# NAVAL POSTGRADUATE SCHOOL

## MONTEREY, CALIFORNIA

# THESIS

**THE BASIC UNDERWATER DEMOLITION/SEAL ACCESSION CALCULATOR MODEL: DETERMINING THE OPTIMAL NUMBER OF JUNIOR OFFICER ACCESSIONS TO MEET END-STRENGTH GOALS**

by

David A. Hooper

March 2011

Thesis Advisor:                         Rachel T. Silvestrini
Second Reader:                          W. Matthew Carlyle

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The mission of the Naval Special (NSW) community is to provide a versatile, responsive, and offensively focused force with continuous overseas presence in order to have strategic impact in missions that include special reconnaissance, direct action, unconventional warfare, and combating terrorism. Currently, the NSW community has large manpower gaps within the officer corps especially, at the Lieutenant Commander rank. This gap threatens the operational readiness of the NSW community, which in turn affects our national security. This thesis presents the development of the Basic Underwater Demolition/SEAL (BUD/S) Accession Calculator (BAC) which uses goal programming and Markov chain analysis to determine the optimal number of new accessions needed to enter the BUD/S training program to meet target end-strength goals for company grade ranks. By properly manning the junior ranks the Lieutenant Commander rank can be properly manned. The results demonstrate that the NSW community can closely meet target end-strength goals of 127 and 285 for Lieutenant Junior Grades and Lieutenants, respectively, with the 100 accessions to BUD/S every year. However, as the attrition rate fluctuates the number of accessions change. The most dramatic impact to BUD/S accession requirements is observed when attrition rate increases. Decrease in attrition rate show that small changes to accession requirements occur.

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**THE BASIC UNDERWATER DEMOLITION/SEAL ACCESSION
CALCULATOR MODEL: DETERMINING THE OPTIMAL NUMBER OF
JUNIOR OFFICER ACCESSIONS TO MEET END-STRENGTH GOALS**

David A. Hooper
Lieutenant Junior Grade, United States Navy
B.S., United States Naval Academy, 2006

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March 2011**

Author:          David A. Hooper

Approved by:     Rachel T. Silvestrini
                 Thesis Advisor

                 W. Matthew Carlyle
                 Second Reader

                 Robert F. Dell
                 Chair, Department of Operations Research

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# ABSTRACT

The mission of the Naval Special Warfare (NSW) community is to provide a versatile, responsive, and offensively focused force with continuous overseas presence in order to have strategic impact in missions that include special reconnaissance, direct action, unconventional warfare, and combating terrorism. Currently, the NSW community has large manpower gaps within the officer corps, especially at the Lieutenant Commander rank. This gap threatens the operational readiness of the NSW community, which in turn affects our national security. This thesis presents the development of the Basic Underwater Demolition/SEAL (BUD/S) Accession Calculator (BAC), which uses goal programming and Markov chain analysis to determine the optimal number of new accessions needed to enter the BUD/S training program to meet target-end-strength goals for company grade ranks. By properly manning the junior ranks, the Lieutenant Commander rank can be properly manned. The results demonstrate that the NSW community can closely meet target end-strength goals of 127 and 285 for Lieutenant Junior Grades and Lieutenants, respectively, with the 100 accessions to BUD/S every year. However, as the attrition rate fluctuates the number of accessions change. The most dramatic impact to BUD/S accession requirements is observed when attrition rate increases. Decreases in attrition rate show that small changes to accession requirements occur.

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# LIST OF ACRONYMS AND ABBREVIATIONS

| | |
|---|---|
| ADM | Admiral |
| BAC | BUD/S Accession Calculator |
| BUD/S | Basic Underwater Demolition/SEAL |
| CAPT | Captain |
| DoD | Department of Defense |
| ENS | Ensign |
| FY | Fiscal Year |
| GAMS | General Algebraic Modeling System |
| LATXFER | Lateral Transfer |
| LCDR | Lieutenant Commander |
| LT | Lieutenant |
| LTJG | Lieutenant Junior Grade |
| MILPRS | The Army Manpower Long-Range Planning System |
| NROTC | Naval Reserve Officers Training Corps |
| NSW | Naval Special Warfare |
| OA/BDM | Officer Accession Branch Detail |
| OCS | Officer Commissioning Source |
| OPA | Officer Programmed Authorizations |
| OPMS | Officer Personnel Management System |
| OPTEMPO | Operational Tempo |
| SEAL | Sea, Air, and Land |
| TACCOM | Total Army Competitive Category Optimization Model |
| TIG | Time in Grade |
| TPM | Transition Probability Matrix |
| U.S. NAVY | United States Navy |
| USN | United States Navy |
| USNA | United States Naval Academy |
| YCS | Years Commissioned Service |

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# EXECUTIVE SUMMARY

The mission of the Naval Special Warfare (NSW) community is to provide a versatile, responsive, and offensively focused force with continuous overseas presence in order to have strategic impact in missions that include special reconnaissance, direct action, unconventional warfare, and combating terrorism. Currently, the NSW community has large manpower gaps within the officer corps, especially at the O-4 rank. This gap threatens the operational readiness of the NSW community, which in turn affects our national security. This thesis presents the development of the Basic Underwater Demolition/Seal (BUD/S) Accession Calculator (BAC), which uses goal programming and Markov chain analysis to determine the optimal number of new accessions needed to enter the BUD/S training program to meet target end-strength goals for company grade ranks.

The NSW community plans to recruit 100 accessions every year into the BUD/S training program. The BUD/S training program trains recruits to become SEAL operators, and the junior officer who successfully complete BUD/S become fully qualified SEAL operators. This recruiting goal is believed to allow the NSW community to meet target end-strength goals of 127 and 285 for the ranks of Lieutenant Junior Grade (LTJG) and Lieutenant (LT), respectively, by Fiscal Year (FY17). Given this information, we developed the BAC and used it to determine the distribution of accessions by rank and commissioning source. Rank and commissioning source play a major part to meeting target end-strength goals because different accession sources have different success rates for the BUD/S training program. The BAC verifies that target end-strength goals can be met within 6 percent.

Historical data shows that attrition rates can change from year to year. In addition to studying the distribution of recruits, we developed the BAC to optimize for the number of accessions as well. This allows for the optimal number of accessions to change from the NSW standard of 100 per year. Four alternative scenarios were considered in this thesis. If the attrition rate is increased by 5 and 10 percent, the accessions required to

meet target end-strength goals are 245 and 282, respectively. In addition, if attrition is decreased by 5 and 10 percent, the required accessions needed are 193 and 175, respectively.

Target end-strength goals can be closely met with the 100 accessions that the NSW community is currently recruiting to BUD/S on a yearly basis. However, as attrition is increased or decreased, the number of accessions required to meet target end-strength goals in FY17 fluctuates.

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I would like to thank everybody who made this thesis possible. Most importantly, I would like to thank God who gave me the strength and focus to get this huge accomplishment done. He directly opened doors for me that I did not think were possible and for that I am extremely thankful!

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# I.    INTRODUCTION

## A.    BACKGROUND

Retaining sailors and meeting yearly target end-strengths has been a prominent issue the United States Navy (USN) has had to addresses over the years.  Retention is currently not an overall issue at the macro level today but some sub-communities within the Navy are facing retention and manpower issues (Cahill, 2010).  One of the communities in the USN that is currently suffering from a retention and manpower crisis is the Naval Special Warfare community (NSW), specifically, the Sea, Air, and Land (SEAL) sub-community (Evenson, 2010).  This manpower issue gets overlooked at first glance because the USN comprises 53,071 officers overall, but within the SEAL community there are only 674 officers (Cahill, 2010), which is approximately one percent of the total officer corps of the USN.  However, the NSW community is a major component of the USN and made up of Officer and Enlisted SEAL operators.  The main goal of the NSW community is to provide a versatile, responsive and offensively focused force with continuous overseas presence (U.S. Navy SEAL, 2011).  In addition, the NSW community is a tactical force with strategic impact in missions that include special reconnaissance, direct action, unconventional warfare, combating terrorism, foreign internal defense, information warfare, security assistance, counter-drug operations, personnel recovery and hydrographic reconnaissance (U.S. Navy SEAL, 2011).  These tactical and strategic missions are extremely crucial to the protection of our nation from both a defensive and security perspective.

The rise of low intensity conflicts and unconventional warfare is a cause of a high operational tempo (OPTEMPO) and demand for the NSW community.  This increased demand is the probable cause of retention becoming an even larger problem, especially, within the SEAL officer ranks in recent years.  The NSW community conducted a survey in 2010 to determine why junior officers were leaving the SEAL community.  The survey revealed that the top reason for leaving the navy was "time spent away from home" (Evenson, 2010).  In 2007, a separate survey revealed the number one reason for leaving the navy was "impact of Navy on family" (Evenson, 2010).  These retention issues in the

SEAL Officer community are prominent in the junior ranks of Ensign (ENS), Lieutenant Junior Grade (LTJG), and Lieutenant (LT), which results in large manpower gaps within the Lieutenant Commander (LCDR) ranks (Evenson, 2010). Pivotal operational leadership billets are held in the LCDR ranks, which the SEAL community is struggling to fulfill currently (D. Evenson, personal communication, December 15, 2010). Figure 1 shows the manpower gaps, per rank, in the SEAL community between current inventory and the officer programmed authorizations (OPA).
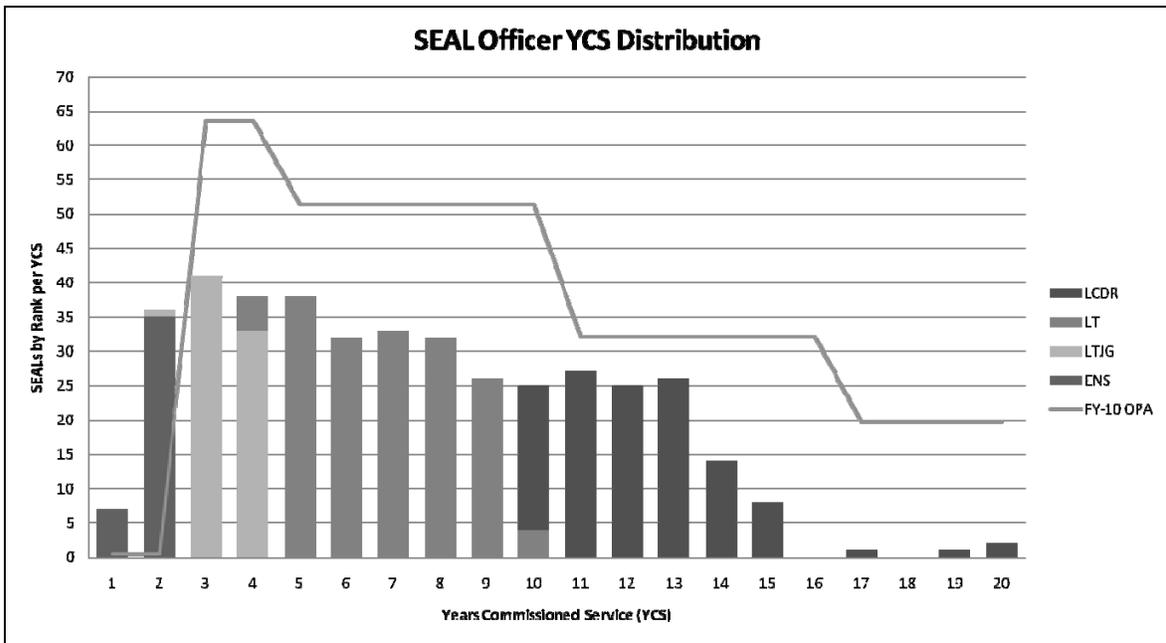


Figure 1.    Manpower Gaps Within SEAL Community

The solid line shows the OPA levels for each Years Commissioned Service (YCS) from 1990 to 2009. The *x*-axis represents YCS starting in 2009 and the *y*-axis is the number of SEALs. For example, in the 1999 year group that corresponds to 11 YCS, there are 27 LCDR SEALs in the community; however, the OPA requirement is 33 LCDR SEALs. The difference of six LCDR SEALs is a shortage for the 11 YCS time period. Figure 1 shows there are significant shortages in the LTJG (approximately 3–4 YCS), LT (approximately 5–9 YCS) and LCDR (approximately 10+ YCS) ranks.

The problem of differences on hand in SEAL officers with OPA requirements is of particular importance because it deals with operational readiness of the SEAL

community.  The LCDR rank is pivotal in the SEAL community because this is one of the main operational leadership billets.  A lack of LCDRs leading Troops within the SEAL teams, results in the NSW community suffering from a mission and operational readiness standpoint.

One way to grow the SEAL base is through accessions to the Basic Underwater Demolition/SEAL (BUD/S) training program. The BUD/S training program is a 6-month program that focuses on physical conditioning, small boat handling, diving physics, basic diving techniques, land warfare, weapons, demolitions, communications, and reconnaissance (U.S. Navy SEAL, 2011).  Junior Officers who successfully make it through the BUD/S training program become fully qualified SEAL operators.  The manpower gaps within the SEAL community will be addressed by determining the optimal number of junior officer accessions needed to enter into the BUD/S training program, thus strengthening the SEAL community and ultimately, their operational readiness.

This thesis uses a mathematical approach to determine the number of junior officer accessions that are needed to enter the BUD/S training pipeline to meet target end-strength goals for a specific goal year.  With the understanding that retention is a current issue within the SEAL community, this thesis looks to help support the SEAL community by helping to close the current manpower gaps in the officer ranks.

The SEAL community is an extremely vital component to the NSW community and Department of Defense (DoD).  Furthermore, the NSW community is an essential and significant part to the success of winning the war on terrorism and other major national security crises.  Although the NSW personnel make up less than 1 percent of the U.S. Navy personnel, they offer big dividends on a small investment.  A fully manned and operationally ready SEAL community allows the NSW to be a continued force in the world crises, war on terrorism, and national security (U.S. Navy SEAL, 2011).

## B.    PROBLEM APPROACH

The SEAL community is currently undermanned at the ranks of LTJG, LT, and LCDR.  The LCDR rank is pivotal from a leadership point of view.  In order for the

LCDR rank to be properly manned, the junior ranks of LTJG and LT must be also properly manned. By determining the optimal number of new junior officer accessions needed to enter the BUD/S training program, target end-strength goals can potentially be met, thus directly impacting the LCDR manning issue. This thesis uses goal programming optimization and Markov chain analysis to determine these accession requirements. Historical data indicates commissioning source and rank play a pivotal part in determining successful completion of the BUD/S training program. Using these tools and historical data, the optimal number of new accessions by rank and source needed to enter the BUD/S training program will be determined.

Currently, the NSW community is trying to reach target-strength goals of 127 LTJGs and 285 LTs by FY17. Senior leaders have determined that this goal is achievable if 200 new accessions per 24-month period are sent to BUD/S. The Markov chain and optimization model developed for this research is used to determine the optimal allocation of officers to the BUD/S training program based on rank and commissioning source in order to meet target end-strength goals. By determining the optimal allocation of new accessions, the SEAL community can potentially meet target end-strength for the ranks of LTJG and LT, which in turn will directly impact the LCDR manning issue. The focus of the accessions for this thesis is on the junior ranks of ENS, LTJG, and LT, due to the different success rates that these ranks have in completing the BUD/S training program. In addition to determining optimal allocation of officers, the mathematical program is used to optimize accession numbers over a range of attrition rates.

## C. RELATED WORK

Bres, Burns, Charnes, and Cooper (1980) developed a goal programming model using discrete time periods to plan for the number of officer accessions needed from various commissioning sources to meet manpower goals for each specific officer inventory requirement. This model was created to effectively deal with "choke points" in specific critical tour ranks within the officer structure. A Markov chain approach was used with various states. The states that were created were warfare community, commissioning source, and YCS. In addition, the transition rates for the transition matrix

were found from historical data. The central application of the model is to determine the operating levels of various commissioning sources and the distribution of officers produced by each source to the different warfare communities, based upon community requirements (Bres, Burns, Charnes, & Cooper, 1980).

Zanakis and Maret (1981) developed a Markov Chain and linear goal programming model to solve manpower problems under various restrictions and conflicting objectives during one specific time period at the marco level. Maret and Zanakis felt it was natural to combine a Markovian approach with mathematical goal programming to solve manpower related problems due to the versatility it provided. The model created referenced a large company needing to meet manpower goals for the next year for specific workers in a specific department. The model had accessions coming from four sources and attritions happening in seven ways. In addition, the transition matrix that was created was created based off historical annual transitions of the personnel. Finally, Maret and Zanakis felt that the extension of time periods would be very useful for long-range manpower planning (Maret & Zanakis, 1981).

Gibson (2007) developed a linear program to help manage the Army Competitive Category within the officer corps of the Army, specifically in the O-4 rank looking at Time in Grade (TIG) and the promotion rates. He developed a model called the Total Army Competitive Category Optimization Model (TACCOM) to decrease the gap rates within the O-4 rank by looking to minimize the weighted deviation from the force requirements over the planning horizon of 40 years. The Army had a similar model that managed Army officers over a seven year time frame called the Officer Personnel Management System (OPMS) XXI. He compared his TACCOM model to the OPMS XXI model that the Army developed to see whether better results were possible. Gibson found that improvements could be made in the rank of Major in the ten-year horizon time frame (Gibson, 2007).

Additional related sources are found in Corbett (1995) for more specific manpower modeling approaches. The three works that were discussed use a similar mathematical modeling approach to manpower issues as this thesis. This thesis takes a similar approach to previous work but with a specific focus on the SEAL officer

community. This thesis uses goal programming and Markov chain analysis in an Excel spreadsheet-based approach in order to determine the optimal number of new accessions needed to enter the BUD/S training program to meet target end-strength goals at the rank LTJG and LT.

## II. OPTIMIZING ACCESSIONS FOR THE SEAL COMMUNITY

### A. MODEL PROCESSES

In this thesis, we use a Markov chain and goal programming optimization model to solve an aspect of the manpower problem that the SEAL community is currently facing. Markov chains are useful for studying manpower problems because they are useful for modeling the flow of people through a particular system; here, we model the BUD/S candidates from accession through graduation to a qualified SEAL. The optimization model uses the BUD/S Accession Calculator (BAC) to determine both the optimal allocation of officers accessed and the number of accessions needed, based on meeting target end-strength goals for FY17.

### B. MARKOV CHAIN

Our Markov chain model has 13 states specified by a (rank/commissioning source pair) and by the vacancy state indicating a vacancy caused by attrition. Vacancy is an absorbing state in our model and we will account for it in our calculations. Each entity (an individual) can only be in one state during any given time period. We model time in equal intervals, or periods, of twenty-four months (two years) each; ENSs and LTJGs promote to the next rank every two years, and the average time a BUD/S candidate stays in the BUD/S training program is around 19 months. By 24 months, all candidates who complete BUD/S move to the SEAL ranks. A portion of the Markov chain states and transitions is shown in Figure 2.
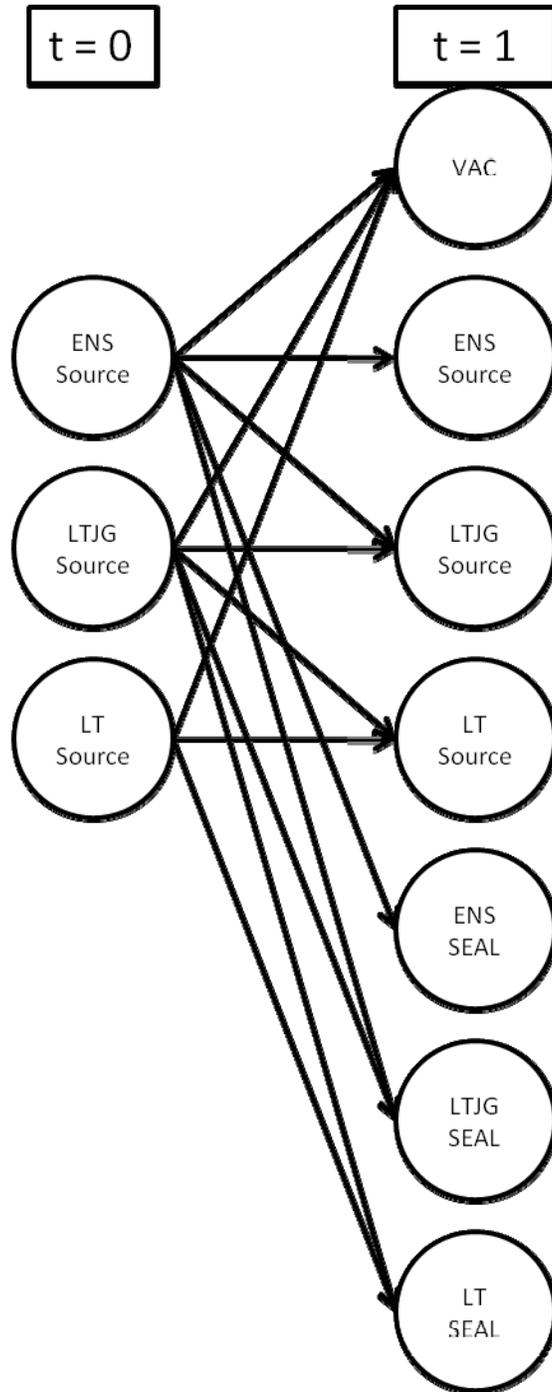
Figure 2.     Generic Markov Chain Diagram

In Figure 2, every state (also known as a node) represents a specific rank and commissioning source. On the right-hand side of Figure 2, there are four additional states that are used to show attrition and completion of the BUD/S training program. Those

states are: ENS SEAL, LTJG SEAL, and LT SEAL.  Movement occurs from one state to another state in one time period.  An example of this would be an ENS entering the BUD/S training program from a given commissioning source.  In 24 months, an ENS can perform one of four movements in one time period: stay in the system, which is illustrated as ENS Source to ENS Source, promote to a LTJG Source while staying in the BUD/S system, promote to ENS SEAL, or promote to LTJG SEAL. Again, this can be done for any rank/commissioning source combination.

The transition matrix, $P$, contains nine states, which are a combination of rank and commissioning sources, three commissioned SEAL ranks and one vacancy node. There are four commissioning sources:  Lateral Transfer (LATXFER), Officer Commissioning Source (OCS), Naval Reserve Officers Training Corps (NROTC), and United States Naval Academy (USNA).  The three ranks are: ENS, LTJG, and LT each with an associated commissioning source.  Finally, there are three states of actual commissioned SEALs based on rank; ENS, LTJG, and LT.  We note that individuals cannot enter the system as a LTJG or LT from the OCS, NROTC, and USNA commissioning sources.  Therefore, there are no LT OCS, LT NROTC, and LT USNA states in our model.  We can define individual moves from one state to another by using the notation $P_{r,s,r',s'}$.  For example, $P_{ENS,ROTC,LTJG,SEAL}$ is the probability an ENS ROTC graduates BUD/S and becomes a LTJG SEAL.

The transition matrix, $P$, was produced using both empirical and theoretical methods.  Historical data that was collected from the NSW community during the time period of October 2002 to October 2009.  This historical data is used to empirically calculate elements in the transition matrix $P$ dealing with accession sources. Transition probabilities are calculated empirically by taking the number of individuals who transitioned from state $r,s$ to $r',s'$ and dividing by the number who started in $r,s$.  The data that was collected was of individual accessions into the BUD/S training pipeline.  In addition to the individual accession data, the NSW community provided historical data of the SEAL inventories for the ranks of ENS, LTJG, and LT for the past nine years. Finally, the OPA requirements for FY09 and FY17 are provided.

Figure 3 illustrates the historical data of SEAL accessions based on commissioning source, not taking rank into account. The graph depicts the source of accession from 2005 to 2010. Over the past six years, accessions have increased for each of the four accession sources: LATXFER, OCS, NROTC, and USNA. Currently, the NSW community is bringing in approximately 100 new accessions per year.
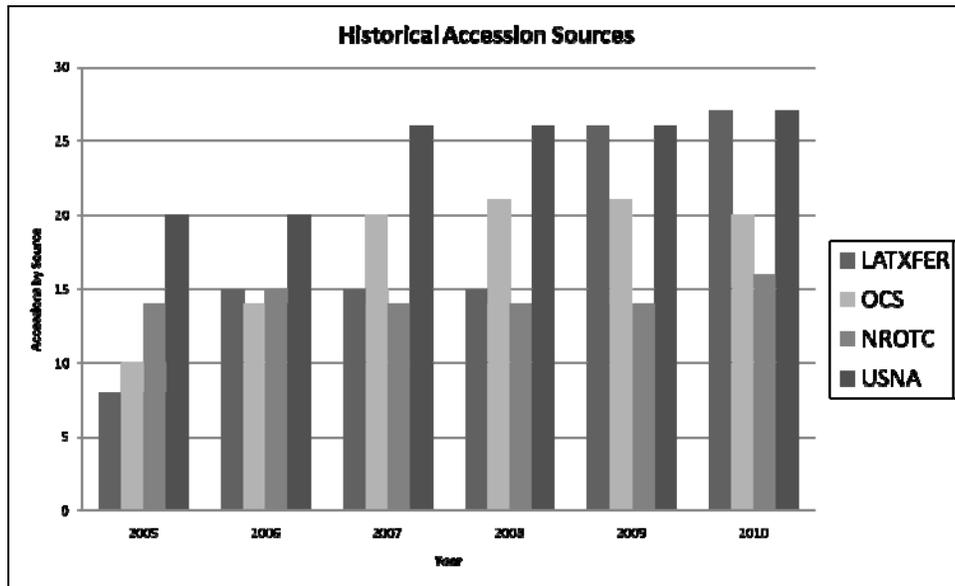


Figure 3.     Historical Accession Sources

Figure 4 illustrates the historical SEAL inventory for the past nine years. The ranks that are being displayed are ENS, LTJG, and LT. The inventory for those three ranks has been relatively constant, with a slight decrease over the past four years.
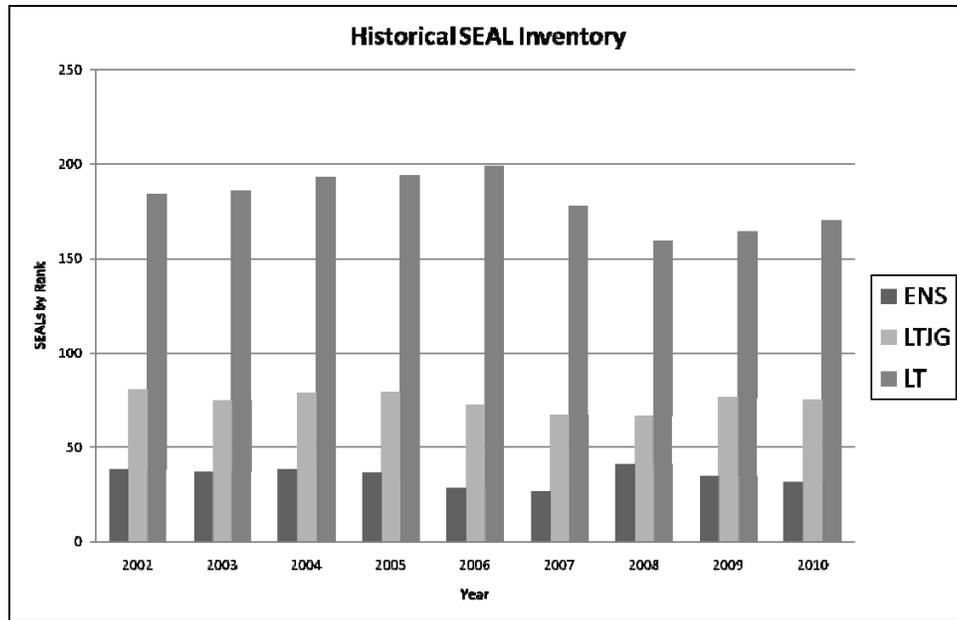
Figure 4.     Historical SEAL Inventory

Figure 5 illustrates the OPA requirements for the SEAL officer community and authorized number of SEALs for the ranks of LTJG, LT, and LCDR.  All of the inventory levels are below authorization with exception of ENS, CAPT and ADM.
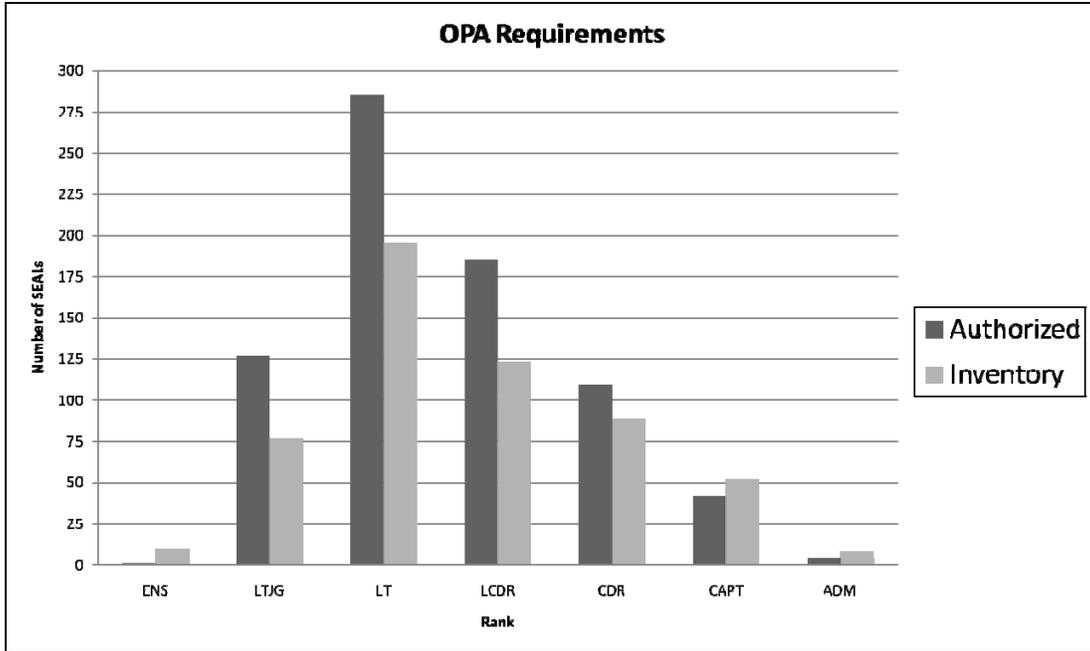
Figure 5.    OPA Requirements

The transition probabilities of the three actual commissioned SEAL officer states were calculated based on Time in Grade (TIG), promotion, and attrition information provided from the NSW community.  We used the geometric distribution to calculate $P_{r,s,r',s'}$ where $r',s'$ is ENS SEAL, LTJG SEAL, and LT SEAL.   The geometric distribution models time until first success out of $n$ Bernoulli trials.  This is appropriate here because success is considered a promotion, so the $P$ elements are calculated based on time until first promotion.  The expected value of the geometric random variable, X, is $E[X] = 1/p$, where $p$ is the probability to promote (e.g., Ross, 2007).   Thus, the transition probability for promotion in SEAL ranks can be calculated as $p = 1/E[X]$, where $E[X]$ is expected TIG.  Note this number must take into account the two-year time step.  For example, $P_{LTJG,SEAL,LT,SEAL}$ is equal to 0.917 as seen in Figure 6.

The transition matrix, $P$, is used to advance individuals in the system through the movement equation given by:

$$N_{t+1} = P'N_t$$

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where $N_t$ is a vector that contains the number of individuals in each node during time step $t$. If all rows in $P$ sum to one, then the number of individuals in the system is preserved and steady state can be calculated. However, if the vacancy node is removed, the number of states is reduced by one and row sums in $P$ are less than one, then the number of individuals in the system will go to zero as time increases. One way to account for new individuals in the system is by expanding the movement equation to the following:

$$N_{r,s,t+1} = \sum_{r',s'} P_{r',s',r,s} N_{r',s',t} + G_{r,s}$$

where $G_{r,s}$ is a vector of number of accessions by rank and commissioning source. For $G_{r,s}$ we can calculate a fraction of new accessions assigned to rank $r$, source $s$, $R_{r,s}$, as $R_{r,s} = G_{r,s} / \lambda$. Figure 6 shows the complete transition matrix with the removed vacancy state (note: the row sums are less than one).

| | ENS LatXfer | ENS OCS | ENS NROTC | ENS USNA | LTJG LatXfer | LTJG OCS | LTJG NROTC | LTJG USNA | LT LatXfer | ENS SEAL | LTJG SEAL | LT SEAL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ENS LatXfer | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.6667 | 0 |
| ENS OCS | 0 | 0 | 0 | 0 | 0 | 0.0357 | 0 | 0 | 0 | 0 | 0.4643 | 0 |
| ENS NROTC | 0 | 0 | 0 | 0 | 0 | 0 | 0.0128 | 0 | 0 | 0 | 0.4615 | 0 |
| ENS USNA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0246 | 0 | 0 | 0.7623 | 0 |
| LTJG LatXfer | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0323 | 0 | 0 | 0.4032 |
| LTJG OCS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 |
| LTJG NROTC | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 |
| LTJG USNA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.92 | 0 |
| LT LatXfer | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0968 | 0 | 0 | 0.129 |
| ENS SEAL | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.932 | 0 |
| LTJG SEAL | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.015 | 0.917 |
| LT SEAL | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.72 |

Figure 6.    Transition Matrix

The rows in Figure 6 represent the "from" state and the columns the "to" state. An example would be that a LTJG LATXFER goes from the LTJG LATXFER state to the LT SEAL state with a probability 0.4032. If 100 LTJG LATXFER individuals entered the BUD/S training pipeline, then approximately 40 would be LT SEALS after two years.

Using the movement matrix, we can illustrate how individuals move through this Markov chain. Assume at time $t = 0$ there are $x$ individuals in the system. $N_0$ is:

|  | $N_0$ |
|---|---|
| ENS LATXFER | 1 |
| ENS OCS | 1 |
| ENS NROTC | 3 |
| ENS USNA | 6 |
| LTJG LATXFER | 2 |
| LTJG OCS | 0 |
| LTJG NROTC | 0 |
| LTJG USNA | 0 |
| LT LATXFER | 3 |
| ENS SEAL | 10 |
| LTJG SEAL | 77 |
| LT SEAL | 196 |

Table 1.     N(0) Vector

$\lambda$ is equal to 200, and $R$ (Table 2) and $G_{r,s}$ is:

| R | Gr,s |
|---|---|
| 0.033241 | 7 |
| 0.1551247 | 31 |
| 0.2160665 | 43 |
| 0.3379501 | 68 |
| 0.1717452 | 34 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0.0858726 | 17 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |

Table 2.     R and $G_{r,s}$ Vectors

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Using the movement matrix we can see how individuals move. $N_1$ represents one time step and is:

| | $N_1$ |
|---|---|
| ENS LATXFER | 7 |
| ENS OCS | 31 |
| ENS NROTC | 43 |
| ENS USNA | 68 |
| LTJG LATXFER | 34 |
| LTJG OCS | 0 |
| LTJG NROTC | 0 |
| LTJG USNA | 0 |
| LT LATXFER | 18 |
| ENS SEAL | 0 |
| LTJG SEAL | 18 |
| LT SEAL | 213 |

Table 3.    N(1) Vector

These displays show how individuals move through our system and advance.

## C.    GOAL PROGRAMMING OPTIMIZATION MODEL

The purpose of the research in this thesis is to determine the optimal accession allocation by rank and commissioning source in order to meet target end-strength goals for FY17. The Markov chain portion of the model depicts the movement of SEAL candidates and SEAL operators through the accession, BUD/S, and finally SEAL company grade ranks. Given any current population for each of the available states in the system, we can use the Markov chain to predict the population for each state at any point in the future. In order to determine the optimal allocation of recruits, to these states in the current period, we develop a goal program that determines the allocation while minimizing penalties for deviating from desired target population levels.

# 1. NPS Formulation

## Indices

| | |
|---|---|
| $r$ | rank {ENS, LTJG, LT} |
| $t$ | time periods |
| $s$ | source {LATXFER, OCS, NROTC, USNA, SEAL} |

## Data

| | |
|---|---|
| $P_{r,s,r',s'}$ | probability that officer of rank $r$ from source $s$ transitions to rank $r'$, source $s'$ in one time period |
| $ts_r$ | target end-strength for SEAL rank $r$ |
| $gMin_{r,s}$ | lower bound on accession by rank and commissioning source |
| $gMax_{r,s}$ | upper bound on accession by rank and commissioning source |
| $avail_{r,t}$ | fraction of total accessions that are of rank r |

## Decision Variables

| | |
|---|---|
| $O_r$ | overage for SEAL of rank $r$ |
| $U_r$ | underage for SEAL of rank $r$ |
| $G_{r,s}$ | growth in rank $r$, source $s$, in each time period |
| $N_{r,s,t}$ | number of officers in rank $r$, from source $s$, in time period $t$ |
| $\lambda$ | total number of accessions in each time period |

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**Formulation**

$$\min \sum_r (O_r + 3U_r) \tag{2.1}$$

$$N_{r,SEAL,4} - O_r + U_r = ts_r \qquad\qquad \forall r \tag{2.2}$$

$$N_{r,s,t+1} = \sum_{r',s'} P_{r',s',r,s} N_{r',s',t} + G_{r,s} \qquad\qquad \forall r,s,t \tag{2.3}$$

$$G_{r,s} - avail_{r,t}\lambda \leq 0 \qquad\qquad \forall r,s = LATXFER,t \tag{2.4}$$

$$\sum_{r,s} G_{r,s} = \lambda \tag{2.5}$$

$$gMin_{r,s} \leq G_{r,s} \leq gMax_{r,s} \qquad\qquad \forall r,s \tag{2.6}$$

$$O_r \geq 0 \qquad\qquad \forall r \tag{2.7}$$

$$U_r \geq 0 \qquad\qquad \forall r \tag{2.8}$$

## 2.     Explanation of the Model

The optimization model developed in this thesis solves initially for the allocation of recruits to BUD/S then subsequently for the number of BUD/S accessions.  This is done by manipulating several decision variables. The overage and underage values determine how far the predicted inventory of SEAL operators is off compared to the target end-strength numbers for FY17 (t=4). The values of $G_{r,s}$ can be used to determine what percentage of new accessions should enter BUD/S by rank and commissioning source, using the relationship $gVector = G_{r,s}/\lambda$.  Where $gVector$ is same as $R$ which was introduced in Chapter II Section B.  We refer to $gVector$ vice $R$ throughout the rest of this thesis.  Note that initially, $\lambda$ is a variable not data representing the total number of new accessions that enter BUD/S during a 24-month time period. After the initial optimization, which is referred to as the base scenario, where $\lambda$ is 200, $\lambda$ becomes a decision variable.

The objective function (2.1) seeks to minimize the total sum of the overages and underages for the ranks of ENS, LTJG, and LT for the source of SEAL. The overages represent the number of current inventory, for each rank, that exceeds the goal end strength for FY17. The underages represent the number of current inventory, for each rank, that falls short of the end strength for FY17. A weight factor of three is applied to the underages, which ensures there is a higher priority on meeting or exceeding the target goals.

The constraints are listed in equations (2.2)–(2.8). The constraint set (2.2) matches the inventory with the target end-strength. The constraint (2.3) provides the updating equation for the Markov chain portion of the model. Note that this equation was previously presented as: $N_{t+1} = P'N_t + G_{r,s}$. The differences between the equations are subtle. The optimization portion of the model occurs after 2010. At 2010, we consider t = 0 for the optimization. For the future time steps, the equation, $N_{r,s,t+1} = P'_{r,s} N_t + G_{r,s}$, uses $G_{r,s}$ to update the number of individuals in each rank and source for each time step. Constraint set (2.4) is used to manipulate the number of LTs and LTJGs in the total number of new accessions for all scenarios in this thesis. The percentages chosen were based off of NSW requirements and by doing so this constraint is forcing the majority of the new accessions to be ENSs, which is in keeping with SEAL recruiting practice. Constraint (2.5) ensures that the total number of officers added in each period is exactly $\lambda$. Constraint set (2.6) makes sure that the category minimums are met and the upper bound on the accessions is not exceeded. Constraints (2.7) and (2.8) are making sure the overage and underage values are positive.

## D.    MODEL VALIDATION

We validated our model using historical data from the NSW community from October 2002 to October 2009. Note that because the time steps were chosen to be 24 months, ENS SEALs are not counted against inventory. ENS TIG is equal to two years; thus, in this model, there are no ENS SEALs, only LTJG and LT SEALS. In reality, an ENS who enters the BUD/S training program and completes prior to two years will be an ENS SEAL for a short time, prior to promotion to LTJG.

Historical $\lambda's$ were calculated by looking at each 24-month time block and determining how many accessions entered into the BUD/S training pipeline during that time period. The vector $R$ was calculated by computing the average number of accessions for each category then dividing the individual average by the sum total of all the individual averages. These two calculated pieces, $\lambda$ and $R$, allowed for an estimated representation of where the accessions where coming from and at what rate they were coming from during the validation years.

NSW historical data, seen in Figure 4, shows the FY09-FY10 inventory for LTJGs was approximately 76 and 164 for LTs. The ranges for these ranks were 61 to 78 for LTJGs and 159 to 194 for LTs. The model calculated FY09 inventories for these two ranks as 58 and 168, respectively. These calculated inventory numbers are acceptable because the range of the inventory for this model is from October 2008 to September 2010, which is FY09–FY10. During this time period, the calculated inventory numbers match closely to the inventory numbers from the NSW historical data.

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# III.  MODEL ANALYSIS

This chapter presents the optimal allocation of BUD/S recruits.  This chapter also explores how the deviation from predicted inventory and target end-strength fluctuates depending on the size of $\lambda$.  For the remainder of this thesis, *gVector* is used instead of $G_{r,s}$.  Recall $G_{r,s}$ can be calculated by multiplying by $\lambda$.  This chapter examines three specific scenarios dealing with $\lambda$.  The first scenario optimizes *gVector* for a fixed $\lambda$ of 200, the second optimizes for both $\lambda$ and *gVector*, and the third optimizes for $\lambda$ and *gVector* while keeping the underage values equal to zero.  These three scenarios provide a deeper insight into how target end-strength is affected by the total number of accessions brought in during a two-year time period.

## A.  BASE SCENARIO

The main goal of this thesis is to determine the distribution of junior officer accessions by commissioning source that need to enter into the BUD/S training program in a 24-month time period in order for target end-strength goals to be met.

Recall that different commissioning sources have different BUD/S success rates and there are caps on certain sources.  Therefore optimization is needed to determine how the SEAL community should focus recruiting efforts to best meet target goals.  The optimization is performed using a $\lambda = 200$, which is a recruiting number determined by analysts in the NSW community.  For this base scenario, $\lambda$ is data but for the next scenarios that deal with a changing $\lambda$, $\lambda$ is treated as a decision variable.  Using this value which represents the number of SEAL candidates entering BUD/S every two years, the optimal allocation of recruits can be determined. The target end-strength goals seen in Table 4 are the values that the NSW community is trying to achieve for FY17.

| Rank | Inventory | FY17 Target End Strength Goals |
|------|-----------|-------------------------------|
| ENS | 10 | 0 |
| LTJG | 77 | 127 |
| LT | 196 | 285 |

Table 4.    Target End-Strength Goals for FY17

The optimal solutions for *gVector* are shown in Table 5.  Table 5 displays the allocation distribution and the approximate number of accessions by source.   The approximate number of accessions is calculated by multiplying $\lambda$ by the optimal percentage of accession.  The solution suggests that the most recruits (74) came from OCS as an ENS and the least came from LATXFER as a LTJG.  A point worth mentioning is that no more than 1.5 percent of the total accessions will be LTs and no more than nine percent will be LTJGs.  In addition, there are restrictions on the number of accessions that can come from a particular accession source.  Those restrictions are 48 ENS LATXFER, 54 ENS USNA, 4 LTJG LATXFER, and 2 LT LATXFER and will be seen throughout this thesis.

| Rank | Percentage of Accessions | Aprrox. # of Accession |
|------|--------------------------|-------------------------|
| ENS LATXFER | 0.240 | 48 |
| ENS OCS | 0.370 | 74 |
| ENS NROTC | 0.100 | 20 |
| ENS USNA | 0.270 | 54 |
| LTJG LATXFER | 0.020 | 4 |
| LT LATXFER | 0.000 | 0 |

Table 5.    BAC Output for Fixed $\lambda$ Scenario

Table 5 displays the output from the BAC, but the optimal output also includes deviations from target end-strength ( *gVector* is the percentage of accessions and $G_{r,s}$ is the approximate number of accessions).  Using the target end-strength goals for FY17 (shown in Table 4), the BAC reports an underage of eight LTJGs and one LT.  These optimal outputs are 6.3 percent and 0.35 percent from goal for LTJGs and LTs, respectively.

The next section looks at the two alternative scenarios that optimize $\lambda$ as well as *gVector*.  The purpose of the changing $\lambda$ analysis is to see how the distribution of accessions is affected along with the degree of deviation from target end-strength.

**B.    ALTERNATIVE SCENARIOS**

The two alternative scenarios deal with optimizing for $\lambda$ as well as *gVector*.  The first scenario has a $\lambda$ that changes while letting underages fluctuate accordingly.  For the second scenario the $\lambda$ changes in addition to keeping the underages equal to zero.  The purpose of having these two scenarios with a changing $\lambda$ is to determine what the accession distribution and deviations from target end-strength will be.  While the SEAL senior leaders have stated that recruits to BUD/S should be 100 per year ($\lambda = 200$ per 24-month time period), the optimization model can be used to see how deviations from this number can be used to meet target end-strength for FY17.  The results of these two scenarios are compared to the base scenario to see the effect of changing the $\lambda$ value from 200.

We see the output from scenarios in which we change $\lambda$ in Table 6.  Table 6 illustrates the percentage of accession and the approximate number of accessions based on accession source for both changing $\lambda$ scenarios.  In addition, the optimal $\lambda$ for each of the scenarios is displayed at the top of the table.

| | $\lambda = 203$ | | $\lambda = 214$ | |
|---|---|---|---|---|
| Rank | Changing $\lambda$ Percentage of Accessions | Approximate # of Accessions | Changing $\lambda$ (Underages = 0) Percentage of Accessions | Approximate # of Assessions |
| ENS LATXFER | 0.237 | 48 | 0.224 | 48 |
| ENS OCS | 0.337 | 68 | 0.429 | 92 |
| ENS NROTC | 0.160 | 32 | 0.094 | 20 |
| ENS USNA | 0.267 | 54 | 0.252 | 54 |
| LTJG LATXFER | 0.000 | 0 | 0.000 | 0 |
| LT LATXFER | 0.000 | 0 | 0.000 | 0 |

Table 6.　　BAC Output for Changing $\lambda$ Scenario

### 1.　　Changing $\lambda$ and *gVector*

The first of the two scenarios deals with a changing $\lambda$ and *gVector*. The optimized $\lambda$ and *gVector* for this scenario is presented in Table 6. The optimal output shows that the distribution of accessions did change compared to the base scenario with a fixed $\lambda$. One key observation that needs to be noted is the size of $\lambda$. The $\lambda$ for this scenario is 203, compared to the fixed $\lambda$ of 200 in the base scenario. This is minute, which suggests the goal for FY17 can possibly be reached with current SEAL strategy assuming transition rates remain relatively constant.

For this scenario, when $\lambda$ increased the percentage of accessions had to adjust optimally to meet target end-strength, while adhering to the BAC's restrictions and constraints. This is evident in the ENS LATXFER and ENS USNA accession sources. $\lambda$ increased for this scenario compared to the base scenario but the approximate number of accessions remains the same for these two accession sources. This happens because both these accession sources have the highest success rates for BUD/S completion compared to the other sources of accession. The BAC maximizes the number of

accessions for sources with higher success rates. By maximizing the number of accessions with the highest success rates, target end-strength is more likely to be achieved.

The BAC reports an underage of five for LTJGs while meeting target end-strength for LTs. These optimal outputs are 3.9 percent and 0 percent from goal for LTJGs and LT, respectively. This is an overall improvement when compared to the base scenario dealing with a fixed $\lambda$ of 200.

### 2.    Changing $\lambda$ and *gVector* (Underages = 0)

The second scenario in this section deals with a changing $\lambda$ and *gVector* while keeping the underages equal to zero. The output for this scenario is shown in Table 6. The size of $\lambda$ increased once again compared to the base scenario. The size of $\lambda$ increased from 200 to 214 and the sources that have the highest success rates at BUD/S are maxed out again for optimality.

The BAC reports an underage of zero for LTJGs while having an overage of 8 LTs. These optimal outputs are 0.0 percent and 2.8 percent from goal for LTJGs and LTs, respectively. This again is an overall improvement when compared to the base scenario dealing with a fixed $\lambda$ of 200. However, the increased size of $\lambda$ from 200 to 214 is a large increase. This increase in $\lambda$ is approximately a 7.0 percent increase in the total number of new accessions required to meet target end-strength optimally.

## C.    MODEL RESULTS

The results illustrate the effect of the $\lambda$ on the deviation from target end-strength and changes the percentage of accession for the different accession sources. When the size of $\lambda$ increases, target end-strength goals are more closely matched.

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# IV. SENSITIVITY ANALYSIS

This chapter presents a sensitivity analysis of the three scenarios that were examined in Chapter III. The specific sensitivity analysis that is conducted deals with the manipulation of the attrition rate. Specifically, the sensitivity analysis is used to study how the optimal value for recruiting total ($\lambda$) and percent allocation ($gVector$) changes when attrition rate is increased or decreased.

There are four attrition rate manipulations: increasing attrition by 10 percent, increasing attrition by 5 percent, decreasing attrition by 10 percent and decreasing attrition by 5 percent. Each case is applied to the three scenarios presented in Chapter III: optimizing $gVector$ for fixed $\lambda$, optimizing $\lambda$ and $gVector$, and optimizing $\lambda$ and $gVector$ while keeping the underage values equal to zero. These manipulation cases were chosen because covering a range of attrition from plus 10 percent to minus 10 percent was seen as practical and would display the true affect of attrition rate on the total number of accessions needed to enter BUD/S to meet target end-strength goals. The graduation rates for BUD/S remains consistent in this chapter, but the total percent of graduation increases or decreases based on attrition. The transition matrix was developed based on historical data but fluctuations can occur. The fluctuation of plus or minus 5 percent can be expected while plus or minus 10 percent is seen as worst- and best-case scenarios.

## A. DIFFERENCE FROM TARGET END-STRENGTH

In this section, we describe the results of the sensitivity analysis by discussing the difference between the target end-strength goal and current optimized inventory for the ranks of LTJG and LT. This sensitivity analysis explores how the attrition rate manipulation cases change the difference between target end-strength and the optimized inventory produced by the BAC.

### 1.    Comparison of Accessions Into BUD/S Program

This section discusses how the $\lambda$ value changes when the attrition rate manipulation cases are put into effect on the three previously discussed scenarios. The total number of new accession into the BUD/S training program every two years is defined as $\lambda$. Currently, the NSW community brings in 200 new accessions on a two year cycle. Figure 7 displays the size of $\lambda$ for the attrition rate manipulation cases for the three scenarios. For the fixed $\lambda$ scenario, a value of 200 is seen for all attrition cases. $\lambda$ is shown to increase as attrition increases and decreases as attrition decreases. This makes sense intuitively.
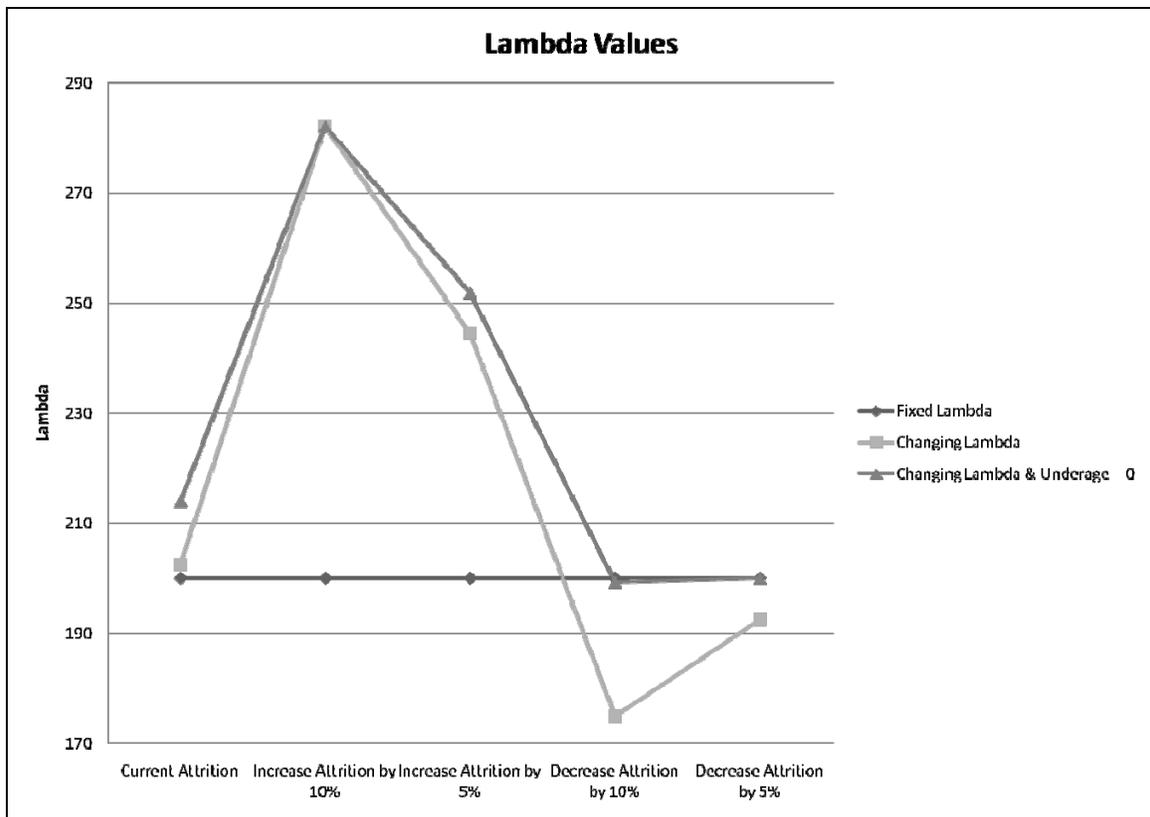


Figure 7.    Optimized $\lambda$ Values

Table 7 presents the results of the optimizations in tabular form. Table 7 shows the three different $\lambda$ scenarios and how $\lambda$ is affected when the attrition rate is increased or decreased. In addition, the current attrition rate is based off of historical data and the size of $\lambda$ can be seen as well for comparison.

| | Current Attrition | Increase Attrition by 10% | Increase Attrition by 5% | Decrease Attrition by 10% | Decrease Attrition by 5% |
|---|---|---|---|---|---|
| Fixed $\lambda$ | 200 | 200 | 200 | 200 | 200 |
| Changing $\lambda$ | 203 | 282 | 245 | 175 | 193 |
| Changing $\lambda$ & Underage = 0 | 214 | 282 | 252 | 199 | 200 |

Table 7.     Optimized $\lambda$ Values

Figure 7 and Table 7 shows that when attrition increases there are more dramatic changes in $\lambda$ than seen with decreasing attrition. The size of $\lambda$ increases when the attrition rate is higher because as more accessions attrite out of the BUD/S system more accessions are required to enter the system in order to meet target end-strength goals. On the other hand, the size of $\lambda$ decreases when the attrition rate is lower because more accessions are successfully completing the BUD/S training program and therefore a smaller $\lambda$ is needed to meet target end-strength. The following sections discuss how the deviation from target end-strength goals is affected for LTJGs and LTs when the attrition rate is manipulated.

## 2.     Comparison of LTJG

The FY17 target end-strength goal for LTJGs is 127. Figure 8 shows the deviation from target end-strength for the three scenarios each with the various attrition rate manipulation cases. The deviation from target end-strength is presented numerically beneath the graphical display. These values are the deviation from target end-strength numbers that display an overage or an underage from the goal of 127 LTJGs.
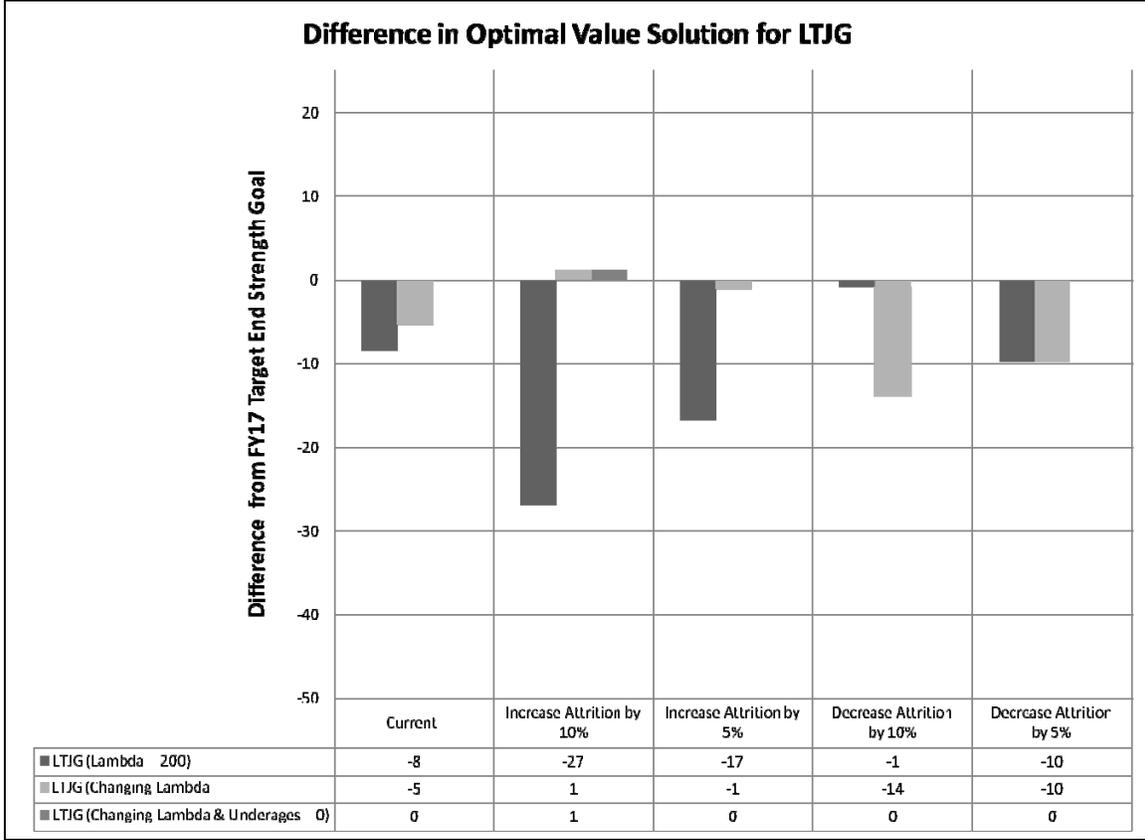
Figure 8.    Difference from Target End-Strength for LTJGs

As we discussed in Chapter III, the optimal solutions for the three scenarios with no attrition rate manipulation were 6.3 percent, 3.9 percent, and 0.0 percent from goal for a fixed $\lambda$ and the changing $\lambda$ scenarios, respectively.    When the attrition rate manipulation cases are taken into effect it can be see that the deviation from target end-strength changes significantly. Table 8 displays the percent from goal values for LTJG for the three scenarios dealing with attrition rate manipulation.

| | Current Attrition | Increase Attrition by 10% | Increase Attrition by 5% | Decrease Attrition by 10% | Decrease Attrition by 5% |
|---|---|---|---|---|---|
| LTJG (Lambda) | -6.3% | -21.3% | -13.4% | -0.8% | -7.9% |
| LTJG (Chaning Lambda) | -3.9% | 0.8% | -0.8% | -11.0% | -7.9% |
| LTJG (Chaning Lamda & No Undererage) | 0.0% | 0.8% | 0.0% | 0.0% | 0.0% |

Table 8.     Percent from Goal Values for LTJGs

By increasing the attrition rate by 10 percent while keeping $\lambda$ fixed at 200, we see that the optimal solution becomes 21.3 percent from goal compared to the 6.3 percent from goal with no increase in the attrition rate.  This is a drastic increase, but this increase happens because the value of $\lambda$ remained at 200 while dealing with a 10 percent attrition rate increase.  On the other hand, when $\lambda$ is allowed to change with a 10 percent increase in the attrition rate, the optimal solution becomes 0.8 percent from goal compared to the 3.9 percent optimal solution for no increase in attrition.  However, (as seen in Figure 7) the size of $\lambda$ increases from 203 to 282 to meet the target end-strength optimally.

As the attrition rate decreases by 10 percent, the optimal solution for a fixed $\lambda$ value of 200 is 0.8 percent from goal, compared to the 6.3 percent from goal for no change in the attrition rate.  The optimal solution with a 10 percent attrition rate decrease becomes 11.6 percent from goal compared to the 3.9 percent optimal solution for no attrition change, when allowing $\lambda$ to change.  However the value of $\lambda$ changes from 203 with no attrition rate manipulation to 175 when decreasing the attrition rate by 10 percent. This decrease in $\lambda$ happens because target end-strength can be met optimally with less accessions.

Figure 8 illustrates that the deviation from target end-strength is much lower for an increase in the attrition rate than deviation for a decrease, when allowing $\lambda$ to change.  This is because more accessions are needed to offset the high attrition rate.  In addition, as the attrition rate decreases, the deviation from target end-strength increases but the size of $\lambda$ decreases because fewer accessions are required to meet the target end-strength goal of 127 LTJGs.

### 3. Comparison of LT

The FY17 target end-strength goal for LTs is 285. Figure 9 shows the deviation from target end-strength for the three scenarios each with the various attrition rate manipulation cases. The deviation from target end-strength is presented numerically beneath the graphical display in Figure 9. These values are the deviation from target end-strength numbers that display an overage or an underage from the goal of 285 LTs.



| | Current | Increase Attrition by 10% | Increase Attrition by 5% | Decrease Attrition by 10% | Decrease Attrition by 5% |
|---|---|---|---|---|---|
| ■ LT (Lambda   200) | -1 | -47 | -24 | 22 | 0 |
| ■ LT (Changing Lambda) | 0 | 0 | 0 | 0 | 0 |
| ■ LT (Changing Lambda & Underage   0) | 8 | 0 | 2 | 22 | 15 |
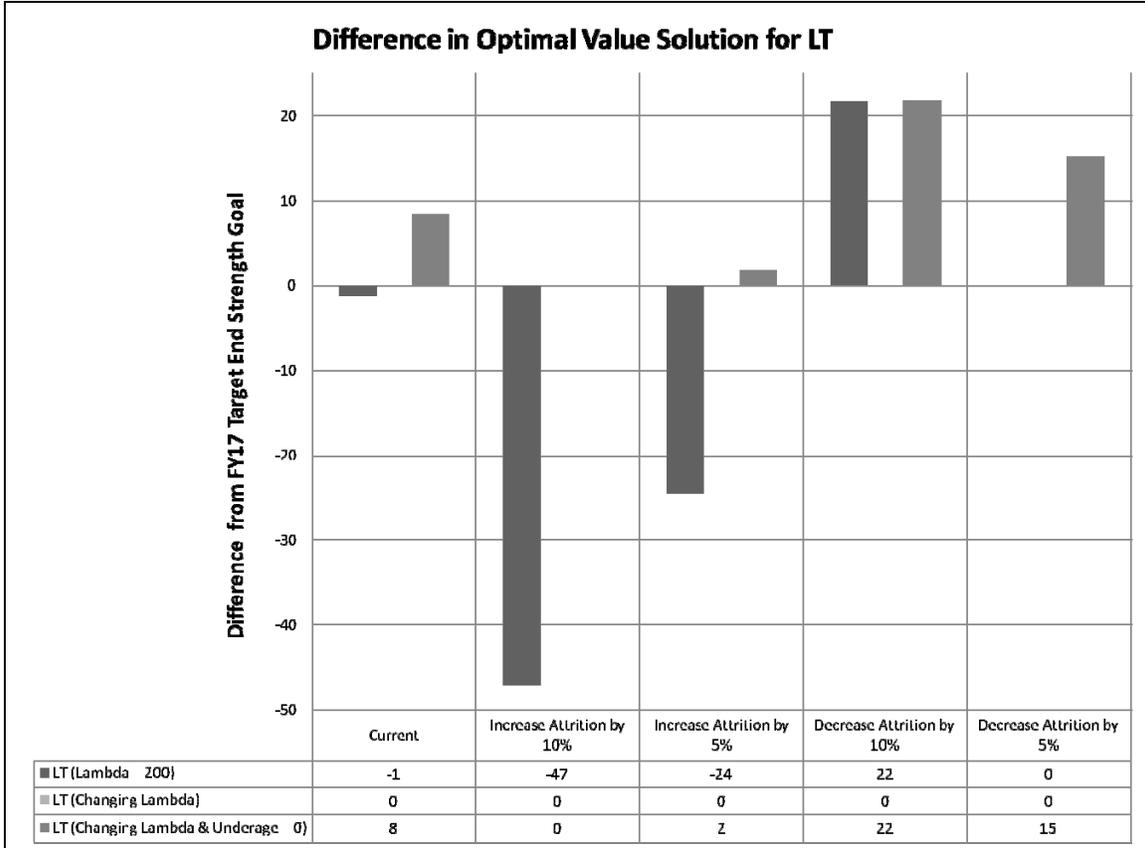
Figure 9.    Difference from Target End-Strength for LTs

The optimal solutions for the three scenarios with no attrition rate manipulation were 0.35 percent, 0.0 percent, and 2.8 percent from goal for a fixed $\lambda$ and the changing $\lambda$ scenarios, respectively (see leftmost column in Figure 9, which represents the initial results). As with LTJGs, it is seen from Figure 9 that as the attrition rate is manipulated, the deviation from target end-strength changes. Table 9 displays the percent from goal values for LT for the three scenarios dealing with attrition rate manipulation.

|  | Current Attrition | Increase Attrition by 10% | Increase Attrition by 5% | Decrease Attrition by 10% | Decrease Attrition by 5% |
|---|---|---|---|---|---|
| LT (Lambda) | -0.35% | -16.49% | -8.42% | 7.72% | 0.00% |
| LT (Changning Lambda) | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% |
| LT (Chaning Lamda & No Undererage) | 2.81% | 0.00% | 0.70% | 7.72% | 5.26% |

Table 9.    Percent from Goal Values for LT

A fixed $\lambda$ of 200 with a 10 percent attrition rate increase has an optimal solution of 16.5 percent from goal compared to the 0.35 percent from goal with no increase in the attrition rate. This is a large increase, however this increase happens because the value of $\lambda$ remained at 200 while dealing with a 10 percent attrition rate increase. On the other hand, when $\lambda$ is allowed to change (along with the underages) with a 10 percent increase in the attrition rate, the optimal solution remains at 0.0 percent from goal or meeting target end-strength. However (as seen in Figure 7), $\lambda$ increases from 203 to 282 for this goal to be met. The increase in $\lambda$ but lack of change in the optimal solution is expected because the increase in $\lambda$ can offset the 10 percent attrition rate and therefore meet target end-strength. When $\lambda$ is allowed to change (while keeping the underages equal to zero) with a 10 percent attrition increase, the optimal solution becomes 0.0 percent compared to 2.8 percent with no change in the attrition rate. For this case, $\lambda$ changes from 214 to 282. This improvement in the optimal solution but increase in $\lambda$ happens again because $\lambda$ can increase as necessary to offset the high attrition rate.

By decreasing the attrition rate by 10 percent, we see that the optimal solution for a fixed $\lambda$ value of 200 is 7.7 percent from goal compared to the 0.35 percent from goal for no change in the attrition rate. This increase from goal happens because with a decrease in attrition more accessions are successfully completing BUD/S. In addition, when the attrition rate is decreased by 10 percent there is an overage of 22 LTs for a fixed $\lambda$ compared to an underage of 1 LT for no change in the attrition rate. Target end-strength is being met but just surpassing the goal compared to target end-strength not being met.

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Allowing $\lambda$ to change (along with the underages); the optimal solution with a 10 percent decrease in attrition becomes 0.0 percent from goal which is equal to the optimal solution for no change in the attrition rate. However the value of $\lambda$ changes from 203 with no attrition rate manipulation to 175 when decreasing the attrition rate by 10 percent. This decrease in $\lambda$ while keeping the optimal solutions equal happens because as attrition decreases more accessions successfully complete BUD/S and therefore less accessions are needed to meet target end-strength.

We see the optimal solution becomes 7.7 percent when decreasing the attrition rate by 10 percent, compared to 2.8 percent when no change in the attrition rate occurs for the scenario dealing with a changing $\lambda$ (while keeping the underages equal to zero). For this attrition rate case, $\lambda$ changes from 214 to 199. This non-improvement in the optimal solution but decrease in $\lambda$ happens due to the fact there is an overage of 22 LTs when a decrease of 10 percent occurs compared to an underage of 8 LTs when no manipulation of the attrition rate occurs.

By increasing and decreasing attrition by 5 percent we see that the optimal solutions improve when increasing the attrition rate and worsen when decreasing the attrition rate. In addition, the $\lambda$ values increase as the attrition rate is increased and decrease when the attrition rate is decreased.

## B. ACCESSION SOURCE PERCENTAGE ANALYSIS

We will focus on allocation of accessions based on commissioning source and rank in this section. The results in this section present optimal *gVector* values for each of the three scenarios in Chapter III, while manipulating the attrition rates.

### 1. Optimal *gVector*, Fixed $\lambda$ ($\lambda$=200)

In this section, we look at how the accession source percentages change as the attrition rate is manipulated for a fixed $\lambda$. Figure 10 shows the percentage of accessions for each of the attrition rate manipulation cases for a fixed $\lambda$.
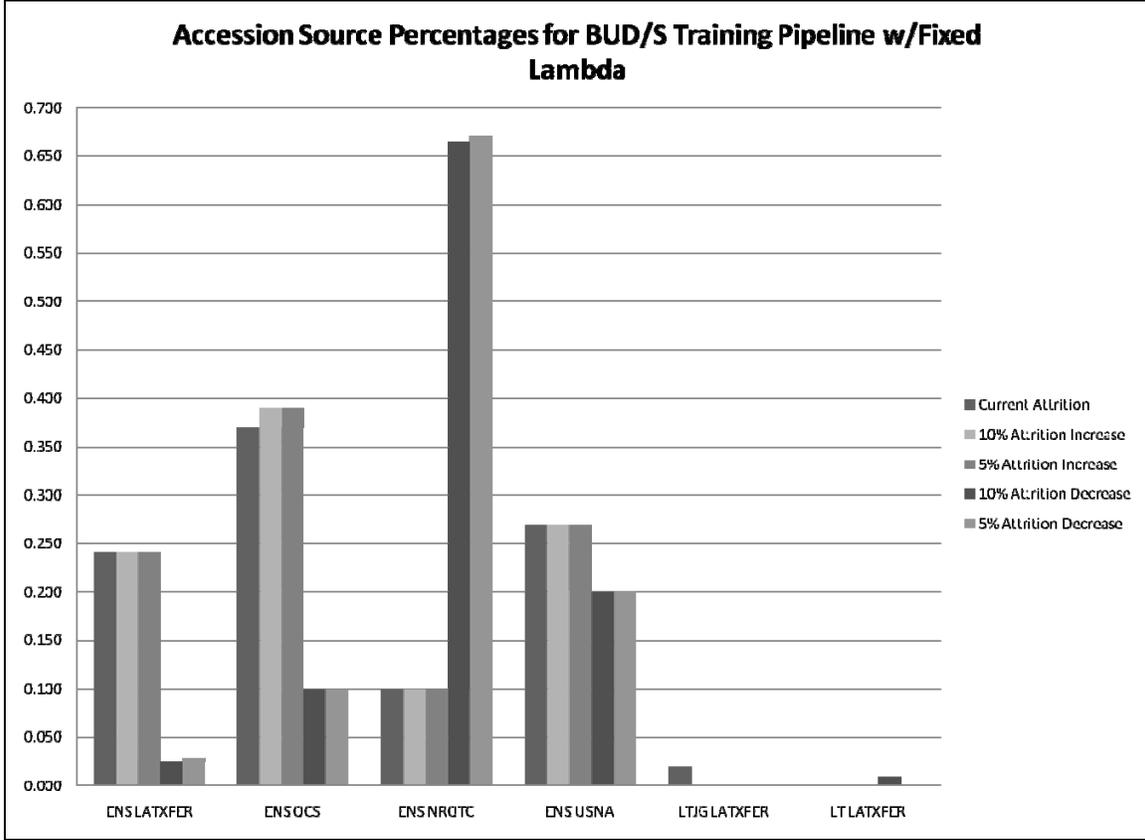
Figure 10.    Accession Source Percentage Breakdown for Fixed $\lambda$

Table 10 presents the approximate number of accessions for the source of accession. Table 10 is calculated by multiplying the accession source percentage by the fixed $\lambda$ of 200.

|  | Current Attrition | Increase Attrition by 10% | Increase Attrition by 5% | Decrease Attrition by 10% | Decrease Attrition by 5% |
|---|---|---|---|---|---|
| ENS LATXFER | 48 | 48 | 48 | 5 | 6 |
| ENS OCS | 74 | 78 | 78 | 20 | 20 |
| ENS NROTC | 20 | 20 | 20 | 133 | 134 |
| ENS USNA | 54 | 54 | 54 | 40 | 40 |
| LTJG LATXFER | 4 | 0 | 0 | 0 | 0 |
| LT LATXFER | 0 | 0 | 0 | 2 | 0 |

Table 10.　　Approximate Number of Accessions

Figure 10 and Table 10 demonstrate that as the attrition rate is increased the accession sources that have the highest completion rate are larger. An example is the ENS LATXFER, ENS USNA, and LTJG LATXFER accession sources. The maximum allowed for these three accession sources are 48, 54, and 4, respectively. Table 10 shows that these three accession sources max out when attrition is high. This happens because the BAC is optimizing to meet target end-strength and uses the sources of accession with the highest success rate first. In addition, ENS OCS has a larger number of accessions than ENS NROTC because ENS OCS has a higher success rate at BUD/S. Conversely, as the attrition rate is decreased, the accession sources that had the highest success rate no longer max out but meet the minimum number of accessions required, while the accession sources that had the lowest success rate increases drastically. This happens because as attrition is decreased more accessions are completing and, therefore, the BAC wants the accession sources that have the fastest completion time.

## 2.　　Optimal $\lambda$ and *gVector*

We examine how the accession source percentages change as the attrition rate is manipulated for a changing $\lambda$ in this section. Figure 11 shows the percentage of accessions for each of the attrition rate manipulation cases for a changing $\lambda$.
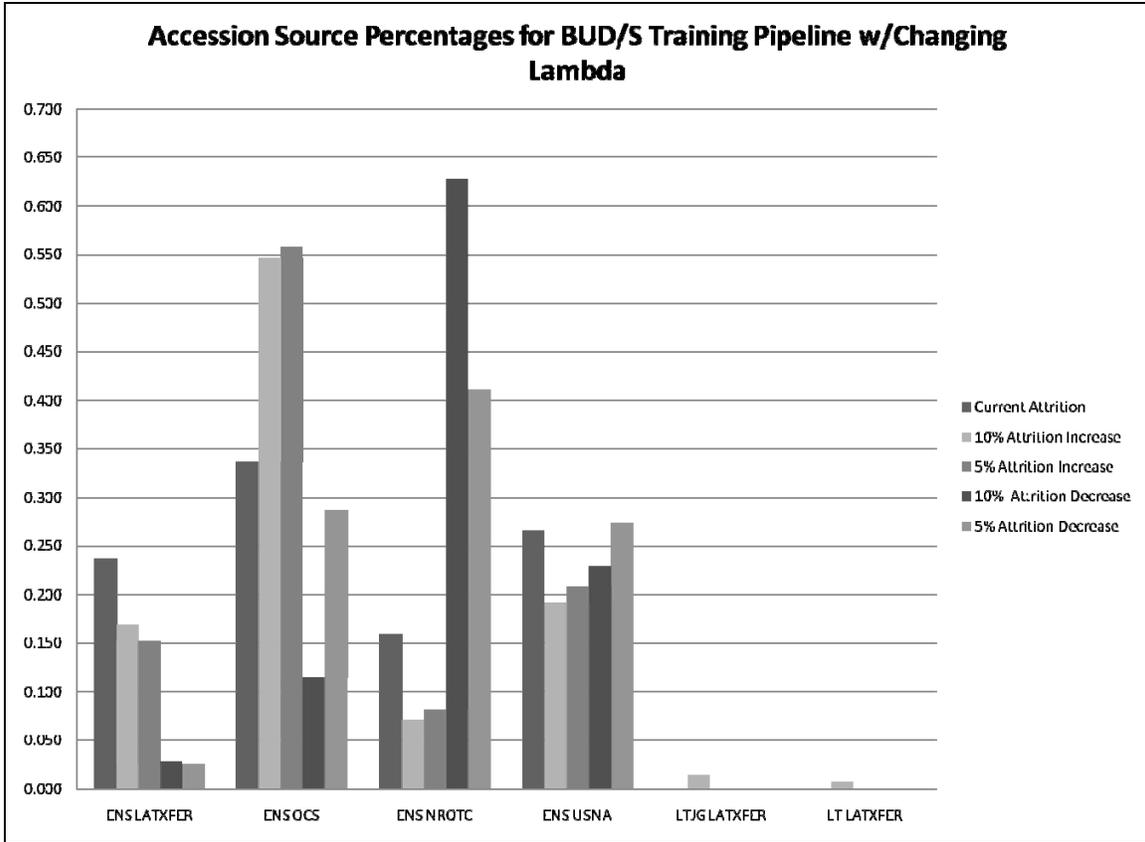
Figure 11.    Accession Source Breakdown for Changing $\lambda$

Table 11 is the approximate number of accessions for the source of accession. Table 11 is calculated by multiplying the accession source percentage by the $\lambda$ value for the given attrition rate manipulation case. Recall the $\lambda$ values for this scenario across the different attrition rate manipulation case are 203, 282, 245, 175, and 193, respectively.

Figure 11 and Table 11 reveal that when the attrition rate decreases, the sources of accession that have the highest completion rate are no longer being maxed out to capacity and the maximization switches to the sources that have the fastest completion time. This is because when attrition is high it is important to maximize every accession from successful sources in order to meet target end-strength optimally. On the other hand, as attrition is decreased and the $\lambda$ value is less than 200, the sources with the highest success rates are no longer as important because the decrease in the attrition rates

neutralized the differences and time is of the essence. Additionally, when the attrition rate is decreased by 5 percent, there is a balance in the distribution of accession sources. This happens because $\lambda$ is close to 200.

|  | Current Attrition | Increase Attrition by 10% | Increase Attrition by 5% | Decrease Attrition by 10% | Decrease Attrition by 5% |
|---|---|---|---|---|---|
| ENS LATXFER | 48 | 48 | 37 | 5 | 5 |
| ENS OCS | 68 | 154 | 136 | 20 | 55 |
| ENS NROTC | 32 | 20 | 20 | 110 | 79 |
| ENS USNA | 54 | 54 | 51 | 40 | 53 |
| LTJG LATXFER | 0 | 4 | 0 | 0 | 0 |
| LT LATXFER | 0 | 2 | 0 | 0 | 0 |

Table 11.    Approximate Number of Accessions With Changing $\lambda$

### 3.    Optimal *gVector* and $\lambda$ with (Underages = 0)

We explore how the accession source percentages change as the attrition rate is manipulated for a changing $\lambda$ while the underages equal zero in this section. Figure 12 shows the percentage of accessions for each of the attrition rate manipulation cases for a changing $\lambda$ while underages equal zero.

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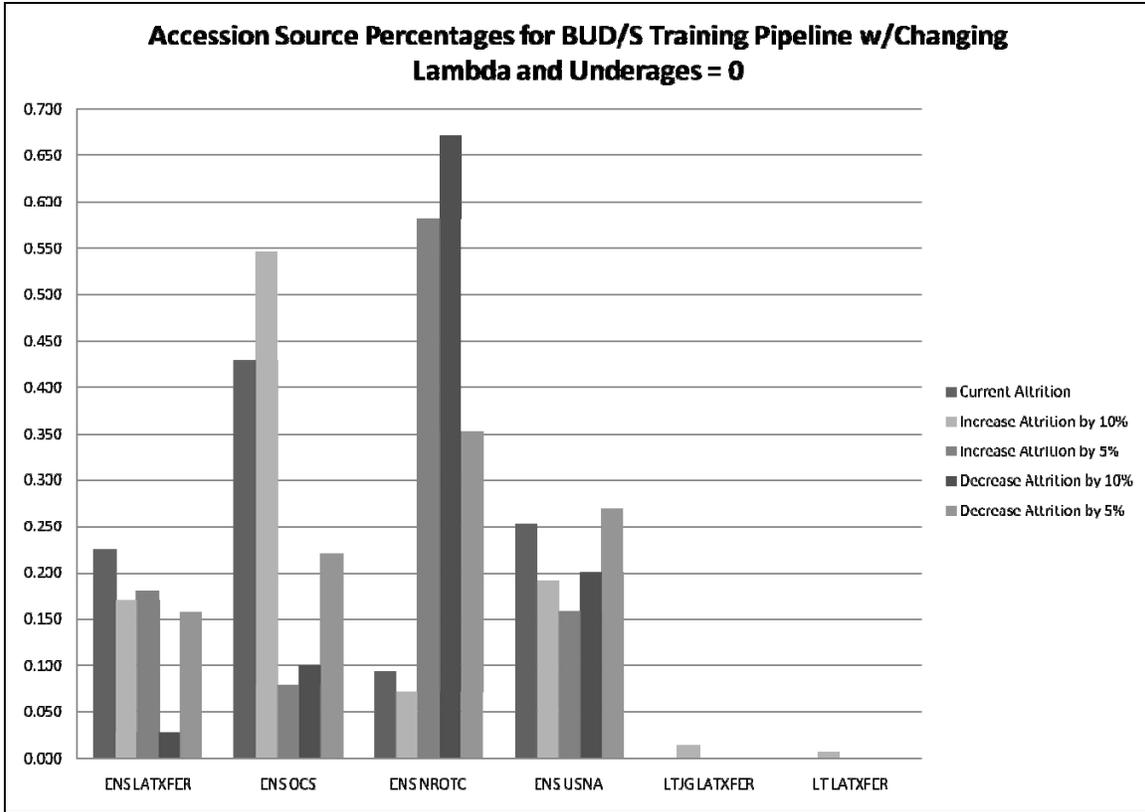
Figure 12.    Accession Source Percentages for Changing $\lambda$ While Underages Equal Zero

Table 12 is the approximate number of accessions for the each source of accession. Table 12 is calculated by multiplying the accession source percentage by the $\lambda$ value for the given attrition rate manipulation case.  Recall the $\lambda$ values for this scenario across the different attrition rate manipulation case are 214, 282, 252, 199, and 200, respectively.

Figure 12 and Table 12 show similar events that happen as the previous two sections.  However, one striking difference is that when the attrition rate decreases by 5 percent, there is a true maximized balance between the sources of accessions.  This is because the $\lambda$ size is 200 and is matching the current two year accession number that the NSW community is bringing in while decreasing the attrition rate by 5 percent. Essentially, it is distributing among the sources of accessions finding the balance between the most successful completion rates and quickest completion time of BUD/S.

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| | Current Attrition | Increase Attrition by 10% | Increase Attrition by 5% | Decrease Attrition by 10% | Decrease Attrition by 5% |
|---|---|---|---|---|---|
| ENS LATXFER | 48 | 48 | 46 | 5 | 31 |
| ENS OCS | 92 | 154 | 20 | 20 | 44 |
| ENS NROTC | 20 | 20 | 146 | 134 | 71 |
| ENS USNA | 54 | 54 | 40 | 40 | 54 |
| LTJG LATXFER | 0 | 4 | 0 | 0 | 0 |
| LT LATXFER | 0 | 2 | 0 | 0 | 0 |

Table 12.    Approximate Number of Accessions With Changing $\lambda$ While Underages Equal Zero

# V. CONCLUSIONS AND FUTURE RESEARCH

This thesis uses a mathematical approach to determine the number of junior officer accessions and distribution of those accessions by rank and commissioning source that are needed to enter the BUD/S training program to meet target end-strength. The mathematical approach is a combination of goal programming and Markov chain analysis.

Currently, the NSW community plans to recruit 100 accessions every year to the BUD/S training program to meet target end-strength goals of 127 and 285 for the ranks of LTJG and LT, respectively, for FY17. These 100 accessions come from various sources with various distributions. Using historical data to drive the model and optimizing allocation of recruits, target end-strength goals can be met within 6 percent for LTJGs and approximately 0 percent for LTs. The optimal allocation by rank and commissioning source is shown in Figure 13.

| Rank | Percentage of Accessions | Aprrox. # of Accession |
|---|---|---|
| ENS LATXFER | 0.240 | 48 |
| ENS OCS | 0.370 | 74 |
| ENS NROTC | 0.100 | 20 |
| ENS USNA | 0.270 | 54 |
| LTJG LATXFER | 0.020 | 4 |
| LT LATXFER | 0.000 | 0 |

Figure 13.    BAC Optimal Allocation by Rank and Commissioning Source

Manipulating the attrition rate directly affect the percent of deviation from target end-strength for the ranks of LTJG and LT (seen in Table 13). The higher the attrition rate, the higher the percent deviation from target end-strength. However, when attrition rate is lowered to 5 and 10 percent target end-strength is met or exceeded. When the

attrition rate is decreased, the percent from deviation is a positive deviation in some cases, which demonstrates that target end-strength is surpassing the goal versus being undermanned.

| | Current Attrition | Increase Attrition by 10% | Increase Attrition by 5% | Decrease Attrition by 10% | Decrease Attrition by 5% |
|---|---|---|---|---|---|
| LTJG ($\lambda$= 200) | -6.30% | -21.26% | -13.39% | -0.79% | -7.87% |
| LT ($\lambda$= 200) | -0.35% | -16.49% | -8.42% | 7.72% | 0.00% |

Table 13.    Percent Deviation From Target End-Strength for LTJG and LT for Fixed $\lambda$

Sensitivity analysis results found that on average target end-strength goals can be met within 1 percent of goal compared to the 6 percent from goal with accessions equal to 100. However, this improvement in meeting target end-strength requires $\lambda$ to change. Table 14 shows the changes in $\lambda$ and it can be seen that as attrition rate increases, the size of $\lambda$ increases. Conversely, as the attrition rate decreases, the value of $\lambda$ decreases. Target end-strength goals can be met or closely met at the 1 percent deviation level but the downside of that is that the number of accessions needed fluctuates.

| | Current Attrition | Increase Attrition by 10% | Increase Attrition by 5% | Decrease Attrition by 10% | Decrease Attrition by 5% |
|---|---|---|---|---|---|
| Changing $\lambda$ | 203 | 282 | 245 | 175 | 193 |
| Changing $\lambda$ & No Underages | 214 | 282 | 252 | 199 | 200 |

Table 14.    $\lambda$ Values for Attrition Rate Manipulation Cases

There are several areas for future work that may improve the BAC and this thesis. One area is that the BAC only evaluates target end-strength goals for LTJGs and LTs. An improvement may be to add the LCDR rank as well. Allowing the BAC to optimize to meet target end-strength goals for LTJG, LT, and LCDR would be a great tool for the

NSW community. The second area that could be addressed would be to allow $\lambda$ to change for each time period. Currently, the BAC uses the same size $\lambda$ for each of the time periods up to the goal year of FY17. By adjusting the size of $\lambda$, the BAC would be addressing the logical growth of the NSW community on a yearly basis. Additionally, time series and/or regression modeling can be used to create forecasted attrition and retention rates rather than using historical averages.

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