Optimization of One-Dimensional Aluminum Foam Armor Model for Pressure Loading

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**Optimization of One-Dimensional Aluminum Foam Armor Model for Pressure Loading**

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**Abstract:**
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Break down

Optimization of One-Dimensional Aluminum Foam Armor Model for Pressure Loading

• Pressure loading
  • i.e. blast

• Aluminum Foam
  • i.e. cellular materials

• Optimization
  • Minimizing acceleration
The Blast Problem

- Rapid expansion of a shockwave with high T and P emanating from a blast source
- Impact from a shock *deforms* and *accelerates* the receiving structure

How to mitigate the effects of the blast?
Cellular materials

- Material that contains a repeated base cell structure
  - Foams
  - Honeycombs
- Characterized by
  - Low density
  - Cannot support tensile loadings
- Near ideal compressive deformation behavior:
  - Transmit minimal load
  - Absorb significant energy
Problem Setup

• **Objective:**
  - To protect a mass from the acceleration effects of a pressure loading

• **Modeling:**
  - Pressure represented as a square pulse
  - Foam material properties are modeled as a function of relative density
  - System is modeled as series of springs and masses
  - System is unconstrained (inertial BCs)
Material Properties

• According to Hanssen (2002), material parameters can be described with calibration functions.

• Stress as a function of strain is then a function of the parameters:

\[
\{\sigma_p, \alpha, \frac{1}{\beta}, \gamma\} = C_0 + C_1 \left(\frac{\rho_f}{\rho_0}\right)^n
\]

\[
\sigma = \sigma_p + \gamma \frac{\varepsilon}{\varepsilon_d} + \alpha \ln\left(\frac{1}{1 - \left(\frac{\varepsilon}{\varepsilon_d}\right)^{1/\beta}}\right)
\]

\[\sigma_p\]

\[\varepsilon_y\]
System Model

- System model has an elastic-plastic spring based on the material properties of the given element joining each mass.
- There are $n+1$ masses:
  - $n$ for the armor
  - $1$ for the protected mass
Equations of Motion

\[ m_i \ddot{y}_i = F_L - F_R = A(\sigma_L - \sigma_R) \]

\[ m_i = \frac{\rho_i L}{\rho_0 n} A \rho_0 = x_i \frac{L}{n} A \rho_0 \quad \quad \omega^2 = \frac{T^2 \sigma_{y_0}}{\rho_0 L^2} \quad \quad \ddot{y}_i = \frac{\ddot{u}_i L}{T^2} \]

\[ \ddot{u}_i = \frac{A T^2 \sigma_{y_0} n}{x_i A \rho_0 L^2} \left( \frac{\sigma_{i-1}}{\sigma_{y_0}} - \frac{\sigma_i}{\sigma_{y_0}} \right) = \omega^2 \frac{n}{x_i} \left( \frac{\sigma_{i-1}}{\sigma_{y_0}} - \frac{\sigma_i}{\sigma_{y_0}} \right) \]

\[ KE_{ND} = \frac{x_i}{2n \omega^2} \dot{u}^2 \quad IE_{ND} = \frac{1}{n} \int_{\varepsilon_0}^{\varepsilon_f} \frac{\sigma_i}{\sigma_{y_0}} d\varepsilon \quad W_{ND}^{ext} = u_1 \frac{p}{\sigma_{y_0}} \]
Model Parameters/ Convergence

Total system length: \( L = 0.1 \text{m} \)
Pressure Pulse duration: \( T = 150 \mu \text{s} \)
Protected Mass: \( M = 100 \text{kg} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( C_0 )</th>
<th>( C_1 )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_p )</td>
<td>0 (MPa)</td>
<td>720 (MPa)</td>
<td>2.33</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0 (MPa)</td>
<td>42 (MPa)</td>
<td>1.42</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0 (MPa)</td>
<td>251 (MPa)</td>
<td>1</td>
</tr>
<tr>
<td>( 1/\beta )</td>
<td>.1</td>
<td>15.7</td>
<td>3</td>
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</table>

\[
\sigma_{y0} = 276 \text{ MPa} \\
E_0 = 68.9 \text{ GPa} \\
\rho_0 = 2700 \frac{\text{kg}}{\text{m}^3}
\]
LOAD CASE 1: $P = 0.1\sigma_{y0}$

$$\min \{ \max \dot{u}_M \}$$

s.t. \hspace{0.5cm} x_L \leq x_i \leq x_U$$

Graphical Solution: \hspace{0.5cm} 0.6 \hspace{0.5cm} 0.15 \hspace{0.5cm} 0.5423
Higher Load Values

LOAD CASE 2: $P = 0.2\sigma_y 0$

$x_1 = 0.60$ and $x_2 = 0.15$
System Response Comparison

Relative densities of comparison systems

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<th>2</th>
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</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>0.6</td>
<td>1</td>
<td>0.375</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.15</td>
<td>1</td>
<td>0.375</td>
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<tr>
<td>$x_3$</td>
<td>1</td>
<td>1</td>
<td>1</td>
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</table>
Conclusions

• Found optimal design for 2 elements of foam
  • $x_1 = .6$ (highest density)
  • $x_1 = .15$ (lowest density)

• Observed that
  • More energy absorbed does not necessarily give the best result
  • Less work done by the blast can be worse than more work
  • Having more system kinetic energy
QUESTIONS?

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Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the U.S. Army TACOM Life Cycle Command.
BACKUP
Material properties

• Yield stress and elastic modulus depend on
  • Relative density (density of foam/density of solid)
  • Distribution constant (open cell vs closed cell)
• Compaction strain is only a function relative density

\[ \frac{\sigma_p}{\sigma_{y_0}} = 0.3\phi^{3/2} \left( \frac{\rho_f}{\rho_0} \right)^{3/2} + (1 - \phi) \left( \frac{\rho_f}{\rho_0} \right) \]

\[ \frac{E_f}{E_0} = 0.3\phi^2 \left( \frac{\rho_f}{\rho_0} \right)^2 + (1 - \phi) \left( \frac{\rho_f}{\rho_0} \right) \]

\[ \varepsilon_d = 1 - 1.4 \left( \frac{\rho_f}{\rho_0} \right) \]