Development of a dynamic visco-elastic vehicle-soil interaction model for rut depth, energy and power determinations

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Presentation Outline

A) Review of Soil Model governing equations

B) Development of pedo-transfer functions (terrain database to engineering properties)

C) Ethan Allen Firing Range (EAFR) terrain database conversion

D) Dynamic Soil Model example (Stryker tire with EAFR soil)
A) Soil Model Governing Equations

1) Boussinesq/Cerruti vertical stress distribution
2) Soil response - Vertical visco-elasto-plastic displacement
3) Shear stress/deformation
   - Slip (longitudinal), Turning (lateral)
   - Bulldozing
Disturbed width for Wheeled vehicles (4 wheel)

\[
DW = \frac{L}{\sin\left(\frac{L}{TR - B/2}\right)} \cdot \left(\sin\left(\frac{\pi}{2} - \frac{W_r}{C_{\alpha r}} \frac{V^2}{g \cdot TR}\right) - \sin\left(\frac{\pi}{2} + \frac{W_r}{C_{\alpha r}} \frac{V^2}{g \cdot TR} - \frac{L}{TR - B/2}\right)\right) + TW
\]
Disturbed width for tracked vehicles

\[ DW = \sqrt{\left(\frac{TL}{2} + \frac{v^2 \cdot TL}{4g\mu_t \cdot TR}\right)^2 + \left(TR - \frac{B}{2} + \frac{TW}{2}\right)^2} - \left(TR - \frac{B}{2} - \frac{TW}{2}\right) } \]
Compression Model Approaches

“Essentially, all models are wrong, but some are useful.” --- George Box

- Wheel numerics – Dimensionless numbers
- Bekker equation
- Boussinesq stress and Compression Index
- FEM or DEM
Boussinesq Vertical Stress Distribution due to Vertical Force

$$\sigma_z = \frac{3Pz^3}{2\pi[(x^2 + y^2 + z^2)]^{5/2}}$$

from Feda, 1978
Boussinesq Stress Distribution

- Assumes soil follows the theory of elasticity
- Assumes contact area of the tire is a rectangular plate
- Modified equation with Frohlich’s concentration factor

\[
\sigma_z = \frac{\nu W z^\nu}{2\pi(x^2 + y^2 + z^2)^{(\nu/2)+1}}
\]

2) Soil response - Vertical visco-elasto-plastic displacement

- Compression index
- Influence of soil type and moisture
- Rebound
- Time dependent sinkage
\[ \rho = \left[ \rho_k + S_T (S_1 - S_k) \right] + C \log \left( \frac{\sigma_a}{\sigma_k} \right) \]

where \( S_T, S_1 \) and \( S_k \) are the moisture factors

from Larson et al., 1980
Elasto-plastic deformation (rebound)
3) Shear stress/deformation

- Lateral/longitudinal stress-strain, shear deformation modulus (Janosi/Wong)
- Bulldozing (soil displacement) analysis, passive lateral earth pressure
Lateral shear deformation modulus
slip displacement

\[ \tau = \tau_{\text{max}} \cdot \left(1 - e^{-j/K}\right) = \left(c + \sigma \cdot \tan \phi\right) \cdot \left(1 - e^{-j/K}\right) \]

- Where \( \tau \) is the shear stress,
- \( j \) is the shear displacement,
- \( c \) is the internal cohesion of the soil,
- \( \phi \) is the angle of internal friction of the soil,
- \( K \) is defined as the shear deformation modulus (Wong, 2001).
“Bulldozer” forces
(passive lateral earth pressure)

\[ P = b \left( \frac{1}{2} \gamma Z^2 N_\phi + 2 c Z \sqrt{N_\phi} \right) \]

\[ N_\phi = \tan^2 \left( 45 + \frac{\phi}{2} \right) \]

from McKyes, 1989
B) Development of pedo-transfer functions

Engineering parameters needed by soil model

- compression index
- rebound index
- time constant
- cohesion
- friction angle
- shear deformation modulus
- density
### Ethan Allen Firing Range Database (what we are starting with)

<table>
<thead>
<tr>
<th>Hex code</th>
<th>Decimal code</th>
<th>Terrain Condition</th>
<th>Soil Type</th>
<th>Moisture content code</th>
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<th>RCI 6-12</th>
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Terrain database descriptors (off-road soil conditions):

- Terrain Condition - Fine-grained soil NFG, coarse-grained soil NCG, road, other.
- Soil Type - Uses Unified Soil Classification System (USCS) CL, CH, ML
- Moisture content code – NOR (normal) and SLP (slippery) for fine-grained soil, normal for coarse-grained soil
- RCI 0-6 - Soil strength 0-6 inches, RCI for fine-grained soils, CI for coarse-grained soils.
- RCI 6-12 - Soil strength 6-12 inches, RCI for fine-grained soils, CI for coarse-grained soils.
Pedo-transfer function

• Need to transfer existing terrain database descriptors to soil engineering properties.
## Unified Soil Classification System (USCS)

<table>
<thead>
<tr>
<th>MAJOR DIVISIONS</th>
<th>GROUP SYMBOLS</th>
<th>TYPICAL NAMES</th>
<th>FIELD IDENTIFICATION PROCEDURES (excluding particles larger than 3 inches and basing fractions on estimated weights)</th>
<th>INFORMATION REQUIRED FOR DESCRIBING SOILS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse-grained Soils</td>
<td>GW</td>
<td>Well-graded gravels, gravel-sand mixtures, little or no fines</td>
<td>Wide range in grain sizes and substantial amounts of all intermediate particle sizes</td>
<td>For undisturbed soils add information on stratification, degree of compactness, cementation, moisture conditions, and drainage characteristics.</td>
</tr>
<tr>
<td>Sands</td>
<td>GP</td>
<td>Poorly graded gravels or gravel-sand mixtures, little or no fines</td>
<td>Predominantly one size or a range of sizes with some intermediate sizes missing</td>
<td>Give typical name: Indicate approximate percentage of sand and gravel, maximum size, angularity, surface condition, and hardness of the coarse grains, local or geologic name, and other pertinent descriptive information, and symbol in parentheses. An example:</td>
</tr>
<tr>
<td>Clayey sands, sand-clay mixtures</td>
<td>GC</td>
<td>Silty gravels, gravel-sand-silt mixtures</td>
<td>Nonplastic fines or fines with low plasticity (for identification procedures see CL below)</td>
<td>Example:</td>
</tr>
<tr>
<td>Well-graded sands, gravelly sands, little or no fines</td>
<td>SW</td>
<td>Predominantly one size or a range of sizes with some intermediate sizes missing</td>
<td>Nonplastic fines or fines with low plasticity (for identification procedures see CL below)</td>
<td>Example:</td>
</tr>
<tr>
<td>Sands with little or no fines</td>
<td>SM</td>
<td>Silty sands, sand-silt mixtures</td>
<td>Nonplastic fines or fines with low plasticity (for identification procedures see CL below)</td>
<td>Example:</td>
</tr>
<tr>
<td>Clayey sands, sand-clay mixtures</td>
<td>SC</td>
<td>Plastic fines (for identification procedures see CL below)</td>
<td>Plastic fines (for identification procedures see CL below)</td>
<td>Example:</td>
</tr>
<tr>
<td>Fine-grained Soils</td>
<td>ML</td>
<td>Inorganic silts and very fine sands, rock flour, silty or clayey fine sands or clayey silts with slight plasticity</td>
<td>None to slight</td>
<td>Dry Strength (Crushing Characteristics)</td>
</tr>
<tr>
<td>Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays</td>
<td>CL</td>
<td>Medium to high</td>
<td>None to very slow</td>
<td>Medium</td>
</tr>
<tr>
<td>Organic silts and organic silty clays of low plasticity</td>
<td>OL</td>
<td>Slight to medium</td>
<td>Slow</td>
<td>Slight</td>
</tr>
<tr>
<td>Inorganic clays, micaceous or diatomaceous fine sandy or silty soils, elastic silts</td>
<td>MH</td>
<td>Slight to medium</td>
<td>Slow to none</td>
<td>Slight to medium</td>
</tr>
<tr>
<td>Inorganic clays of high plasticity, fat clays</td>
<td>CH</td>
<td>High to very high</td>
<td>None</td>
<td>High</td>
</tr>
<tr>
<td>Organic clays and silts of medium to high plasticity</td>
<td>OH</td>
<td>Medium to high</td>
<td>None to very slow</td>
<td>Slight to medium</td>
</tr>
<tr>
<td>Peat and other highly organic soils</td>
<td>Pt</td>
<td>Predominantly one size or a range of sizes with some intermediate sizes missing</td>
<td>Predominantly one size or a range of sizes with some intermediate sizes missing</td>
<td>Predominantly one size or a range of sizes with some intermediate sizes missing</td>
</tr>
</tbody>
</table>
USCS soil particle size and USDA soil texture
The centroid percent Sand, Silt, and Clay and particle size for 9 USCS Classes

<table>
<thead>
<tr>
<th>USCS Soil Type</th>
<th>%Clay Avg</th>
<th>%Silt Avg</th>
<th>%Sand Avg</th>
<th>Average Grain Size (mm)</th>
<th>Clay Index (0-5)</th>
<th>Grain Size Index (0-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH</td>
<td>73.3</td>
<td>13.3</td>
<td>13.3</td>
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<td>5.00</td>
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<tr>
<td>CL</td>
<td>47.2</td>
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<td>0.76</td>
<td>3.21</td>
<td>2.32</td>
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<tr>
<td>MH</td>
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<td>15.2</td>
<td>0.41</td>
<td>2.38</td>
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<tr>
<td>ML</td>
<td>13.6</td>
<td>61.5</td>
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<td>0.67</td>
<td>0.93</td>
<td>2.63</td>
</tr>
<tr>
<td>SC</td>
<td>26.2</td>
<td>8.8</td>
<td>65.0</td>
<td>1.69</td>
<td>1.79</td>
<td>1.04</td>
</tr>
<tr>
<td>SM</td>
<td>8.8</td>
<td>26.2</td>
<td>65.0</td>
<td>1.70</td>
<td>0.60</td>
<td>1.04</td>
</tr>
<tr>
<td>SW/SP-SC</td>
<td>6.9</td>
<td>2.3</td>
<td>90.8</td>
<td>2.36</td>
<td>0.47</td>
<td>0.75</td>
</tr>
<tr>
<td>SW/SP-SM</td>
<td>2.3</td>
<td>6.9</td>
<td>90.8</td>
<td>2.36</td>
<td>0.16</td>
<td>0.75</td>
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<tr>
<td>SW &amp; SP</td>
<td>1.7</td>
<td>1.7</td>
<td>96.7</td>
<td>2.51</td>
<td>0.11</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Ethan Allen Firing Range terrain database over 1000 soil units (rows). Ignoring the elevation and non-soil units, a set of 13 unique soil units were identified as shown below.

<table>
<thead>
<tr>
<th>Terrain</th>
<th>Soil Type</th>
<th>Moisture</th>
<th>RCI</th>
</tr>
</thead>
<tbody>
<tr>
<td>NFG</td>
<td>CL</td>
<td>SLP</td>
<td>80</td>
</tr>
<tr>
<td>NFG</td>
<td>GM</td>
<td>NOR</td>
<td>150</td>
</tr>
<tr>
<td>NFG</td>
<td>ML</td>
<td>NOR</td>
<td>100</td>
</tr>
<tr>
<td>NFG</td>
<td>ML</td>
<td>NOR</td>
<td>65</td>
</tr>
<tr>
<td>NFG</td>
<td>ML</td>
<td>NOR</td>
<td>43.5</td>
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<td>ML</td>
<td>NOR</td>
<td>30</td>
</tr>
<tr>
<td>OTH</td>
<td>PT</td>
<td>NOR</td>
<td>17.5</td>
</tr>
<tr>
<td>NFG</td>
<td>SM</td>
<td>NOR</td>
<td>130</td>
</tr>
<tr>
<td>NFG</td>
<td>SM</td>
<td>NOR</td>
<td>110</td>
</tr>
<tr>
<td>NFG</td>
<td>SM</td>
<td>NOR</td>
<td>65</td>
</tr>
<tr>
<td>NCG</td>
<td>SP</td>
<td>NOR</td>
<td>130</td>
</tr>
<tr>
<td>NFG</td>
<td>CL</td>
<td>SLP</td>
<td>130</td>
</tr>
<tr>
<td>NFG</td>
<td>SC</td>
<td>SLP</td>
<td>80</td>
</tr>
</tbody>
</table>
Pedo-transfer functions

\[ \varphi = 24.03 - 3.06 \cdot (Clay\_Index) + 1.03 \cdot (Grain\_Size\_Index) + 0.0047 \cdot (RCI) \]

\[ c = 8.65 + 12.15 \cdot (Clay\_Index) - 8.94 \cdot (Grain\_Size\_Index) + 0.0070 \cdot (RCI) \]

\[ C = 0.2175 + 0.01245 \cdot (\%Clay) - 0.000131 \cdot (\%Clay)^2 \]

\[ K = 21.74 - 3.22 \cdot (Clay\_Index) + 0.47 \cdot (GSI) \]

\[ \rho = 1.68 - 0.10 \cdot (Clay\_Index) - 0.04 \cdot (GSI) \]

\[ \text{Rebound\_Constant} = 0.011 + 0.004 \cdot (GSI) \]

\[ \rho_K = 1.544 - 0.00556 \cdot (\%Clay) - 3.468 \times 10^{-5} \cdot (\%Clay)^2 \]

\[ S_T = 0.003461 + 1.742 \times 10^{-4} \cdot (\%Silt) - 2.980 \times 10^{-6} \cdot (\%Silt)^2 \quad (C, M) \]

\[ S_T = 0.003217 + 3.251 \times 10^{-4} \cdot (\%Clay) - 5.385 \times 10^{-6} \cdot (\%Clay)^2 \quad (S) \]
## C) Predicted Soil Engineering Properties for the EAFR Terrain Codes

<table>
<thead>
<tr>
<th>Terrain</th>
<th>Soil Type</th>
<th>Moisture</th>
<th>RCI</th>
<th>Clay Index</th>
<th>Grain Size Index</th>
<th>Friction Angle (º)</th>
<th>c (kPa)</th>
<th>K (cm)</th>
<th>C (g/cm³)</th>
<th>Bulk Density,ρ (g/cm³)</th>
<th>Rebound Constant (g/cm³)</th>
<th>Time Constant (s)</th>
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<td>CL</td>
<td>SLP</td>
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<td>0.36</td>
<td>1.48</td>
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<td>80</td>
<td>1.79</td>
<td>1.04</td>
<td>20.0</td>
<td>21.6</td>
<td>15</td>
<td>0.45</td>
<td>1.45</td>
<td>0.015</td>
<td></td>
</tr>
</tbody>
</table>

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D) Dynamic Soil Compression Model

- A 1 cm$^3$ soil element, 5 cm below the soil surface, positioned along the tire centerline

- EAFR terrain - CL (USCS), 80 psi (RCI), SLP (moisture)

- Simple Stryker tire (22.25 kN normal load) traveling at 3 m/s with a tire inflation pressure of 276 kPa (40 psi). (uniformly distributed rectangular surface load)
EAIFR Soil Engineering Properties

- Terrain - NFG
- Soil Type - CL
- Moisture - SLP
- RCI - 80
- Clay Index - 3.21
- Grain Size Index - 2.32
- Friction Angle (°) - 17.0
- c (kPa) - 27.4
- K (cm) - 10
- C (g/cm³) - 0.51
- Bulk Density, \(\rho\) (g/cm³) - 1.25
- Rebound Constant (g/cm³) - 0.020
- Time Constant (s) – 0.2
Stryker Soil Stress at 0.05 m - Centerline - (3 m/s)
Number of Wheel Passes

Soil Element Vertical Displacement (cm)

1.5 m/s Travel Speed
3.0 m/s Travel Speed
6.0 m/s Travel Speed

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Number of Wheel Passes vs Change in Energy (J)

- 1.5 m/s Travel Speed
- 3.0 m/s Travel Speed
- 6.0 m/s Travel Speed
Number of Wheel Passes

Peak Power (W)

1.5 m/s Travel Speed
3.0 m/s Travel Speed
6.0 m/s Travel Speed
Benefits of New Soil Model

• Theoretical (soil deformation, energy, power)
• Not dependent of scale
• Real-time modeling
• Include visco-elastic (time constant/rebound)
• Uses existing terrain database with pedo-transfer functions.
Questions

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