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CAPILLARY WAVE SPECTRA FROM SOUND REVERBERATION MEASUREMENTS

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CAPILLARY WAVE SPECTRA FROM

SOUND REVERBERATION MEASUREMENTS

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ABSTRACT

There is a unique relationship between sound wave scattering from a rough surface and its elevation spectrum. This relationship allows the use of available underwater sound reverberation measurements in the wave number range 0.025 to 2.5 radians/cm (5 to 55 radians/sec) to be used to develop sea surface capillary wave spectra. As an aside, optically measured surface slope spectra are converted to elevation spectra and these suggest the possible variability of capillary wave height spectra as well as a "cut-off" wavelength due to viscous effects.

Footnote to ABSTRACT

*Director, Systems Evaluation Division

Measurements of sea surface underwater sound reverberation, which depends upon sea surface elevation spectra, may, of course, be used to deduce those spectra. This paper summarizes the wave spectra aspects of such an analysis[1] and compares this with currently extant spectral representations.

To review briefly, the Neumann-Pierson spectrum[2,3] for long, energy-containing gravity waves is given by

$$[E_z(\omega, v)]_2 = C_1 \omega^{-6} \exp(-2g^2 \omega^{-2} v^{-2}) \quad (1)$$

in which $[E_z(\omega, v)]_2$ is the two-dimensional elevation spectrum of surface wave heights, z; ω is sea surface wave radian frequency; g is acceleration of gravity; v is wind speed; and C_1 is an arbitrary constant equal to 4.8×10^4 cm^2 sec^{-5} in the applicable range of $\omega \leqslant \pi$ radians/sec.

At higher wave frequencies, Phillips[4] gives on dimensional grounds the relationship

$$[E_z(\omega)]_2 = C_2 g^2 \omega^{-5} \quad (2)$$

for an equilibrium range in which C_2 is a dimensionless constant with the value 7.4×10^{-3} in the applicable range $4 < \omega < 12$ radians/sec.

In sufficiently deep water the (dispersion) relation[5] between ω and wavenumber k is

$$\omega^2 = gk + \gamma k^3/\rho \quad (3)$$

where γ is surface tension and ρ is water density. Furthermore, ω and k are related as

$$k \equiv \omega/c \qquad (4)$$

where c is wave phase angle velocity. If in Eq. 3 $\gamma k^3/\rho \ll gk$ then gravity waves predominate, but when $\gamma k^3/\rho = O(gk)$ then capillary waves are effective. At $\gamma k^3/\rho = gk$, $k \cong 3.7$ radians/sec, i.e., $\omega \cong 83$ radians/sec, at which point the minimum phase velocity $c_{min} \cong 23$ cm/sec.

Kuo[6] has shown that relative underwater sound power reverberation J_S depends upon sea surface elevation as

$$J_S = 4k_r^4 \sin^4\phi \, [E_Z(2k_r \cos\phi, v)]_2 \qquad (5)$$

referred to 1 yd^2 of sea surface and normalized to a distance of 1 yd from the surface. In Eq. 5, k_r is sound wavenumber (= sound radian frequency/speed of sound), ϕ is the grazing angle of the sound at the surface and $[E_Z(k_r, \phi, v)]_2$ is the surface elevation spectrum dependent upon k_r, ϕ and v. To be useful in estimating spectra, reverberation measurements must have ϕ large enough so that sea surface sublayer turbulence does not scatter appreciably and small enough so that specular reflections from the surface are ineffective. In fact, for $30 \text{ deg} < \phi < 50 \text{ deg}$, these conditions are met. The analysis[1] of somewhat over 2400 acoustic reverberation measurements showed for $2.5 \leqslant v \leqslant 7.5$ m/sec, and $0.025 \leq k \leq 2.5$ radians/cm that

$$[E_Z(k, v)]_2 = C_3 \, v^{2.36} \, k^{-3.67} \quad . \qquad (6)$$

With v in m/sec, k in radians/cm, C_3 has a value 4×10^{-5} and units such that the units of $[E_Z(k, v)]_2$ are cm^2. The $k^{-3.67}$ dependence in Eq. 6

is close to the k^{-4} dependence suggested by Phillips[5] for both gravity and capillary waves. The analysis[1] indicates also that outside $2.5 \leqslant v \leqslant 7.5$ m/sec, the elevation spectrum of sea surface is substantially constant at the values given by Eq. 6 at these extremes.

As it is possible to relate $[E_Z(k)]_2$ and $[E_Z(\omega)]_2$ through the relationship

$$[E_Z(k)]_2 = [E_Z(\omega)]_2 \ (d\omega/dk) \tag{7}$$

where $d\omega/dk$ comes from Eq. 3, the spectra of Neumann-Pierson (Eq. 1), of Phillips (Eq. 2) and the present contribution (Eq. 6) are shown in Fig. 1 over their appropriate ranges. The Neumann-Pierson spectrum is not limited to $2.5 \leq v \leq 7.5$ m/sec. The agreement among these seems adequate to endow Eq. 6 with some merit.

As a part of the analysis of sound reverberation, the optically measured laboratory water surface slope (z') spectra of Cox[7] were converted[1] from their one-dimensional form $(\omega/2\pi)[E_{Z'}(\omega/2\pi, v)]_1$ to the form $[E_Z(k, v)]_2$. The results of this conversion are shown also in Fig. 1. The converted data of Cox are interesting in that they suggest much larger spectral densities in the vicinity of c_{min} than the acoustic data of the open sea yield. In the laboratory measurements by Cox, the apparatus was both long and deep enough that the spectral densities are not boundary limited and this, taken with the reasonable agreement of these laboratory data at their low wavenumber extreme with the trends and magnitudes which Eqs. 1, 2, and 5 yield, suggests a mechanism for wave growth. The disparity between laboratory and open sea acoustic measurements might be explained by the fact that the laboratory water surface was kept especially clean, and contamination of open sea surface inhibits roughness as Cox and Munk[8] have shown. Furthermore, the acoustic data are at marginally large enough wavenumber to show the clean water effect even if it existed.

Another interesting aspect of Cox's data is the suggestion of rapid decrease of elevation spectral density in the vicinity of $k = 20$ to 50 radians/cm, somewhat analogous to the k^{-7} variation of turbulence theory[9] near thermalization of eddies.

Thus the sea surface sound reverberation data analysis confirms the trends of both the Neumann-Pierson and Phillips spectra and predicts a wind speed dependence of spectral density for short gravity and capillary waves as $v^{2.36}$. The laboratory optical data indicate that for very clean surfaces some additional texture may result in capillary wave elevation spectra, but the sound data do not indicate this texture for the open sea. The optical data also suggest that the "cut-off" wavelength $\lambda = 2\pi k^{-1}$ is of the order of $2\pi(30 \text{ radians/cm})^{-1} \cong$ 2 mm or so, as Phillips[10] has suggested.

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FIGURE

1. Two-Dimensional Elevation Spectrum vs Wavenumber. Data are of Neumann-Pierson (N-P), Phillips (OMP), Cox (CSC) and the Present Contribution (JJM) Over Applicable Ranges.

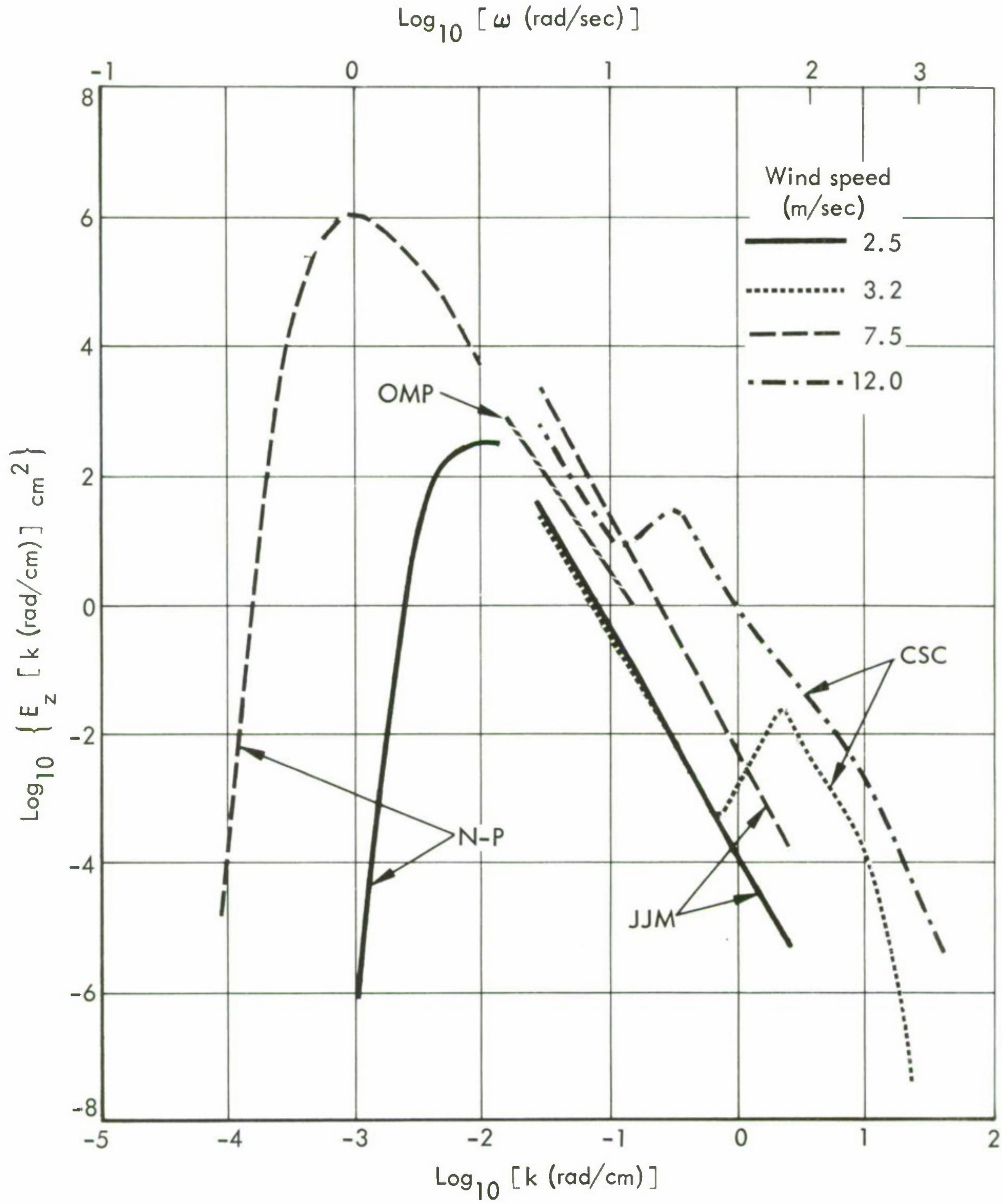

FIGURE 1. Two-Dimensional Elevation Spectrum vs Wavenumber. Data are of Neumann-Pierson (N-P), Phillips (OMP), Cox (CSC) and the Present Contribution (JJM) Over Applicable Ranges.

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