

## **Inverse Modeling of Coastal Tides**

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### **LONG TERM GOALS**

The principal goal of this project is to develop efficient, relocatable, tidal data assimilation schemes that make use of all available information, including data (e.g., altimetry, current moorings, coastal radar) and dynamics to constrain barotropic and internal tides (especially tidal currents) in coastal areas and shallow seas. A secondary objective is to develop a better understanding of how uncertainties and approximations in the hydrodynamic equations limit the degree to which particular data types (e.g., spatially dense estimates of tidal elevations from altimetry, or surface currents from coastal radar) can constrain tidal currents and elevations in shallow water environments.

### **OBJECTIVES**

I have three objectives in the current project period. The first is to adapt the computationally intensive methods developed by Egbert, Bennett, and Foreman [1994] for inverse tidal modeling at the global scale, into a relocatable inversion package that may be routinely applied to smaller scale (regional/coastal) barotropic tidal modeling. The second objective is to apply these methods to study tidal currents off the west coast of North America, with particular emphasis on the Oregon coastal zone where extensive current meter and coastal radar data are available. A third objective is to begin development of methods for tidal inversion in a three-dimensional stratified ocean. These methods will be tested on coastal radar and current meter data from the Oregon coast.

### **APPROACH**

We have used a rigorous variational approach to data assimilation scheme, based on minimizing a penalty functional defined in terms of weighted misfits to available tidal data, and the hydrodynamical equations. The weighting of data and dynamics is defined by rational estimates of errors in the two types of information. To solve the inverse problem we use variants on the explicit representer approach described by Bennett [1992]. A major challenge (and a focus of much of my effort) has been to develop more efficient solution methods, to allow for larger data sets, higher resolution, and routine rapid application to smaller scale problems. To develop a three-dimensional tidal inversion, we are beginning with simple linear stratified dynamics (e.g., Clarke and Brink, 1985) for which full representer calculations are feasible (at least in research mode). We are also using a non-linear primitive equations model, the Princeton Ocean Model (POM) for tidal modeling studies.

# Report Documentation Page

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## WORK COMPLETED

We have completed development and documentation of a self-contained efficient barotropic tidal inversion package, which we refer to as the OSU Tidal Inversion Software, or OTIS. OTIS is a relocatable implementation of the TOPEX/POSEIDON altimetry tidal inversion scheme based on methods described in Egbert, Bennett & Foreman [1994; EBF], Egbert and Bennett (1996) and Egbert (1997). Major changes to the software have been made to significantly improve computational efficiency, and ease of use for regional tidal modeling constrained by altimetry data. As in EBF, a modified or reduced basis representer approach is used. That is, representers are calculated for a subset of data locations, and a solution to the variational problem is sought within the space of linear combinations of calculated representers. With this approach it is quite feasible to fit all available altimetry data in a modest sized area (e.g., 20 degrees by 20 degrees, with data at several thousand locations) using a few hundred representers per constituent. Over the past year a new and much more efficient approach to the representer calculation has been developed. With the new approach, representers are calculated by solving the linearized frequency domain shallow water equations (SWE) after factoring the matrix of coefficients for the elevation wave equation. Once the equations are factored (this must be done only once for each tidal constituent) the representer computations are very rapid. This new approach allows for a decrease in computational time (relative to the time stepping approach used for the representer calculation of EBF) by a factor of 100 or more. Table 1 compares run times required for representer calculations with the old approach of EBF (time stepping on a 32 node CM-5), with the new approach using a single node of an IBM SP2 (120Mflops/sec). Combined with the reduced basis representer approach the new approach allows solution of moderate size problems on a high-end workstation with approximately 500 Mb of memory.

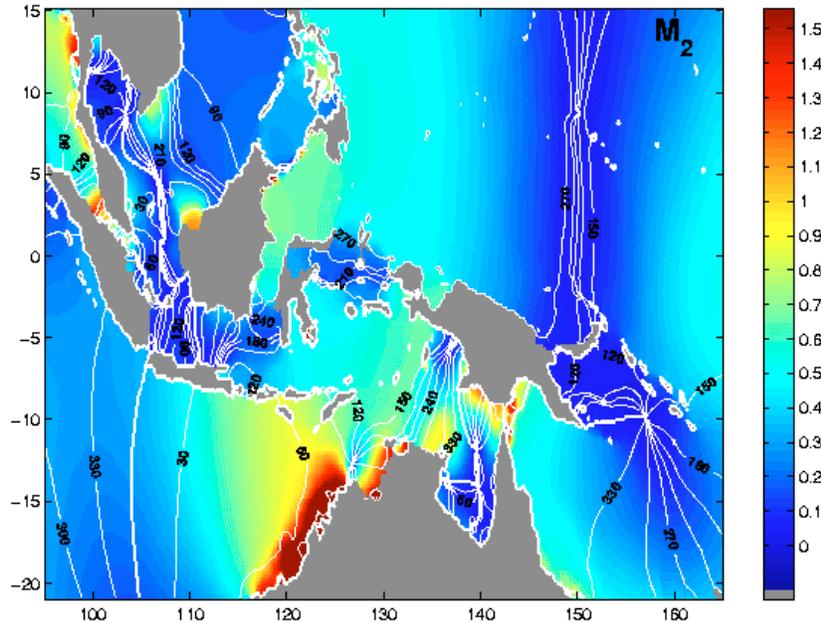
	IBM RS6000 CPU time		CM5 CPU time
	Time Stepping Solution	Direct Solution	Time Stepping Solution
One representer for West Coast of USA	36 hours	2 minutes	30 minutes
30 representers for West Coast of USA	-	5 minutes	15 hours
One representer for Indonesia	-	40 minutes	1 hour
188 representers for Indonesia	-	2 hours	188 hours

Table 1: summary of computational resources needed for representer calculation using direct factorization of the SWE coefficient matrix.

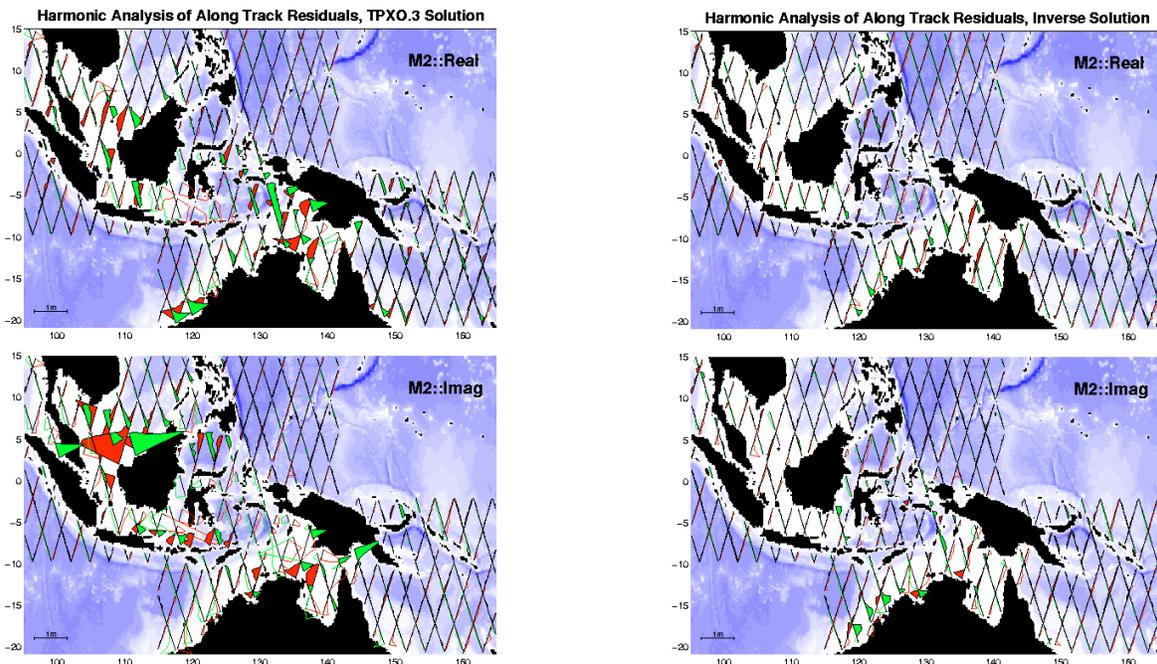
Various other improvements and additions to our inversion software have been included in OTIS. These include a TOPEX/POSEIDON altimetry data base appropriate for tidal modeling, programs to extract and reduce altimetry data for a particular region/set of constituents, programs and data needed to create finite difference bathymetric, code for calculation of posterior error covariances for the inverse solution, and various MATLAB routines for displaying and editing final and intermediate results. A document describing use of the package has also been prepared. Documentation and tar files containing source code and databases are available over the internet at

<http://www.oce.orst.edu/po/research/tide>.

As an example application of OTIS we have developed an 8 constituent tidal model for the seas around Indonesia and the Philippines, including the Yellow and South China Seas. This regional scale model has a resolution of 1/6 degree, and fits all TOPEX/Poseidon data in the area (for the first 154 orbit cycles). The elevation component for the M2 constituent of the inverse solution is plotted in Figure 1. A comparison of T/P residuals at the aliased M2 frequency for the regional and global solutions is given in Figures 2.



*Figure 1: Amplitude and phase for M2 elevation; Indonesian Seas inverse solution on 1/6 degree grid*



*Figure 2: Residuals in TOPEX/Poseidon data at M2 frequency using global TPXO.3 solution to remove tides (left) and using the regional Indonesian inverse solution (right). The new solution is dramatically improved.*

We have also constructed regional scale tidal solutions for the west coast of North America, at 1/6 degree resolution, the coast of Oregon at 1 km resolution, the Mediterranean Sea at 1/12 degree resolution, and the area surrounding the Hawaiian Islands, again at 1/12 degree resolution. For the Oregon Coast we have assembled times series for all available current moorings, and computed major tidal constituents. We are in the process of comparing these results to the currents predicted by the barotropic inversion. The representer calculation programs have been modified to allow calculation of representers for velocity data, and a barotropic inversion incorporating velocity data will soon be completed. Programs for harmonic analysis of coastal radar data have been written, and initial analysis of these data are underway. We have also been working with visiting Russian researchers A. Kurapov and G. Kivman to test the value of finite element modeling to improve resolution of barotropic velocity fields for the Oregon Coast. Depending on our experiences with these initial tests, further efforts in this direction may be pursued.

Finally, we have begun work on three-dimensional tidal inversion methods. Equations and boundary conditions for representer calculations for a linear stratified model have been worked out. Plans are to implement this calculation initially on the NRL CM-500E that has recently been transferred to COAS. To get a better feel for three dimensional currents in the area and the importance of various terms in the non-linear dynamical equations we will do numerous runs of the Princeton Ocean Model (POM) for an Oregon Coast model domain. A grid has been set up with 15 sigma levels, and initial test runs are underway.

## **RESULTS**

This year the most important results of this project relate to the methodology of data assimilation. First, we have demonstrated that a rigorous variational approach to data assimilation can be made quite efficient, at least for a frequency domain problem. The methods developed are in fact more efficient than simpler approaches such as nudging (e.g., Kantha, 1996; Tierney et al., 1998) or conjugate gradients minimization of a simplified penalty functional (e.g., Kivman, 1998). The representer approach also allows for relatively straightforward computation of posterior error bars for tidal fields using Monte Carlo methods. This calculation is particularly efficient with the direct factorization approach developed in this project. The most costly part of the calculation is factoring the large sparse coefficient matrix for the linearized shallow water equations. This only has to be done once; the same factored matrix can be used for all random realizations in the Monte Carlo simulations.

Our new tidal data assimilation scheme demonstrates another methodological development that could be more generally applicable to other data assimilation problems. It is generally simpler to specify a plausible error covariance for the first order system of shallow water equations (momentum and mass conservation), than for the second order wave equation that may be derived from these equations. However, reducing to a state space of scalar fields (e.g., only elevation or pressure), can offer significant computational advantages, particularly for direct factorization methods. We have demonstrated for the tide problem a simple strategy for specifying error statistics for the vector "primitive equations" while doing the heavy calculations using a second order scalar differential equation. With this approach we are able to do all representer calculations using only elevation components, but still unambiguously recover velocities (and indeed to use velocities as data). We expect this general idea to be especially useful for three-dimensional tidal assimilation problems where the need to minimize program memory usage is more pressing.

## IMPACT/APPLICATIONS

The OSU tidal inversion software (OTIS) has been turned into an efficient, general, relocatable system for modeling barotropic tides. The system is now documented well, relatively easy to use, and can be run with only modest computing resources. It is available to all interested parties over the world wide web. We have just released OTIS, so we cannot claim significant application of the software outside our own group yet. However we anticipate that the software should be useful for regional and local tidal modeling for a broad range of researchers. Initial interest in the software has been high. A visit to NRL SSC will be made in the next month to describe the methods, and demonstrate use of OTIS to NRL researchers.

## TRANSITIONS

The methods developed in this project have already had significant application and impact. Other groups working on tidal modeling have adopted our representer approach (e.g., LeProvost et al., 1997; Kivman and Kurapov, 1998) to tidal data inversion. Perhaps more importantly, some of the efficient solution methods developed during this project have also been applied to other oceanographic assimilation problems (e.g., Bennett et al., (1996) have used similar data space conjugate gradients methods; see also Bennett (1998)).

## RELATED PROJECTS

With NSF/NASA funding I am collaborating with R. D. Ray of NASA/GSFC on a study of global tidal dynamics using satellite altimetry data. Through this funding I am participating as a co-investigator on the upcoming Jason mission. A major goal of this project is to improve understanding of the geographical distribution and mechanisms of tidal energy dissipation, focusing in particular on the role of internal tides in the energy budget. Improvements to inversion methods and software will be to some extent transferable from this NSF project to ONR funded research on coastal tidal modeling, and vice-versa. We are also participating in the data assimilation efforts for the OSU NOPP funded coastal oceanography project. Our contribution to this effort will be to provide a tidal modeling component, both to remove tidal signals from poorly sampled data before assimilation, and to provide a tidal component in the nowcasts and forecasts produced. We are also working with Profs. J. Allen and R. Miller on developing practical assimilation methods for the coastal problem.

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