REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)* 25-05-2011	**2. REPORT TYPE** Conference Paper	**3. DATES COVERED** *(From - To)*

4. TITLE AND SUBTITLE

Analysis of Air Breathing Hall Effect Thruster

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
L. Pekker and M. Keidar

5d. PROJECT NUMBER

5f. WORK UNIT NUMBER
50260542

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory (AFMC)
AFRL/RZSA
10 E. Saturn Blvd.
Edwards AFB CA 93524-7680

8. PERFORMING ORGANIZATION REPORT NUMBER

AFRL-RZ-ED-TP-2011-189

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory (AFMC)
AFRL/RZS
5 Pollux Drive
Edwards AFB CA 93524-7048

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S NUMBER(S)
AFRL-RZ-ED-TP-2011-189

12. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution unlimited (PA #11127).

13. SUPPLEMENTARY NOTES
For presentation at the 42nd AIAA Plasmadynamics and Laser Conference, held in Honolulu, HI, 27-30 June 2011.

14. ABSTRACT

The principle idea of using air breathing electric propulsion for a vehicle flying at orbital speed on the edge of Earth's atmosphere is examined. In this paper, we present a simple model of a Hall Effect thruster in which the propellant is ambient air. The required lengths of the thruster chamber, the magnetic fields, the thrust, and other parameters of an ideal air breathing Hall Effect thruster are calculated as a function of the flying altitude of the vehicle. We demonstrate that an air breathing Hall thruster is indeed capable of providing the needed thrust for near space satellites for the altitudes of 80 – 110 km.
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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** Marcus P. Young
a. REPORT **Unclassified**	**b. ABSTRACT** **Unclassified**	**c. THIS PAGE** **Unclassified**	SAR	16	**19b. TELEPHONE NUMBER** *(include area code)* N/A

Analysis of Air Breathing Hall Effect Thruster

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The principle idea of using air breathing electric propulsion for a vehicle flying at orbital speed on the edge of Earth's atmosphere is examined. In this paper, we present a simple model of a Hall Effect thruster in which the propellant is ambient air. The required lengths of the thruster chamber, the magnetic fields, the thrust, and other parameters of an ideal air breathing Hall Effect thruster are calculated as a function of the flying altitude of the vehicle. We demonstrate that an air breathing Hall thruster is indeed capable of providing the needed thrust for near space satellites for the altitudes of 80 – 110 km.

Nomenclature

A = mass of a heavy particle, in au

B = strength of the magnetic field

D_{diff} = ambient gas diffusion coefficient

E = strength of the electric field

E_{jet} = energy flow leaving the Hall thruster with the plasma jet

E_{anode}^{loss} = heat losses at the anode

E_{wall}^{loss} = energy losses at the wall

E_{ioniz}^{cost} = ionization cost

e = electron charge

F_{drag} = drag force

f_{drag} = drag factor

j_e = electron current density

j_i = ion current density

H = altitude of satellite orbit

L = length of the Hall thruster channel

$\ln \Lambda$ = Coulomb logarithm

M = mass of a heavy particle

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M_i = ion mass

m_e = mass of an electron

n_e = electron number density

n_{gas} = number density of ambient gas

n_i = ion number density

n_{pl} = number density of plasma

P = electric power put into the discharge

r_a = inner radius of the opening of the air breathing Hall thruster

r_b = outer radius of the opening of the air breathing Hall thruster

r_D = Debye radius

r_L = electron cyclotron radius

S = effective cross-section of the satellite due to drag force

T_e = electron temperature

$T_{e,\,jet}$ = electron temperature in the plasma jet

T_{gas} = ambient gas temperature

$T_{i,\,jet}$ = ion temperature in the plasma jet

$Thrust$ = thrust

V_B = Bohm velocity

$V_{e,drift}$ = electron drift velocity

V_{Te} = electron thermal velocity

V_{Tgas} = thermal velocity of ambient gas

V_0 = the first cosmic velocity, 7.8 km/s

γ = coefficient of secondary electron emission

ΔV = delta-V (change in the velocity)

λ_{gas} = ambient gas mean free path

μ_e = electron mobility

σ_{gas} = collision cross-section of neutral particles

σ_{ioniz} = ionization cross-section

τ_{e-life} = electron lifetime in Hall thruster

τ_{ioniz} = ionization time

τ_{wall} = characteristic time for gas molecule collisions with wall

$\tau_0 = L / V_0$ = time that a gas molecule is flying through the thruster chamber

ν_{ei} = electron-ion collision frequency

ϕ_0 = the Bohm sheath potential

φ = applied voltage

ω_{He} = electron cyclotron frequency

I. Introduction

This work is motivated by increasing interest in military and civil spacecrafts flying at the altitudes of 70 - 120 km [1]. Since the drag at such altitudes is still significant, the thrusters of such a satellite should work continuously maintaining the orbit altitude. This demands that a significant amount of propellant be stored on-board satellites that use ordinary thrusters for propulsion. However, for such spacecrafts, using air breathing thrusters, in which the propellant is ambient air, looks very attractive, particularly for long-duration missions; air breathing thrusters allow significantly increased payload-to-weight ratio for such satellites.

In a recent paper [2], Diamant has proposed a two-stage cylindrical Hall Effect thruster for air breathing electric propulsion, in which the first stage is an electron cyclotron resonance ionization stage and the second stage is a cylindrical Hall thruster. The author built such a thruster and demonstrated its operation in xenon gas. As follows from his assumptions, his two-stage cylindrical Hall thruster is able to work at a 220 km orbit with ambient air passively compressed by a factor of 500 [1]. Achieving such a large passive gas compression seems to be difficult. In the BUSEC conception of an air breathing Hall thruster [2], the compression of air is achieved by a diffuser. It is not clear what compression ratio of the ambient air can be achieved by using such a diffuser at orbits of about 100 km, where the gas mean free paths are tens of centimeters and comparable with the dimensions of the thruster. Furthermore, the use of a diffuser can significantly increase the drag because gas is reflected diffusely in space [3].

In the present work we propose a simple configuration of an air breathing Hall thruster, (shown in Figure 1), in which the incoming air flow is ionized and accelerated directly in the Hall thruster chamber without

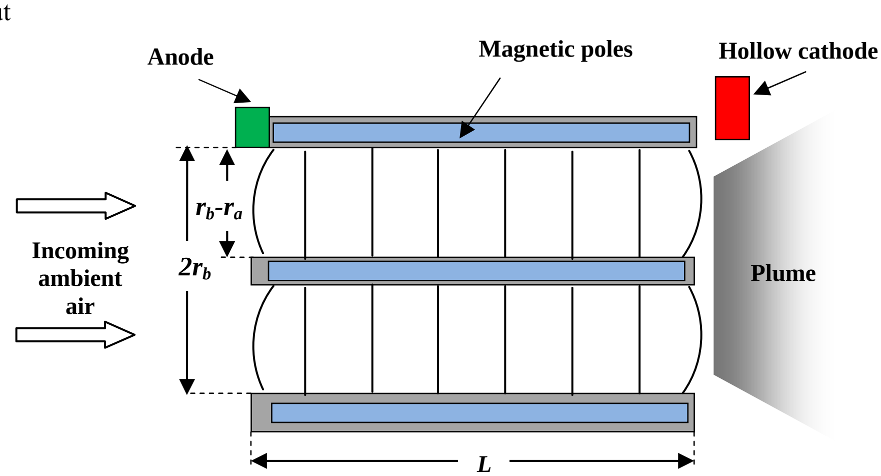

Figure 1. Schematic of air breathing Hall thruster (not to scale).

preliminary compression as in [1, 2]. Such a design leads to a decrease of the possible drag, however its feasibility warrants careful consideration, which is the subject of this paper. The description of the model and numerical results are provided in Sections II and III, respectively, and the conclusions are given in Section IV.

II. Description of the Model

The following assumptions are made in the model: (1) no drag due to ambient gas passing through the Hall thruster chamber, illustrated in Figure 1, corresponding to a condition in which the ambient gas freely flows through the thruster chamber without a significant interactions with the wall; (2) the gas jet leaving the thruster chamber is fully ionized providing the maximum thrust; (3) the plasma in the Hall thruster is quasineutral, $n_i = n_e = n_{pl}$, corresponding to a condition in which r_D is smaller than the characteristic dimensions of the thruster; in our case, Figure 1, r_D must be smaller than $r_b - r_a$; (4) the electron cyclotron radius is much smaller than the Hall thruster gap, $r_L < r_b - r_a$, as is required for confinement of electrons in the Hall thruster.

The first assumption is well satisfied when the characteristic time required for a gas molecule to reach the wall and transfer its momentum, τ_{wall}, is larger than the time required for the molecule to freely fly through the thruster chamber, $\tau_0 = L / V_0$, i.e.

$$\frac{\tau_{wall} \cdot V_0}{L} > 1 \quad . \tag{1}$$

In the case of free molecular flow, when the gas mean free path is larger than $r_b - r_a$, Figure 2, τ_{wall} can be estimated as

$$\tau_{wall} = \frac{r_b - r_a}{V_{Tgas}} \quad , \tag{2}$$

and in the case of collisional molecular flow, $r_b - r_a < \lambda_{gas}$, as

$$\tau_{wall} = \frac{(r_b - r_a)^2}{D_{diff}} = \frac{(r_b - r_a)^2}{\lambda_{gas} \cdot V_{Tgas}} = \frac{n_{gas} \cdot \sigma_{gas} \cdot (r_b - r_a)^2}{V_{Tgas}} \quad . \tag{3}$$

Here, we have substituted $(n_{gas} \cdot \sigma_{gas})^{-1}$ for λ_{gas}. Combining Eqs. (2) and (3) we obtain

$$\tau_{wall} = Max\left[\frac{r_b - r_a}{V_{Tgas}}, \frac{n_{gas} \cdot \sigma_{gas} \cdot (r_b - r_a)^2}{V_{Tgas}}\right]. \tag{4}$$

The second assumption is fulfilled when the ionization time of a gas molecule,

$$\tau_{ioniz} = \frac{1}{V_{Te} \cdot n_e \cdot \sigma_{ioniz}}, \tag{5}$$

is much smaller than τ_0. Let us estimate τ_{ioniz}. The equation describing the mass conservation law for the heavy particles can be written as

$$V_0 \cdot n_{gas} = n_{pl} \cdot \sqrt{V_0^2 + \frac{2 \cdot e \cdot \varphi}{M}}, \tag{6}$$

where the left-hand side in Eq. (6) is the flux of incoming gas and the right-hand side is the ion flux leaving the chamber. In Eq. (6) we have assumed that the downstream plasma in the thruster chamber is quasineutral and fully ionized.

It should be stressed that in the model we assume the ambient gas is pure nitrogen. This is a reasonable assumption since the air contains 78 percent of nitrogen and 21 percent of oxygen and their electron impact ionization cross sections are similar [4]. In the model, we neglect the dissociation of nitrogen assuming that $M = M_i$. Since for electron energies between 20 – 80 eV the ionization and dissociation cross sections of molecular nitrogen are close and the ionization cross section of molecular nitrogen is much larger than the ionization cross section of atomic nitrogen [5], neglecting of the dissociation of nitrogen should not lead to a significant error and is reasonable for our simple model.

Since $e \cdot \varphi$ is usually much larger than the kinetic energy of an ambient gas molecule at the entrance plane of the Hall thruster (for nitrogen molecules it is about 8.75 eV), the n_{pl} can be written as

$$n_{pl} = n_e = \frac{V_0 \cdot n_{gas}}{\sqrt{\frac{2 \cdot e \cdot \varphi}{M}}}. \tag{7}$$

Substituting Eq. (7) and V_0 as 7.8 km/s into an Eq. (5), we obtain that condition $\tau_0 >> \tau_{ioniz}$ is fulfilled when

$$47.6 \cdot L \cdot n_{gas} \cdot \sigma_{ioniz}(T_e) \cdot \left(\frac{\varphi}{A \cdot T_e} \right)^{1/2} > 1 \ , \tag{8}$$

where T_e in Eq. (8) is measured in eV.

On the other hand, the electron lifetime in the Hall thruster, τ_{e-life}, must be larger than the electron ionization time; otherwise, typical electrons will leave the thruster channel before producing any ionization events; τ_{e-life} can be estimated as

$$\tau_{e-life} = \frac{L}{V_{e,drift}} = \frac{L}{\mu_e \cdot E} \cdot \left(1 + \frac{\omega_{He}^2}{\nu_{ei}^2} \right), \tag{9}$$

It worth noting that in Eq. (9) we have assumed the classical electron transport across the magnetic field, and neglected the collisions of electrons with neutral particles (since a fully ionized plasma is assumed); recall that in the case of a high magnetic field, electron transport in Hall thrusters can approach the classical limit [6]. Substituting in Eq. (9)

$$\mu_e = \frac{e}{m_e \cdot \nu_{ei}} , \quad \nu_{ei} = 2.1 \cdot 10^{-12} \cdot \ln \Lambda \cdot n_{pl} \cdot T_e^{-3/2} , \quad \omega_{He} = 1.76 \cdot 10^7 \cdot B , \quad E = \varphi / L , \tag{10}$$

and taking into account that ω_{He} / ν_{ei} in Hall thrusters is much larger than one, we obtain

$$\tau_{e-life} = 1.5 \cdot 10^{15} \cdot \frac{L^2 \cdot B^2}{\sqrt{\varphi \cdot A} \cdot \ln \Lambda \cdot n_{gas} \cdot T_e^{-3/2}} , \tag{11}$$

where B is measured in Gauss, and T_e in eV. Now, taking into account that τ_{e-life}, Eq. (11), must be smaller than τ_{ioniz}, Eq. (5), we obtain that the following condition on L must also be satisfied to fulfill assumption 2:

$$4 \cdot 10^{10} \cdot L \cdot B \cdot T_e \cdot \left(\frac{\sqrt{\varphi \cdot A} \cdot \ln \Lambda}{\sigma_{ioniz}(T_e)} \right)^{-1/2} > 1 \ . \tag{12}$$

Thus, we have shown that if the conditions given by Eqs. (8) and (12) are well satisfied, then the gas in the air breathing Hall thruster is well ionized.

Now let us examine the third assumption of the model: that the plasma is quasineutral. Substituting n_{pl} from Eq. (7) into an equation for the Debye radius [7],

$$r_D = 7.43 \cdot 10^3 \cdot \left(\frac{T_e}{n_{pl}} \right)^{1/2} , \tag{13}$$

and assuming that the gas is fully ionized, we obtain the following equation for the Debye radius:

$$r_D = 10^4 \cdot \left(\frac{T_e \cdot \sqrt{\varphi}}{n_{gas} \cdot \sqrt{A}} \right)^{1/2} . \tag{14}$$

Thus, if the Debye radius, Eq. (14), is much smaller than the characteristic dimension of the thruster,

$$r_b - r_a > 10^4 \cdot \left(\frac{T_e \cdot \sqrt{\varphi}}{n_{gas} \cdot \sqrt{A}} \right)^{1/2} , \tag{15}$$

the plasma in the Hall thruster can be considered quasineutral; T_e in Eqs. (13) – (15) is measured in electron volts.

Assumption 4 of the model requires that the characteristic electron cyclotron radius be much smaller than the thruster gap. Substituting in an expression for electron cyclotron radius V_{Te} for electron transfer velocity, we obtain

$$\frac{r_b - r_a}{r_L} = 42 \cdot \frac{(r_b - r_a) \cdot B}{T_e^{1/2}} > 1 , \tag{16}$$

where the electron temperature is in electron volts and the magnetic field is in Gauss.

Assuming that all three assumptions of the model are fulfilled, the thrust of an air breathing Hall thruster can be estimated as

$$Thrust = \pi \cdot (r_b^2 - r_a^2) \cdot M \cdot V_0 \cdot n_{gas} \cdot \sqrt{\frac{2 \cdot e \cdot \varphi}{M}} \tag{17}$$

and the electric power supplied to the discharge as

$$P = \pi \cdot (r_b^2 - r_a^2) \cdot \left(j_e + j_i\right) \cdot \varphi = \pi \cdot (r_b^2 - r_a^2) \cdot \left(e \cdot n_{pl} \cdot V_{e,drift} + e \cdot n_{gas} \cdot V_0\right) \cdot \varphi \ , \qquad (18)$$

where $\pi \cdot (r_b^2 - r_a^2)$ is the opening area of the thruster, and $M \cdot V_0 \cdot n_{gas}$ and $\Delta V = \sqrt{2 \cdot e \cdot \varphi / M}$ in Eq. (17) are correspondingly the mass flux entering the thruster and the increase in the velocity of heavy particles due to their acceleration in the Hall thruster chamber. Since the third assumption of the model is that the plasma is quasineutral, in Eq. (18) in the expression for j_e we have used n_{pl} instead of n_e. Substituting in Eq. (17) $V_{e,drift} = \mu_e \cdot E \cdot \nu_{ei}^2 / \omega_{He}^2$, n_{pl} from Eq. (7), and V_0 = 7.8 km/s and using Eqs. (10), we obtain that

$$j_e = 6.01 \cdot 10^{-35} \cdot \frac{\ln \Lambda \cdot T_e^{-3/2} \cdot A \cdot n_{gas}^2}{B^2 \cdot L} \ , \qquad (19)$$

$$j_i = 1.25 \cdot 10^{-15} \cdot n_{gas} \ , \qquad (20)$$

where T_e is in eV, B in Gauss.

Assuming diffuse reflection of gas molecules from the satellite walls [5], the drag force can be estimated as,

$$F_{drag} = M \cdot V_0^2 \cdot n_{gas} \cdot S \ . \qquad (21)$$

It should be stressed that since the gas entering the thruster does not produce drag, the first assumption of the model, the opening area of the thruster, $\pi \cdot (R_b^2 - R_a^2)$, is not included in S.

One of the important parameters governing the discharge in a Hall thruster is the electron temperature. Now let us obtain an equation for electron temperature for an air breathing Hall thruster. An energy conservation equation in the Hall thruster chamber can be written as

$$\pi \cdot (R_b^2 - R_a^2) \cdot \left(e \cdot n_{pl} \cdot V_{e,drift} + e \cdot n_{gas} \cdot V_0\right) \cdot \varphi = E_{wall}^{loss} + E_{jet} + E_{anode} \ , \qquad (22)$$

where the left-hand side of Eq. (22) is the electric power supplied to the discharge, Eq. (18), while the right-hand of this equation describes the heat losses at the Hall thruster walls and at the anode, and the energy flow leaving the Hall thruster with the plasma jet. Assuming no secondary electron emission from the wall, $\gamma = 0$, this corresponds to the case of minimal heat loss to the wall; equations for E_{wall}^{loss}, E_{jet}, and E_{anode} can be written as

$$E_{wall}^{loss} = 2\cdot\pi\cdot(R_a + R_b)\cdot L\cdot\left[n_{pl}\cdot V_B\cdot e\cdot\left(\phi_0 + E_{ion}^{\cos t}\right) + 2\cdot T_e\cdot n_{pl}\cdot\exp\left(-\frac{e\cdot\phi_0}{T_e}\right)\cdot\sqrt{\frac{T_e}{2\cdot\pi\cdot m_e}}\right], \quad (23)$$

$$E_{jet} = \pi\cdot(R_b^2 - R_a^2)\cdot V_0\cdot n_{gas}\cdot e\cdot\left(\varphi + T_{i,\,jet} + T_{e,\,jet} + E_{ioniz}^{\cos t}\right), \quad (24)$$

$$E_{anode} = \pi\cdot(R_b^2 - R_a^2)\cdot n_{pl}\cdot V_{e,drift}\cdot\frac{3}{2}\cdot e\cdot T_e\,. \quad (25)$$

In Eq. (23) we have used the Bohm plasma sheath theory [8] and in Eq. (25) have taken into account the heat flux that the electrons bring to the anode, and neglected the (typically small) ion flux to the anode. The first term in the square brackets on the right-hand side of Eq. (23) describes the heat flux to the wall due to the recombination process plus the kinetic energy flux that ions bring to the wall, and the second term describes the heat flux that electrons bring to the wall. The plasma sheath potential drop, ϕ_0, and the Bohm velocity, V_0 [8], are

$$\phi_0 = \frac{1}{2}\cdot T_e\cdot\ln\left(\frac{2\cdot\pi\cdot m_e}{M}\right), \quad (26)$$

$$V_B = \sqrt{\frac{T_e}{M}}\,. \quad (27)$$

Since the ion and electron temperatures in the plasma jet (in the plume, Figure 1) are typically a few electron volts and much smaller than the ionization cost, which is 20 – 40 eV, we will further neglect $T_{i,\,jet}$ and $T_{e,\,jet}$ in Eq. (24). Substituting the reduced equations for E_{wall}^{loss}, E_{jet}, E_{anode} into Eq. (22), after some algebra we obtain a final equation for the electron temperature:

$$\begin{aligned} &4.9\cdot 10^{-20}\cdot\frac{A\cdot n_{gas}\cdot\ln\Lambda}{L^2\cdot B^2}\cdot\left(\frac{\varphi}{T_e}\right)^{3/2}\cdot\left(1 - \frac{3\cdot T_e}{2\cdot\varphi}\right) = \\ &= T_e^{1/2}\cdot\frac{E_{ioniz}^{\cos t}}{L}\cdot\left(\frac{\varphi}{T_e}\right)^{1/2} + \frac{1}{(r_b - r_a)}\cdot 1.5\cdot T_e^{1/2}\cdot\left(E_{ion}^{\cos t} + \left[2 + \ln\left(17\cdot A^{1/2}\right)\right]\cdot T_e\right) \end{aligned}\,. \quad (28)$$

Thus, knowing the parameters of the air breathing Hall thruster (L, r_a, r_b, B, φ) and parameters of the ambient air (A, $\sigma_{ion}(T_e)$, σ_{gas}, n_{gas}, T_{gas}, E_{ioniz}^{cost}) using Eq. (28) we can calculate T_e, then check the assumptions made in the model, Eqs. (1), (8), (12), (15) and (16), and finally calculate the thrust, Eq. (17), the power, Eq. (18), the ion and electron currents, Eqs. (19) and (20), and other parameters of the air breathing Hall thruster at a given satellite orbit.

III. Numerical Results

In the numerical results presented in this section the collision cross section for molecular nitrogen gas, σ_{gas}, was taken as $4.4 \cdot 10^{-19}$ m^2 [9]; this corresponds to

$$\lambda_{gas} = \frac{1.63 \cdot 10^{-5} \cdot T[K]}{P[Pa]} \quad . \tag{29}$$

The ionization cross section of molecular nitrogen as a function of electron temperature, Figure 2, was calculated using data [5], and $\ln \Lambda$ was taken as 13.5; the length and the inner and outer radii of the Hall thruster, were chosen as 0.5, 0.03, and 0.05 m respectively. The calculations have been performed for two applied voltages, 3000 and 30000 Volts, and for satellite orbits of 80 and 90 km.

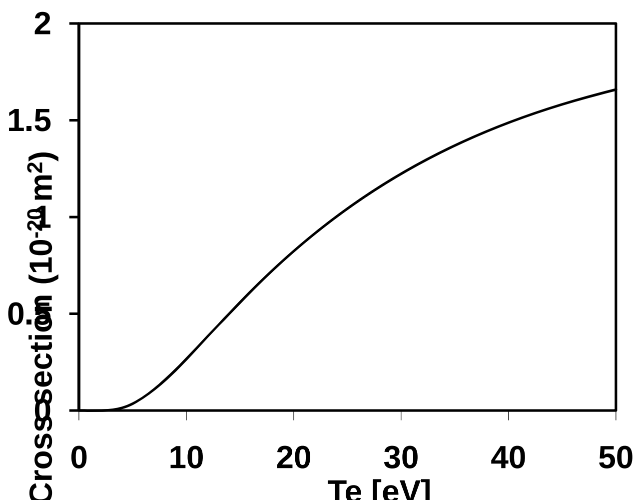

Figure 2. Ionization cross section of molecular nitrogen

Figure 3 shows the drag factor $f_{drag} = \tau_{wall} \cdot V_0 / L$ calculated for 80, 90, 100, and 110 km satellite orbits. As one can see, the first assumption of the model is well fulfilled only for the 80 km satellite orbit. However, since the thrust of air breathing Hall thrusters, Eq. (17), is assumed to be much larger than the maximal drag $\pi \cdot (R_b^2 - R_a^2) \cdot M \cdot V_0^2$ that might be produced by the air flowing through the Hall chamber, this assumption is not critical for the air breathing Hall thruster concept. It is worth noting that the gas flow through the thruster is free-molecular, Eq. (2), for $H = 90$ km and larger, and is collisional, Eq. (3), for $H = 80$ km and smaller. Obviously, for different thruster geometries the transition between the free molecular and the collisional flows can take place at different orbits.

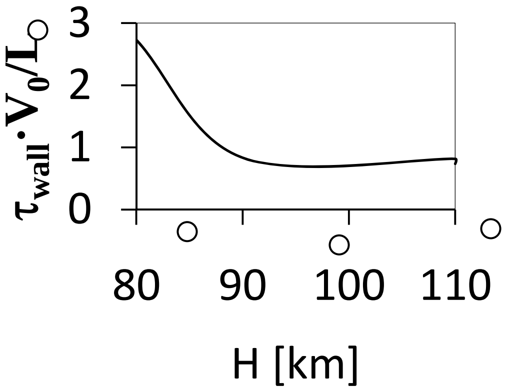

Figure 3. Drag factor for the air breathing Hall thruster: L = 0.5 m, r_b = 0.05 m, and r_a = 0.03 m

Figures 4 and 5 show the ratios of τ_0 / τ_{ioniz}, $\tau_{e-life} / \tau_{ioniz}$, and $(r_b - r_a)/r_L$, Eqs. (8), (12), and (16), calculated for the satellite orbits of 90 and 80 km respectively and the applied voltages of 3 kV and 30 kV; they correspond to the second and forth assumptions of the model. As one can see for the satellite orbit of 90 km these assumptions are satisfied for magnetic fields in the ranges of 10 – 30 and 20 – 70 Gauss correspondingly for $\varphi = 3$ kV and 30 kV; and for the orbit of 80 km in the ranges of 30 – 120 G for $\varphi = 3$ kV and for $\varphi = 30$ kV in all region of magnetic fields, 50 – 300 G. The third assumption of the model, Eq. (15), and $\omega_{He} / \nu_{ei} >> 1$ condition are very well satisfied in all ranges of magnetic field, potential, and orbit considered in the paper.

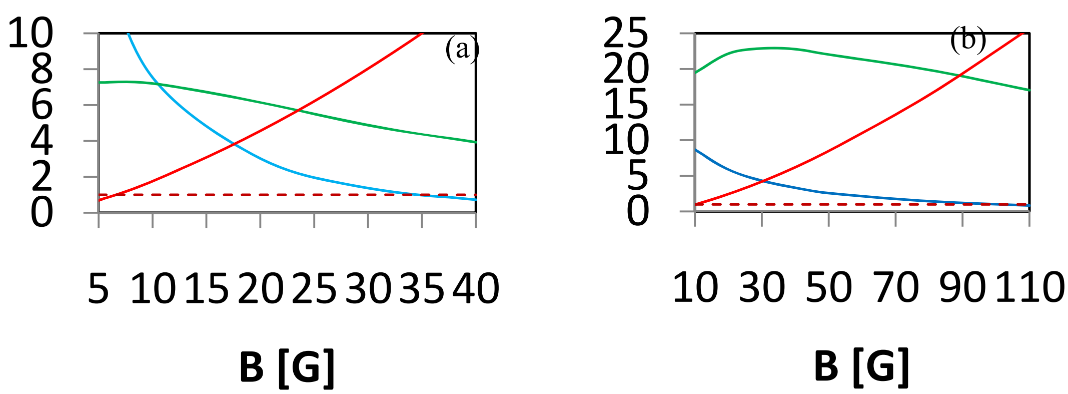

Figure 4. The τ_0 / τ_{ioniz} - blue line, $\tau_{e-life} / \tau_{ioniz}$ - green line, $(r_b - r_a)/r_L$ - red line calculated for $H = 90\, km$ and $\varphi = 3\, kV$ - (a) and $\varphi = 30\, kV$ - (b); the broken line correspond to unity.

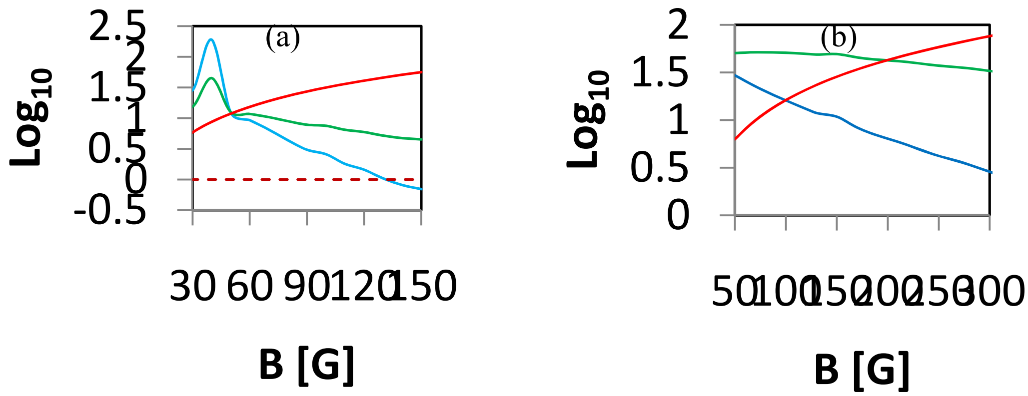

Figure 5. The τ_0 / τ_{ioniz} **- blue line,** $\tau_{e-life} / \tau_{ioniz}$ **- green line,** $(r_b - r_a)/r_L$ **- red line calculated for** $H = 80\,km$ **and** $\varphi = 3\,kV$ **- (a) and** $\varphi = 30\,kV$ **- (b).**

Since the air breathing Hall thruster plasma in our model is assumed to be fully ionized, the thrust, Eq. (20), is determined only by the ambient gas number density and the applied voltage, and the ion current, Eq. (17), only by the ambient gas number density. The model gives $j_i = 8.9 \cdot 10^4$ and $4.8 \cdot 10^5$ A/m^2 for $H =$ 90 and 80 km respectively; and *Thrust* = 18.63, 58.94, 100.5, and 317.9 N correspondingly for $H =$ 90 km, $\varphi =$ 3 and 30 kV and $H =$ 80 km, $\varphi =$ 3 and 30 kV. The obtained results are very understandable: with a decrease in the satellite orbit, the ambient gas density increases, leading to an increase in the ion current and the thrust; and an increase in the applied voltage leads to larger ΔV and, therefore, larger thrust.

Figures 6 and 7 show the electron temperature, power, and j_i / j_e vs. magnetic field for $H = 90$ and 80 km respectively. In these figures we have selected magnetic fields for which the model assumptions are satisfied. As was expected with an increase in the applied voltage, the electron temperature increases, and with an increase in the magnetic field it decreases, as shown in Figures 6a, 6b and 7a and 7b. Since the ion current in the model is independent of the magnetic field (ions are not magnetized), while the electron current decreases with an increase in the magnetic field (they are magnetized), the electrical power ratio decreases and the ratio of j_i to j_e increases with an increase in B, as illustrated in Figures 6c - 6f and 7c - 7f. Since the electron component of the total current sharply decreases with magnetic field, as in Figures 6e, 6f, 7c, 7f, P flattens for large magnetic fields, as in Figures 6c, 6d, 7c, 7d.

We have also examined the air breathing Hall thruster with the selected L, r_a, and r_b for the satellite orbits of 100 and 110 km. Although we could not select B and φ to satisfy the model assumptions at such high altitude, we believe that changing the geometry of the thruster, for example increasing the length of the thruster, see Eqs. (8) and (12), may allow this.

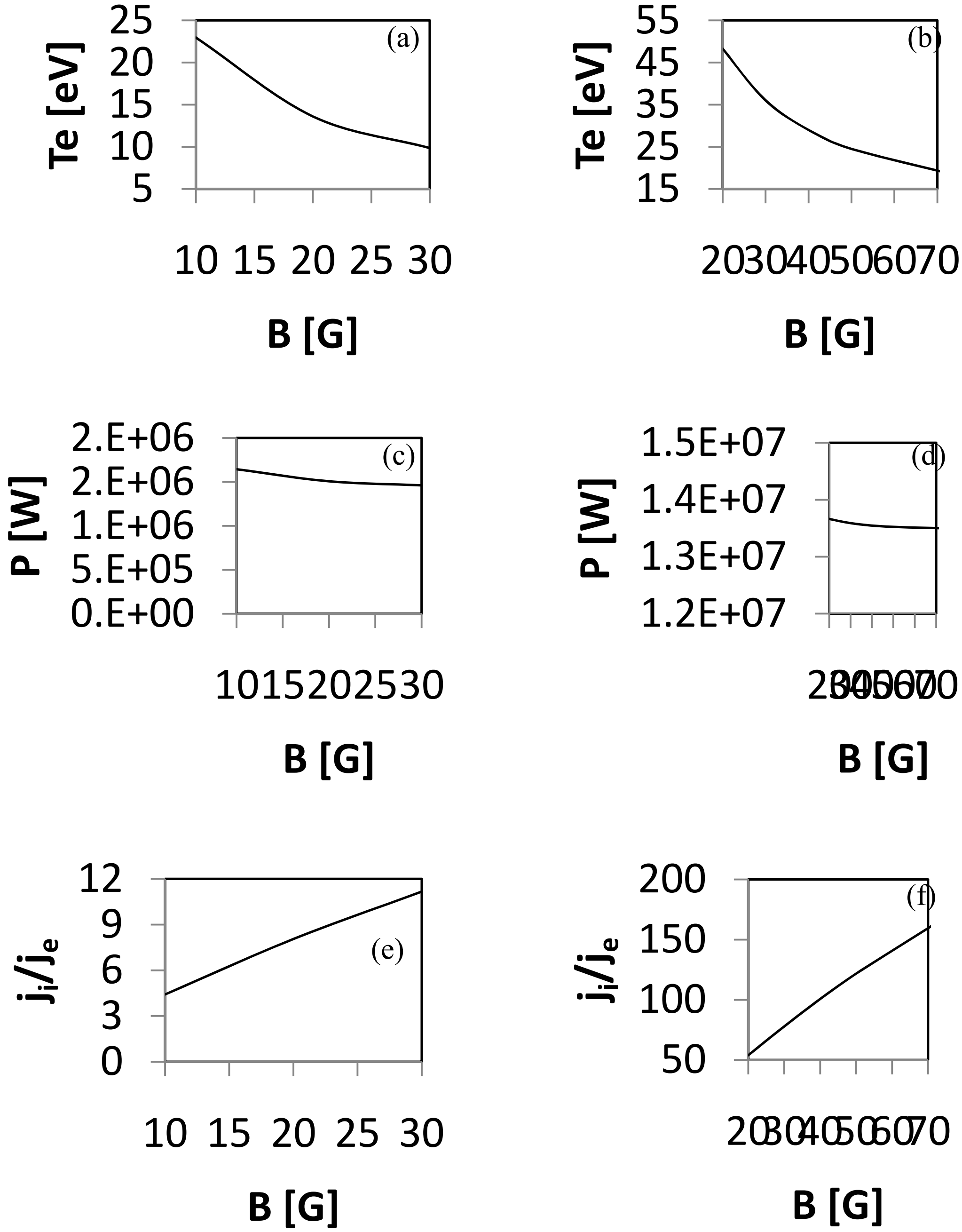

Figure 6. Parameters of the air breathing thruster for H = 90 km: plots (a), (c) and (e) correspond to $\varphi = 3\,kV$ and plots (b), (d) and (f) to $\varphi = 30\,kV$.

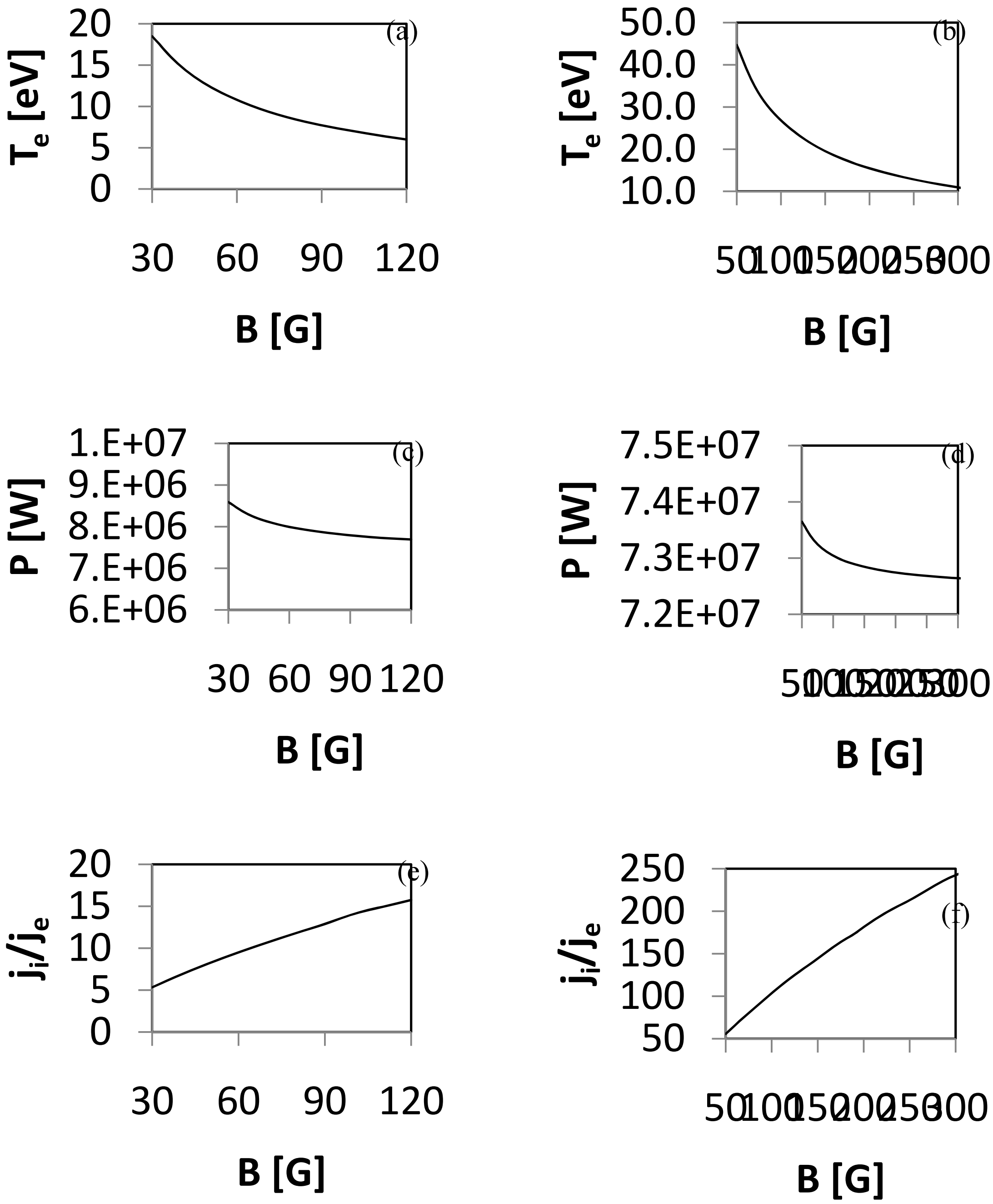

Figure 7. Parameters of the air breathing thruster for H = 80 km: plots (a), (c) and (e) correspond to $\varphi = 3\,kV$ and plots (b), (d) and (f) to $\varphi = 30\,kV$.

IV. Conclusions

The idea of using air breathing electrical propulsion for a vehicle flying at orbital speed on the edge of Earth's atmosphere has been examined for a thruster based on the Hall Effect. We have shown that, conceptually, such a thruster can indeed work effectively at the orbits 80 – 90 km, producing tremendous thrust. We believe that by changing the thruster geometry, we could find reasonable conditions allowing increasing the flying orbit of an air breathing satellite up to 100 – 110 km. It should be stressed that in the model we have assumed that the plume is fully ionized, corresponding to the maximum possible thrust and power. However, if we soften this condition, assuming that the plume can be only partially ionized, the parameters of the thruster become less "extreme" and may lead to use of the thruster at higher orbital altitude.

In the future we are planning to simulate the work of air breathing Hall Effect thrusters at orbital speed to numerically verify the estimations obtained in this paper.

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