

## Estimating Surface Currents from SAR

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### LONG-TERM GOALS

The long-term goal of this program is to be able to utilize SAR imagery to estimate surface currents over the ocean. With the availability of a large number of commercial SAR satellite sensors, it may be possible to have an operational capability to generate surface current maps around the world's oceans that could be complimentary to the operational altimeters.

### OBJECTIVES

There are two objectives in this program.

- (1) Develop a capability to use SAR imagery to estimate surface currents.
- (2) Develop a capability to assimilate the SAR estimates into current models and/or other observations to generate a wide-area surface current estimate

### APPROACH

#### *(1) Estimating surface currents from SAR*

Two approaches will be investigated under this program. One will be the inversion of radar cross section (RCS) modulations into the underlying surface current gradient that has caused those modulations. This will be done by inverting the wave action balance equation that determines changes in wave spectra due to interaction with surface currents. This capability has been generated for internal wave signatures by another ONR program (the Non-Linear Internal Wave Initiative). Under this program, it will be modified for larger-scale current patterns. The second approach will be to generate a two-dimensional map of surface wave spectra from the SAR, and use the spatial changes in the spectra to estimate surface currents. We have already developed tools to estimate wave spectra under a NOAA/NESDIS program, however that algorithm will be modified under this program to include non-linear transfer functions. We anticipate that the translation of wave spectral changes into surface currents will also be done via inversion of the wave action balance equation

#### *(2) Assimilation into current models*

It is not clear if SAR imagery will ever be able to provide the wide-area view of surface currents required by operational users. Thus we anticipate that we may need to assimilate the SAR-derived

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currents into a large field-of-view by using existing models of current fields. We propose using the DELFT3D model for which we have done previous inversion work. We will modify this previous work so that we can determine the model inputs required to reproduce the SAR-derived currents, then use the model to generate the large field-of-view.

## WORK COMPLETED

We have finished the derivation of the adjoint expressions for the wave action balance equation for estimating the two-dimensional surface current field from observations of radar cross section in a SAR image and we have implemented it in software. We are currently testing the code which will finish the first half of Objective 1

## RESULTS

The wave action balance question (shown in one dimension for simplicity) is

$$\frac{\partial A}{\partial t} + (u + c_g) \frac{\partial A}{\partial x} - k_x \frac{\partial u}{\partial x} \frac{\partial A}{\partial k_x} = F_s \quad (1)$$

where A is the wave action, u is the surface current in the x-direction,  $c_g$  is the wave group velocity,  $k_x$  is the wave number in the x-direction, and  $F_s$  represents the sources and sinks of the wave action (normally a wind growth term, a energy dissipation from wave breaking term, and a non-linear dissipation term). This represents the changes in wave spectra across a scene due to the interaction of the waves with a current pattern. We have developed a model for predicting the radar cross section (RCS) of a SAR image from the map of 2D wave spectra, so the problem to solve is to find the current pattern,  $u(x)$ , such that the resulting wave spectra when put through the RCS model generate the “best fit to the SAR image.

The metric we will minimize is:

$$J = \iiint (O(x) - M(x))^2 + D \left[ \frac{\partial A}{\partial t} + (u + c_g) \frac{\partial A}{\partial x} - k_x \frac{\partial u}{\partial x} \frac{\partial A}{\partial k_x} - F_s \right] \quad (2)$$

where D is a Lagrangian multiply to make sure that the wave spectra satisfy the wave action balance equation (this is become the adjoint of A),  $M(x)$  is the model for RCS based on the wave spectrum at location x, and  $O(x)$  are the observations of RCS (i.e. the SAR image). We want to find  $u(x)$  that minimizes J, and we want to do that via a search operation. This means that we need to know the gradient of J with respect to u. We have worked through the variational calculus for this to show that:

$$\frac{\partial J}{\partial u} = D \frac{\partial A}{\partial x} + \frac{\partial}{\partial x} \left[ D k_x \frac{\partial A}{\partial k_x} \right] \quad (3)$$

where D must satisfy the so-called adjoint equation

$$\frac{\partial D}{\partial t} + (u + c_g) \frac{\partial D}{\partial x} - k_x \frac{\partial u}{\partial x} \frac{\partial D}{\partial k_x} = -2[O(x) - M(x)] \frac{\partial M}{\partial A} - \frac{\partial F_s}{\partial A} D \quad (4)$$

which is just the wave action balance equation with a different right-hand side. Eq. (4) is a standard derivation from Eq. (2); Eq. (3) is unique to the surface current problem. In general we would proceed by starting with a guess for  $u(x)$ , solving Eq. (1) to generate the wave action,  $A$ , everywhere, then solving Eq. (4) to generate the adjoint,  $D$ , everywhere, then plugging  $A$  and  $D$  into Eq. (3) to generate the gradient at each location to use in generating the next guess for  $u(x)$ . However, our RCS model,  $M(x)$ , is a two scale scattering model, which means that there is a Bragg wave scattering component combined with a modulation of the Bragg waves by the long-scale waves that is dependent on integral properties of the wave spectrum. This means that the term,  $\partial M/\partial A$  in Eq. (4) is dominated by values near the Bragg wavelength. This is illustrated in Figure 1 which shows an image of  $\partial M/\partial A$  (top) compared to an image of  $A$  (bottom). This means to first order we only need to solve Eq. (4) for these waves (i.e. for the spectral locations where  $\partial M/\partial A$  is large). These waves have large relaxation rates and thus do not depend significantly on the transport terms in the wave action balance equations. We can therefore approximate the solution to Eq. (4) as

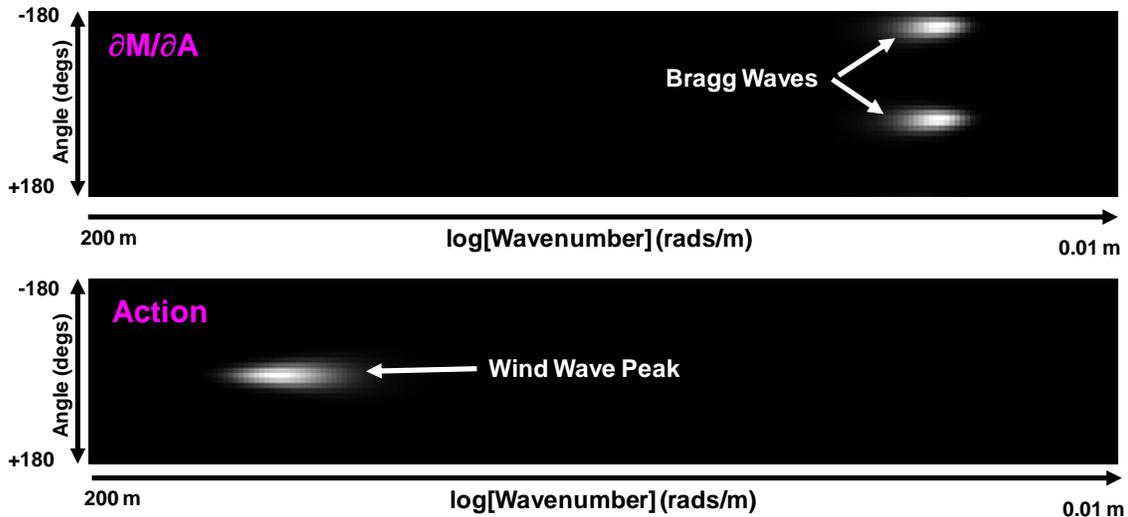
$$D \approx \frac{-2[O(x) - M(x)] \frac{\partial M}{\partial A}}{\frac{\partial F_s}{\partial A}} \quad (5)$$

Note that this is the exact solution for a Bragg scattering model for RCS and essentially tells us how to convert errors in the RCS comparison,  $O(x)-M(x)$ , into gradient estimates (using Eq. (5) put into Eq. (3)). We can then take one step further, and note that the long-wave component of  $M(x)$  is dominated by the action peak (i.e. the bright region in the bottom image of Figure 1) whose waves have small relaxation rates and thus just spread the error term “upwave” from where we calculate it to perturb the estimate of surface current.

This approach has been implemented and is currently being tested.

## IMPACT/APPLICATIONS

If successful, the resulting algorithms will generate an operational capability to estimate surface currents from commercial SAR systems.



*Figure 1: Top image show  $\partial M/\partial A$  which is a measure for how much a change in one spectral location will change the model output. This term is dominated by the region around the Bragg wave (bright blobs in the top image) since the model is dominated by Bragg scattering. Note that if we only used Bragg scattering, the top image would be two dots. Bottom image shows the action (i.e. the wave spectrum) which is dominated by the wind wave peaks. Thus this spectral regions dominates the changes in the model due to changes in the integral measures of the wave spectrum.*

## RELATED PROJECTS

The proposed research will start with algorithms for inverting the wave action balance equation to estimate currents that have been developed under the ONR Non-Linear Internal Wave Initiative. The proposed research will also start with algorithms for assimilating currents into DELFT3D that were developed under a previous ONR program entitled "DELFT3D Assimilation".