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# Methodology for Variable Fidelity Multistage Optimization Under Uncertainty

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## Nomenclature

| | |
|---|---|
| $\mathbf{x}$ | = Set of decision variables |
| $\mathbf{x}^{(j)}$ | = Decision (design) variables for $j^{th}$ stage |
| $x_i$ | = $i^{th}$ member of decision variable set |
| $\hat{\boldsymbol{x}}^{(j)}$ | = Realizations of $j^{th}$ stage decision variables |
| $\mathbf{x}^{(j)*}$ | = Expected values for $j^{th}$ stage decision variables at stages prior to $j$ |
| $\boldsymbol{\xi}$ | = Set of uncertain parameters |
| $\boldsymbol{\xi}^{(j)}$ | = Set of uncertain parameters to be observed between stages $j$ and $(j+1)$ |
| $\boldsymbol{\xi}^{(j)\prime}$ | = Modification to $j^{th}$ stage uncertainty with additional information |
| $\xi_i$ | = $i^{th}$ member of uncertain parameter set |
| $\hat{\boldsymbol{\xi}}^{(j)}$ | = Realizations of $j^{th}$ stage uncertain parameters |
| $\mathcal{N}(\mu, \sigma^2)$ | = Normal distribution with mean $\mu$ and a standard deviation $\sigma$ |
| $f$ | = Objective function |
| $g$ | = Constraint function |
| $\mathbb{E}[\cdot]$ | = Expectation operator |
| $\mathbb{P}[\cdot]$ | = Probability |
| $\alpha$ | = Reliability-Based Design Optimization (RBDO) feasibility target |
| $\alpha^{(j)}$ | = Multistage RBDO feasibility target at $j^{th}$ decision stage |
| $I_{sp}$ | = Specific impulse |

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## I. Introduction

Until relatively recently, optimization processes and methods for complex systems were primarily focused on maximizing a system's individual parameters of performance. However, in the modern Department of Defense (DoD) acquisition environment, the research and development of complex engineering designs, such as aircraft and spacecraft emphasizes system optimization for high reliability, ease of maintainability, and reduced risk of cost/schedule growth, while attempting to quantify the inherent uncertainty present in developing complex systems. Modern optimization research in the areas of Optimization Under Uncertainty (OUU) and Reliability-Based Design Optimization (RBDO) holds the promise of enabling these types of system optimizations.

Ensuring system-level optimization of these non-performance-related system attributes requires the integration of advanced numerical methods into the design and analysis processes to improve both the results and the efficiency of these acquisition processes. For high-cost complex systems that require years of development before reaching production, the incorporation of uncertainty methods into the design, analysis, and optimization processes promises significant payoffs. However, broad implementation of uncertainty methods poses unique challenges in the context of analysis of alternatives and technology development/acquisition. The goal of this research was to identify the key optimization challenges, formulate approaches for overcoming them, and create a new methodology for variable-fidelity multistage optimization under uncertainty of representative engineering design problems. The objective of this paper is to present the problem formulations that the new method will be required to solve, a review of previous optimization under uncertainty research, the approach that was selected for the new method, initial test results, and a summary of the engineering application to which the final methodology will be applied.

### A. System Engineering and Acquisition of Complex Systems

In order to identify possible areas where current optimization methods fall short, a review of system engineering practices within DoD acquisition was performed. The system acquisition timeline in DoD is generally divided into several phases. The activities, documentation, and decisions which occur during each of the phases are fairly consistent, even when the specific details of the different systems are not. The different timeline phases of a typical DoD system acquisition can be seen in Figure 1.

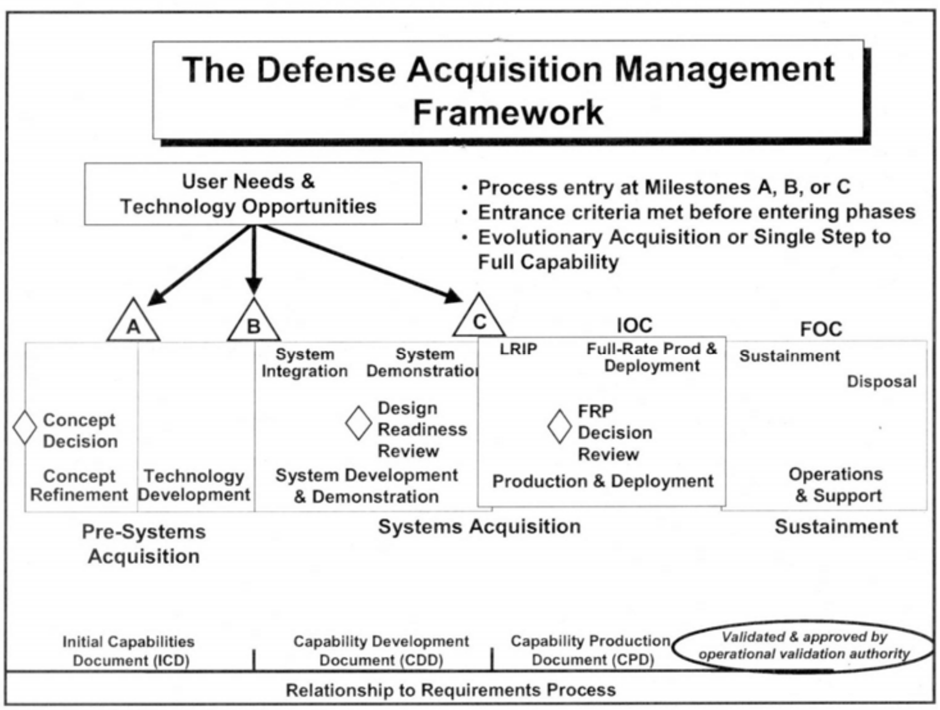


**Figure 1. DoD system acquisition phases[1]**

Depending on technology maturity, an official acquisition program-of-record may start at milestones A, B, or C shown in the figure. However, in this research we will consider the acquisition of a new system capability to typically start in the 1st phase, Concept Refinement/Concept Decision. This is also known as pre-milestone A. During this phase several potential technical solutions which may be capable of providing the desired capability are

identified, and early conceptual designs are generated. Trade studies are performed to understand the capabilities, limitations, and approximate acquisition costs for each of the candidate systems. If one or more of the concepts appear capable of providing the desired capability, preparation for a more rigorous Analysis of Alternatives (AOA) is initiated to identify system requirements, key performance parameters, and the modeling approaches/tools needed to evaluate the candidates objectively. At this point an AOA is authorized, performed, and the results are documented for presentation to the Milestone Decision Authority (MDA). The MDA makes the official decision on whether to go forward with the potential acquisition into the next phase, in this case, Technology Development. During this phase the key technologies that are on the critical path to develop the leading or alternative concept systems are funded for development in order to mature them to a point at which a go/no-go decision can be made. Depending on what happens during this phase one or more of the leading system concepts may be authorized to proceed into the next acquisition phase, System Development and Demonstration. During this phase, systems such as x-planes and/or subsystem demonstrators are built and tested to demonstrate the key technologies and possibly down-select to a single system concept to acquire. During these first three phases, engineering design models of various levels of fidelity (uncertainty) for each candidate system are exercised. For the purposes of this research, we use the term fidelity to describe how well the system models capture the key performance/cost characteristics of the concept. Our current definition of fidelity encompasses both fidelity and resolution, as they are described in Figure 2.

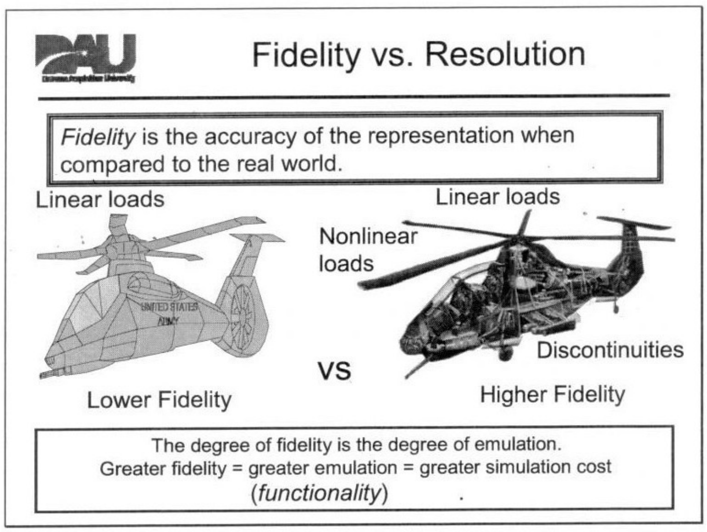


**Figure 2. Model fidelity[2]**

During each of the acquisition phases, a formal System Engineering Process is followed, which consists of a functional decomposition followed by a system synthesis. Together these legs are known as the system engineering V, due to the logic flow diagram which describes the process, as shown in Figure 3. Internal design/analysis iterations can be seen in the figure to take place where the recursive arrows are shown labeled “trades” and “analyze”. Typically as a design shows promise at earlier phases, it receives higher fidelity analysis and additional design iterations at the subsequent phases of Preliminary Design and Detailed Design, as represented by the 2nd and 3rd development phases of Figure 1. During these subsequent phases, the concept design is completed to deeper and deeper levels of the system work breakdown structure. Conceptual designs may only look at top level specifics of the major subsystem design, and iterative design cycles go deeper until all components have complete detailed designs. In order to use high fidelity analysis tools on a concept’s design, the specific design variable values of the design must be appropriately detailed. As a result, during concept comparison, a balance must be maintained during design iterations between design detail and tool fidelity, in order to answer the relevant questions in the shortest time, at the least cost necessary to efficiently screen out the inferior designs/concepts. The importance of this balancing act can be illustrated by the breadth of designs considered during the Republic of Korea Air Force acquisition of their most recent trainer aircraft, the T-50, shown in Figure 4.

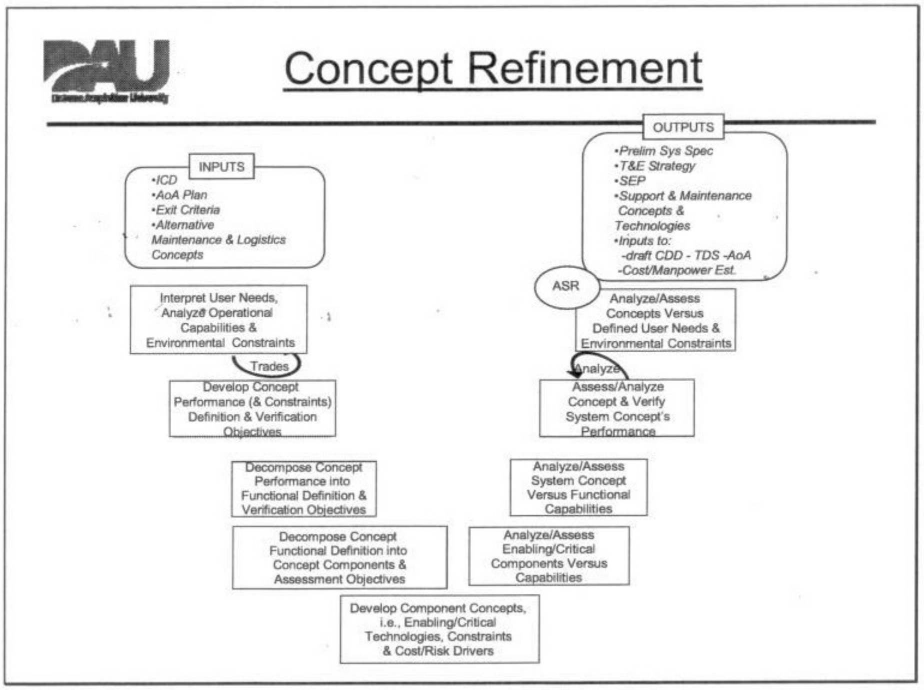


**Figure 3. System engineering process during concept refinement phase[2]**

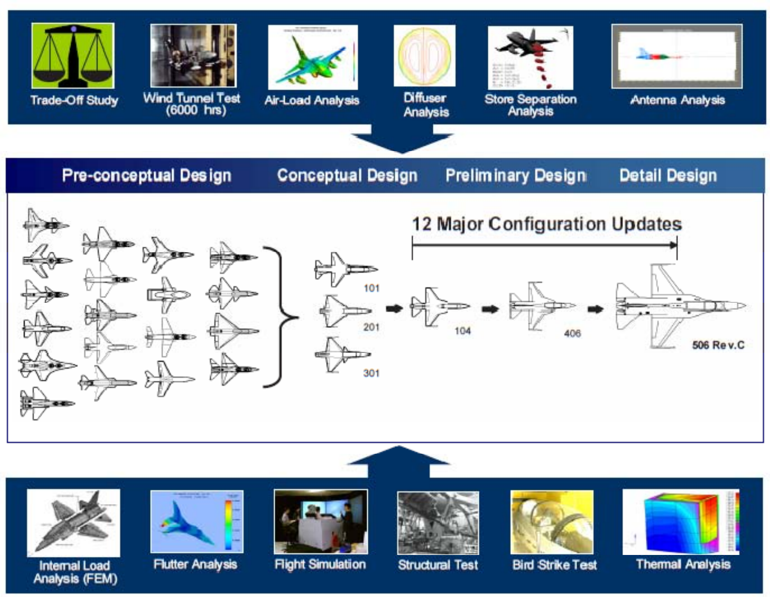


**Figure 4. T-50 design evolution during acquisition[3]**

## B. Specific Challenges for OUU Methods

Current engineering practice during the early analysis-of-alternatives and technology development phases of a system acquisition, as described previously, pose several specific challenges for any candidate OUU method that would be utilized for optimization assessments. The most important of these challenges were viewed as the baseline capability requirements for the envisioned multi-stage OUU algorithm.

1. Presence of design variables of mixed types-continuous, discrete, and categorical
2. Multiple objective functions-often competing
3. Uncertainty in optimization parameters of both aleatory and epistemic types
4. Uncertainties present at the input design variables level, intermediate model parameters level, and output level within an integrated system model
5. Uncertainties present in externally imposed system constraint function and in system performance requirements, implemented as threshold/objective values for the KPPs
6. Design problems that rarely allow description of a unitary set of discrete equations solvable by traditional derivatives-based optimization codes
7. Decision-making practice that relies on several discrete time-based events (multiple stage decisions) when down selecting among technology alternatives
8. Uncertainties (represented here as probability distributions) in the modeled parameters that vary over time as the overall system design evolves, improved fidelity tools are used, and experimental data is incorporated into knowledge of the design

## Optimization Literature Review

A literature review was conducted of OUU methods applicable to complex system acquisition problems. A subset of these methodology papers appears particularly relevant, and forms a basis from which to develop a new methodology.

1. *Audet and Dennis: Mixed Variable Generalized Pattern Search (MGPS)* [4]

This work extended Torczon's Generalized Pattern Search (GPS) algorithm for deterministic optimization problems. Their algorithm extension was in the form of modifications to allow optimization of mixed variable programming problems containing continuous, discrete, and catagorical design variable types. In addition the authors created separate Poll and Search steps in the continuous variable algorithm to achieve global optimization and convergence efficiency. Lastly, an Extended Poll step was created to search the continuous variables of discrete neighbors that are close in objective value to the current incumbent design point.

2. *Sriver and Chrissis: Mixed Generalized Pattern Search with Ranking and Selection (MGPS-RS)* [5-6]

This work extended the Audet and Dennis GPS for Mixed Variables algorithm by adding a Ranking and Selection statistical selection algorithm for application to linearly constrained stochastic optimization problems. This created the MGPS-RS framework in which "iterates converge almost surely to limit points that stationary point conditions …over a mixed variable domain." It also incorporated a surrogate function which utilized previous samples to generate an efficient surrogate function to improve convergence time of the algorithm.

3. *Walston: Stochastic Multi-Objective Mesh Adaptive Direct Search (SMOMADS)* [7]

This work in turn extended Sriver and Chrissis's MGPS-RS with the addition of the Mesh Adaptive Direct Search (MADS) method from Abramson, Audet, and Dennis to handle problems with nonlinear constraints. This implementation also utilized user interactive aspiration/reservation levels as well as scalarization techniques to enable stochastic optimization for multi-objective functions. An algorithm was included for multi-objective optimal computing budget allocation (MOCBA). The implementation was tested on several problems with aleatory uncertainty (stochastic noise) added to the objective functions.

4. *Romero and Chen: Spatially Correlated Uncertainty and Continuous Variable Ordinal Optimization* [8-10]

Romero and Chen developed an algorithm implementation to apply to single objective continuous variable OUU problems. This work developed several interesting features. The implementation utilized an adaptive ordinal optimization method combined with Coordinate Pattern Search (CPS) method. In order to significantly improve convergence efficiency, they combined three unique features: spatially-correlated uncertainty sampling, usage of threshold and objective values in the response function, and the Point of First Separation (PFS) truncation of iterations. The method implementation was shown for an engineering application with uncertainty present in input parameters that was propagated into the objective function.

*5. Nam and Mavris: Multi-Stage Reliability-Based Design Optimization (MSRBDO)* [3, 11]

These authors compared the qualities of Reliability-Based Design Optimization (RBDO) and Stochastic Programming with Recourse (SPwR) to engineering practice in acquisition. As a result of this comparison they proposed an algorithm with characteristics of both. The goal is optimal selection of first stage design variables amongst alternatives under the anticipation of reductions in uncertainty at defined future stages. As a result the proposed algorithm would incorporate a finite number of planned stages with varying levels of design maturity and freedom. It also incorporated delayed final decisions using periodic down selects instead of a single up-front optimal selection. The authors tested an initial algorithm on a single objective Two Stage RBDO (TSRBDO) example problem with uncertain parameters present in the problem's constraint function. The authors suggest that MSRBDO

# II. Methodology Discussion

## A. Variable Fidelity Multi-Stage Optimization Under Uncertainty Methodology

After the review of the applicable research literature, five key capabilities for the MS-OUU method to implement were identified.

1. Efficient algorithms for handling large numbers of design parameters
2. Multi-objective function capable
3. An ability to optimize mixed-variable problems
4. Capability to process varying fidelity in a particular parameter between the optimization stages
5. Ability to accommodate threshold and objective constraints for key performance parameters

Table 1 below shows the latter optimization methods discussed in the previous section against the five key needs for the proposed MS-OUU method.

**Table 1. Key desired characteristics vs. previous methods**

| CHARACTERISTICS | Walston | Romero & Chen | Nam & Mavris |
|---|---|---|---|
| **Multi-objective problems** | **YES** | **NO** | **NO** |
| **Threshold/objective levels** | * | ** | **NO** |
| **Multi-stage uncertainty (epistemic)** | **NO** | **NO** | **YES** |
| **Mixed variables** | **YES** | **NO** | **NO** |
| **Efficiency-driven** | **NO** | **YES** | **NO** |
| **Location of uncertainty** | **Objective (Response) Function** | **Input Parameters (Propagated into Response Function** | **Constraint Functions** |
| | ***Use of Reservation/Aspiration levels in scalarization equation to bound Pareto frontier limits of interactive search*** | *****Use of threshold/objective values in ordinal ranking to identify Point of First Separation to eliminate lowest performing design point candidate*** | |

## B. Initial Test Problem

The initial implementation of the proposed method is an expansion of MS-RBDO to allow the change of uncertain parameters with time, exercised in a modification of the Nam-Mavris test problem used in Ref. 11. Expressing the test problem's modification from a pure deterministic optimization one may be useful here. The initial deterministic problem can be expressed as:

$$\min_{\boldsymbol{x}} f(x_1, x_2, x_2) = {x_1}^2 + {x_2}^2 + {x_3}^2 \tag{1}$$

$$subject\ to\ \xi_1 x_1 + \xi_2 x_2 + x_3 \geq 5 \tag{2}$$

where $(x_1,x_2) \in [0,\infty)^2$ , $x_3 \in [0,2]$, and here $\boldsymbol{\xi}$ represents the constraint function constants $\xi_1$=2 and $\xi_2$=3. Reformulating this, with $\boldsymbol{\xi}$ now representing the set of epistemic uncertainty parameters, specifically $\xi_1 \sim \mathcal{N}(2, 1^2)$ and $\xi_2 \sim \mathcal{N}(3, 1^2)$ , equations (2) & (3) represent a classic RBDO problem.

$$\min_{\boldsymbol{x}} f(x_1, x_2, x_2) = {x_1}^2 + {x_2}^2 + {x_3}^2 \tag{3}$$

$$s.t. \mathbb{P}[\xi_1 x_1 + \xi_2 x_2 + x_3 \geq 5] \geq \alpha \tag{4}$$

Nam and Mavris then split the problem into two sequential design decision stages, with the design variables partitioned into 1st stage decisions, $\mathbf{x}^{(1)} = (x_1,x_2)$, and 2nd stage decisions, $\mathbf{x}^{(2)} = (x_3)$. Similarly, the uncertain variables were partitioned into 1st stage realized uncertain parameters, $\boldsymbol{\xi}^{(1)} = (\xi_1)$, and 2nd stage realized parameters, $\boldsymbol{\xi}^{(2)} = (\xi_2)$. This set up two sequential optimization problems. The first decision stage selects the optimal members of $\mathbf{x}^{(1)}$, which is followed by a realization of the values for $\boldsymbol{\xi}^{(1)} \rightarrow \hat{\boldsymbol{\xi}}^{(1)}$. The optimal second stage decision variable then selected, followed by the realization of the values for the parameters in set $\boldsymbol{\xi}^{(2)}$. The realization of the final uncertainty parameter yields the final value of the objective function, and whether the constraint function inequality was met. The 1st stage optimization is expressed in Eqns. (5) and (6).

$$\min_{\boldsymbol{x}^{(1)}} \mathbb{E}\left[f\left(\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)*}, \boldsymbol{\xi}^{(1)}, \boldsymbol{\xi}^{(2)}\right)\right] = {x_1}^2 + {x_2}^2 + {x_3}^2 \tag{5}$$

$$s.t. \mathbb{P}[\xi_1 x_1 + \xi_2 x_2 + x_3 \geq 5] \geq \alpha^{(1)} \tag{6}$$

Then the 2nd stage optimization is represented by Eqns. (7) and (8).

$$\min_{\boldsymbol{x}^{(2)}} \mathbb{E}\left[f\left(\hat{\boldsymbol{x}}^{(1)}, \boldsymbol{x}^{(2)}, \hat{\boldsymbol{\xi}}^{(1)}, \boldsymbol{\xi}^{(2)}\right)\right] = {x_1}^2 + {x_2}^2 + {x_3}^2 \tag{7}$$

$$s.t. \mathbb{P}[\hat{\xi}_1 x_1 + \xi_2 x_2 + x_3 \geq 5] \geq \alpha^{(2)} \tag{8}$$

To this point, the problem formulations reflect a process where epistemic uncertainty is reduced during a single stage to a deterministic value. The authors of Ref. 11 discuss the possibility that during a decision stage an epistemic uncertainty parameter could realize a reduction in its uncertainty, though this wasn't further examined. In addition, they discussed the possibility that a realized uncertainty during an early stage could preclude the selection of later stage design variables which still meet the feasibility targets for that stage. In that case, they discuss the addition of logic during later stage optimization which would maximize the feasibility of meeting the constraint function in lieu of stopping the problem. This appears to be a rational approach which would enable a decision maker to reexamine the problem (acquisition) constraints, although the additional logic for the 2nd stage optimization was eliminated here for the sake of brevity. In any case, here an addition is necessary to denote a modified uncertainty parameter, shown here as $\boldsymbol{\xi}^{(2)'}$ . The modified 2nd stage problem is now reflected by Eqns. (9) and (10), where ${\xi_2}' \sim \mathcal{N}(3, 0.5^2)$.

$$\min_{\boldsymbol{x}^{(2)}} \mathbb{E}\left[f\left(\hat{\boldsymbol{x}}^{(1)}, \boldsymbol{x}^{(2)}, \hat{\boldsymbol{\xi}}^{(1)}, \boldsymbol{\xi}^{(2)'}\right)\right] = {x_1}^2 + {x_2}^2 + {x_3}^2 \tag{9}$$

$$s.t. \mathbb{P}[\hat{\xi}_1 x_1 + {\xi_2}' x_2 + x_3 \geq 5] \geq \alpha^{(2)} \tag{10}$$

Generally, we presume that for this research, epistemic uncertainty parameters will undergo reductions in uncertainty as measured by decreasing standard deviation, σ, for the symmetric unbounded probability distribution functions.

## C. An Engineering Application Problem

An engineering application problem has been selected to exercise the optimization method. It is important that the engineering design space for the optimization problem should include technology alternatives that are not all at the same initial level of uncertainty (or fidelity). During the prelude to most acquisitions, alternative technology solutions are proposed which often vary markedly in the levels of uncertainty in the technology. The problem selected for the application of the new optimization methodology is a Single Stage To Orbit (SSTO) expendable launch vehicle (ELV). Three different nozzle technologies of widely differing Technology Readiness Levels (TRL) have been incorporated into the vehicle design space and will provide the primary exercise of the variable fidelity optimization portion of the code.

SSTO vehicles have been discussed almost exclusively in the context of reusable launch vehicles (RLV). There is very little discussion in recent literature of SSTO designs which are expendable. In the light of the current demand-constrained launch market, an engineering design study of such a vehicle appears timely. When compared to an RLV, an SSTO ELV design enables achieving a higher propellant mass fraction for a vehicle, since no reentry or long duration in-space support systems are required. To put the relative technology needs for a SSTO vehicle into context, it is useful to examine the ELV propellant mass fractions of the 1$^{st}$ stages for selected historical launch vehicles, which are shown Figure 5.

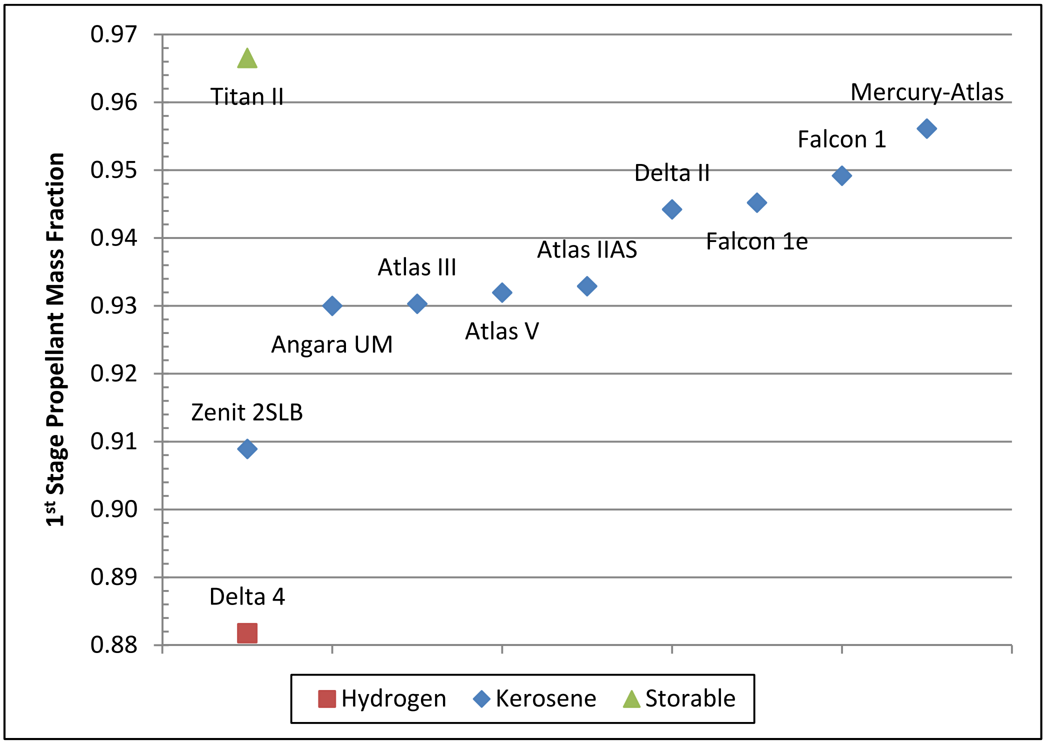


Figure 5. Historical 1st stage propellant mass fractions of TSTO vehicles

One issue that nominally could hamper utilization of an expendable SSTO launch architecture is the question of how to provide in-space propulsion for a wide range of orbital destinations. One benefit of modern multistage ELV's is that the upper stages of the vehicles, such as the Atlas Centaur, are excellent orbit transfer systems for moving the payload from a Low Earth Orbit (LEO) parking orbit to a geosynchronous transfer orbit (GTO) or from a GTO into the destination geosynchronous/geostationary orbit. Given the recent and continuing developments of multiple unmanned vehicle systems for International Space Station (ISS) cargo resupply such as the European ATV, the Japanese HTV, and the commercial Cygnus and Dragon systems in the U.S., and the resurgence of the concept of propellant depots in space, it seems reasonable to assume the future availability of reusable orbit transfer vehicles (OTV). Recently, MDA Corp and Intelsat announced a contract to demonstrate the GEO refueling of an Intelsat communication satellite with 1,000 kg of propellant by a MDA Space Infrastructure Services (SIS) robotic servicer

vehicle. The future realization of such an orbital servicer capability could provide the vital link in a launch architecture which includes SSTO ELV payload delivery to a short duration LEO parking orbit.

The launch vehicle thrust and performance requirements for the selected design reference mission have been initially identified by using the rocket equation and a nominal design payload class of 3,000 lbs, along with the estimation that the parking orbit requires the launch vehicle to deliver an ideal velocity change of 30,000 ft/sec. The resulting vehicle Gross Lift-Off Weights (GLOW) for several different values of vehicle propellant mass fraction and mission-effective $I_{sp}$ are shown in Figure 6. Plotted on the same chart for comparison are the values of mission average $I_{sp}$ vs. liftoff weight for the 1$^{st}$ stages of several historical ELVs. The 1$^{st}$ stage weights of the historical vehicles do not include the weights of any upper stages or payloads, and the 1$^{st}$ stage mission averaged $I_{sp}$ is

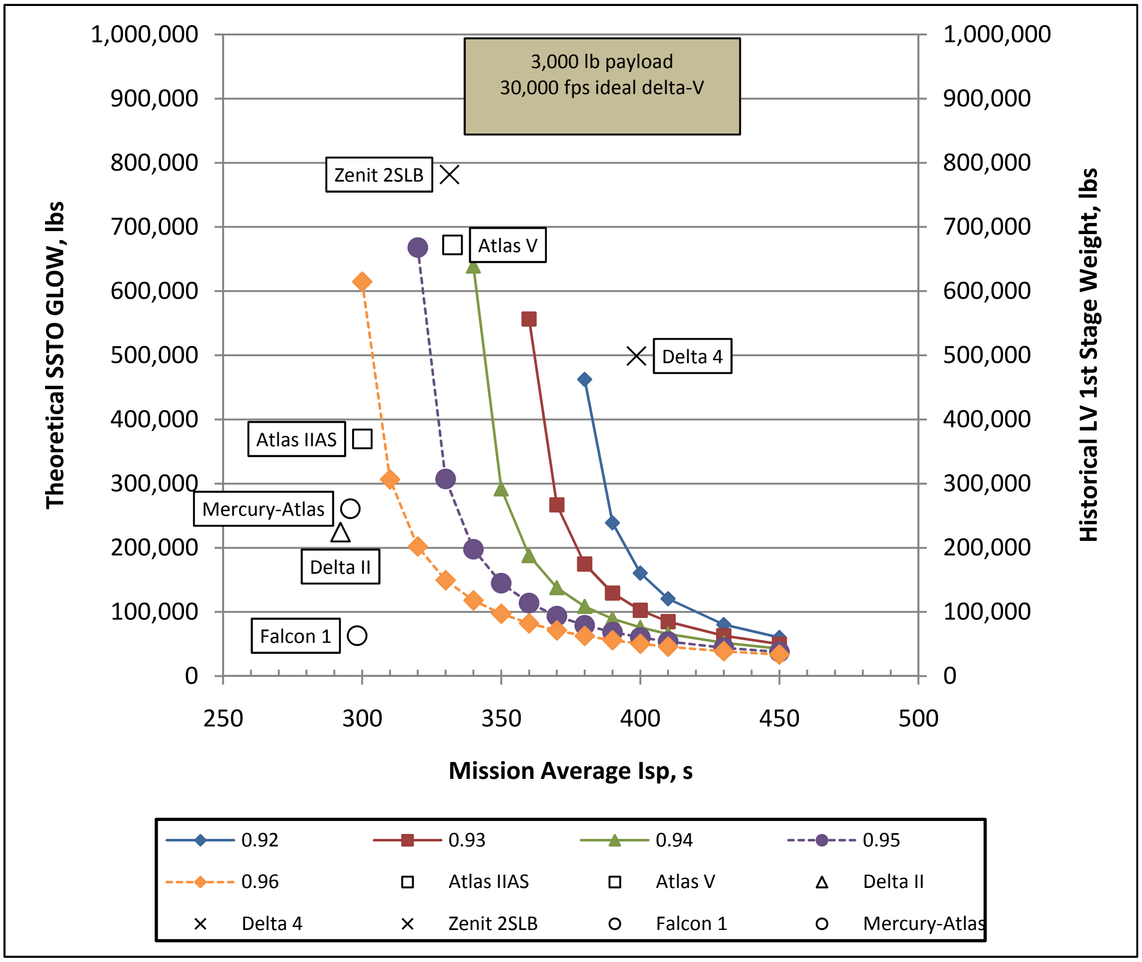


Figure 6. SSTO Mission effective $I_{sp}$ vs. GLOW

calculated by a weighted average of sea-level and vacuum $I_{sp}$, with weighting factors of 20% and 80% respectively. The rapid growth in the required GLOW with either a decrease in mission average $I_{sp}$, or a decrease in the vehicle's achieved propellant mass fraction illustrates an important need in quantifying the performance uncertainties in such a launch concept.

## D. Results

The results of the initial MS-OUU methodology when applied to the modified Nam-Mavris test problem will be presented in the final version of the paper.

## III. Future Work

In the near future, the authors intend to further develop the initial method to allow application to multiple objective optimization problems similar to those examined in Ref. 7. To address the slow convergence characteristics of Walston's method, the ordinal ranking and spatially correlated sampling approaches of Romero and Chen is being modified for application to the Ranking & Selection components. The resulting method will be benchmarked against several test problems previously exercised in our reviewed research. The test problems have been nominally selected and their characteristics are listed in Table 2. The results of the benchmarking exercise will be used to identify any desired final changes to the existing code to increase the convergence efficiency. The final method will then be applied to the SSTO launch vehicle engineering application described previously.

Some additional work may be necessary to enable application of the method to a broader range of typical space/missile vehicle engineering and acquisition problems, since the lead author plans to deploy the new methodology at his organization.

**Table 2. Test problem characteristics**

| Problem | Min/ max | Design variables | Number of objective functions | Type of objective functions | Constraint functions | Equality constraint functions | Linear constraint functions | Design variable bounds | Notes |
|---|---|---|---|---|---|---|---|---|---|
| Poloni | max | 2 | 2 | nonlinear | 4 | 0 | 4 | 4 | Complex Pareto frontier |
| Dias Γ2 | min | 30 | 2 | linear & nonlinear | 60 | 0 | 60 | 60 | Concave Pareto frontier, large # of design variables |
| Fonseca F1 | min | 2 | 2 | nonlinear | 4 | 0 | 4 | 4 | Fold in objective space, concave |
| DTLZ7 | min | 2 | 2 | linear & nonlinear | 4 | 0 | 4 | 4 | Separate dispersed convex Pareto frontier |
| Disk Brake | max | 4 | 2 | nonlinear | 13 | 0 | 10 | 8 | Mixed variable, convex Pareto frontier |
| Nam-Mavris | min | 2 | 1 | nonlinear | 5 | 0 | 5 | 4 | RBDO, MSRBDO |

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# Methodology for Variable Fidelity Multistage Optimization Under Uncertainty (MS-OUU)

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# Variable Fidelity MS-OUU
## Outline

- Research Objective
- Overview
  - DoD System Acquisition Phases
  - Key Optimization Challenges
  - Previous Research
- Capability Gaps
- Proposed Multistage Methodology
- Future Work
- Summary

# Variable Fidelity MS-OUU

## Research Objective

- **Conducting Ph. D. Dissertation Research on Multistage Optimization Under Uncertainty (MS-OUU)**
  - **Early phases of Department of Defense complex system acquisition would appear to be able to be profit from an application of Optimization Under Uncertainty methods**
  - **How can we adapt current methods to these early phases of system acquisition?**
    - **Much of research originates in different disciplines for solving different problems than those of early concept exploration and system acquisition**
      - **Operations Research**
      - **Detailed Structural Design**
      - **Stewardship of Well-Understood Strategic Systems**

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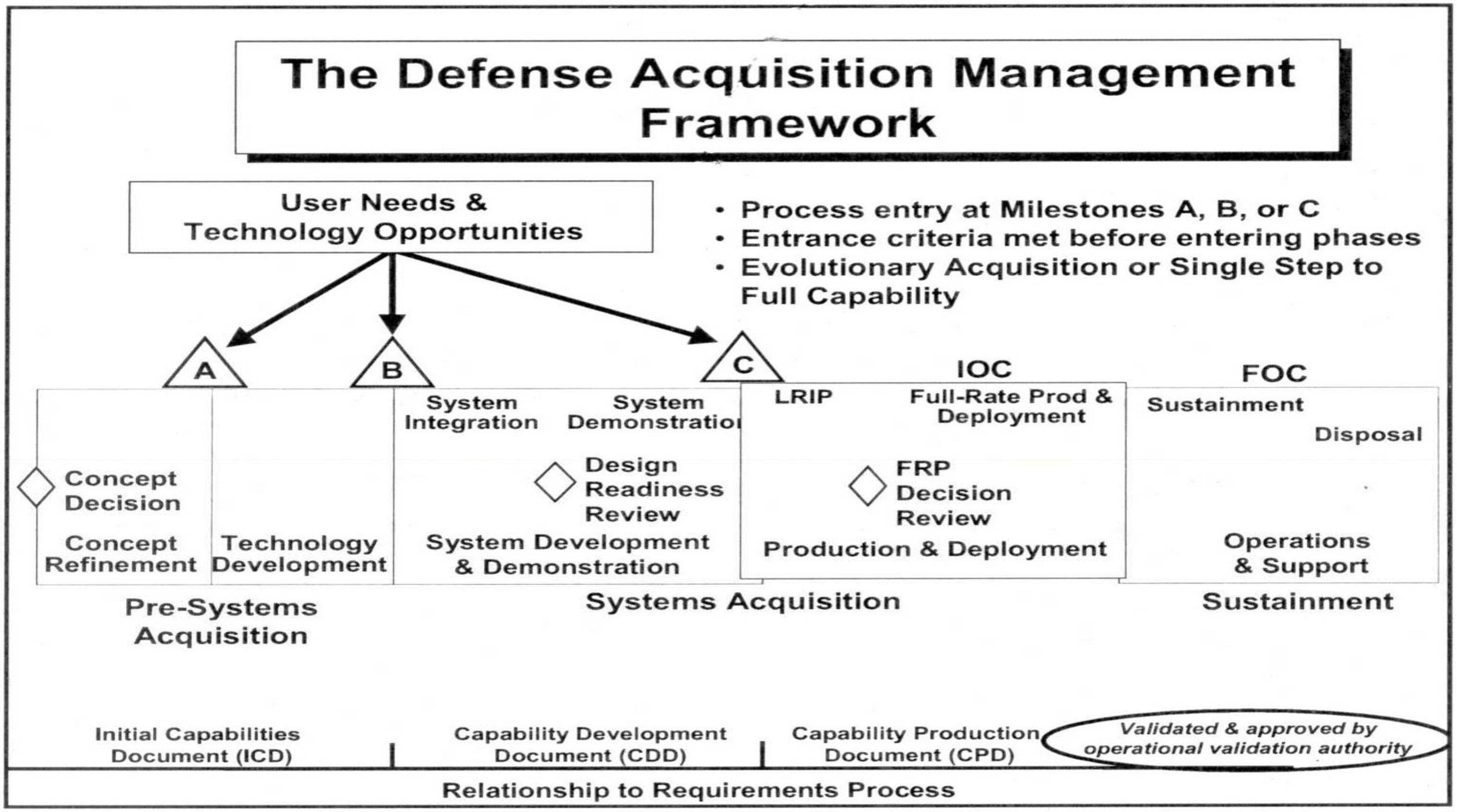
## Overview-DoD System Acquisition Phases



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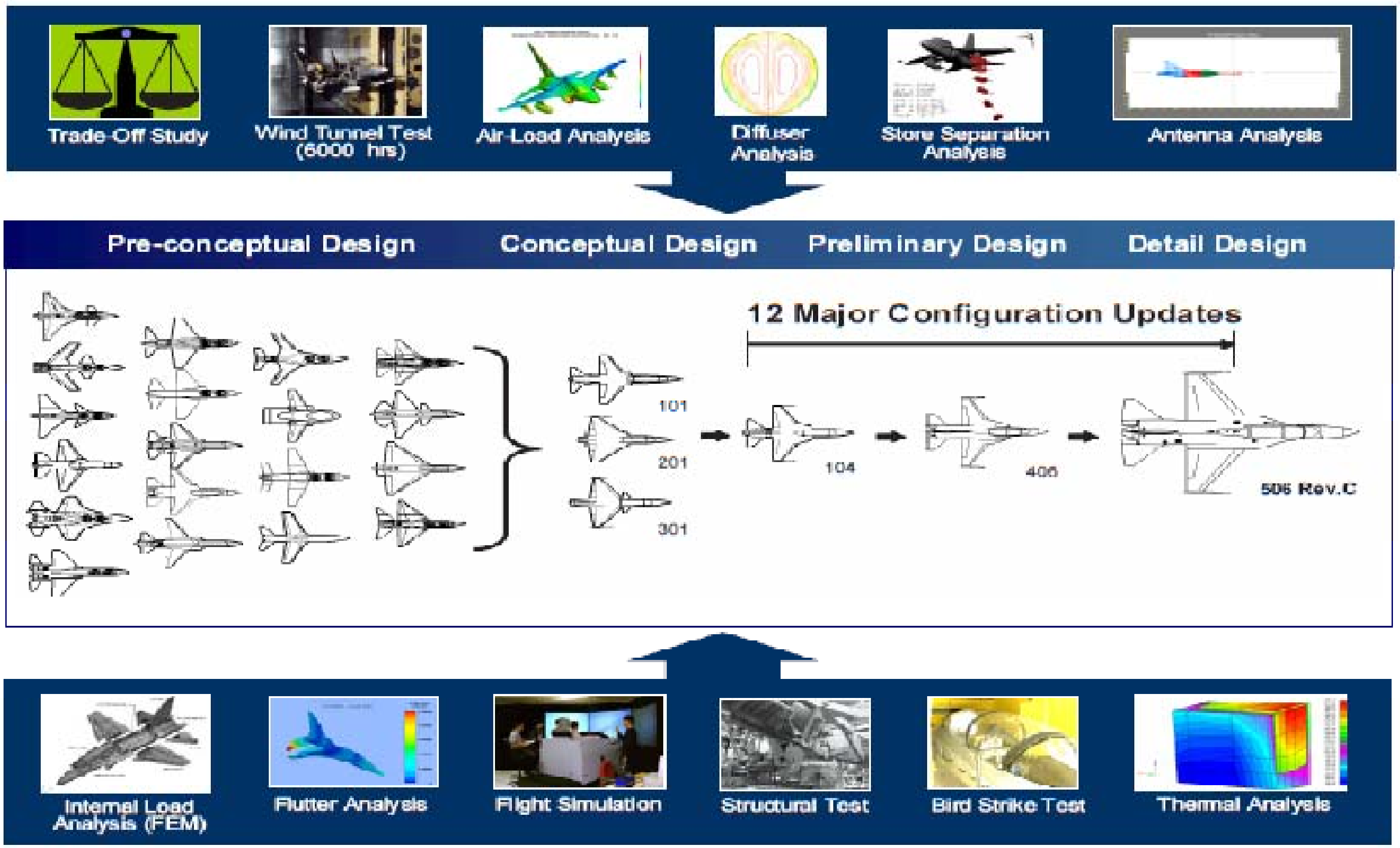
## Overview-System Acquisition M&S Burden



# Variable Fidelity MS-OUU
## Overview-Key Optimization Challenges

1) Presence of design variables of mixed types-continuous, discrete, and categorical
2) Multiple objective functions-often competing
3) Uncertainty in optimization parameters of both aleatory and epistemic types
4) Uncertainties present at the input design variables level, intermediate model parameters level, and output level within an integrated system model
5) Uncertainties present in externally imposed system constraint function and in system performance requirements, implemented as threshold/objective values for the KPPs
6) Design problems that rarely allow description of a unitary set of discrete equations solvable by traditional derivatives-based optimization codes
7) Decision-making practice that relies on several discrete time-based events (multiple-stage decisions) when down selecting among technology alternatives
8) Uncertainties in the modeled parameters that vary over time as the overall system design evolves, improved fidelity tools are used, and experimental data is incorporated into knowledge of the design

# Variable Fidelity MS-OUU

## Overview-Previous Research

1. **Audet and Dennis:** *Mixed Variable Generalized Pattern Search (MGPS)*
2. **Sriver and Chrissis:** *Mixed Generalized Pattern Search with Ranking and Selection (MGPS-RS)*
3. **Walston:** *Stochastic Multi-Objective Mesh Adaptive Direct Search (SMOMADS)*
4. **Romero and Chen:** *Spatially Correlated Uncertainty and Continuous Variable Ordinal Optimization*
5. **Nam and Mavris:** *Multi-Stage Reliability-Based Design Optimization (MSRBDO)*

# Variable Fidelity MS-OUU

## Overview-Previous Research

| CHARACTERISTICS | Walston | Romero & Chen | Nam & Mavris |
|---|---|---|---|
| Multi-objective problems | YES | NO | NO |
| Threshold/objective levels | * | ** | NO |
| Multi-stage uncertainty (epistemic) | NO | NO | YES |
| Mixed variables | YES | NO | NO |
| Efficiency-driven | NO | YES | NO |
| Location of uncertainty | Objective (Response) Function | Input Parameters (Propagated into Response Function | Constraint Functions |
| | **Use of Reservation/Aspiration levels in scalarization equation to bound Pareto frontier limits of interactive search* | ***Use of threshold/objective values in ordinal ranking to identify Point of First Separation to eliminate lowest performing design point candidate* | |

# Variable Fidelity MS-OUU

## Capability Gaps of Previous Methods

After the review of the applicable research literature, five key capabilities for the MS-OUU method to implement were identified.

1. Efficient algorithms for handling large numbers of design parameters
2. Multi-objective function capable
3. An ability to optimize mixed-variable problems
4. Capability to process varying fidelity in a particular parameter between the optimization stages
5. Ability to accommodate threshold and objective constraints for key performance parameters

### Nomenclature

**x** = Set of decision variables
**$x^{(j)}$** = Decision (design) variables for $j^{th}$ stage
**$x_i$** = $i^{th}$ member of decision variable set
$\hat{x}^{(j)}$ = Realizations of $j^{th}$ stage decision variables
**$x^{(j)*}$** = Expected values for $j^{th}$ stage decision variables at stages prior to $j$
**ξ** = Set of uncertain parameters
**$\xi^{(j)}$** = Set of uncertain parameters to be observed between stages $j$ and ($j$+1)
**$\xi^{(j)\prime}$** = Modification to $j^{th}$ stage uncertainty with additional information
**$\xi_i$** = $i^{th}$ member of uncertain parameter set
$\hat{\xi}^{(j)}$ = Realizations of $j^{th}$ stage uncertain parameters

# Variable Fidelity MS-OUU
## Proposed Multistage Methodology

### Nomenclature continued

| | |
|---|---|
| $N(\mu, \sigma^2)$ | = Normal distribution with mean μ and a standard deviation σ |
| $f$ | = Objective function |
| $g$ | = Constraint function |
| E(-) | = Expectation operator |
| P(-) | = Probability |
| α | = Reliability-Based Design Optimization (RBDO) feasibility target |
| $\alpha^{(j)}$ | = Multistage RBDO feasibility target at $j^{th}$ decision stage |

# Variable Fidelity MS-OUU

## Proposed Multistage Methodology

**Problem Evolution: deterministic to multistage RBDO**

**Deterministic problem with $\xi_1$=2, $\xi_2$=3,**

**($x_1$,$x_2$) are elements in $[0,\infty)^2$, and $x_3$ is element in [0,2]**

$$\min_x f(x_1, x_2, x_3) = x_1^{\ 2} + x_2^{\ 2} + x_3^{\ 2} \qquad (1)$$

$$\text{s.t. } \xi_1 x_1 + \xi_2 x_2 + x_3 \geq 5 \qquad (2)$$

**RBDO problem with uncertain parameters $\xi_1 \sim N(2,1^2)$, $\xi_2 \sim N(3,1^2)$**

$$\min_x \mathrm{E}\left[f(x_1, x_2, x_3)\right] = x_1^{\ 2} + x_2^{\ 2} + x_3^{\ 2} \qquad (3)$$

$$\text{s.t. } \mathrm{P}\left[\xi_1 x_1 + \xi_2 x_2 + x_3 \geq 5\right] \geq \alpha \qquad (4)$$

# Variable Fidelity MS-OUU

## Proposed Multistage Methodology

**Problem Evolution continued**

**MSRBDO problem with uncertain parameters $\xi_1 \sim N(2,1^2)$, $\xi_2 \sim N(3,1^2)$; 1st stage optimization**

$$\min_{\vec{x}^{(1)}} \mathrm{E}\left[ f(\vec{x}_1^{(1)}, \vec{x}_2^{(2)*}, \vec{\xi}^{(1)}, \vec{\xi}^{(2)}) \right] = x_1^{\;2} + x_2^{\;2} + x_3^{\;2} \qquad (5)$$

$$\text{s.t. } \mathrm{P}\left[ \xi_1 x_1 + \xi_2 x_2 + x_3 \geq 5 \right] \geq \alpha^{(1)} \qquad (6)$$

**After realization of 1st stage uncertainty, and 1st stage decision variables, MSRBDO problem 2nd stage optimization**

$$\min_{\vec{x}^{(2)}} \mathrm{E}\left[ f(\hat{\vec{x}}_1^{(1)}, \vec{x}_2^{(2)}, \hat{\vec{\xi}}^{(1)}, \vec{\xi}^{(2)}) \right] = x_1^{\;2} + x_2^{\;2} + x_3^{\;2} \qquad (7)$$

$$\text{s.t. } \mathrm{P}\left[ \hat{\xi}_1 x_1 + \xi_2 x_2 + x_3 \geq 5 \right] \geq \alpha^{(2)} \qquad (8)$$

# Variable Fidelity MS-OUU

## Proposed Multistage Methodology

**Problem Evolution continued**

**MSRBDO problem with varying uncertainty. 1st stage optimization remains same as previous problem**

$$\min_{\vec{x}^{(1)}} \mathrm{E}\left[ f(\vec{x}_1^{(1)}, \vec{x}_2^{(2)*}, \vec{\xi}^{(1)}, \vec{\xi}^{(2)}) \right] = x_1^{\ 2} + x_2^{\ 2} + x_3^{\ 2} \qquad (5)$$

$$\text{s.t. } \mathrm{P}\left[ \xi_1 x_1 + \xi_2 x_2 + x_3 \geq 5 \right] \geq \alpha^{(1)} \qquad (6)$$

**However, now the 2nd stage uncertainty parameter was reevaluated, along with 1st stage realizations, and 2nd stage optimization:**

$$\min_{\vec{x}^{(2)}} \mathrm{E}\left[ f(\hat{\vec{x}}_1^{(1)}, \vec{x}_2^{(2)}, \hat{\vec{\xi}}^{(1)}, \vec{\xi}'^{(2)}) \right] = x_1^{\ 2} + x_2^{\ 2} + x_3^{\ 2} \qquad (9)$$

$$\text{s.t. } \mathrm{P}\left[ \hat{\xi}_1 x_1 + \xi_2' x_2 + x_3 \geq 5 \right] \geq \alpha^{(2)} \qquad (10)$$

# Variable Fidelity MS-OUU

## Future Work

- **Complete incorporation of the selected additional algorithms necessary to meet method’s requirements**
  - **Mixed generalized pattern search (linear constraints)**
  - **Mesh adaptive direct search (nonlinear constraints)**
  - **Ordinal ranking and selection**
  - **Spatially-correlated uncertainty and Point of First Separation iteration completion**
- **Apply the final method**
  - **Selected test problems (next slide)**
  - **SSTO ELV engineering application**

# Variable Fidelity MS-OUU
## Future Work-Test Problems

| Problem | Min/ max | Design variables | Number of objective functions | Type of objective functions | Constraint functions | Equality constraint functions | Linear constraint functions | Design variable bounds | Notes |
|---|---|---|---|---|---|---|---|---|---|
| Poloni | max | 2 | 2 | nonlinear | 4 | 0 | 4 | 4 | Complex Pareto frontier |
| Dias Γ2 | min | 30 | 2 | linear & nonlinear | 60 | 0 | 60 | 60 | Concave Pareto frontier, large # of design variables |
| Fonseca F1 | min | 2 | 2 | nonlinear | 4 | 0 | 4 | 4 | Fold in objective space, concave |
| DTLZ7 | min | 2 | 2 | linear & nonlinear | 4 | 0 | 4 | 4 | Separate dispersed convex Pareto frontier |
| Disk Brake | max | 4 | 2 | nonlinear | 13 | 0 | 10 | 8 | Mixed variable, convex Pareto frontier |
| Nam-Mavris | min | 2 | 1 | nonlinear | 5 | 0 | 5 | 4 | RBDO, MSRBDO |

# Variable Fidelity MS-OUU

## Summary

- **Research Objective**
  - Develop methodology for new Optimization Under Uncertainty (OUU) algorithm applicable to the early system acquisition phases involving uncertain technologies.
- **Investigated previous research on optimization under uncertainty methods**
- **Identified capability gaps and major requirements for a new method applicable to early system acquisition**
- **Tested the selected method for accommodating multiple decision stages and varying uncertainty**