Maxwellian distribution. It approaches the Maxwellian distribution in the limit as $\kappa \to \infty$. Typical values of $\kappa$ for space plasmas are in the range 2–6. In the last several years, many authors studied electrostatic and electromagnetic waves in spatially homogeneous and weakly inhomogeneous magnetoplasmas using different types of kappa distributions for the equilibrium state.\(^1,3–17\) The presence of substantially larger number of suprathermal particles in comparison with the Maxwellian was shown to have important distinguishing effects on the spectral properties of the excited waves and on the wave-particle interactions, in particular. In this paper, we present for the first time the stability properties of the electrostatic ion cyclotron modes in current-carrying Lorentzian (kappa) plasma with anisotropic electron and ion temperatures and compare them with those in Maxwellian plasma. The comparative study is based on the numerical solutions of the linear dispersion relations for the two types of plasma. Numerical analysis allows the investigation of parameter regimes that are not accessible to tractable analytical treatments and thus allows a more comprehensive quantitative comparison. Quasilinear estimates of the resonant ion heating rates due to ion-cyclotron turbulence in the two types of plasma are also presented for comparison.

Electrostatic ion-cyclotron instability, which considers waves propagating at large angles to the ambient magnetic field with frequency near the ion cyclotron frequency, can arise from various free energy sources such as field-aligned currents,\(^18–23\) electron beams,\(^24\) ion beams,\(^25,26\) or combined effects of ion beams, and counterstreaming electrons.\(^27\) We concentrate here on the field-aligned current, which is a common feature in many space and laboratory plasmas, as the free energy source for the ion-cyclotron instability.

Current-driven electrostatic ion-cyclotron instability plays an important role in the generation of ion cyclotron turbulence and in the concomitant anomalous ion heating observed in space and laboratory plasmas. The previous theoretical and numerical studies of the instability assumed that the electron and the ion distributions in velocity space are Maxwellians. Drummond and Rosenbluth\(^18\) first developed the theory of this instability for Maxwellian plasma with isotropic electron and ion temperatures and showed that, for $T_e \approx T_i$, the critical value of the electron drift relative to the stationary ions (i.e., the critical current) for the onset of the instability is much smaller than that for the onset of the electrostatic ion-acoustic instability, which considers waves...
providing parallel to the magnetic field. Kindel and Kenne\textsuperscript{19} extended the theoretical analysis of Drumm\forte and Rosenbluth\textsuperscript{18} and showed that the ion cyclotron waves are unstable to smaller currents for a broad range of $T_e/T_i$. The effects of temperature anisotropies, which are also important features of collisionless plasmas, on the excitation of the current-driven electrostatic ion-cyclotron instability in Maxwellian plasma were first investigated by Lee\textsuperscript{21} who followed the theoretical analysis of Kindel and Kenne\textsuperscript{19}. Later, Okuda and Ashour-Abdalla\textsuperscript{22,23} investigated the effects of temperature anisotropies over a wider range of parameters by numerically solving the dispersion relation. For a review of these previous studies, see Ref. 28.

In Sec. II, we present the mathematical model and the linear dispersion relations for the current-driven ion cyclotron modes in Maxwellian and Lorentzian (kappa) plasmas. In Sec. III, we present the approximate analysis of the dispersion relations in order to gain some preliminary understanding of the differences in the stability properties of the ion cyclotron modes in the two types of plasma. In Sec. IV, we present the numerical solutions of the dispersion relations. In Sec. V, we calculate the resonant ion heating rates due to ion-cyclotron turbulence in the two types of plasma, within the framework of the quasilinear theory, and discuss their differences. The paper is concluded with a summary in Sec. VI.

II. LINEAR DISPERSION RELATION FOR CURRENT-DRIVEN ION CYCLOTRON MODES

The starting point of our study is the linear dispersion relation for obliquely propagating electrostatic ion cyclotron modes in current-carrying plasma with anisotropic electron and ion temperatures. For this, we consider spatially homogeneous, nonrelativistic, collisionless plasma in which electrons are drifting along the uniform ambient magnetic field $B_0$ with velocity $V_0$ relative to the stationary ions. Consider normal modes of the form $exp[i(k \cdot r − \omega t)]$ with frequency $\omega$ and propagation vector $k$, we assume $|k \cdot V_0|/\Omega_e \ll 1$, $k \cdot V_\perp/\Omega_e \ll 1$ and $k \cdot V_\parallel/\Omega_e \ll 1$, where $\Omega_e = eB_0/(m_e c)$ is the electron cyclotron frequency, $V_\parallel$ and $V_\perp$ are the electron thermal speeds defined as $V_\parallel = T_e/(m_e c)$, $k_\parallel = (k \cdot B_0)/B_0$, and $k_\perp$ is the perpendicular (to $B_0$) component of $k$. We further assume that $\omega_{pe}/\Omega_e \ll 1$, where $\omega_{pe}$ is the electron plasma frequency. These assumptions imply that the electrons are strongly magnetized particles so that their dynamic response to the electric field perturbation is practically one-dimensional (parallel to $B_0$). For the ion response, however, we retain both the parallel and the perpendicular dynamics. Then, according to the linearized Vlasov theory, the dispersion relation for obliquely propagating electrostatic waves is given by\textsuperscript{29}

\begin{equation}
1 + \frac{4\pi e^2}{m_e k^2} \int \frac{dv}{\omega - k_\parallel V_\parallel} \left[ k_\parallel \frac{\partial}{\partial V_\parallel} F_i(v^2_{\perp}, v_\parallel) + \frac{4\pi e^2}{m_i k^2} \right] \times \sum_{n = -\infty}^{+\infty} \int dv \frac{J_n^2(\mu)}{\omega - k_\parallel V_\parallel - n\Omega_i} \left[ k_\perp \frac{\partial}{\partial V_\perp} + n\Omega_i \frac{\partial}{\partial V_\perp} \right] F_i(v^2_{\perp}, v_\parallel) = 0,
\end{equation}

for $\Im \omega > 0$. Here $F_z(v^2_{\perp}, v_\parallel)$ is the unperturbed (equilibrium) distribution function for the charged particle species $\alpha = e, i$, $\Omega = eB_0/(m_{\alpha} c)$, $J_n(\mu)$ is the Bessel function of order $n$, $\mu = k_\parallel V_\parallel/\Omega_i$, and $k_\perp^2 = k^2 - k_\parallel^2$. For analytic continuation to $\Im \omega \leq 0$, the Landau contour has to be used for carrying out the $v_\parallel$ integration. The two types of equilibrium distribution function, considered here for the derivation of the linear dispersion relation, are the following.

A. Bi-Maxwellian

\begin{equation}
F_z(v^2_{\perp}, v_\parallel) = \frac{n_0}{\pi^{3/2} \theta_{z0}^2 \theta_{zk}^2} \exp \left( -\frac{v^2_{\perp}}{\theta_{z0}^2} \right) \exp \left( -\frac{u^2_{\parallel}}{\theta_{zk}^2} \right),
\end{equation}

where $u_{\alpha} = v_\parallel - V_{\alpha 0}$ with $V_{\alpha 0} = V_0$ for the electrons and $V_{\alpha 0} = 0$ for the ions, $\theta_{z0}(\xi) = \beta V_{\alpha 0}$ is related to the particle temperature $T_{\alpha}(\xi)$ by $\theta_{z0}(\xi) = 2\pi \theta_{z0}^2(\xi)/m_{\alpha} V_{\alpha 0}$, and $F_z$ is normalized to the particle density $n_0$ (same for both electrons and ions). The definitions of $T_{\parallel0}$ and $T_{\perp0}$ are

\begin{equation}
n_0 T_{\parallel0} = 2\pi m_e \int dv_\perp dv_\parallel n_0^2 F_z^2(v^2_{\perp}, v_\parallel),
\end{equation}

\begin{equation}
n_0 T_{\perp0} = 2\pi m_i \int dv_\perp dv_\parallel n_0^2 F_z^2(v^2_{\perp}, v_\parallel).
\end{equation}

Substituting $F_z(v^2_{\perp}, v_\parallel)$, given by Eq. (2), into Eq. (1) and performing the velocity space integrations, we obtain the well-known dispersion relation

\begin{equation}
D(k, \omega) \equiv 1 - \frac{\omega_{pe}^2}{k^2 \theta_{z0}^2} Z_M(\zeta_e) - \frac{\omega_{pe}^2}{k^2 \theta_{zk}^2} \sum_{n = -\infty}^{+\infty} \Lambda_n(b_i)
\times \left[ Z_M(\zeta_m) - 2\pi \theta_{z0}^2 \frac{k_\parallel \Omega_i}{k_\parallel \theta_{z0}^2} Z_M(\zeta_m) \right] = 0.
\end{equation}

Here, $\omega_{pe}^2 = 4\pi e^2 n_0/m_{\alpha}$, $\Lambda_n(b_i) = I_n(b_i) \exp(-b_i)$, $I_n$ is the modified Bessel function of the first kind, $b_i = k_\parallel^2 \theta_{zk}^2/(2\Omega_i^2)$, $\zeta_e = (\omega - k_\parallel V_0)/(k_\parallel \theta_{z0})$, and $\zeta_m = (\omega - n\Omega_i)/(k_\parallel \theta_{z0})$, where $k_\parallel > 0$ is assumed. $Z_M(\zeta)$ is the well-known plasma dispersion function\textsuperscript{30} associated with the Maxwellian $v_\parallel$-distribution and it is defined as

\begin{equation}
Z_M(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} ds \exp(-s^2) s^{-\zeta},
\end{equation}

for $\Im \zeta > 0$ and as the analytic continuation of this for $\Im \zeta \leq 0$. The prime notation on $Z_M(\zeta)$ denotes its derivative with respect to the argument.

B. Kappa-Maxwellian

\begin{equation}
F_z(v^2_{\perp}, v_\parallel) = \frac{n_0 f(k_\perp)}{\pi^{3/2} \theta_{z0}^2 \theta_{zk}^2} \left( 1 + \frac{u^2_{\parallel}}{k_\perp^2 \theta_{zk}^2} \right)^{-k_\perp (1)} \exp \left( -\frac{v^2_{\perp}}{\theta_{z0}^2} \right),
\end{equation}

where $f(k_\perp) = \Gamma(k_\perp + 1)/[k_\perp^2 \Gamma(k_\perp + 1/2)]$, $\Gamma(x)$ being the gamma function. As before, $u_{\alpha} = v_\parallel - V_{\alpha 0}$ with $V_{\alpha 0} = V_0$.
for the electrons, $V_{\omega 0} = 0$ for the ions, and $F_x$ is normalized to the particle density $n_0$ (same for both electrons and ions), while $\theta_{\perp}^2$ and $\theta_{\parallel}^2$ are related to the particle "temperatures" $T_{\parallel}$ and $T_{\perp}$, as defined by Eqs. (3) and (4), according to $\theta_{\parallel}^2 = [(2k_\parallel - 1)/k_\parallel](T_{\parallel}/m_\parallel) \equiv [(2k_\parallel - 1)/k_\parallel] \theta_{\parallel}^2$ and $\theta_{\perp}^2 = 2T_{\perp}/m_\perp \equiv 2V_{\perp}^2$, provided $k_\parallel > 1/2$. As $k_\parallel \rightarrow \infty$, $F_x$ [given by Eq. (7)] asymptotically approaches the bi-Maxwellian [given by Eq. (2)]. In Eq. (7), the particle distribution in $v_{\perp}$—space is modeled by the kappa distribution function, while the distribution in $v_{\parallel}$—space is modeled by the Maxwellian distribution. The physical arguments for not considering a kappa distribution in $v_{\perp}$—space are the following. First, in the presence of an ambient magnetic field, the mechanism that produces the kappa type distribution in velocity space is most likely to be more effective in $v_{\parallel}$—space. Second, the most important physical aspect of the kappa distribution, which distinguishes it from the Maxwellian distribution, is the resonant wave-particle interaction (Landau and cyclotron resonances) between the excited waves and the enhanced population of suprathermal charged particles that are moving along the ambient magnetic field. It may be mentioned, however, that a kappa distribution in $v_{\perp}$—space does not pose a serious mathematical difficulty. With it, the velocity integral in the ion term involving $\theta_{\parallel}^2$ and $(\partial F_x/\partial v_{\perp})$ cannot be expressed in terms of any known mathematical function; but, it can be numerically evaluated quite easily. The integral approaches its Maxwellian counterpart, $\Lambda_n(b_i)$, in the limit as $k_\parallel \rightarrow \infty$, and its numerical values for finite values of $k_\parallel$ are not too different from those of $\Lambda_n(b_i)$.

Substituting $F_x(v_{\parallel}, v_{\perp})$, given by Eq. (7), into Eq. (1) and performing the velocity space integrations, the dispersion relation is obtained as

$$D(k, \omega) \equiv 1 - \frac{\omega^2}{k^2 \theta_{\parallel}^2} Z_{k_\parallel}(\zeta_\parallel) - \frac{\omega^2}{k^2 \theta_{\perp}^2} \sum_{n=-\infty}^{\infty} \Lambda_n(b_i) \times \left[ Z_{k_\parallel}(\zeta_\parallel) - \frac{2n\Omega_i \theta_{\perp}^2}{k_\parallel \theta_{\parallel}^2} \right] = 0,$$

where $\Lambda_n(b_i) = I_n(b_i) \exp(-b_i)$, $I_n$ is the modified Bessel function of the first kind, $b_i = k_i^2 \theta_{\parallel}^2 / (2\Omega_i^2)$, $\zeta_\parallel = (\omega - k_\parallel V_0)/(k_\parallel \theta_{\parallel}^2)$, and $\zeta_\perp = (\omega - n\Omega_i)/(k_\parallel \theta_{\parallel}^2)$ with $k_i > 0$. The dispersion relation for kappa-Maxwellian plasma is formally equivalent to that for the bi-Maxwellian plasma, except that $\theta_{\parallel}^2 = (T_{\parallel}/m_\parallel) [(2k_\parallel - 1)/k_\parallel]$ and $Z_{k_\parallel}(z)$ is the modified plasma dispersion function associated with the kappa distribution function in $v_{\parallel}$—space. The modified plasma dispersion function is defined by

$$Z_{k_\parallel}(\zeta) = \frac{f(\kappa_\parallel)}{\sqrt{\pi}} \int_{-\infty}^{\zeta} ds (s-\zeta)(1+s^2/\kappa_\parallel)^{\kappa_\parallel+1},$$

for $\text{Im} \, \zeta > 0$ and by the analytic continuation of this for $\text{Im} \, \zeta \leq 0$. The prime notation on $Z_{k_\parallel}(\zeta)$ denotes its derivative with respect to the argument. Aside from the multiplication factor, this function is analogous to the plasma dispersion function that was first introduced and discussed by Summers and Thorn7 and was later further analyzed by Mace and Hellberg31 and Summers et al.32

III. APPROXIMATE STABILITY ANALYSIS

Here, we present the approximate analytical solutions of the dispersion relations in order to gain some preliminary understanding of the ion-cyclotron instability in the two types of plasmas. Considering $\omega = Re \omega + i \text{Im} \, \omega$ and making a Taylor series expansion of $D(k, \omega)$ around $\omega = Re \, \omega$ while assuming $\text{Im} \, \omega \ll Re \, \omega$ and $Im \, D \ll Re \, D$, we find to lowest order,

$$Re \, D(k, \omega = Re \, \omega) = 0,$$

$$Im \, \omega = \frac{\text{Im} \, D(k, \omega = Re \, \omega)}{\partial Re \, D(k, \omega = Re \, \omega)/\partial \omega}.$$}

These two equations determine $Re \, \omega$ and $\text{Im} \, \omega$. Generally speaking, unstable solutions are found when $Re \, \omega/k_\parallel < V_0$ (i.e., $Re \, \zeta < 0$) so that $(\partial F_x/\partial v_{\perp})|_{\omega=Re \, \omega/k_\parallel > 0}$ (Landau growth) and, simultaneously, when (a) $Re \, \zeta_{\parallel i} > 1$ for cyclotron modes with $Re \, \omega = m\Omega_i$; and (b) $Re \, \zeta_{\perp i} \gg 1$ for all $n \neq m$ such that the Landau growth rate due to the drifting (current-carrying) electrons exceeds the total ion damping (both Landau and cyclotron damping) rate. For the approximate stability analysis, we follow the commonly used analytical approach of Drummond and Rosenbluth18 i.e., we consider $|\zeta_{\parallel} | < 1$ and $|\zeta_{\perp} | \gg 1$ for all $n$ and retain the leading terms in the power series expansion of the electron plasma dispersion function and the asymptotic expansion of the ion plasma dispersion function. The power series and the asymptotic expansions of $Z_{\kappa}(\zeta)$ are30

$$Z_{\kappa}(\zeta) = i \sqrt{\pi} \exp(-\zeta^2) - 2\zeta + 4\zeta^3/3 - \cdots, \quad |\zeta| \ll 1,$$

$$Z_{\kappa}(\zeta) \approx i \sqrt{\pi} \exp(-\zeta^2) - \frac{1}{\zeta} - \frac{1}{2\zeta^3} - \cdots, \quad |\zeta| \gg 1,$$

while the power series and the asymptotic expansions of $Z_{\kappa}(\zeta)$ for integer values of $\kappa_\parallel$ are3,17,18

$$Z_{\kappa}(\zeta) = i \sqrt{\pi} \left( \frac{1 + \zeta^2/\kappa_\parallel}{1 + \zeta^2/\kappa_\parallel} \right)^{\kappa_\parallel+1} \left( 1 - \frac{2\kappa_\parallel + 3}{3\kappa_\parallel} \zeta^2 + \cdots \right), \quad |\zeta| \ll 1,$$

$$Z_{\kappa}(\zeta) = i \sqrt{\pi} \left( \frac{1 + \zeta^2/\kappa_\parallel}{1 + \zeta^2/\kappa_\parallel} \right)^{\kappa_\parallel+1} \left( 1 - \frac{2\kappa_\parallel}{3\kappa_\parallel} \zeta^2 + \cdots \right), \quad |\zeta| \ll 1.$$}

For noninteger (including half-integer) values of $\kappa_\parallel$, the asymptotic expansion has small correction terms,31,32 which may be neglected.

A. Bi-Maxwellian plasma

Restricting ourselves to the fundamental ion cyclotron mode, i.e., $Re \, \omega \approx \Omega_i$, we retain only the $n = 1$ term in the ion sum. Using the leading terms of Eqs. (12) and (13) in Eq. (5), we find

$$Re \, D(k, \omega = Re \, \omega) = 0,$$

$$Im \, \omega = - \frac{\text{Im} \, D(k, \omega = Re \, \omega)}{\partial Re \, D(k, \omega = Re \, \omega)/\partial \omega}.$$
\[ \text{Re } D(k, \omega = Re \omega) \simeq 1 + \frac{2\omega_{pe}^2}{k^2 \theta_{||}^2} - \frac{2\omega_{pe}^2 \theta_{||}^2}{k^2 \theta_{\perp}^2} \frac{\Omega}{Re \omega - \Omega_i} \Lambda_1(b_i), \]  
(16)

\[ \text{Im } D(k, \omega = Re \omega) \simeq 2i\sqrt{\pi} \frac{\omega_{pe}^2}{k^2 \theta_{||}^2} \frac{Re \omega - k_i V_0}{k_i \theta_{||}} \times \Lambda_1(b_i) \exp(-\eta_i^2). \]  
(17)

Then, referring to Eqs. (10) and (11), we obtain
\[ \frac{Re \omega}{\Omega_i} = 1 + \frac{T_{||} \Lambda_1(b_i)}{T_{||} \frac{Re \omega - (1 - T_{||}/T_{||})\Omega}{k_i V_{||}}} , \]  
(18)

\[ \frac{Im \omega}{\Omega_i} = \sqrt{\frac{\pi}{2} \frac{T_{||} \Lambda_1(b_i)}{T_{||} \frac{Re \omega - (1 - T_{||}/T_{||})\Omega}{k_i V_{||}}} - \frac{T_{||} \Lambda_1(b_i)}{k_i V_{||}} \times \left[ Re \omega - (1 - T_{||}/T_{||})\Omega \right] \Lambda_1(b_i) \exp(-\eta_i^2) + \frac{T_{||} \Lambda_1(b_i)}{k_i V_{||}} \times \frac{Re \omega - (1 - T_{||}/T_{||})\Omega}{k_i V_{||}} \right] , \]  
(19)

where \( \eta_i = (Re \omega - \Omega_i)/(\sqrt{2}k_i V_{||}) \). We have related \( \theta_{||}^2 \) to \( T_{||} \) and introduced the Debye length, \( \lambda_{De} \), defined by \( \lambda_{De}^2 = T_{||} \). The onset condition for instability (\( Im \omega > 0 \)) is
\[ \frac{V_0}{V_{||}} > \frac{Re \omega}{k_i V_{||}} + \frac{T_{||} Re \omega - (1 - T_{||}/T_{||})\Omega}{k_i V_{||}} \Lambda_1(b_i) \exp(-\eta_i^2) \right] . \]  
(20)

The results presented in Eqs. (16)–(25) are new and they reduce to the corresponding results for bi-Maxwellian plasma [Eqs. (16)–(20)] when \( \kappa_e, \kappa_i \rightarrow \infty \).

The approximate analytical solutions indicate that the stability properties of the current-driven ion cyclotron modes in both types of plasma (bi-Maxwellian and kappa-Maxwellian) depend on \( m_i/m_e, \omega_{pe}/\Omega_e, V_0, k_i, k_{\perp}, \) electron to ion temperature ratio \( (T_{ei}/T_{ei}) \), and ion temperature anisotropy \( T_{ei}/T_{ei} \). Dependence on \( \omega_{pe}/\Omega_e \) arises through the term \( \omega_{pe}^2/(k^2 \theta_{||}^2) \) in the dispersion relations as \( \omega_{pe}^2/(k^2 \theta_{||}^2) \propto (k^2 \lambda_{De}^2)^{1} = (m_i/m_e)(\omega_{pe}/\Omega_e)^2/(k^2 \theta_{||}^2) \), where \( \rho_i \) is the ion gyroradius. The analytical solutions also indicate that Re \( \omega \), Im \( \omega \), and the onset condition for instability in kappa-Maxwellian plasma can be quite
different from those in bi-Maxwellian plasma depending on the choice of the values of $\kappa_e$ and $\kappa_i$. Comparison shows that $\text{Re} \, \omega$ is reduced in kappa-Maxwellian plasma, while both the electron term that drives the instability when $V_0 > \text{Re} \, \omega / k_{||}$ and the ion cyclotron damping term in $\text{Im} \, \omega$ are enhanced in kappa-Maxwellian plasma. The enhancement of the electron-drive term in kappa-Maxwellian plasma is due to the reduction of $\text{Re} \, \omega$ and the multiplying factors involving $\kappa_e$. The enhancement of the ion cyclotron damping term in kappa-Maxwellian plasma is predominantly due to its power-law dependence on $\eta_i > 1$ in contrast with its exponential dependence on $\eta_i > 1$ in bi-Maxwellian plasma. For the same reason, the threshold value of $V_0$ for the onset of the instability in kappa-Maxwellian plasma is enhanced over the corresponding value in bi-Maxwellian plasma.

We shall not pursue the analytical solutions any further as they are valid only for the restricted values of the aforementioned parameters for which the assumed conditions ($|z_x| < 1$ and $|z_m| \gg 1$ for all $n$) can be satisfied. Outside the range of validity, the analytical solutions are not only quantitatively inaccurate; they even give erroneous dependence on $k_{||}$, for example. For an accurate and comprehensive study of the entire unstable spectrum of the current-driven ion-cyclotron modes, it is, therefore, necessary to solve the dispersion relations numerically. Indeed, numerical analysis shows that the assumed conditions ($|z_x| < 1$ and $|z_m| \gg 1$ for all $n$) are rather difficult to satisfy, particularly when the maximum values of the growth rates occur.

IV. NUMERICAL SOLUTIONS OF THE DISPERSION RELATIONS

We have developed a numerical code that evaluates the plasma dispersion functions $Z_\text{M}(z)$, $Z_\kappa(z)$ and solves the dispersion relations to find the complex values of $\omega(= \omega_r + i \omega_i)$ as a function of the aforementioned variable parameters. Here, we consider the electron-proton plasma $(m_e / m_i = 1/1836)$ and concentrate on the solutions with $\omega_r \approx \Omega$. For kappa-Maxwellian plasma, $\kappa_e = \kappa_i = 3$ is assumed. The numerical results, graphically presented in Figs. 1–9, 13, and 14, are obtained by assuming $\omega_{pe} / \Omega_e = 1/15$, which is consistent with the assumption $\omega_{pe} / \Omega_e \ll 1$ used in the derivation of the dispersion relations. The important effects of $\omega_{pe} / \Omega_e$ on the stability properties are discussed later in this section by means of Figs. 10–12.

Figure 1 shows the threshold values (critical values) of $V_0$ (normalized to $V_{c||}$ and denoted by $V_C$, as a function of $T_{e||} / T_{e\perp}$ and $T_{i||} / T_{i\perp}$, for $\omega_{pe} / \Omega_e = 1/15$. The solid curves represent results for bi-Maxwellian plasma and the dashed curves represent results for kappa-Maxwellian ($\kappa_e = \kappa_i = 3$) plasma. The curves are labeled by the selected values of $T_{ei} / T_{ei}$ and $T_{ei} / T_{ei}$. The curves are labeled by the selected values of $T_{ei} / T_{ei}$ and $T_{ei} / T_{ei}$. The threshold values are obtained by minimizing $V_0 / V_{c||}$ for $\omega_{pe} / \Omega_e = 1/15$ and selected values of $T_{ei} / T_{ei}$ and $T_{ei} / T_{ei}$. The solid curves represent results for bi-Maxwellian plasma and the dashed curves represent results for kappa-Maxwellian ($\kappa_e = \kappa_i = 3$) plasma. The curves are labeled by the selected values of $T_{ei} / T_{ei}$ and $T_{ei} / T_{ei}$.
corresponding values of $Re \ W^{*}$ also indicates that, for a fixed value of $\kappa$, $Im \ W$ decreases sharply as $V_{0}$ decreases down to a value which is nearly independent of $\kappa$. The curves are labeled by the selected values of $T_{\parallel}/T_{\perp}$ and $T_{\parallel}/T_{\perp}$.

Maxwellian plasma. The top panel of Fig. 1 shows that, for a fixed value of $T_{\parallel}/T_{\perp}$, the value of $V_{C}$ in both types of plasmas decreases sharply as $T_{\parallel}/T_{\perp}$ increases and then settles down to a value which is nearly independent of $T_{\parallel}/T_{\perp}$. It also indicates that, for a fixed value of $T_{\parallel}/T_{\perp}$, the value of $V_{C}$ decreases as $T_{\parallel}/T_{\perp}$ increases. This is shown more explicitly in the bottom panel of Fig. 1. The results for bi-Maxwellian plasma are in agreement with those obtained earlier. The new results in Fig. 1 are that a larger $V_{C}$ (i.e., larger current) is needed to excite the instability in kappa-Maxwellian plasma for all $T_{\parallel}/T_{\perp}$ and for all $T_{\parallel}/T_{\perp}$ above a certain value, which increases with decreasing $T_{\parallel}/T_{\perp}$.

Figure 2 shows the maximum values of $Im \ W(=Im \omega/\Omega_{i})$ maximized with respect to $k_{\parallel}$, $k_{\perp}$ and denoted by $(Im \ W)_{max}$ as a function of $V_{0}(=V_{0}/V_{e})$ for selected values of $T_{\parallel}/T_{\perp}$ and $T_{\parallel}/T_{\perp}$. Figure 3 shows the corresponding values of $Re \ W(=Re \omega/\Omega_{i})$, denoted by $(Re \ W)_{max}$, as a function of $V_{0}(=V_{0}/V_{e})$ for the same values of $T_{\parallel}/T_{\perp}$ and $T_{\parallel}/T_{\perp}$. As expected, for all values of $T_{\parallel}/T_{\perp}$ and $T_{\parallel}/T_{\perp}$, and for both types of plasma, $(Im \ W)_{max}$ decreases as $V_{0}$ decreases toward the threshold values. However, the new and interesting result is that, for all values of $T_{\parallel}/T_{\perp}$ and $T_{\parallel}/T_{\perp}$, $(Im \ W)_{max}$ in kappa-Maxwellian plasma is larger than that in bi-Maxwellian plasma when $V_{0}$ is much larger than the threshold value; decreases more rapidly as $V_{0}$ decreases; and then becomes smaller as $V_{0}$ approaches the threshold value $V_{C}$. This is consistent with the results presented in Fig. 1, namely, $V_{C}$ is larger for kappa-Maxwellian plasma. Figure 3 shows that $(Re \ W)_{max}$ is smaller in kappa-Maxwellian plasma than in bi-Maxwellian plasma for all values of $V_{0} > V_{C}$, $T_{\parallel}/T_{\perp}$, and $T_{\parallel}/T_{\perp}$. It should be pointed out that the approximate analytical solutions in Sec. III do not show the dependence, albeit weak, of the real frequency on $V_{0}$. As Fig. 2 indicates the maximum linear growth rate of the instability can be somewhat large when $T_{\parallel}/T_{\perp}$, $T_{\parallel}/T_{\perp}$ are large (> 1) and simultaneously when $V_{0}$ is also large (e.g., $V_{0} = 1$). This is particularly true for the kappa-Maxwellian plasma. The linear analysis still remains valid as $(Im \ W)_{max}/(Re \ W)_{max} \leq 0.2$ under those conditions. All it suggests is a faster onset of the nonlinear processes. However, according to Fig. 1, the instability can be excited with a much smaller $V_{0}(=1)$ when $T_{\parallel}/T_{\perp} > 1$ and $T_{\parallel}/T_{\perp} > 1$. For example, (1) when $T_{\parallel}/T_{\perp} = 1$ and $T_{\parallel}/T_{\perp} = 5$, $V_{C,\kappa} \approx 0.28$ and $V_{C,\kappa} \approx 0.32$; (2) when $T_{\parallel}/T_{\perp} = 5$ and $T_{\parallel}/T_{\perp} = 1$, $V_{C,\kappa} \approx 0.14$ and $V_{C,\kappa} \approx 0.17$; and (3) when $T_{\parallel}/T_{\perp} = 5$ and $T_{\parallel}/T_{\perp} = 5$, $V_{C,\kappa} \approx 0.12$ and $V_{C,\kappa} \approx 0.14$. Here, $V_{C,\kappa}$ refers to the threshold value $V_{C}$ for bi-Maxwellian (kappa-Maxwellian) plasma. The maximum linear growth rates of the instability are quite small when $V_{0}$ is above and near these small threshold values (see Fig. 2) and,
for such values of $V_0$, the linear stability analysis is certainly appropriate. In the following presentation of the various characteristics of the unstable ion-cyclotron modes in the two types of plasma, we have used $V_0 = 1$ in order to cover a large range of values of $T_{\|}/T_{\perp}$ and $T_{\perp}/T_{\|}$ for which the instability occurs (see Fig. 1).

The top panel of Fig. 4 shows $(\text{Im} W)_{\text{max}}$, defined as above, as a function of $T_{\|}/T_{\perp}$ for selected values of $T_{\perp}/T_{\|}$. It is evident that, for a fixed $T_{\|}/T_{\perp}$, $(\text{Im} W)_{\text{max}}$ increases with $T_{\|}/T_{\perp}$ (rate of increase being larger for a larger $T_{\|}/T_{\perp}$) and that it increases more rapidly in kappa-Maxwellian plasma. The bottom panel shows the values of $k_{\parallel}/k_{\perp}$ (denoted by $(\text{Kratio})_{\text{max}}$), for which $(\text{Im} W)_{\text{max}}$ are obtained, as a function of $T_{\|}/T_{\perp}$ for the same selected values of $T_{\perp}/T_{\|}$. It shows that the angle (between $k$ and $B_0$) of propagation of the maximum unstable modes becomes smaller with increasing values of $T_{\|}/T_{\perp}$ (i.e., the modes propagate more parallel to $B_0$) and that this change in the angle of propagation occurs more rapidly in kappa-Maxwellian plasma. Figure 4 also indicates that, for a fixed $T_{\|}/T_{\perp}$, both $(\text{Im} W)_{\text{max}}$ and $(\text{Kratio})_{\text{max}}$ increase substantially as $T_{\|}/T_{\perp}$ increases. These are shown more explicitly in the top and bottom panels of Fig. 5. Numerical analysis further shows that the magnitudes of $(\text{Im} W)_{\text{max}}$ and $(\text{Kratio})_{\text{max}}$ as a function of $T_{\|}/T_{\perp}$ and $T_{\perp}/T_{\|}$ in both types of plasma depend on the choice of the value of $V_0 > V_C$.

Figures 6–9 show the spectral ($\omega$ vs $k$) behavior of the unstable ion-cyclotron modes for the selected values of $T_{\|}/T_{\|}$ and $T_{\perp}/T_{\perp}$. In these figures, $K_{\text{para}}(\equiv K_{\|}) = k_{\|}/\rho_i$ and $K_{\text{perp}}(\equiv K_{\perp}) = k_{\perp}/\rho_i$, where $\rho_i$ is the ion gyroradius. Figures 6 and 7 show $\text{Re} \ W$ and $\text{Im} \ W$ versus $K_{\|}$ for $K_{\perp} = 0.08$ and various values of $T_{\|}/T_{\|}$ and $T_{\perp}/T_{\perp}$. They show that, for a given $T_{\|}/T_{\perp}$ (or $T_{\|}/T_{\|}$), $\text{Re} \ \omega$ moves closer to $\Omega_i$ and the growth rate $(\text{Im} \ \omega)$ decreases (suggesting larger critical drift) as $T_{\|}/T_{\|}$ (or $T_{\|}/T_{\|}$) decreases. Conversely, $\text{Re} \ \omega$ moves away from $\Omega_i$ and the growth rate increases (suggesting smaller critical drift) as $T_{\|}/T_{\|}$ (or $T_{\|}/T_{\|}$) increases. These conclusions hold for both bi-Maxwellian and kappa-Maxwellian plasmas. However, as the figures show, there are significant quantitative differences between the unstable spectra in the two types of plasma. Magnitudes of these differences depend on the values of $T_{\|}/T_{\|}$ and $T_{\perp}/T_{\perp}$ for the assumed values of $K_{\|}$, $K_{\perp}$, and $V_0$. The real frequencies are smaller in kappa-Maxwellian plasma than in bi-Maxwellian plasma for all values of $K_{\perp}$. On the other hand, the growth rates are larger in kappa-Maxwellian plasma than in bi-Maxwellian plasma beyond some values of $K_{\perp}$. Additionally, the unstable spectra in kappa-Maxwellian plasma extend to comparatively larger values of $K_{\perp}$. Figures 8 and 9 show $\text{Re} \ W$ and $\text{Im} \ W$ versus $K_{\|}$ for $K_{\perp} = 0.8$ and values of
$T_{e\|}/T_{i\parallel}$ and $T_{e\perp}/T_{i\perp}$ same as those in Figs. 6 and 7. They show similar (to Figs. 6 and 7) functional dependence of the unstable spectra on $T_{e\parallel}/T_{i\parallel}$, $T_{e\perp}/T_{i\perp}$, and similar differences between the unstable spectra in the two types of plasma as a function of $K_{\parallel}$. Both the $K_{\perp}$- and the $K_{\parallel}$-dependence of $\Im W$ and their differences in the two types of plasmas may be understood in terms of the combined effects of the electron drive (Landau growth) and the ion cyclotron damping.

For heavier-ion plasma, qualitatively similar, but quantitatively different due to increased ion mass, results are obtained. We do not present those results here. Instead, more interesting effects of the parameter $\omega_{pe}/\Omega_e$ on the stability properties of the ion-cyclotron modes are presented next. Figure 10 shows the variation of $V_C$ (threshold value of $V_0$ normalized to $V_{e\parallel}$) with respect to $\omega_{pe}/\Omega_e = \sigma$ (Sigma) for $T_{e\parallel}/T_{i\parallel} = 1$ and different values of $T_{e\perp}/T_{i\perp}$. When $\sigma$ is less than 0.02, $V_C$ is large exceeding unity. As $\sigma$ increases, $V_C$ decreases sharply and then settles down to a value, which is nearly independent of $\sigma$. Furthermore, the values of $V_C$ are larger for smaller values of $T_{e\perp}/T_{i\perp}$ and for all values of $\sigma$. The same functional behavior is found for both types of plasma. However, as evident from the figure, there are appreciable quantitative differences between the two types of plasma. The top and bottom panels of Fig. 11, which are obtained for $V_0 = 1$, $T_{e\parallel}/T_{i\parallel} = 1$, and selected values of $T_{e\perp}/T_{i\perp}$, show that $(\Im W)_{\max}$ and $(\Re W)_{\max}$, defined as above, decrease as $\sigma$ decreases. The rate of decrement increases as $T_{e\perp}/T_{i\perp}$ increases, and the instability disappears when $\sigma$ is smaller than a certain value that depends on $T_{e\perp}/T_{i\perp}$. The disappearance of the instability has to do with the fact that the electron drive decreases while the ion damping increases as $\sigma$ decreases. Finally, Fig. 12 shows the dependence of $(\text{Kratio})_{\text{max}}$, defined as above, on $\sigma$ for $V_0 = 1$, $T_{e\parallel}/T_{i\parallel} = 1$, and selected values of $T_{e\perp}/T_{i\perp}$. For both types of plasma, the angle (between $k$ and $B_0$) of propagation of the maximum unstable mode decreases, i.e., the mode propagates more parallel to $B_0$, with increasing $\sigma$. Quantitative differences between the two types of plasma are evident from the figures. Similar dependences on $\sigma$ are found for other values of $T_{e\parallel}/T_{i\parallel}$, but they are not shown here. We have chosen $\sigma = 1/15$ in the presentation for illustration purposes only as $V_C$ tends to be insensitive to $\sigma$ around this value.

V. QUASILINEAR ION HEATING RATES

It has been argued that quasilinear plateau formation cannot stabilize the current-driven ion-cyclotron instability because of the slowly convecting nature of the modes and growth continues until other nonlinear mechanisms provide...
Quasilinear analysis, therefore, does not yield correct estimates of the ion heating rates. Nevertheless, for the purposes of comparison, we present here the quasilinear estimates of the ion heating rates due to ion-cyclotron turbulence in bi-Maxwellian and kappa-Maxwellian plasmas.

The quasilinear response of the magnetized ions to unstable electrostatic field fluctuations is described by

$$
\frac{\partial}{\partial t} F_i(v_{\perp}, v_{\parallel}, t) = -\frac{1}{m_i} \sum_{n=-\infty}^{\infty} \int dk \left| \delta F_i(k, t) \right|^2 
\times \left( k_\parallel \frac{\partial}{\partial v_\parallel} + n \Omega_L \frac{\partial}{\partial v_\perp} \right) \omega_k - v_\parallel - n \Omega_L 
\times \left( k_\parallel \frac{\partial}{\partial v_\parallel} + n \Omega_L \frac{\partial}{\partial v_\perp} \right) F_i(v_{\perp}, v_{\parallel}, t),
$$

(26)

for small $\text{Im} \omega_k \geq 0$. Here $\mu = k_\perp v_{\perp}/\Omega_L$, $\omega_k = \omega(k_\perp, k_\parallel)$ is the complex solution of the linear dispersion relation, and $\delta F_i(k, t)$ is the potential fluctuation, which, within the framework of the quasilinear theory, evolves adiabatically in time according to

$$
\frac{\partial}{\partial t} \delta F_i(k, t) = 2 \text{Im} \omega_k(t) \delta F_i(k, t).
$$

(27)

We assume that the fundamental harmonic is the dominant mode and so keep $n = 1$ term only. Then, in the resonant region of velocity space ($\text{Re} \omega_k - k_\parallel v_\parallel - \Omega_L = 0$), Eq. (26) can be approximated by

$$
\text{Im} W_{\max} = V_0 = 1,
$$

(ImW)$_{\max}$

maximized with respect to $k_\parallel$ and $k_\perp$ and denoted by $V_c$, as a function of $\alpha_{\perp}/\Omega_L = \sigma$ (Sigma) for $T_\parallel/T_\perp = 1$ and selected values of $T_\perp/T_\parallel$. The solid curves represent results for bi-Maxwellian plasma and the dashed curves represent results for kappa-Maxwellian ($\kappa_\perp = \kappa_\parallel = 3$) plasma. The curves are labeled by the selected values of $T_\perp/T_\parallel$. The solid curves represent results for bi-Maxwellian plasma and the dashed curves represent results for kappa-Maxwellian ($\kappa_\perp = \kappa_\parallel = 3$) plasma. The curves are labeled by the selected values of $T_\perp/T_\parallel$. The solid curves represent results for bi-Maxwellian plasma and the dashed curves represent results for kappa-Maxwellian ($\kappa_\perp = \kappa_\parallel = 3$) plasma. The curves are labeled by the selected values of $T_\perp/T_\parallel$. The solid curves represent results for bi-Maxwellian plasma and the dashed curves represent results for kappa-Maxwellian ($\kappa_\perp = \kappa_\parallel = 3$) plasma. The curves are labeled by the selected values of $T_\perp/T_\parallel$.
under the small Im $\omega_k$ assumption. Here, $\delta(x)$ is the Dirac $\delta$ function. Multiplying Eq. (28) by $m v^2 / 2$, using Eqs. (2) and (7) for $F_i$, on the right-hand side (for estimation purposes), and integrating over velocities, we find

$$\frac{d}{dt} I_{k||} = \sqrt{\frac{2 \pi m_i^{\frac{3}{2}}}{T_{i||}}} \int \frac{dk}{k_{||}} \left| \frac{\phi(k, t)}{k_{||}^2} \right|^2 \Lambda_i(b_i) I(k_{||}, k_i),$$

(29)

where

$$I(k_{||}, k_i) = \left( \frac{\text{Re} \omega_k - \Omega_i}{\Omega_i} + \frac{T_{i||}}{T_{i||}} \right) \exp(-\eta^2_{k||}) \equiv I_{BM},$$

(30)

for bi-Maxwellian plasma, and

$$I(k_{||}, k_i) = f(k_i) \left( \frac{2k_i}{2k_i - 1} \right)^{3/2} \left( 1 + \frac{\eta_{k||}^2}{k_i - 1/2} \right)^{-(k_i + 1)} \times \left( 1 + \frac{\eta_{k||}^2}{k_i - 1/2} \right)^{-1} \frac{\text{Re} \omega_k - \Omega_i}{\Omega_i}$$

$$+ \frac{2k_i - 1}{2k_i} \frac{T_{i||}}{T_{i||}} \equiv I_{KM},$$

(31)

for kappa-Maxwellian plasma, provided $k_i > 1/2$. Here $\eta_{k||} \equiv (\text{Re} \omega_k - \Omega_i) / \sqrt{2k_i|V_{i||}|}$. Figure 13 shows $I(k_{||}, k_i)$ as a function of the unstable values of $K_i(= k_i \rho_i)$, for $K_i(= k_i \rho_i) = 0.8$, $\Omega_i / \sigma = 1/15$, $V_0(= V_0 / V_{i||}) = 1$, $T_{i||} / T_{i||} = 1$, and selected values of $T_{i||} / T_{i||}$. The solid curves represent $I_{BM}$ given by Eq. (30) for bi-Maxwellian plasma and the dashed curves represent $I_{KM}$ given by Eq. (31) for kappa-Maxwellian plasma. The figure shows that $I_{KM} > I_{BM}$ over almost the entire unstable $k_i$ spectrum. This is basically due to the power-law dependence of $I_{KM}$ on $\eta_{k||}$, in contrast with the exponential dependence of $I_{BM}$ on $\eta_{k||}$, when $|\eta_{k||}| > 1$. Similar results are found for other values of $T_{i||} / T_{i||}$.

Next, multiplying Eq. (28) by $m_i v_{i||}^2 / 2$, using Eqs. (2) and (7) for $F_i$, on the right-hand side (for estimation purpose), and integrating over velocities, we find

$$\frac{d}{dt} T_{i||} = \sqrt{\frac{2 \pi m_i v_{i||}^2}{T_{i||}}} \int \frac{dk}{k_{||}} \left| \frac{\phi(k, t)}{k_{||}^2} \right|^2 \Lambda_i(k_i) J(k_{||}, k_i),$$

(32)

where

$$J(k_{||}, k_i) = (\text{Re} \omega_k / \Omega_i - 1) I_{BM} \equiv J_{BM},$$

(33)

for bi-Maxwellian plasma, and

$$J(k_{||}, k_i) = (\text{Re} \omega_k / \Omega_i - 1) I_{KM} \equiv J_{KM},$$

(34)

for kappa-Maxwellian plasma. Figure 14 shows $J(k_{||}, k_i)$ as a function of the unstable values of $K_i(= k_i \rho_i)$, for $K_i(= k_i \rho_i) = 0.8$, $\Omega_i / \sigma = 1/15$, $V_0(= V_0 / V_{i||}) = 1$, $T_{i||} / T_{i||} = 1$, and selected values of $T_{i||} / T_{i||}$. The solid curves represent $J_{BM}$ given by Eq. (33) for bi-Maxwellian plasma and the dashed curves represent $J_{KM}$ given by Eq. (34) for kappa-Maxwellian plasma. The figure shows that $J_{KM} > J_{BM}$ over almost the entire unstable $k_i$ spectrum. Once again, this is basically due to the power-law dependence of $J_{KM}$ on $\eta_{k||}$ in contrast with the exponential dependence of $J_{BM}$ on $\eta_{k||}$, when $|\eta_{k||}| > 1$. Similar results are found for other values of $T_{i||} / T_{i||}$.

Furthermore, we may estimate that

$$\frac{dT_{i||}}{dt} \approx \frac{I_{BM}}{2J_{BM}} \equiv \frac{1}{2} \text{Re} \omega_k / \Omega_i,$$

(35)

for bi-Maxwellian plasma and

$$\frac{dT_{i||}}{dt} \approx \frac{I_{KM}}{2J_{KM}} \equiv \frac{1}{2} \text{Re} \omega_k / \Omega_i,$$

(36)

for kappa-Maxwellian plasma.
for kappa-Maxwellian plasma. Although the right-hand side of Eqs. (35) and (36) is formally the same, they are quite different. In particular, numerical solutions of the linear dispersion relations show that \((\text{Re } \omega_k - \Omega_i)/\Omega_i \ll 1\) for both types of plasma, but \((\text{Re } \omega_k - \Omega_i)/\Omega_i\) has smaller values for kappa-Maxwellian plasma, when \(V_0, T_{e0}/T_{i0}\), and \(T_{e||}/T_{i||}\) are not too large [see Figs. 6–9]. Equations (35) and (36) then suggest that, under these conditions, ions are heated preferentially in the perpendicular direction and this effect is enhanced in kappa-Maxwellian plasma. These conclusions can also be derived from Figs. 13 and 14 where \(J_{BM}, J_{KM}, J_{BBM}, \) and \(J_{KBM}\) are shown.

VI. SUMMARY

We have presented for the first time the linear stability properties of the current-driven electrostatic ion-cyclotron modes in kappa-Maxwellian plasma where the parallel velocity distributions of electrons and protons are modeled by kappa distributions and the perpendicular velocity distributions are modeled by Maxwellian distributions with parallel temperatures different from perpendicular temperatures. In particular, dependence of the stability properties on various parameters, such as \(V_0, k, T_{e0}/T_{i0}, T_{e||}/T_{i||}, \) and \(\omega_{pe}/\Omega_i\), has been discussed in some details. For comparison, we have also presented the corresponding results for bi-Maxwellian plasma where both the parallel and the perpendicular velocity distributions are modeled by Maxwellian distributions with parallel temperatures different from perpendicular temperatures. Quasilinear estimates of resonant ion heating rates due to ion-cyclotron turbulence in the two types of plasma have also been presented for comparison. In order to gain some preliminary understanding of the linear instability in kappa-Maxwellian plasma and its comparison with that in bi-Maxwellian plasma, we have included approximate analytical solutions of the dispersion relations. But, more emphasis has been given on the numerical solutions of the dispersion relations in order to provide accurate and comprehensive comparison over a wide range of values of the various plasma parameters. Numerical analysis has been carried out for electron-proton plasma. Qualitatively similar, but quantitatively different due to increased ion mass, results are expected for heavier-ion plasma.

Linear stability properties of the current-driven electrostatic ion cyclotron modes, as well as the quasilinear ion heating rates due to ion-cyclotron turbulence, in bi-Maxwellian and kappa-Maxwellian plasmas are qualitatively similar. However, there are quantitative differences, which can be significant, depending on the values of \(\kappa_i\) and \(\kappa_e\). The important conclusions of the comparative study are summarized as follows.

(1) The threshold (critical) value of the electron drift speed (i.e., current), above which the current-driven ion-cyclotron instability is excited, is larger in kappa-Maxwellian plasma for all values of \(T_{e||}/T_{i||}\) and for all \(T_{e0}/T_{i0}\) above certain value, which increases with decreasing \(T_{e||}/T_{i||}\) (see Fig. 1).

(2) Above and near the threshold (critical) value of the electron drift speed, the maximum growth rates are smaller in kappa-Maxwellian plasma. But, as \(V_0\) is increased, the maximum growth rates become larger in kappa-Maxwellian plasma. Additionally, the unstable spectra in kappa-Maxwellian plasma extend to comparatively larger values of \(K_{e0} = k_{e0} \rho_i\) and \(K_{i0} = k_{i0} \rho_i\) (see Figs. 2, 6–9). The frequencies of the unstable modes are smaller in kappa-Maxwellian plasma (see Figs. 3, 6–9). These conclusions are valid for all values of \(T_{e||}/T_{i||}\) and \(T_{e0}/T_{i0}\) for which the instability can be excited with a given \(V_0\).

(3) The angle (between \(k\) and \(B_0\)) of propagation of the maximum unstable mode decreases with increasing \(T_{e||}/T_{i||}\) and \(T_{e0}/T_{i0}\) in both bi-Maxwellian and kappa-Maxwellian plasmas. In other words, as \(T_{e||}/T_{i||}\) and \(T_{e0}/T_{i0}\) increase, the maximum unstable mode propagates more parallel to \(B_0\). However, the change in the angle of propagation occurs more rapidly in kappa-Maxwellian plasma (see bottom panels of Figs. 4 and 5). The magnitude of the change in the angle of propagation as well as the magnitude of the differences between the bi-Maxwellian and kappa-Maxwellian plasmas depends on the value of \(V_0 > V_c\).

(4) The threshold (critical) value of the electron drift speed, above which the current-driven ion-cyclotron instability is excited, decreases sharply as \(\omega_{pe}/\Omega_i\) increases and then settles down to a value, which is nearly independent of \(\omega_{pe}/\Omega_i\), for both bi-Maxwellian and kappa-Maxwellian plasmas (see Fig. 10). For both types of plasma, the maximum value of \(\text{Im } W(\equiv \text{Im } \omega/\Omega_i)\) and the corresponding value of \(\text{Re } W(\equiv \text{Re } \omega/\Omega_i)\) decrease as \(\omega_{pe}/\Omega_i\) decreases. The rate of decrement increases as \(T_{e||}/T_{i||}\) increases, and the instability disappears when \(\omega_{pe}/\Omega_i\) is smaller than a certain value, which depends on \(T_{e||}/T_{i||}\) (see Fig. 11). For both types of plasma, the angle (between \(k\) and \(B_0\)) of propagation of the maximum unstable mode decreases, i.e., the mode propagates more parallel to \(B_0\), with increasing \(\omega_{pe}/\Omega_i\) (see Fig. 12). Quantitative differences between the two types of plasma are evident from the figures.
Quasilinear estimates of the resonant ion heating rates suggest that both the perpendicular and the parallel heating rates are larger in kappa-Maxwellian plasma. This is basically due to the power-law dependence (as opposed to the exponential dependence) on $v^2$ of the distribution of resonant ions in kappa-Maxwellian plasma [see Eqs. (29)–(34)]. They also suggest that, under certain conditions, ions are heated preferentially in the perpendicular direction in both bi-Maxwellian and kappa-Maxwellian plasmas, but this effect is enhanced in kappa-Maxwellian plasma.

The present study is based on nonrelativistic kappa distributions. For some space plasma, highly energetic particles are better modeled by relativistic kappa distributions. The reader is referred to the works by Xiao\textsuperscript{35} and Xiao \textit{et al.}\textsuperscript{36–38} for relativistic kappa distributions.

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12\textsuperscript{R. L. Mace, Phys. Plasmas 10, 2181 (2003).
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