HOW THE DEGREE OF ACCURACY OF AN INERTIAL MEASUREMENT UNIT (IMU) INFLUENCES THE MISS DISTANCE OF A GUN- LAUNCHED PRECISION MUNITION

by

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September 2011

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ABSTRACT

Precision Munition projectiles guide to an area to hit their target. The projectile must read position in-flight and measure deviations from the intended flight path. This allows the projectile to correct and maintain the intended trajectory. An Inertial Measurement Unit (IMU) device measures the relative movement of a projectile throughout flight and measures the deviation from the intended path, enabling the projectile to course correct. The purpose of this thesis is to understand the degree to which the precision of the IMU influences the delivery accuracy of a gun-launched munition. This research will model the influences of gyro bias stability and acceleration bias stability and quantify their effects.
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LIST OF ACRONYMS AND ABBREVIATIONS

AMC – Access Processor Module
ARDEC – Armament Research, Development and Engineering Center
ARES – Aeroballistic Rapidly Evolving Simulation
CEP – Circular Error Probability
CG – Center of Gravity
CGCS – Common Guidance Common Sense
CP – Center or Pressure
DoD – Department of Defense
g’s – Gravitational Forces
GPS – Global Position System
IMU – Inertial Measurement Unit
\( I_{xx} \) = Mass Moment of Inertia
\( I_{yy} \) = Rotational Moment of Inertia
\( I_{zz} \) = Polar Moment of Inertia
\( m \) – Meter
\( mm \) – Millimeter
\( mrad \) - Milliradian
MET – Meteorology
\( mg \) – Milli-gravity
MMS – Meteorological Measuring Set
MMS-P – Meteorological Measuring Set – Profiler (MMS-P)
MMW – Millimeter Wave
Std Dev – Standard Deviation
TI – Texas Instruments
QE – Quadrant Elevation
UAV – Unmanned Ariel Vehicle
EXECUTIVE SUMMARY

The purpose of this thesis is to understand the degree to which the precision of the IMU influences the delivery accuracy of a gun-launched munition. The research models the influences of gyro bias stability and acceleration bias stability on IMU accuracy and quantifies their effects to answer the research question, “How does the accuracy of the Inertial Measurement Unit affect miss distance?” Using simulation, the perfect-modeled trajectory of a fin-stabilized 155mm artillery projectile allowed comparison of accuracy with and without IMU bias stability error. The simulations revealed that a 1 degree/hour, 1 milli-gravity (mg) IMU resulted in a 95.18% improvement in accuracy vs. a “standard” 75 degree/hour, 9 mg IMU. While tightening the specification to deliver a 1 degree/hour creates challenges in design and development, it significantly increases the accuracy of the projectile and delivers economies of scale that make it less costly.
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I. INTRODUCTION

Gun-launched precision munitions have been in development since mid-1970. Precision munition projectiles guide to an area to hit their target. The projectile must know its position in-flight, and measure deviations from its intended flight path. This allows corrective action to happen that returns the projectile to the intended trajectory. An Inertial Measurement Unit (IMU) device measures the relative movement of a projectile throughout flight and measures the deviation from the intended path, enabling the projectile to course correct. The ability of an IMU to measure deviation from the path is critical to the successful delivery of a precision guided munition. The precision, or accuracy, of an IMU has both financial and social implications for the delivery of precision guided munitions.

In 2001, the U.S. Army started a Common Guidance program to lower the production cost of IMU’s (Panhorst, LeFevre, & Rider, 2005) by developing a Common Guidance performance specification for both gun-launched projectiles and missiles. By leveraging economies of scale via larger production runs, the goal was to cut the cost of a typical IMU by one-third. Because of the large discrepancy in gyro and accelerometer bias stability requirements between gun and missile systems, leaders of gun-launched programs feared that satisfying the missile requirements “over engineered” the IMU, leading to overpaying for a device that provided more accuracy than needed. One Army leader wanted a quantified answer to the question what does 1 degree/hour buy me?

The purpose of this thesis is to understand the degree to which the precision of the IMU influences the delivery accuracy of a gun-launched munition. This research will model the influences of gyro bias stability and acceleration bias stability and quantify their effects. Chapter I provide the background and an overall roadmap of this thesis. Chapter II contains the literature review, which includes exploring the need for precision; the architectural makeup of a precision munition to include functional and form decomposition; the external influences
that have an impact on miss distance; the need of an IMU; and finally, the accuracy requirement for the purposes of this thesis. Chapter III outlines the research methodology and develops a model for answering the primary research question. Chapter IV provides the data analysis and results by showing how to interpret the outcome of the analytical output and discussing the significance of the results. Chapter V provides a summary of key points as well as areas for further research.
II. LITERATURE REVIEW

A. NEED FOR PRECISION

The *New Webster’s Dictionary* of the English Language defines precision as “the state of being precise as to meaning; exactness; accuracy” (1988, p. 402).

Bailey says precision is “often used when describing the capabilities of a system or the effects created by that system. Some define precision predominately in terms of accuracy at a point target or in terms of the consistent close grouping of shots” (Bailey, 2004, p. 11).

Bailey (2004) goes on to point out:

A new lexicon has emerged to describe munitions, with *precision* meaning that a munition is self-locating and maneuvers to attack its target with sufficient accuracy. A *smart* munition is one that can search, detect, acquire, and provide its own terminal guidance to the target. A *discriminating* munition is one that can do all of the above but also select a certain type of target and attack it successfully. (p. 11)

Precision is critical for avoiding collateral damage. Lucas defines collateral damage as “the killing or injuring of non-combatant civilians and the destruction or damage to civilian property that is not being used for a military purpose” (Lucas, 2003, p. 1).

A common misperception is that the use of precision munitions prevents collateral damage because precision munitions always hit their intended target. However, “even “precise” weapons can land at precisely wrong locations and cause incidents of unintended suffering” (Roblyer, 2003, p. 5). Even when programmed to follow a ballistic path in case of a computer malfunction, a precision munition can veer radically off course due to a folding canard that can stick in a half-open position. There have been instances where projectiles have actually circled back and landed behind the launch position. Understanding
factors that affect the precision of guided projectiles is critical to minimizing collateral damage and the negative social and political aspects of operations. Bailey suggests that (Bailey, 2004):

In conventional war, physical targets such as headquarters, guns, and missiles are likely to be the immediate priority. In peace operations, the most valuable targets are the minds of leaders and the local population and international opinion. The highest-payoff targets are therefore likely to be those that affect perceptions and “play well” in the media. The intent of fires is less likely to be to destroy or neutralize per se, although these may be the necessary physical effects selected, than to produce a moral effect upon the will of the various actors and influence their subsequent behavior. Weapons effects are therefore measured not so much in terms of fragmentation efficiency, lethal distance, or depth of penetration as by the emotional impact of the graphic image created and its global distribution through the media to electorates and decision makers. (p.432)

Defining the precision needed by a guided munition has implications across its architecture framework. It determines how accurately it will hit its target, to what extent it will avoid unintended targets, and largely dictates cost, time and complexity of development.

B. ARCHITECTURE FRAMEWORK OF A GUN-LAUNCHED PRECISION MUNITION

1. Functional Decomposition

Figures 1 and 2 provide the functional decomposition of a generic gun-launched Precision Munition. These figures serve as a template for structuring the design and development to deliver these functions. Modifications depend on the specific requirements that drive the design.
Figure 1. Gun-Launched Precision Munition Functional Decomposition

Figure 2. Gun-Launched Precision Munition Functional Decomposition (continued)
a. The functions of a Gun-Launch Munition Provide:

Protection – munitions are designed to survive 10-year storage. They must be fully functional for ten years.

Fuzing – fuzing of precision munitions has two functions: 1) to keep the munition safe prior to launch and 2) to arm the munition to detonate, as desired, at the target.

Target Detection – a precision munition must detect the target or target coordinates in order to defeat it. It must be able to distinguish a legitimate target from background clutter in the presence of countermeasures.

Structure – a munition must have structural integrity to survive gun-launch. The structure also allows the munition to fly to range and perform the maneuver functions required to engage the target.

Gun-Launch – this function propels the munition down range and provides enough impulse to cycle the gun properly so it is available for subsequent firings.

Flight Control – the flight control function provides the maneuverability needed for the munition to course correct. These corrections help the munition to properly adjust to varying weather and launch conditions and to compensate for the mass properties and aeroballistics characteristics of the projectile itself.

End-Effects – to defeat a target, the munition must function to deliver the desired end-effects. The operational concept will dictate the end-effect requirement.

In-flight Diagnostics – the complexity of precision munitions warrant the inclusion of a function to record activities of the “as is” condition. This function can provide the user with real-time feedback to understand current mission performance, and archive data to help improve the design of future munitions.
Data Interpretation – a precision munition must have a “brain” to function properly. This “brain” must take real-time data, analyze it, and “course correct” in real time.

Data Insertion – the more information the munition has regarding the launch conditions, the better the performance. Data insertion prior to launch provides the munition with the current firing conditions.

2. Form Decomposition

Figures 3 and 4 illustrate the form decomposition of a generic precision munition. The form decomposition maps directly from the functional decomposition. These figures serve as a template for potential solution space of how the functions of a precision munitions perform. Modifications depend on the approach taken to meet constraints and stated needs.

Figure 3. Gun-Launched Precision Munition Form Decomposition
a. **Considerations of Form Include:**

**Development Timeline** – one special consideration in developing the form of a precision munition is the timeline for developing the munition. The development cycle of defense programs often suffer frequent interruptions that lengthen development time. This can complicate the form selection, because as the development cycle lengthens, technology continues to evolve, leading to potential obsolescence of the technology designed to make the munition work. A good form must provide the modularity and flexibility to allow for upgrades as technology changes.

**Limitation** – the selection of form has limiting factors that often result in modeling and simulation trade studies. One is the selection to “Sense Targets.” The form of the sensor will significantly influence the ability of the munition to discriminate between a target, the background, and countermeasure.
Interface Issues – Munitions are typically launched from multiple platforms, and the interface issues include compatibility with new cannon or gun systems as well as “legacy” systems. A typical interface issue is seen when a new munition will be fired from an existing fielded weapon, and the weapon cannot be modified.

Design and Approval Process – new systems must meet requirements of several safety review boards (e.g., Insensitive Munitions Board, Army Fuze Safety Review Board, etc.). Therefore, the form of certain components and subsystems need to meet the requirements for certification by these boards. This leads to a common practice of incorporating proven subsystems such as safe and arm (S&A) mechanisms to avoid the extensive testing needed to validate a new design.

C. EXTERNAL FACTORS INFLUENCING PRECISION OF GUN-LAUNCHED MUNITIONS

A host of external factors influences the precision of a gun-launched munition, including weather conditions at the gun position and target location, mass properties of the projectile, and aeroballistics characteristics of the projectile.

1. Weather Conditions

Army Field Manual FM3–09.15 (2007) outlines the tactics, techniques and procedures followed by artillery units when launching conventional artillery ammunition or a precision munition under different meteorological conditions. The key is to understand the meteorological (MET) conditions, not only at the gun position but also at the target impact point if possible. The MET conditions vary over time, and the ability to collect data on these changing conditions significantly influences the delivery accuracy of gun-fired ammunition. As stated in the FM:
Since MET is one of the five requirements for accurate and predicted fires it is considered part of the precision fires system of systems. MET sections provide data to enhance first round accuracy, effective downwind predictions, intelligence preparation of the battlefield, and forecast capabilities of the staff weather officer. The commander and staff who include MET in the planning process should always use the most accurate MET data available, as it will benefit the most. The planning process focuses on what data is needed, who needs it, and how will they acquire it. (Headquarters, Department of the Army, 2007, pp. 1-1)

There are two-type MET collectors preferred by field artillery: the Metrological Measuring Set – Profiler (MMS-P) and the Metrological Measuring Set (MMS). The MMS-P provides local forecast information, while the MMS gathers vertical data by launching an instrumented balloon that records conditions (wind speed and direction, barometric pressure, air temperature, etc.) as it ascends. MMS can gather data up to 30000 feet; however, typical artillery engagements only require data up to about 10000 feet (Headquarters, Department of the Army, 2007).

2. Launch Conditions

Ammunition and propellant temperature play a role in determining the muzzle velocity. Typical temperature extremes for precision munitions can vary from +85°C to –51.1°C. (US Army ARDEC, AMRDEC, 2006) Failure to compensate for muzzle velocity variation due to temperature can result in larger miss distances. Accuracy is also influenced by projectile jump. A projectile rarely leaves the muzzle aligned perfectly with the bore of the gun. (Carlucci & Jacobson, 2008) This misalignment will cause the external forces to act asymmetrically on the projectile and erode its accuracy.

To eliminate the effects of gravity on the simulated trajectories all simulations were fired due North from a fixed launch location in Yuma, Arizona. This kept all gravitational effects constant and eliminated them as a variable.
3. Aeroballistics Characteristics

The external shape of the projectile plays a role in the range and accuracy of the projectile. Fins that protrude into the airflow will increase drag. Asymmetries of the projectile surface can impart unequal forces on either side of the projectile and increase drag.

“Shorter, blunt-nosed projectiles are higher drag shapes than longer, more streamline shapes” (McCoy, 1999, p. 70).

Static wind tunnel tests and dynamic ballistic range tests help design engineer’s choose the appropriate shape that will fulfill the projectile accuracy requirement. The forces that act on a projectile and contribute to the accuracy include (McCoy, 1999, pp. 33–36):

- Drag – opposes the forward velocity of the projectile
- Lift – tends to pull the projectile in the direction the nose is pointed, causing it to climb if pointed up, or dive if pointed down
- Magnus – produced by unequal pressures on opposite sides of a spinning projectile

Drag, Lift, and Magnus are a function of Mach number at which the projectile is traveling and varies during the flight in a nonlinear manner. They combine to determine the maximum range the projectile can travel.

4. Mass Properties

The mass properties of the projectile affect its stability and accuracy. The center of gravity (CG) is the point in which all mass can be concentrated for analysis; the result is a location, the CG, where an equivalent force and moment pair can be located to represent the distributed forces acting on the projectile.

The center of pressure (CP) is a point on the projectile that all of the pressure forces can be equivalently concentrated to represent the pressure distribution on the projectile. For fin-stabilized projectiles, the CP is usually behind the CG, making fin-stabilized projectiles unconditionally stable (Carlucci & Jacobson,
Forces and mass distribution about the projectile result in moments that contribute to projectile accuracy. Moments that act upon the projectile include (McCoy, 1999):

- **Rolling Moment** – canted fins tend to impart a roll rate to the projectile and help smooth forces that act on the projectile due to physical mechanical asymmetry or miss-alignment.

- **Spin Damping Moment** – opposes spin of the projectile, causing axial spin to decay. The interaction of spin damping with roll moment determines the quasi steady-state spin rate for a given mach number. The spin rate varies with projectile velocity.

- **Overturning Moment** (also known as the Pitch Moment) – the moment that tends to bring the nose of the projectile back to the flight path should it deviate due to an external influence. This is true for fin-stabilized projectiles only, because on projectiles without fins the moment would cause the nose to turn away from the flight path.

- **Magnus Moment** – results from the moment arm of the Magnus force about the CP.

The moments acting on the projectile determine the overall stability during flight. If the projectile is not statically stable, these moments will cause flight motion to grow uncontrolled and ultimately cause the projectile to tumble to the ground.

**D. NEED FOR AN INERTIAL MEASUREMENT DEVICE (IMU)**

An IMU is a device that measures the acceleration and rotational changes of a projectile that is in flight along a trajectory. By measuring deviation in angle rate and acceleration along the trajectory, the IMU provides data so the projectile can “course correct” and maneuver itself back to its intended flight path. Output signals from the IMU are mathematically integrated and corrective instructions are relayed to the flight control system to make trajectory corrections. Measuring and correcting projectile flight are done with sensors and controls (Chen &

Accelerometers are used to sense the magnitude of acceleration, but acceleration is a vector having direction as well as magnitude. For this reason a set of gyroscopes, or simply gyros, are used to maintain the accelerometers in a known orientation with respect to a fixed, non-rotating coordinate system, commonly referred to as inertial space. (pp. 176–177)

“The accelerometers and gyros in the IMU perform sensing functions and provide acceleration and angle rate data to the guidance computer on-board the projectile.” (Lin, 1991, p. 179) Accelerometers and gyros are mounted in 3-axis configuration; they measure changes in movement in the forward (down range), right (cross range) and down (towards the earth) directions as well as in the pitch, yaw and roll planes. Figure 5 illustrates the six degrees-of-freedom.

![Figure 5. The six degrees of freedom forward/back, up/down, left/right, pitch, yaw, roll (From Six Degrees of Freedom, 2011)](image)

Like any electro-mechanical device, an IMU is built within certain tolerances. Imperfections introduce error, and error contributes to the IMU accuracy and the projectile miss distance. Many factors determine the accuracy of an IMU, and this research focused on gyro bias stability and accelerometer bias stability. Gyro fixed bias stability is measured in rotational degrees per hour
($^\circ$/hr). Accelerometer fixed bias stability is measured in milli-g’s (mg), where g’s are the gravitational pull experience during acceleration.

E. ESTABLISHING THE ACCURACY REQUIREMENT

Various munition delivery options have various accuracy requirements, as illustrated in Figure 6.

![Figure 6. IMU Performance Demands (After Panhorst et al., 2004)](image)

**Meets Accuracy & Size Needs for >90% DoD Precision Weapons**

Figure 6. IMU Performance Demands (After Panhorst et al., 2004)

Gun-launched precision munitions require less accuracy from an IMU than tactical missiles require because of their shorter flight time (200 seconds for gun-launched vs. 1000 seconds for missiles) (Barbour, Hopkins, & Kourepenis, 2011). Gun-launched precision munitions generally require approximately 75–100 degree/hour accuracy, whereas tactical missiles generally require 1 degree/hour. The intent of the Common Guidance Common Sense (CGCS) program was to reduce the cost of the IMU through economies of scale by building a new IMU device for use by both tactical missiles and gun-launched
precision munitions. This would increase the size of IMU production runs, thereby lowering the overall production cost. The CGCS IMU performance specification was a compilation of accuracy requirements expressed by all the major manufacturers of missiles and precision munitions. This resulted in a specification for 1 degree/hour accuracy for both missiles and precision munitions, which was tighter than the 75 degree/hour requirement for just precision munitions. This tighter accuracy requirement led to a perception that gun-launched munitions were overpaying for accuracy they did not need, leading a senior Army official to ask, “What does 1 degree/hour do for me?” (Machak, 2006) This original research will quantify an answer to this question. By fixing the external contributors on miss distance of a precision munition, IMU performance will be isolated to quantify IMU bias stability errors as it influences a precision munitions miss distance.

F. SUMMARY

The literature review reveals the importance of the proper precision/accuracy requirement in the development of gun-launched munitions. The precision of the munition is largely a function of the accuracy of the IMU, making the IMU a critical component of the overall architecture framework. The research that follows builds a model that isolates and quantifies the contribution of IMU accuracy to the overall precision of the munition, and provides a model for future study.
III. RESEARCH METHODOLOGY

A. HOW DOES THE ACCURACY OF THE IMU AFFECT MISS DISTANCE?

The purpose of this research is to quantify the effect of IMU error on projectile miss distance. A model was developed to isolate the effects of bias stability errors of the IMU to determine the errors effect on miss distance.

An Inertial Measurement Unit (IMU) uses six primary sensors to determine changes in the relative motion of a projectile in flight: three gyros and three accelerometers. Gyros measure changes in the relative angular rotation while accelerometers measure changes in the relative linear acceleration. Gyro bias is typically measured in angular degrees/hour, and accelerometers bias is measured in milli-g’s. To illustrate gyro bias, consider the following: An operating IMU is placed on a table and left to run. It is placed such that its x-axis is facing exactly due north and it is located on the equator, thereby eliminating the contribution of the earth’s rotation on the vertical gyro. After one hour, an IMU with a 1 degree/hour gyro bias will still be pointed north, even though its coordinate system has drifted from due north 1 degree in either direction. In the case of gun-launched precision munitions, studies show that the required accuracy is much looser than that required by missile applications: – 75 degrees/hour vs. 1 degree/hour, respectfully, as shown in Figure 6. The biggest factor driving the bias stability requirement difference is time of flight, with gun-launched munition having a shorter time of flight than missile systems. The simulation will model miss distance (from the perfect trajectory) due to IMU bias stability error, which will enable quantification to answer the research question.

B. RESEARCH APPROACH

For this research, a generic 155mm gun-launched munition was simulated with no external errors or mass property effects to create a “perfect trajectory” from which the effects of IMU accuracy on the projectile miss distance were analyzed. This perfect trajectory case was simulated using the ARDEC
Aeroballistics Rapidly Evolving Simulation (ARES). The perfect trajectory was then duplicated using MATLAB®, and a subroutine was written to emulate IMU accuracy errors. Once properly modeled, a Monte Carlo simulation consisting of 5000 trajectories provided the median miss distance for each IMU case, and the mean and standard deviation of the miss distance in both the North (down range) and East (cross range) direction. The Circular Error Probability (CEP) resulted from mapping the impact distances from the true trajectory. According to Encyclopedia Britannica:

CEP uses the mean point of impact of projectile test firings, usually taken at maximum range, to calculate the radius of a circle that would take in 50 percent of the impact points. Bias measures the deviation of the mean impact point from the actual aim point. (http://www.britannica.com/EBchecked/topic/118330/circular-error-of-probability)

The results will provide a miss distance at maximum range that compares performance variation due only to IMU bias stability error, discounting external factors and mass properties. External influences, such as wind, temperature, atmospheric pressure, and gravity that normally act on a projectile in-flight were held constant, since the intent was to isolate the IMU bias stability contribution on miss distance. For the same reason, mass properties of the projectile were excluded.

The Basic Finner is a generic airframe that is scalable to any caliber, and the aeroballistic characteristics are well-studied, making it easy to model. The source of the Basic Finner characteristics used for this research results from research sponsored by Air Force Armament Laboratory and conducted at the University of Notre Dame. (Nicolaides, Eikenberry, Ingram, & Clare, 1968) The Notre Dame research provided a new non-linear analysis method and provided an accurate representation of various motions of projectile in-flight and correct values for the various static and dynamic stability coefficients to represent a typical 155mm, non-spinning artillery projectile. The characteristics used for the simulation emulate that of a standard M795 projectile. The muzzle velocity was
800 m/s, and the trajectory varied from 600 to 1000 Army mils weapon elevation (33.75 to 56.25 degrees from horizontal) to provide dispersion as a function of range.

The projectile configuration chosen was 155mm, slow rolling artillery projectile. The muzzle velocity is that of a standard M795 projectile, 800m/s. The trajectory angle of fire was varied from 600 to 1000mils to study miss distance effects at various ranges. A 2-degree fin-cant was assumed to impart a slow roll in the airframe. Because a projectile never leaves the gun perfectly, a two radian/second tipoff was introduced occurring at 45 degrees, up and to the right. Table 1 summarizes the initial conditions inputs for the perfect trajectory analysis.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Azimuth</td>
<td>0 degrees (North)</td>
<td>Determine Gravitation effects used for the perfect trajectory</td>
</tr>
<tr>
<td>Muzzle Velocity</td>
<td>800 m/s</td>
<td>Comparable to a standard M795 projectile</td>
</tr>
<tr>
<td>Initial spin rate</td>
<td>0 radians/sec</td>
<td>Smooth bore cannon</td>
</tr>
<tr>
<td>Tipoff</td>
<td>2 radians/sec</td>
<td>Based on observed gun-launch data</td>
</tr>
<tr>
<td>Tipoff Direction</td>
<td>45 degrees, up and to the right</td>
<td></td>
</tr>
<tr>
<td>Atmosphere</td>
<td>Standard</td>
<td>Aeroballistic coefficients effects</td>
</tr>
<tr>
<td>Meteorological</td>
<td>None</td>
<td>Perfect Trajectory</td>
</tr>
</tbody>
</table>

Table 1 Perfect Trajectory Initial Conditions

The list in Table 2 show the characteristics used for the 155mm projectile. The values are typical of a standard M795 projectile which allows for performing the analysis on an airframe will known flight characteristics.
<table>
<thead>
<tr>
<th>Reference Diameter</th>
<th>155mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>219.21 kg</td>
</tr>
<tr>
<td>CG Location</td>
<td>5.5 caliber from nose</td>
</tr>
<tr>
<td>Mass Moment (Ixx)</td>
<td>0.708391082 kg-m/m</td>
</tr>
<tr>
<td>Rotational Moment (Iyy) = Polar Moment (Izz)</td>
<td>36.35466996</td>
</tr>
</tbody>
</table>

Table 2 Perfect Trajectory Initial Conditions

Finally, the angle of gun-launch from the horizontal is the Quadrant Elevation (QE) of launch. The chosen QEs ensured that, for these airframe characteristics, the projectile would achieve its maximum range. QE is defined in mils. There is $2000\,\pi$ milliradian (mrad) in a circle, meaning there are 6283.185 mrad in a circle. [http://en.wikipedia.org/wiki/Angular_mil#Definitions_of_the-angular_mil](http://en.wikipedia.org/wiki/Angular_mil#Definitions_of_the-angular_mil) Army mils are calculated rounding $\pi$ to a value of 3.2. Therefore, there are 6400 mils in an Army mils circle. Table 3 summarizes the QEs used for this analysis.

<table>
<thead>
<tr>
<th>QE</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mils</td>
<td>Degrees</td>
</tr>
<tr>
<td>600</td>
<td>33.75</td>
</tr>
<tr>
<td>700</td>
<td>39.375</td>
</tr>
<tr>
<td>800</td>
<td>45.00</td>
</tr>
<tr>
<td>900</td>
<td>50.625</td>
</tr>
<tr>
<td>1000</td>
<td>56.25</td>
</tr>
</tbody>
</table>

Table 3 Quadrant Elevations
After running the perfect trajectory in the ARES code, the resulting trajectory was replicated in MATLAB®. A subroutine was developed to run IMU bias stability error within MATLAB®. The MATLAB® code was run with IMU errors set at zero to ensure that the algorithm was properly replicating the ARES perfect trajectory. Once confirmed, MATLAB® ran the various error conditions as specified by table 4. The error values were derived from the Common Guidance program. (US Army ARDEC, AMRDEC, 2006) The IMU gyro and accelerometer stability bias errors simulation represent the goals of the three phases of the Common Guidance program. Specifically, the values modified included:

- Gyro Rate Bias Stability - deg/hr
- Gyro Angular Random Walk – deg/hr
- Accelerometer Bias Stability – mg

A Monte Carlo simulation of 5000 runs produced results with each axis of the IMU gyro and accelerometers assigned errors randomly based on a normal distribution specified by the Common Guidance specification. The standard deviation on the errors was also investigated with separate cases being run with errors run a 1-sigma and 3-sigma values. Table 4 shows the simulation run matrix.

<table>
<thead>
<tr>
<th>QE (mils)</th>
<th>1 deg/hr &amp; 1 mg</th>
<th>20 deg/hr &amp; 4 mg</th>
<th>75 deg/hr &amp; 9 mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
</tr>
<tr>
<td>700</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
</tr>
<tr>
<td>800</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
</tr>
<tr>
<td>900</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
</tr>
<tr>
<td>1000</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
<td>1σ/3σ</td>
</tr>
</tbody>
</table>

Table 4 Perfect Trajectory Initial Conditions
Finally, MATLAB® generated the Circular Error Probability (CEP) circle by calculating where 50% of the impact points compare in relation to the perfect trajectory impact.

C. IMU DISCUSSION

Accelerometer and gyros sensors are mounted in 3-axis configuration. The sensors measure changes in movement in the forward, right, and down direction. For the simulation the x direction is north, the y direction is east, and the z direction is down toward the center of the earth. Since an independent sensor measures each axis of relative motion, the error of each axis varied for every simulation run. By randomly choosing the IMU sensor errors based on product manufacturer specifications, each independent axis replicates a real world scenario since there is no way to know what sensor will wind up on which axis of measurement.

D. BASIC FINNER DISCUSSION

ARDEC conducted wind tunnel tests of a scaled version of the Basic Finner design with 2-degree cant on the tail fins resulted in a record of the static aerodynamics of the design. These results appropriately scaled to simulate the 155mm projectile for the perfect trajectory simulation. The dynamic damping moments resulted from testing done on the Basic Finner design. (Dunn, 1989), (Regan, 1964), (Jenke, 1976) The ARDEC ARES code simulated the scaled projectile utilizing a flat-earth 6-degree of freedom model. Upon completion of the simulation the projectile position, velocity, acceleration, Euler angles, and body rates as a function of time were stored for use in MATLAB®/Simulink. Figure 7 shows a representation of the Basic Finner and in order to model IMU error effects the simulation output included Center of Gravity accelerations and body rates for subsequent IMU trade studies, as well as a trajectory that included realistic dynamic effects and aerodynamics.
Figure 7. Basic Finner External Configuration (From Nicolaides et al., 1968)

E. PERFECT TRAJECTORY

The projectile simulated here was the Basic Finner projectile as described previously. It was scaled to 155 mm diameter. The static aerodynamics were measured in the U.S. Army Armament Research, Development and Engineering Center (ARDEC) Wind Tunnel Facility (Appendix A), and the dynamic aerodynamics were estimated from test data generated using the Basic Finner (Dunn, 1989; Regan, 1964; Jenke, 1976). (Note: Since this research was conducted on a canted fin-stabilized, slow rolling projectile Magnus effect is minimal and was not simulated.) The initial conditions for the trajectories were as follows:

QE = variable between 600 mils and 1000 mils
AZ = 0 deg
Muzzle velocity = 800 m/s
Tipoff magnitude = 2 rad/s
Tipoff direction = 45 degrees, up and to the right
Standard Atmosphere as built into MATLAB®/Simulink

The projectile characteristics included:

Reference diameter = 0.155 m

Projectile mass = 219.2121083 kg

Center of gravity = 5.5 calibers

Axial moment of inertia = 0.708391082 kg m^2

Transverse moment of inertia = 36.35466996 kg m^2

F. MATLAB® SUBROUTINE

ARDEC engineers in the Aeroballistics Division at the ARDEC, Picatinny Arsenal, NJ created the Aeroballistic Rapidly Evolving Simulation (ARES). It leverages the flexibility and ease of use of MATLAB®/Simulink and the Aerospace Blockset for:

- high fidelity 6DOF flight simulation of experimental projectiles
- trajectory prediction
- trajectory matching and data analysis

The benefits of the MATLAB®/Simulink combination include:

- ease of customization to a specific projectile
- ease of visualization of predicted trajectory data
- ease of input and visualization of test data
- greater portability than lower level languages

The workspace allows MATLAB® and Simulink to work together

- MATLAB® script sets initial conditions and parameters in the workspace
- Simulink has access to those variables during execution
ARDEC engineers modified a MATLAB® subroutine that would use the original data tables from ARES and allow the introduction of the Gyro and Accelerometer bias stability errors. The subroutine converted IMU biases and errors from degrees/hour to radians/second and created a normal distribution for each of the biases. The code generated a random bias stability input based on the Common Guidance specification for the gyros and accelerometers using a normal Gaussian distribution. This resulted in the calculation of random forward, right, and down deviations of the projectile center of gravity in space. Plotting the compilation of these positions throughout the flight gave the ultimate impact location. Once simulated, the bias stability error was held constant for the duration of the simulation run. Upon initiation of a new run, a new bias stability error was selected at random.

G. SUMMARY

By using a well-understood, generic airframe, and eliminating the effect of other external factors, a MATLAB® simulation provided the means to model a perfect trajectory output and isolate IMU bias stability error to quantify its effect on gun-launched precision. The interpretation of the simulation results provides the answer to the research question. Chapter IV will provide the output of the simulation runs and an explanation and interpretation of the results.
IV. DATA ANALYSIS AND RESULTS

A. MATLAB® RESULTS

The research yielded plots that showed impact points about the perfect trajectory output resulting from IMU bias stability error. Figure 8 represents the data presented as impact points about the true trajectory point and represent miss distance in the North and East direction. The normal distribution curves show the mean miss distance in each direction. The CEP is computed by finding the radius of a circle that encompasses 50% of the impact points for a given weapon elevation. Table 5 summarizes the findings of the simulation runs when IMU error is introduced. The data output of this research was reviewed by ARDEC Senior Scientist, Dr. Carlucci, who reviewed the method of achieving results and concurred with the consistency of the output.
Figure 8. An Example Simulation Output
### Table 5  Perfect Trajectory Initial Conditions

<table>
<thead>
<tr>
<th>QE</th>
<th>1deg/hr and 1mg</th>
<th>20deg/hr and 4mg</th>
<th>75deg/hr and 9mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>600mils</td>
<td>1.7543m</td>
<td>11.8470m</td>
<td>35.4615m</td>
</tr>
<tr>
<td>700mils</td>
<td>2.1543m</td>
<td>15.1116m</td>
<td>45.4455m</td>
</tr>
<tr>
<td>800mils</td>
<td>2.6601m</td>
<td>18.5371m</td>
<td>55.8079m</td>
</tr>
<tr>
<td>900mils</td>
<td>3.0603m</td>
<td>21.2143m</td>
<td>63.5117m</td>
</tr>
<tr>
<td>1000mils</td>
<td>3.3217m</td>
<td>22.6314m</td>
<td>67.3433m</td>
</tr>
</tbody>
</table>

B. **MATLAB® DATA RUNS**

Figures 9, 10, and 11 show the results of IMU accuracy at maximum range. The plots show the data for IMU error as a 1-sigma error, meaning that 67% of the sensors chosen to make this IMU are within the specified error value. The miss distance for the 1 deg/hr, 1 mg IMU is better than either the 20 deg/hr, 4 mg and the 75 deg/hr, 9 mg IMU. The 1 deg/hr, 1 mg IMU shows a 95.18% improvement over the 75 deg/hr, 9 mg IMU.
Figure 9. Maximum Range Accuracy (900 mils, 75 deg/hr, 9 mg)
Figure 10. Maximum Range Accuracy (900 mils, 20 deg/hr, 4 mg)
Table 6 shows the range achieved by the perfect trajectory for each of the quadrant elevations analyzed. The intent of choosing these QE values was to discover the miss distance at maximum range. As seen from the table, this range occurs at approximately 900 mils QE. If the projectile were to fly in a
vacuum, maximum range would result from perfect parabolic flight at a launch elevation 45 degrees (800 Army mils) from the horizontal. (Carlucci & Jacobson, 2008) Since the simulation takes into account aeroballistic effects such as lift and drag, and muzzle jump, the maximum range is achieved with a slightly higher QE than the theoretical.

<table>
<thead>
<tr>
<th>QE</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>600mils</td>
<td>30,335m</td>
</tr>
<tr>
<td>700mils</td>
<td>32,149m</td>
</tr>
<tr>
<td>800mils</td>
<td>33,177m</td>
</tr>
<tr>
<td>900mils</td>
<td>33,247m</td>
</tr>
<tr>
<td>1000mils</td>
<td>32,200m</td>
</tr>
</tbody>
</table>

Table 6  Perfect Trajectory Initial Conditions

Presented in Appendix B are the remaining simulation outputs, specifically the runs for the 600, 700, 800, and 1000 mils runs for the remaining 1-sigma as well as the 600, 700, 800, 900, and 1000 assuming the bias stability as a 3-sigma error value.

Figure 12 provides an explanation of the simulation output and the data used to draw conclusions. At the top of the slide is the title that indicates the conditions for the simulation output; QE, gyro bias and accelerometer bias stability error. On the right side of the diagram are the miss distances in the North and East Directions. The histogram distribution for each direction results from the 5000 Monte Carlo trajectories run by the simulation. The mean is a critical piece of data. In each direction, the mean should be close to zero, indicating that the Monte Carlo results of the MATLAB® simulations are
duplicating the results of the perfect trajectory. The standard deviation is the distance from the trajectory where 67% of the data points lie. The left side of the Figure 12 show the simulated impact points. The red dot in the middle is the impact point of the perfect trajectory, in this case 33,247 meters North of the gun-launch position. The coordinates of the impact scale are set to zero for the perfect trajectory to make miss distance from the perfect easier to calculate. Finally, MATLAB® generates the CEP Circle with a radius distance that captures 50% of the impact locations of the perfect trajectory.

Figure 12. Data Interpretation
C. SPREADSHEET DATA TABLE

Tables 7 and 8 summarize the data for the 1-sigma case and 3-sigma case, respectively.

<table>
<thead>
<tr>
<th>QE</th>
<th>1deg/hr and 1mg</th>
<th>20deg/hr and 4mg</th>
<th>75deg/hr and 9mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>600mils</td>
<td>1.7543m</td>
<td>11.8470m</td>
<td>35.4615m</td>
</tr>
<tr>
<td>700mils</td>
<td>2.1543m</td>
<td>15.1116m</td>
<td>45.4455m</td>
</tr>
<tr>
<td>800mils</td>
<td>2.6601m</td>
<td>18.5371m</td>
<td>55.8079m</td>
</tr>
<tr>
<td>900mils</td>
<td>3.0603m</td>
<td>21.2143m</td>
<td>63.5117m</td>
</tr>
<tr>
<td>1000mils</td>
<td>3.3217m</td>
<td>22.6314m</td>
<td>67.3433m</td>
</tr>
</tbody>
</table>

Table 7  Perfect Trajectory Initial Conditions

<table>
<thead>
<tr>
<th>QE</th>
<th>1deg/hr and 1mg</th>
<th>20deg/hr and 4mg</th>
<th>75deg/hr and 9mg</th>
</tr>
</thead>
<tbody>
<tr>
<td>600mils</td>
<td>0.5838m</td>
<td>3.9448m</td>
<td>11.8514m</td>
</tr>
<tr>
<td>700mils</td>
<td>0.7171m</td>
<td>5.0361m</td>
<td>15.1715m</td>
</tr>
<tr>
<td>800mils</td>
<td>0.8867m</td>
<td>6.1704m</td>
<td>18.6616m</td>
</tr>
<tr>
<td>900mils</td>
<td>1.0173m</td>
<td>7.0633m</td>
<td>21.1036m</td>
</tr>
<tr>
<td>1000mils</td>
<td>1.1062m</td>
<td>7.5481m</td>
<td>22.4726m</td>
</tr>
</tbody>
</table>

Table 8  Perfect Trajectory Initial Conditions

The 1-sigma vs. 3-sigma excursion on IMU bias stability quantified the difference in accuracy if the specification requirement were misinterpreted.
D. DATA DISCUSSION

It is not surprising that the data showed miss distance growing as IMU bias stability error increased; however, it was an unexpected finding that in the 1000 mils case, miss distance continued to increase even when achieving a range that was lower than maximum. This is because the time of flight for the 1000 mils cases is longer than for the 900 mils case, and the simulation integrates the IMU errors for a longer period, thereby increasing the miss distance. Since the 1000 mils case is less that maximum range, it seems there are two methods to achieve that particular range, in this case either 700 or 1000 mils. The miss distance would be less in the 700 mils case; however, depending on the tactical situation it might be determined that 1000 mils is a better solution even the miss distance may be greater. The field command would choose the appropriate solution depending on the tactical situation.

The data for the 900 mils, 1-sigma error, shows that for the 75 degree/hr, 9 mg condition, the miss distance is 63.5117 meters. For the 1 degree/hr, 1 mg conditions, the miss distance is 3.0603 meters. Calculating the improvement shows that with the more accurate IMU improves miss distance by 95.18%

\[ \left| \frac{(3.0603 - 63.5117)}{63.5117} \right| \times 100 = 95.18\% \]

Therefore, the answer to "How does the accuracy of the IMU affect miss distance?" is that a 1 degree/hour IMU is 95.18% more accurate than a 75 degree/hour. This is a significant improvement in accuracy, which would result in much better target accuracy and less collateral damage.

Comparing the data for the 1-sigma versus 3-sigma case shows that with a miss distance of 1.0173 meters, there is a 200.83% degradation in accuracy of the 1-sigma interpretation over the 3-sigma error value.

\[ \left| \frac{(3.0603 - 1.0173)}{1.0173} \right| \times 100 = 200.83\% \]
The interpretation of IMU accuracy requirement in this case doubles the miss distance. Having 99% of IMUs meet the requirement versus 67% could significantly increase unit price.

E. SUMMARY

The data reveal that the more accurate the IMU the smaller the miss distance. The excursion to investigate the impact of 1-sigma versus 3-sigma interpretation of the specification requirements showed a significant degradation in miss distance. Table 9 summarizes the results.

<table>
<thead>
<tr>
<th>Evaluation Criteria</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>75°/hr, 9mg vs. 1°/hr, 1 mg</td>
<td>95.18% Improvement in performance</td>
</tr>
<tr>
<td>1-sigma vs. 3-sigma specification interpretation</td>
<td>200.83% degradation in performance</td>
</tr>
</tbody>
</table>

Table 9 Summary of Simulation Results

Isolating IMU contribution to miss distance of a gun-launched precision munition provided a quantified answer to differences in IMU accuracy. It justifies the benefit of designing to a tighter specification, while revealing the importance of clearly specifying the meaning of requirements in the performance specification. Chapter V summarizes the key points of the research and outlines areas for further research.
V. CONCLUSION

A. KEY POINTS AND RECOMMENDATIONS

Conducting simulations that isolated the IMU bias stability error on miss distance showed that the 1 degree/hour specification resulted in a 95.18% improvement in the accuracy gun-launched munition in hitting a target. Although the analysis was limited to a 155 mm, fin-stabilized artillery projectile, and discounted external factors, mass properties, and metal part misalignments, it provided a model to quantify the effect of IMU stability bias error on miss distance. The research suggests that, while tightening the specification to 1 degree/hour made it more challenging to develop and design IMUs the tighter specifications significantly improved gun-launched munition precision, which would minimize collateral damage while still supporting a lower per unit cost for IMU production.

B. AREAS FOR FURTHER RESEARCH

The methodology used for this research was a strong step in understanding the effect of various factors on the precision of gun-launched projectiles. This model could be expanded for continued research, which might include:

1) Apply the model to a spin-stabilized projectile, including the magnus aeroballistic effects
2) Consider the external influences on the projectile, such as investigating global weather patterns to determine average environmental conditions and including them in the model.
3) Investigate mass property fluctuations by conducting a metal parts tolerance stack assessment could determine the effects of mass offset and misalignments on miss distance.
4) Conduct a cost assessment of specification requirements. The 1-sigma versus 3-sigma excursion showed a 200% increase in miss distance.
Meeting the 3-sigma condition could potentially result in more IMU rejects for not meeting specification, and quantifying the cost impact of this could be of interest.

C. SIGNIFICANCE OF THE RESEARCH

This research quantified the significance of driving gun-launched IMU requirements to tighter specification for better performance as well as cost savings. In addition, it quantifies the outcome of what happens when specifications are not clearly written and left open to interpretation.

Choosing generic setup conditions to conduct this research provided a platform to develop a subroutine within MATLAB® that serves as the foundation for IMU evaluation. This research results in a tool for the investigation of additional IMU error parameters and performance evaluation. It serves as the foundation for development of future accuracy requirements for gun-launched precision munitions.


Jenke, L. M. (1976). Experimental roll-damping, magnus, and static-stability characteristics of two slender missile configurations at high angles of attack (0 to 90) and mach numbers 0.2 through 2.5. Arnold Air Force Station: Arnold Engineering Development Center.


Regan, F. J. (1964). Roll damping moment measurements for the basic finner at subsonic and supersonic speeds. White Oak, MD: U.S. Naval Ordinance Laboratory.


APPENDIX A

Table 10 through 13 shows the static aerodynamic data used for the Basic Finner simulation runs derived from testing conducted in the ARDEC wind tunnel facility.

### Axial Force Coefficient

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<tr>
<th>AoA/Mach</th>
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### Normal Force Coefficient

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Table 10: Static Aeroballistic Data – Axial Force

Table 11: Static Aeroballistic Data – Normal Force
Table 13 and 14 show the derived pitch and roll damping coefficients used for the Basic Finner simulation runs.

### Roll Moment Coefficient

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Table 12 Static Aeroballistic Data – Roll Moment

### Pitch Damping Coefficient

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Table 13 Pitch Damping Coefficient

### Roll Damping Coefficient

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Table 14 Roll Damping Coefficient Data
APPENDIX B

Output of all MATLABR Simulation results:

Figure 13.  Accuracy Data (600 mils,75 degree/hour, 9 mg, 1-sigma)
Figure 14. Accuracy Data (600 mils, 20 degree/hour, 4 mg, 1-sigma)
Figure 15.  Accuracy Data (600 mils, 1 degree/hour, 1 mg, 1-sigma)
Figure 16.  Accuracy Data (700 mils, 75 degree/hour, 9 mg, 1-sigma)
Figure 17. Accuracy Data (700 mils, 20 degree/hour, 4 mg, 1-sigma)
Figure 18. Accuracy Data (700 mils, 1 degree/hour, 1 mg, 1-sigma)
Figure 19. Accuracy Data (800 mils, 75 degree/hour, 9 mg, 1-sigma)
Figure 20. Accuracy Data (800 mils, 20 degree/hour, 4 mg, 1-sigma)
Figure 21. Accuracy Data (800 mils, 1 degree/hour, 1 mg, 1-sigma)
Figure 22. Accuracy Data (1000 mils, 75 degree/hour, 9 mg, 1-sigma)
Figure 23. Accuracy Data (1000 mils, 20 degree/hour, 4 mg, 1-sigma)
Figure 24.  Accuracy Data (1000 mils, 1 degree/hour, 1 mg, 1-sigma)
Figure 25. Accuracy Data (600 mils, 75 degree/hour, 9 mg, 3-sigma)
Figure 26. Accuracy Data (600 mils, 20 degree/hour, 4 mg, 3-sigma)
Figure 27. Accuracy Data (600 mils, 1 degree/hour, 1 mg, 3-sigma)
Figure 28. Accuracy Data (700 mils, 75 degree/hour, 9 mg, 3-sigma)
Figure 29.  Accuracy Data (700 mils, 20 degree/hour, 4 mg, 3-sigma)
Figure 30. Accuracy Data (700 mils, 1 degree/hour, 1 mg, 3-sigma)
Figure 31. Accuracy Data (800 mils, 75 degree/hour, 9 mg, 3-sigma)
Figure 32. Accuracy Data (800 mils, 20 degree/hour, 4 mg, 3-sigma)
Figure 33. Accuracy Data (800 mils, 1 degree/hour, 1 mg, 3-sigma)
Figure 34. Accuracy Data (900 mils, 75 degree/hour, 9 mg, 3-sigma)
Figure 35. Accuracy Data (900 mils, 20 degree/hour, 4 mg, 3-sigma)
Figure 36. Accuracy Data (900 mils, 1 degree/hour, 1 mg, 3-sigma)
Figure 37. Accuracy Data (1000 mils, 75 degree/hour, 9 mg, 3-sigma)
Figure 38. Accuracy Data (1000 mils, 20 degree/hour, 4 mg, 3-sigma)
Figure 39. Accuracy Data (1000 mils, 1 degree/hour, 1 mg, 3-sigma)
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    Picatinny Arsenal, NJ