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Estimating Ergodic Capacity of Cooperative Analog Relaying under Different Adaptive Source Transmission Techniques

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ABSTRACT — Upper bounds on link spectral efficiency of amplify-and-forward cooperative diversity networks with independent but non-identically distributed wireless fading statistics are studied by deriving the Shannon capacity of three distinct adaptive source transmission techniques: (i) constant power with optimal rate adaptation (ORA); (ii) optimal joint power and rate adaptation (OPRA); and (iii) fixed rate with truncated channel inversion (TCIFR). Asymptotic capacity bound is also derived which show that optimal rate adaptation with constant power policy provides roughly the same ergodic capacity as the optimal joint power and rate adaptation policy at high mean signal-to-noise ratios (SNRs). Different previous related studies, we advocate a simple numerical procedure for unified analysis of ergodic channel capacity in a myriad of fading environments. This framework allows us to gain insights as to how fade distributions and dissimilar fading statistics across the diversity paths affect the Shannon capacity, without imposing any restrictions on the fading parameters.

I. MOTIVATION

While the range, capacity, and reliability of a radio link can always be improved by increasing transmit power, this solution is limited by several practical considerations — battery-life, power amplifier size/weight, interference to co-channel and neighboring nodes, and probability-of-intercept requirements, all render this strategy ineffective. Besides, while multiple-input-multiple output (MIMO) architectures could drastically improve the range and reliability of beyond line-of-sight and/or over-the-horizon communication links without increasing transmit-power — the cost, weight, and poor aero-dynamics of antenna arrays may prohibit their use on both unmanned vehicles and dismounted tactical war-fighters. Moreover, fixed transmission methods that are designed to provide the required quality of service in the “worst-case” scenario are very inefficient when better channel conditions prevail.

In multi-vehicle cooperative operations, networked nodes in a tight cluster can coordinate both their transmissions and/or receptions to mimic a space-time processing system as if they were part of a single antenna array platform (e.g., [1]-[2]). Thus cooperative airborne networking could significantly increase the range and reliability of a long-haul inter-cluster communication, thereby improving platform endurance with enhanced LPI/LPD capability, without using an antenna array. Several standardization groups such as IEEE 802.16 and IEEE 802.11 have also incorporated cooperative relaying into their emerging wireless standards (e.g., Mobile Multihop Relaying

Group has defined a multihop relay architecture in the baseline IEEE 802.16j standard).

An intermediate (relay) node in a cooperative airborne network (CAN) may either amplify what it receives (in case of amplify-and-forward relaying) or digitally decodes, and re-encodes the source message (in case of decode-and-forward relaying) before re-transmitting it to the destination node. In this article, we shall focus on the amplify-and-forward relaying scheme because it does not require “sophisticated” transceivers at the relays (although our framework is also applicable for digital relaying once the moment generating function is found or available). While this protocol can achieve full diversity using a virtual antenna array, there is a loss of spectral efficiency due to its inherent half-duplex operation. But this penalty could be “recovered” by combining the cooperative diversity with a link adaptation mechanism wherein the power level, signal constellation size, coding rate or other transmission parameters are adapted autonomously in response to fluctuations in the channel conditions.

But the art of adaptive link layer in a cooperative wireless network is still in its infancy especially when optimized in a cross-layer design paradigm. For instance, majority of the literature on cooperative diversity are limited to both fixed signaling rate and constant transmit power for all nodes. It is also not very clear as to how link adaptation (e.g., distributed power control) could be performed with limited channel side-information (CSI), and what are the benefits of jointly optimizing the upper layer protocols with an adaptable PHY layer? While the problem of optimal power allocation in a cooperative network has been studied recently in [3]-[5], its solution requires the knowledge of CSI of all links (i.e., large overhead) and moreover, source rate-adaptation was not considered. As a consequence, it is distinctly different than the adaptive source transmission policies of [6]. Motivated by these observations, [7] derived bounds for the Shannon capacity of a non-regenerative link-adaptive cooperative diversity system with limited CSI, in which the rate and/or power level at the source node is adapted according to the channel condition (i.e., only feedback of the effective SNR at the destination node is required to be available at the source node) while the relays simply amplify and forward the signals. Nevertheless, their analysis relies on a bounding technique, and more critically, it is limited to Rayleigh fading. It is much more realistic to model the channel gain of each link in a CAN as a Nakagami-m or a Rice random variable due to the increased likelihood of the presence of a strong specular component in an airborne platform.

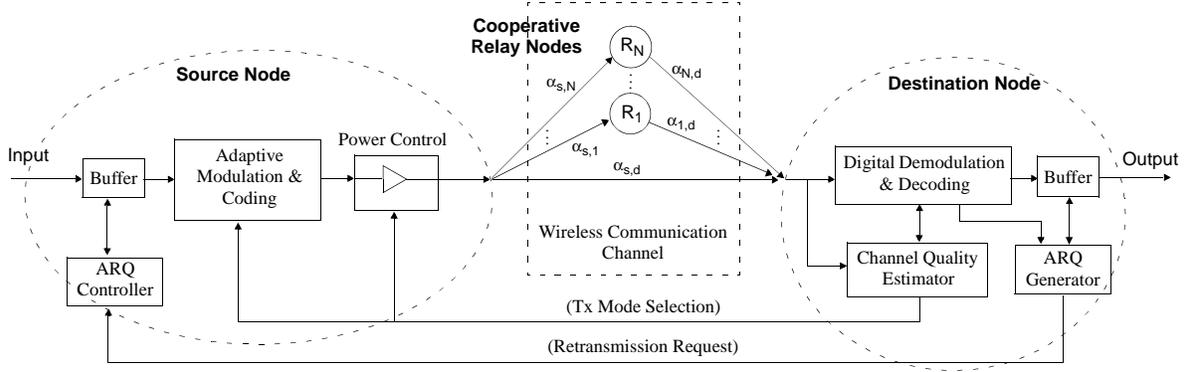


Fig. 1: Link-adaptive cooperative diversity system for ensuring the connectivity and network stability in airborne networks.

However, it has been noted recently that “... Although the performances of wireless relaying systems over fading channels have been extensively evaluated in terms of outage probability and error rate, there have been few studies on ergodic capacity of fading relay channels...” [8, pp. 2286]. In fact, the lack of significant contributions on ergodic capacity analysis can be attributed to the difficulty in evaluating the probability density function (PDF) of the end-to-end SNR in closed-form. In [7], the authors circumvented this difficulty by evaluating upper and lower bounds on the capacity instead, while [8] resorted to the method of moments. Nevertheless, both their analyses and results are limited to Rayleigh channels.

On the contrary, the moment generating function (MGF) of the end-to-end SNR is much easier to compute or may be readily available (i.e., since it is extensively used for outage probability and error rate analysis!). Hence, in this article we employ a novel method which we refer to as ‘cumulative distribution function (CDF) approach’ (in conjunction with a fixed-Talbot method [9]) and develop a general yet computationally efficient numerical procedure for evaluating the ergodic capacity of CANs under three distinct adaptive source transmission policies in a myriad of stochastic channels.

II. SYSTEM MODEL

Fig. 1 depicts the cooperative wireless network under consideration. The source node S communicates with the destination node D via a direct-link and through N amplify-and-forward relays, $R_i, i \in \{1, 2, \dots, N\}$, in two transmission phases. During the initial phase, S transmits signal x to D and relays R_i where the fading coefficients between S and D , S and the i -th relay node R_i , and R_i and D are denoted by $\alpha_{s,d}$, $\alpha_{s,i}$ and $\alpha_{i,d}$, respectively. In the second phase, the N relays transmit the received signal after amplification via orthogonal transmissions (e.g., TDMA in a round-robin fashion and/or FDMA). The i -th relay amplifier gain is chosen as [1]

$$G_i = \sqrt{E_s / (E_s |\alpha_{s,i}|^2 + N_o)}, \quad (1)$$

where E_s denotes the average symbol energy, while N_o corresponds to the noise variance.

Suppose maximum-ratio combining is employed at the destination node, the total SNR is given by

$$\gamma_T = \gamma_{s,d} + \sum_{i=1}^N \frac{\gamma_{s,i} \gamma_{i,d}}{1 + \gamma_{s,i} + \gamma_{i,d}} \cong \gamma_{s,d} + \sum_{i=1}^N \gamma_i = \gamma_{TB}, \quad (2)$$

where $\gamma_{a,b} = |\alpha_{a,b}|^2 E_s / N_o$ denotes the instantaneous SNR of link $a-b$, while the instantaneous SNR of a two-hops path can be accurately approximated as the harmonic mean of the link SNRs. The total SNR can be upper and lower bounded as

$$\gamma_{s,d} + \frac{1}{2} \sum_{i=1}^N \tilde{\gamma}_i = \gamma_{LB} \leq \gamma_T \leq \gamma_{UB} = \gamma_{s,d} + \sum_{i=1}^N \tilde{\gamma}_i, \quad (3)$$

where $\tilde{\gamma} = \min(\gamma_{s,i}, \gamma_{i,d})$.

If $\gamma_{s,d}$, $\gamma_{s,i}$, $\gamma_{i,d}$ are independent random variables, then it is straight-forward to show that the MGF of γ_{TB} is given by

$$\phi_{\gamma}(s) = \phi_{\gamma_{s,d}}(s) \prod_{i=1}^N \phi_{\gamma_i}(s). \quad (4)$$

For instance, the MGF of $\gamma_i = \gamma_{s,i} \gamma_{i,d} / (\gamma_{s,i} + \gamma_{i,d})$ for a Rayleigh channel with independent but non-identically distributed (i.n.d) fading statistics is well-known, and it is given by

$$\phi_{\gamma_i}(s) = [(1/\Omega_{s,i} - 1/\Omega_{i,d})^2 + (1/\Omega_{s,i} + 1/\Omega_{i,d})s] / \Delta^2 + \frac{2s}{\Delta^3 \Omega_{s,i} \Omega_{i,d}} \ln \left(\left(s + \Delta + \frac{1}{\Omega_{s,i}} + \frac{1}{\Omega_{i,d}} \right)^2 \frac{\Omega_{s,i} \Omega_{i,d}}{4} \right), \quad (5)$$

where $\Omega_{a,b} = E[\gamma_{a,b}]$ corresponds to the mean link SNR, and

$$\Delta = \sqrt{(1/\Omega_{s,i} - 1/\Omega_{i,d})^2 + 2s(1/\Omega_{s,i} + 1/\Omega_{i,d}) + s^2}.$$

If a closed-form expression for γ_i is not available (e.g., Nakagami- m channel with i.n.d fading statistics), but does exist for $\gamma_i = \min(\gamma_{s,i}, \gamma_{i,d})$, we may then develop capacity bounds using the inequality used in [7], viz.,

$$\phi_{\gamma_{s,d}}(s) \prod_{i=1}^N \phi_{\gamma_i}(s/2) \leq \phi_{\gamma}(s) \leq \phi_{\gamma_{s,d}}(s) \prod_{i=1}^N \phi_{\gamma_i}(s). \quad (6)$$

In this case, the MGF of γ_i for 2-hops path may be derived as

$$\phi_{\gamma_i}^-(s) = \sum_{\substack{k \in \{(s,i), (i,d)\} \\ j \neq k}} \int_0^\infty e^{-sx} f_{\gamma_k}(x) [1 - F_{\gamma_j}(x)] dx. \quad (7)$$

For instance, it is not difficult to show that the MGF of γ_i for Nakagami- m fading with i.n.d fading statistics is given by

$$\begin{aligned} \phi_{\gamma_i}^-(s) &= \sum_{\substack{k \in \{(s,i), (i,d)\} \\ j \neq k}} \frac{\Gamma(m_k + m_j)}{\Gamma(m_k)\Gamma(m_j)} \left(\frac{\Omega_j m_k}{s\Omega_j\Omega_k + \Omega_j m_k + \Omega_k m_j} \right)^{m_k} \\ &\times \frac{1}{m_k} {}_2F_1 \left(1 - m_j, m_k; 1 + m_k; \frac{(s\Omega_k + m_k)\Omega_j}{s\Omega_j\Omega_k + \Omega_j m_k + \Omega_k m_j} \right). \quad (8) \end{aligned}$$

Once the MGF of γ_i or γ_i is found, we can compute the outage probability (i.e., its CDF) efficiently using a fixed-Talbot (multi-precision Laplace transform inversion) method, viz.,

$$\begin{aligned} F_X(x) &\cong \frac{r}{M} \sum_{k=1}^{M-1} \operatorname{Re} \left\{ e^{xs(\theta_k)} \phi(s(\theta_k)) \frac{1 + j\sigma(\theta_k)}{s(\theta_k)} \right\} \\ &+ \frac{1}{2M} \phi_X(r) e^{rx}, \quad (9) \end{aligned}$$

where $r = 2M/(5x)$, $\sigma(\theta_k) = \theta_k + (\theta_k \cot(\theta_k) - 1) \cot(\theta_k)$, $\theta_k = k\pi/M$, and $s(\theta_k) = r\theta_k(j + \cot(\theta_k))$.

In Section III, we will show that the ergodic capacity of ORA and OPRA adaptive transmission policies can be expressed in terms of the complementary CDF of γ_T alone. Hence the ergodic capacity may be evaluated readily for all of the cases discussed above (since the MGF of γ_i or γ_i is available in closed-form). For instance, (11) (or (16)), (15) and (19) in conjunction with (6), (8) and (9) generalizes the results in [7] to Nakagami- m channels. Similarly, precise estimates of the ergodic capacities with different adaptive source transmission techniques in Rayleigh fading can be obtained by using (4), (5) and (9) in (11), (15) and (19).

III. ERGODIC CAPACITY OF FADING CHANNELS

The well-known Shannon-Hartley law tells us that there is an absolute limit on the error-free bit rate R that can be transmitted within a certain channel bandwidth B at a specified SNR. This theoretical limit denotes the channel capacity C . Shannon's noisy channel coding theorem also states that it is not possible to make the probability of error tend to zero if $R > C$ with any code design. Thus it is clear that the metric C plays an important role in the design or appraisal of any communications system (since it serves as an upper limit on the transmission rate for reliable communications over a noisy communication channel).

A. Optimal Power and Rate Adaptation (OPRA)

In OPRA scheme, the transmission power and rate is matched to the varying channel condition through use of a multiplexed multiple codebook design. This leads to the highest achievable capacity with CSI. From [6], we have

$$\begin{aligned} \bar{C}_{OPRA} &= B \int_{\gamma_0}^\infty \log_2(\gamma/\gamma_0) f_\gamma(\gamma) d\gamma \\ &= \frac{\gamma_0 B}{\ln 2} \int_1^\infty \ln(\gamma) f_\gamma(\gamma_0 \gamma) d\gamma \quad (10) \end{aligned}$$

where γ_0 is the optimal cutoff SNR level below which data transmission is suspended. Using integration by parts (with $u = \ln(\gamma)$, $dv = f_\gamma(\gamma) d\gamma$ and $v = -\int_{\gamma_0 \gamma}^\infty f_\gamma(\gamma) d\gamma$), the second line of (10) can be concisely expressed as

$$\bar{C}_{OPRA} = \frac{B}{\ln 2} \int_1^\infty \frac{F_\gamma^c(\gamma_0 \gamma)}{\gamma} d\gamma, \quad (11)$$

where $F_\gamma^c(\gamma) = 1 - F_\gamma(\gamma) = \int_\gamma^\infty f_\gamma(x) dx$ is the complementary cumulative distribution function (CCDF) of γ .

To achieve the capacity (11), side information of the channel fade level (i.e., CSI) tracked at the receiver must be conveyed to the transmitter on the feedback path for power and rate adaptation in accordance with the time-varying nature of the channel fading. When $\gamma < \gamma_0$, no data is transmitted, and thus the optimal policy suffer an outage probability given by

$$P_{out} = F_\gamma(\gamma_0), \quad (12)$$

which equals to the probability of no transmission. The optimal cutoff SNR must satisfy

$$F_\gamma^c(\gamma_0) - \gamma_0 \left[1 + \int_{\gamma_0}^\infty \gamma^{-1} f_\gamma(\gamma) d\gamma \right] = 0. \quad (13)$$

The integral term in (13) can be evaluated readily using the identity (B.3) when the MGF is available. Furthermore, asymptotic analysis of (13) shows that $\gamma_0 \rightarrow 0$ when the mean SNR $\Omega \rightarrow 0$ because $F_\gamma(x) \rightarrow 1$ and $f_\gamma(x) \rightarrow 0$ (the effect of $\Omega \rightarrow 0$ can be predicted from the normalized PDF or the normalized CDF curve when its argument $x \rightarrow \infty$). When $\Omega \rightarrow \infty$, $F_\gamma(x) \rightarrow 0$ because this is equivalent to computing the CDF when its argument $x \rightarrow 0$. It is also well-known that $\phi_\gamma(\cdot) \rightarrow 0$ as $\Omega \rightarrow \infty$. Hence, $\gamma_0 \rightarrow 1$ as $\Omega \rightarrow \infty$. Hence, γ_0 (can be determined by solving (13) numerically) always lies in the interval $[0, 1]$ regardless of the fading channel model and number of relay nodes employed.

B. Optimal Rate Adaptation with Fixed Transmit Power (ORA)

When only the rate is adapted with the changing channel condition, the channel capacity is given by

$$\bar{C}_{ORA} = \frac{B}{\ln 2} \int_0^\infty \ln(1 + \gamma) f_\gamma(\gamma) d\gamma \quad (14)$$

The above also corresponds to the channel capacity when CSI is available only at the receiver. Since it is generally difficult or expensive to acquire fast, reliable feedback from the receiver to the transmitter, (14) is a useful capacity bound that is applicable to many situations encountered in practical operating con-

ditions. Once again, using integration by parts, (14) can be concisely expressed as

$$\bar{C}_{ORA} = \frac{B}{\ln 2} \int_0^\infty \frac{F_\gamma^c(\gamma)}{1+\gamma} d\gamma \quad (15)$$

It is interesting to point out that the new form (15) allows us to show that the capacity increases with the increasing diversity order regardless of the fading channel model or diversity combining technique employed. Gunther [11, p. 401] suggested that while this is intuitive, it not easy to prove this trend mathematically for the ORA adaptation policy. However, recognizing that the CDF $F_\gamma(\gamma)$ will decrease with increasing

order of diversity for any given γ , the numerator term $F_\gamma^c(\gamma)$ in (15) will be much closer to unity in comparison with that of no-diversity or lower diversity order cases. Thus, we have shown that the capacity increases with diversity order as expected.

It is also interesting to highlight that by rewriting (11) as

$$\bar{C}_{OPRA} = \frac{B}{\ln 2} \int_0^\infty \frac{F_\gamma^c(\gamma_0(1+\gamma))}{1+\gamma} d\gamma, \quad (16)$$

and noting that $\gamma_0 \rightarrow 1$ as $\Omega \rightarrow \infty$ and $F_\gamma^c(1+\gamma) \approx F_\gamma^c(\gamma)$, one may conclude that the discrepancy between the capacities of ORA and OPRA adaptation policies will diminish as Ω increases (because (15) asymptotically approaches (16) in this limiting case).

C. Channel Inversion with Fixed Rate (CIFR)

In CIFR policy, the transmitter adapts its power to maintain a constant SNR at the receiver and uses fixed-rate modulation and fixed-code designs. This technique is the least complex to implement given that reliable channel estimates are available at the transmitter. The zero-outage capacity formula for this adaptation policy is given by [6]

$$\bar{C}_{CIFR} = B \log_2 \left(1 + \frac{1}{\int_0^\infty \gamma^{-1} f_\gamma(\gamma) d\gamma} \right) \quad (17)$$

However, when the channel experience deep fades, the penalty in transmit power requirement with the CIFR policy will be enormous because channel inversion needs to compensate for the deep fades. To overcome this, a truncated channel inversion and fixed rate policy (TCIFR) was considered in [6] where the channel fading is only inverted above a fixed cutoff fade depth γ_0 . The data transmission is ceased if γ falls below γ_0 . In this case, it is easy to show that the outage probability is $P_{out} = F_\gamma(\gamma_0)$ and the channel capacity is given by

$$\bar{C}_{TCIFR} = B \log_2 \left(1 + \frac{1}{\int_{\gamma_0}^\infty \gamma^{-1} f_\gamma(\gamma) d\gamma} \right) F_\gamma^c(\gamma_0). \quad (18)$$

Next, utilizing identity (B.3), (18) can be re-stated as

$$\bar{C}_{TCIFR} = B \log_2 (1 - 1/\nabla) F_\gamma^c(\gamma_0) \quad (19)$$

where $\nabla = \frac{1}{\pi} \int_0^\infty \text{Re}\{\phi_\gamma(-j\omega) \text{Ei}(-j\omega\gamma_0)\} d\omega$. Observe that,

if the MGF of γ is known in closed-form, then the above integral can be evaluated efficiently via Gauss-Chebyshev quadrature method (for a wide-range of fading channel models and diversity combining techniques employed including maximal-ratio-combining and selection diversity).

APPENDIX A

While the exponential-integral $\text{Ei}(x)$ is usually defined for real $x < 0$, in the following we will show that this function is also well-defined even if its argument is purely imaginary. This is particularly interesting because our unified expression for the Shannon capacity with truncated channel inversion policy and the transcendental equation for computing the optimal cut-off SNR γ_0 for the OPRA policy are expressed in terms of $\text{Ei}(-jc)$ where $c > 0$ is real.

Letting $q = \pm jy$ (y is real), we have

$$\text{Ei}(-q) = - \int_q^\infty \frac{e^{-t}}{t} dt = - \int_1^\infty \frac{e^{-qt}}{t} dt. \quad (A.1)$$

Utilizing the Euler identity $e^{\pm jy} = \cos(y) \pm j \sin(y)$, (A.1) can be restated in terms of the more familiar sine-integral and cosine-integral, viz.,

$$\begin{aligned} \text{Ei}(\mp jy) &= - \int_1^\infty \frac{1}{t} \cos(yt) dt \pm j \int_1^\infty \frac{1}{t} \sin(yt) dt, \\ &= \text{ci}(y) \mp j \text{si}(y) \end{aligned} \quad (A.2)$$

with the aid of [10, Eqs. (3.721.2) and (3.721.3)]. The sine-integral and cosine-integral may be computed efficiently via rapidly converging series representations:

$$\text{si}(y) = - \int_y^\infty \frac{\sin(t)}{t} dt = - \frac{\pi}{2} + \sum_{k=1}^\infty \frac{(-1)^{k+1} y^{2k-1}}{(2k-1)(2k-1)!}, \quad (A.3)$$

$$\text{ci}(y) = - \int_y^\infty \frac{\cos(t)}{t} dt = C + \ln(y) + \sum_{k=1}^\infty \frac{(-1)^k y^{2k}}{2k(2k)!}, \quad (A.4)$$

where $C = 0.57721566\dots$ is Euler's constant. Alternatively, the quantity $\text{Ei}(-jc)$ can be evaluated in MATLAB using command line “`cosint(c) - j(-pi/2 + sinint(c))`”.

APPENDIX B

Let $\phi_X(s) = \int_0^\infty e^{-sx} f_X(x) dx$ and $\Phi_X(j\omega) = \int_0^\infty e^{j\omega x} f_X(x) dx$ denote the MGF (moment generating function) and the CHF (characteristic function) of random variable $X \geq 0$ respectively. Thus, the CHF is related to the MGF as $\Phi_X(j\omega) = \phi_X(-j\omega)$. The PDF of X (i.e., can be expressed as an inverse Fourier transform of its CHF) is given by

$$f_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_X(-j\omega) e^{-j\omega x} d\omega. \quad (\text{B.1})$$

Let us express the CHF of random variable X in its polar form

$\Phi_X(j\omega) = |\Phi_X(j\omega)| e^{j\theta(\omega)}$. Hence (B.1) may be re-stated as

$$\begin{aligned} f_X(x) &= \frac{1}{2\pi} \int_0^{\infty} \Phi_X(j\omega) e^{-j\omega x} d\omega + \frac{1}{2\pi} \int_{-\infty}^0 \Phi_X(j\omega) e^{-j\omega x} d\omega \\ &= \frac{1}{2\pi} \int_0^{\infty} [\Phi_X(j\omega) e^{-j\omega x} + \Phi_X(-j\omega) e^{j\omega x}] d\omega \\ &= \frac{1}{\pi} \int_0^{\infty} |\Phi_X(j\omega)| \left[\frac{e^{j(\theta(\omega)-\omega x)} + e^{-j(\theta(\omega)-\omega x)}}{2} \right] d\omega \quad (\text{B.2}) \\ &= \frac{1}{\pi} \int_0^{\infty} |\Phi_X(j\omega)| \cos(\theta(\omega) - \omega x) d\omega \\ &= \frac{1}{\pi} \int_0^{\infty} \text{Re} \{ \Phi_X(j\omega) e^{-j\omega x} \} d\omega \end{aligned}$$

and consequently, we can simplify the integral

$$\begin{aligned} \int_1^{\infty} \gamma^{-1} f_\gamma(\gamma_0 \gamma) d\gamma &= \frac{1}{\pi} \int_0^{\infty} \text{Re} \left\{ \Phi_\gamma(j\omega) \left(\int_1^{\infty} \frac{e^{-j\omega \gamma_0 \gamma}}{\gamma} d\gamma \right) \right\} d\omega \quad (\text{B.3}) \\ &= -\frac{1}{\pi} \int_0^{\infty} \text{Re} \{ \phi_\gamma(-j\omega) \text{Ei}(-j\omega \gamma_0) \} d\omega \end{aligned}$$

with the aid of (A.1) and (B.2).

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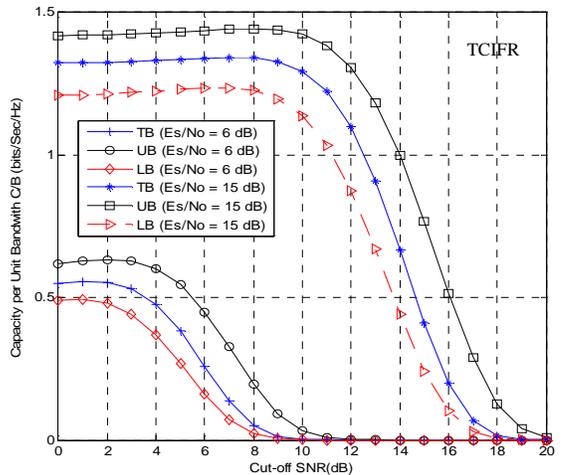
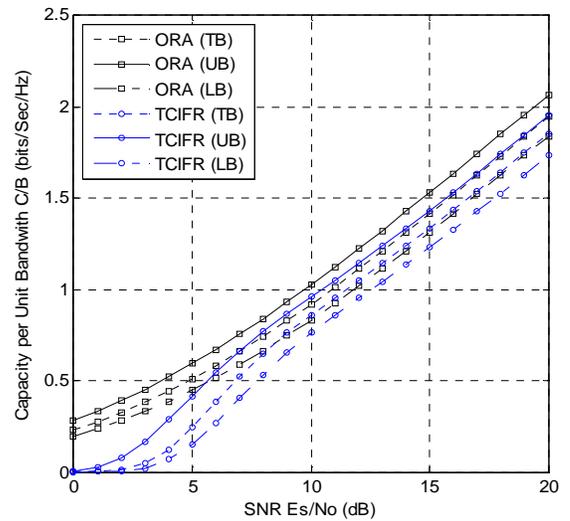


Fig. 2: Channel capacity comparison of ORA and TCIFR in i.n.d Rayleigh fading ($\gamma_0 = 6\text{ dB}$, $\Omega_{s,1} = \Omega_{2,d} = E_s/N_0$, $\Omega_{s,d} = 0.2E_s/N_0$ and $\Omega_{s,2} = \Omega_{1,d} = 0.5E_s/N_0$).