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HUMAN PREDICTIVE REASONING FOR GROUP INTERACTIONS

Marcus B. Perry

The University of Alabama
Box 870226
Tuscaloosa AL 35487-0226

Patrick J. Vincent

Northrop Grumman IT
2555 University Boulevard
Fairborn OH 45324

Jeremy D. Jordan

Anticipate & Influence Behavior Division
Sensemaking & Organizational Effectiveness Branch

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AIR FORCE RESEARCH LABORATORY
711TH HUMAN PERFORMANCE WING,
HUMAN EFFECTIVENESS DIRECTORATE,
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433
AIR FORCE MATERIEL COMMAND
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//SIGNED//
PAUL D. FAAS
Work Unit Manager
Sensemaking & Organizational
Effectiveness Branch

//SIGNED//
DAVID G. HAGSTROM
Anticipate & Influence Behavior Division
Human Effectiveness Directorate
711th Human Performance Wing
Air Force Research Laboratory

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[1]Marcus B. Perry, [2]Patrick J. Vincent [3]Jeremy D. Jordan

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[1]The University of Alabama
Box 870226
Tuscaloosa AL 35487-0226

[2]Northrop Grumman IT
2555 University Boulevard
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14. ABSTRACT

In this effort, two approaches to predicting attitude and behavior of human groups are developed. One approach employs a novel statistical model for assessing group-level and individual-level factors on group constructs of interest. The second approach is more probabilistic in nature, and involves the use of a Markov chain with Bayesian updates in order to capture changes in group constructs as a function of changes in important environmental variables. These models were developed as alternative approaches to capturing the dynamics between group constructs of interest and important predictors of these group constructs. It is demonstrated how each model developed in this effort could be instantiated into the National Operational Environment Model (NOEM).

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

PREFACE

The Human Predictive Reasoning for Group Interactions research effort was sponsored by the Air Force Research Laboratory's (AFRL), Sensemaking and Organizational Effectiveness Branch (711 HPW/RHXS) under Task Order #14 of the Technology for Agile Combat Support (TACS) contract (FA8650-D-6546). The period of performance for the research effort extended from 10 January 2008 to new contract date 9 August 2010. This report documents the results of research activities conducted as part of this task order.

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1.0 SUMMARY

The ultimate goal of this project was to develop a model for instantiated into the National Operational Environment Model (NOEM) that permits the capture of changes in group attitudes and behavior as functions of changing environmental variables. The 'behavior module' in NOEM is expected to publish affinity measures between group entities over time. The word 'affinity' here implies a 'natural liking to' or 'attraction to', so that if a certain in-group has a high level of affinity towards a given out-group, the in-group has a natural attraction towards that out-group. For example, a population might transition from an "unsupportive" state to a "supportive" state, say, towards the indigenous government, in a relatively short period of time if it perceives that its quality of life will be improved by, say, the election of a new political figure. The model developed in this effort effectively captures the dynamic nature of collective and/or individual behavior as a function of changing variables in the operating environment (OE). In addition to instantiation into the NOEM, the methods and procedures outlined in this report can be used as stand-alone methods for conducting *causal* or *correlation* studies (the former implying a properly designed experiment is used in the process of data collection) involving groups. For example, one might be interested in determining if group-level and/or individual-level characteristics (i.e., factors) provide any predictive power of some collective or individual-level outcome.

There are two primary contributions of this effort. The first is the development of a novel statistical methodology for analyzing group and/or individual-level constructs, where the goal is to estimate the functional relationships between the constructs and one or more characteristics of groups or individuals, including attitudes towards other groups. The model is especially useful in situations where observational studies or designed experiments involving groups and their members are conducted, say using survey response data (e.g., Likert-items). The proposed analysis method employs a random effects model in conjunction with a data-based approach for determining an appropriate transformation on the 'response' variable in order to correct for violations of the underlying model assumptions. We develop an algorithm for use in parameter estimation using the method of maximum likelihood, and suggest an approximate test for determining the statistical significance of the independent variables considered. The proposed model takes into account the non-independence between responses obtained from members

belonging to the same group. That is, members of the same group are assumed to share common goals, values, culture, beliefs, etc., and as such, one might expect the responses obtained from members randomly selected from the same group to be more similar than responses obtained from two individuals that were randomly selected from the entire population of individuals. If this non-independence is neglected by the use of, say, standard analysis of variance (ANOVA) or regression procedures (which, unfortunately, is quite common in practice), the results of the analysis can be misleading. Specifically, collective or group-level independent variables may appear to be statistically significant when they are not, and individual-level independent variables that are in fact significant can go undetected. Once fitted models are obtained using the proposed approach, one can then instantiate these into the NOEM as meta-models.

The second contribution of this effort, the Markov Affinity Model with Bayesian Updates (MAMBU), involves an alternative approach to modeling group constructs (i.e., affinity, trust, etc.) as a function of the changing state of the environment. The model is especially useful when historical data (or subject matter expert knowledge when empirical data is lacking) is available on the conditional distribution of "important" environmental variables, given the behavioral state of the groups under study. The term "important" implies that only those variables that are assumed to affect group attitudes/behavior are considered, perhaps determined *a priori* via an appropriate statistical analysis. The intent of MAMBU is primarily for instantiation into the NOEM. The approach taken by MAMBU models the probability distribution assigned to a group's behavioral state space over time using a discrete time Markov chain, and updates this probability distribution whenever new information becomes available using a Bayesian updating procedure. The purpose of the Markov chain is to serve as a prior probability distribution over the behavioral state space; that is, prior to obtaining any new information. Once new information becomes available (i.e., current environmental conditions are observed), the *posterior* probability distribution assigned to the group's behavioral state space is computed. The posterior distribution represents the probability distribution assigned to the behavioral state space, however, conditional on current environmental conditions. The affinity scores between groups are computed at any time t, and are taken to be an expectation across the posterior probability distribution assigned to the behavioral state space at time t. In addition, to account for the fact that individuals (and hence, groups) retain memory of past events, we geometrically weight the posterior probability distributions (and thus, the affinities) over time. We provide several examples demonstrating how MAMBU is applied and discuss alternative approaches to populating MAMBU using empirical data and subject matter expert knowledge.

2.0 INTRODUCTION

Following that of (Nezlek & Zyzniewski, 1998), we define *groups* as collections of individuals that occur either naturally, such as work groups in organizations, or arbitrarily, such as groups created in experiments. Further, *group-level phenomena* are defined as variables or outcomes that exist only at a group or aggregate level (e.g., unemployment rate, violent death rate, crime rate). In contrast, *individual-level phenomena* are defined as variables or outcomes that exist at the individual level (e.g., age, sex, education).

Prior to the development a behavior model for NOEM, one must first have a means to understanding how group-level (or collective individual) responses are functionally related to relevant variables, where the relevant variables can be cast across both group and individual levels. This can be accomplished via appropriate statistical analyses of observational and/or experimental data that might be available to the analyst. In empirical studies of groups, response data is sampled from individual group members, and thus, it is hierarchical in nature, as pointed out by (Nezlek & Zyzniewski, 1998) and others (e.g., see (Anderson & Ager, 1978), (Draper, 1995), (Faris & Brown, 2003), (Forsyth, 1998), (Hoyle & Crawford, 1994), (Hoyle, Georgesen, & Webster, 2001), (Kenny & Voie, 1985), (Quillian, 1995), (Raudenbush, 1995), (Raudenbush & Willms, 1995)). For example, one might be interested in determining the effect of a host population's unemployment rate and skill level of individuals on the individual or collective attitudes and behaviors towards immigrants. Note that *skill level* is an individual-level factor and *unemployment rate* is an aggregate or group-level factor. Since we are dealing with groups, individuals are then, by definition, *nested* units of observation; that is, nested within groups. Nested data structures, such as that considered in this effort, present several problems from an analysis point of view.

The problem studied in this effort is one involving human group processes. In particular, interest lies in drawing statistical inference on variables that influence human group processes, whether at the individual or group level. In the past, this task has been accomplished primarily via standard ANOVA and/or multiple linear regression methods, such as those discussed in (DeMaris, 2004). However, people that exist within groups (e.g., students in schools, members of churches, residents of villages, members of organizations, etc.) tend to be more homogeneous in their beliefs, goals, culture, etc., than people, say, randomly selected from the entire population of humans. This suggests that group membership induces positive correlation (i.e., non-independence) between responses obtained from any two people belonging to the same group. Unfortunately, this complicates the problem from a statistical analysis point of view, and often renders results obtained from standard statistical analyses questionable.

The problem of non-independent observations has been addressed by several authors in the social science literature, e.g., see (Hoyle, Georgesen, & Webster, 2001), and a variety of solutions have been proposed. One class of solutions attempts to circumvent the statistical problem altogether. Some of these strategies are discussed in (Hoyle & Crawford, 1994). Although these strategies can serve to eliminate the problem of non-independence of observations, their use is limited primarily to laboratory research and research questions that focus on *individual* group members as opposed to group process and behavior. Since our interest lies in drawing inference on collective behavior, we do not view this class of solutions as viable.

Another class of solutions is statistical in nature. A common strategy in this class involves the ANOVA design discussed in (Anderson & Ager, 1978). This method essentially corrects the variance estimates needed for performing correct statistical tests; however, at the cost of statistical power, i.e., some factors can go undetected. A major pitfall of this method is that it does not generally allow for drawing inference at both the group and individual levels. For studies in which group-level effects are present, researchers are advised to study only the group-level, to the exclusion of the individual. Another approach along these lines is (Schiffenbauer, Schulman, & Poe, 1978). Since a defining goal in sociology is the study of both the group and individual, these models are rather limited in their usefulness within the field of sociology.

In another approach, (Kenny & Voie, 1985) proposed a statistical technique that includes both individual and group-level effects. These authors treat the simultaneous study of individuals and groups as an exercise in construct validity, as defined by (Cronbach & Meehl, 1955). Their model provides for the estimation of individual- and group-level correlations, where if individual level correlations are found to exist, a hierarchical ANOVA strategy as recommended by (Myers, 1972) is used. On the other hand, if these correlations are found to be null, then standard ANOVA methods are used at the individual-level. Although the model proposed by (Kenny & Voie, 1985) considers the non-independence of individuals belonging to the same group, their method of analysis relies heavily on the fact that individuals are randomly assigned to groups. This is certainly a plausible assumption in experimental and laboratory situations where randomization poses no problem. However, in cases for which natural groups are studied, randomization in this manner usually does not occur. Another limitation is that their method does not allow for the estimation and test of interactions between group- and individual-level independent variables. This is a significant limitation as many times these interactions are highly significant predictors. Additional limitations of this model are pointed out in (Moritz & Watson, 1998).

A more sophisticated strategy involves hierarchical linear modeling (Byrk & Raudenbush, 1992), which permits correct variance estimates for inference purposes and allows for simultaneous hypothesis testing at both the group and individual levels. This is the approach we take in this effort. In particular, we propose a hierarchical linear model for analyzing group and/or individual-level constructs as functions of group and/or individual-level factors. The model considers the possibility of group-level main effects and interactions, individual-level main effects and interactions, as well as interaction effects involving group-level variables and individual-level variables. Further, it is not required that individuals be randomly assigned to groups. In fact, the proposed method is quite general, and can be used with experimental data, as well as observational data. We derive maximum likelihood estimates for the unknown parameters of the model and propose a method for testing the significance of the factor effects at all levels of analysis. The proposed statistical method has generality well outside the scope of NOEM, to include the statistical characterization and prediction of group attitudes and behavior as a function of relevant factors. However, we recommend a strategy for instantiation into the NOEM using meta-models fit by way of the proposed method.

As an alternative to the above mentioned statistical model, we also developed the MAMBU. MAMBU is our first attempt at modeling 'affinity' between groups within the NOEM framework. It is particularly useful when historical data (or subject matter expert knowledge) is available on the conditional distributions of "important" environmental variables, given the behavioral state of the groups under study. The term "important" implies that only those variables that are assumed to affect group attitudes/behavior are considered, perhaps determined *a priori* via an appropriate statistical analysis. The approach taken by MAMBU models the probability distribution assigned to a group's behavioral state space over time using a discrete time Markov chain, and updates this probability distribution whenever new information becomes available using a Bayesian updating procedure. An advantage to this approach relative to the hierarchical modeling approach discussed above is that the probability distribution of the behavioral state space of the groups is captured at each time t. As a result, action sets can be mapped to the behavioral state space at each time t quite easily (which is desirable within NOEM). Additionally, the statistical analysis required to populate MAMBU is rather straightforward, relative to the analysis required for the proposed hierarchical modeling technique.

3.0 METHODS, ASSUMPTIONS, AND PROCEDURES

3.1 Statistical Model

In this subsection, a novel statistical model that permits correct testing of group-level and individual-level effects on group and/or individual-level constructs is developed, including model specification, estimation and inference.

3.1.1 Model Specification

Consider the following statistical model

$$\mathbf{y}_i = \mathbf{1}_{n_i}\mathbf{z}_i^{'}\beta + \mathbf{X}_i\gamma + \mathbf{W}_i\eta + \mathbf{1}_{n_i} b_i + \varepsilon_i$$

where

- $\mathbf{y}_i$ denotes an $n_i \times 1$ vector of responses obtained from the i^{th} group.
- $\mathbf{z}_i$ denotes a $p \times 1$ vector of group-level covariate values for the i^{th} group.
- β denotes a $p \times 1$ vector of unknown group-level effects.
- $\mathbf{X}_i$ denotes a $n_i \times k$ matrix of individual-level covariate values nested within the i^{th} group.
- γ denotes a $k \times 1$ vector of unknown individual-level effects.
- $\mathbf{W}_i$ denotes a $n_i \times q$ matrix of covariate values corresponding to group-level by individual-level interaction effects.
- η denotes a $q \times 1$ vector of unknown group-level by individual-level interaction effects.
- b_i is a scalar-valued random effect and is assumed to follow a normal distribution with zero mean and variance σ_b^2. Additionally, the b_i's are assumed to be independent over the index *i*.
- ε_i is the error term corresponding to within group variation and is modeled as multivariate normal with zero mean vector and variance-covariance matrix $\sigma_\varepsilon^2\mathbf{I}$.

where n_i denotes the number of observations (i.e., individuals) sampled from the i^{th} group. Lastly, it is assumed that the covariance between b_i and ε_i is zero. This is a common and reasonable assumption in practice.

Note that for the proposed model, we have

$$E(\mathbf{y}_i) = \mathbf{1}_{n_i}\mathbf{z}_i^{'}\beta + \mathbf{X}_i\gamma + \mathbf{W}_i\eta$$

and

$$Var(\mathbf{y}_i) = \mathbf{V}_i = \sigma_b^2 \mathbf{1}_{n_i}\mathbf{1}_{n_i}^{'} + \sigma_\varepsilon^2 \mathbf{I}_{n_i}$$

which implies the following correlation structure

$$Cov(y_{ij}, y_{i^{'}j^{'}}) = \begin{cases} \sigma_b^2 + \sigma_\varepsilon^2 & i = i^{'}; j = j^{'} \\ \sigma_b^2 & i = i^{'}; j \neq j^{'} \\ 0 & i \neq i^{'} \end{cases}$$

suggesting that responses measured from individuals belonging to the same group have covariance σ_b^2, while responses measured from individuals belonging to different groups have zero covariance. The unknown parameter vectors (β, γ, and η), as well as the unknown variance components (σ_b^2 and σ_ε^2) need to be estimated from a sample. For example, one might use a sample of survey response data collected from the individual group members.

The above model can be written more compactly by

$$\mathbf{y}_i = \Lambda_i\lambda + \mathbf{1}_{n_i} b_i + \varepsilon_i$$

where b_i and ε_i are as defined above, and

$$\mathbf{\Lambda}_i = \begin{bmatrix} \mathbf{1}_{n_i}\mathbf{z}_i' & \mathbf{X}_i & \mathbf{W}_i \end{bmatrix}$$

denotes an $n_i \times r$ dimensional matrix, where $r = p + k + q$, where

$$\lambda = \begin{bmatrix} \beta^{'} & \gamma^{'} & \eta^{'} \end{bmatrix}^{'}$$

is a $r \times 1$ unknown parameter vector.

A critical assumption in the above model is that the response vectors (i.e., $\mathbf{y}_i$'s) are independent, each following a multivariate normal distribution with mean vector μ_i and variance-covariance matrix $\mathbf{V}_i$. Unfortunately, for many applications involving the study of groups, this assumption can be grossly violated (e.g., Likert-scale data). As a result, we propose a data-driven approach involving a power transformation on the original response variable to "force" the data to appear normal. Subsequent analysis is then performed in the transformed domain, and then transformed back into the original units for interpretation purposes.

3.1.2 Model Estimation

Suppose that $\mathbf{y}_i$ is as assumed in the above model. That is, the $\mathbf{y}_i$'s are independently distributed as multivariate normal with mean vector μ_i and variance-covariance matrix $\mathbf{V}_i$. Then given the observed vector of responses $\mathbf{y} = \left(\mathbf{y}_1^{'},\ \mathbf{y}_2^{'},\ \ldots,\ \mathbf{y}_N^{'}\right)'$, the log-likelihood function for $\phi = \left(\phi_1,\ \phi_2\right) = \left(\sigma_b^2,\ \sigma_\varepsilon^2\right)'$ and λ is given by

$$\ell\left(\phi, \lambda \mid \mathbf{y}\right) = -\sum_{i=1}^{N} \log_e \left|\mathbf{V}_i\right| - \sum_{i=1}^{N} \left(\mathbf{y}_i - \Lambda_i \lambda\right)' \mathbf{V}_i^{-1} \left(\mathbf{y}_i - \Lambda_i \lambda\right)$$

In what proceeds, note that $\mathbf{V}_i$ can be written as

$$\mathbf{V}_i = \phi_1 \mathbf{V}_{i1} + \phi_2 \mathbf{V}_{i2}$$

where $\mathbf{V}_{i1} = \mathbf{1}_{n_i} \mathbf{1}_{n_i}^{'}$ and $\mathbf{V}_{i2} = \mathbf{I}_{n_i}$. We will first differentiate the likelihood function with respect to the unknown parameters ϕ_u $(u = 1, 2)$, or

$$\frac{\partial \ell\left(\phi, \lambda \mid \mathbf{y}\right)}{\partial \phi_u} = -\sum_{i=1}^{N} tr\left(\mathbf{V}_i^{-1} \mathbf{V}_{iu}\right) + \sum_{i=1}^{N} \rho_{iu}$$

where

$$\rho_{iu} = \left(\mathbf{y}_i - \Lambda_i \lambda\right)' \mathbf{V}_i^{-1} \mathbf{V}_{iu} \mathbf{V}_i^{-1} \left(y_i - \Lambda_i \lambda\right)$$

Note further that we can write

$$tr\left(\mathbf{V}_i^{-1} \mathbf{V}_{iu}\right) = \phi_1 \omega_{iu1} + \phi_2 \omega_{iu2}$$

where

$$\omega_{ium} = tr\left(\mathbf{V}_i^{-1} \mathbf{V}_{iu} \mathbf{V}_i^{-1} \mathbf{V}_{im}\right)$$

for $m = 1,\ 2$. Therefore, if we define the 2×2 matrices $\Omega_i = \left\{\omega_{ium}\right\}$ $\left(i = 1, \ldots, N\right)$, then the likelihood equations for any given λ are

$$\sum_{i=1}^{N} \Omega_i \phi = \sum_{i=1}^{N} \rho_i$$

where

$$\rho_i = \begin{bmatrix} \left(\mathbf{y}_i - \Lambda_i \lambda\right)' \mathbf{V}_i^{-1} \mathbf{1}_{n_i} \mathbf{1}_{n_i}^{'} \mathbf{V}_i^{-1} \left(\mathbf{y}_i - \Lambda_i \lambda\right) \\ \left(\mathbf{y}_i - \Lambda_i \lambda\right)' \mathbf{V}_i^{-1} \mathbf{I}_{n_i} \mathbf{V}_i^{-1} \left(\mathbf{y}_i - \Lambda_i \lambda\right) \end{bmatrix}$$

and thus the estimate for ϕ is obtained iteratively from

$$\tilde{\phi}(\lambda) = \left(\sum_{i=1}^{N} \Omega_i\right)^{-1} \left(\sum_{i=1}^{N} \rho_i\right)$$

The likelihood equation with respect to λ are easily shown to be

$$\left[\sum_{i=1}^{N} \left(\Lambda_i^{'} \mathbf{V}_i^{-1} \Lambda_i\right)\right] \lambda = \sum_{i=1}^{N} \Lambda_i^{'} \mathbf{V}_i^{-1} \mathbf{y}_i$$

so that the estimate for λ given ϕ is

$$\tilde{\lambda}(\phi) = \left[\sum_{i=1}^{N} \left(\Lambda_i^{'} \mathbf{V}_i^{-1} \Lambda_i\right)\right]^{-1} \sum_{i=1}^{N} \Lambda_i^{'} \mathbf{V}_i^{-1} \mathbf{y}_i$$

Consider the case where the response data is non-normal. Suppose there exists a transformation on the $\mathbf{y}_i$'s such that the transformed $\mathbf{y}_i$'s are independent and follow multivariate normal distributions with mean vectors μ_i and variance-covariance matrix $\mathbf{V}_i$. In this effort, we consider the class of power transformations

$$y_{ij} = \begin{cases} \dfrac{y_{ij}^{\theta} - 1}{\theta} & \theta \neq 0 \\ \log_e\left(y_{ij}\right) & \theta = 0 \end{cases}$$

where θ denotes the transformation parameter. Since the transformed $\mathbf{y}_i$'s are assumed to be independent and follow multivariate normal distributions with mean vector μ_i and variance-covariance matrix $\mathbf{V}_i$, the likelihood function for the *untransformed* response is then

$$l(\lambda, \phi, \theta) = (\theta - 1) \sum_{i=1}^{N} \sum_{j=1}^{n_i} \log_e\left(y_{ij}\right) - \frac{1}{2} \sum_{i=1}^{N} \log_e\left|\mathbf{V}_i\right| - \sum_{i=1}^{N} \left(\mathbf{y}_i(\theta) - \Lambda_i \lambda\right)' \mathbf{V}_i^{-1} \left(\mathbf{y}_i(\theta) - \Lambda_i \lambda\right)$$

and the goal is to find the values of θ, λ, and ϕ that maximizes this likelihood function. To accomplish this, one can perform the following steps:

1. Choose a value for θ and an initial value for ϕ, say $\tilde{\phi}^{(0)}$.
2. Compute

   $$V_i^{(0)} = \tilde{\phi}_1^{(0)} 1_{n_i} 1_{n_i}^{'} + \tilde{\phi}_2^{(0)} \mathbf{I}_{n_i}$$

   and

   $$\tilde{\lambda}^{(0)}(\theta) = \left[\sum_{i=1}^{N} \Lambda_i^{'} \left(\mathbf{V}_i^{(0)}\right)^{-1} \Lambda_i\right]^{-1} \sum_{i=1}^{N} \Lambda_i^{'} \left(\mathbf{V}_i^{(0)}\right)^{-1} \mathbf{y}_i(\theta)$$

3. Use $\mathbf{V}_i^{(0)}$ and $\tilde{\lambda}^{(0)}(\theta)$ to evaluate Ω_i, $\rho_i(\theta)$ $(i = 1,\ldots,N)$, then compute

$$\tilde{\phi}^{(1)} = \left(\sum_{i=1}^{N}\Omega_i\right)^{-1}\left(\sum_{i=1}^{N}\rho_i(\theta)\right)$$

4. Return to Step 2 and iterate until some convergence criteria is met.

Thus, one could perform the above steps for a range of values for θ, each time substituting the resulting estimates of λ and ϕ into the likelihood function given above, and retain that value of θ that maximizes this function. We suggest using values of θ in the interval [-1, 1] in increments of, say, 0.50. Values in this set include the inverse, square-root, natural log, and inverse square-root transformations, as well as the case of no transformation (i.e., $\theta = 1$). Values of θ outside of this range are more difficult to interpret in practice.

Note that the transformation proposed in this research requires y_{ij} to be a positive number. However, in cases where y_{ij} is negative, one can include a positive constant *c* so that the transformation becomes

$$y_{ij}(\theta, c) = \begin{cases} \dfrac{\left(y_{ij}+c\right)^{\theta}-1}{\theta} & \theta \neq 0 \\ \log_e\left(y_{ij}+c\right) & \theta = 0 \end{cases}$$

where *c* is chosen so that $y_{ij} + c > 0$ for all *i* and *j*.

Suppose the model has been fit using the iterative procedure outlined above, then the fitted values in the original units are given by

$$\hat{y}_{ij} = \exp\left\{\frac{\log_e\left(1+\hat{\theta}\Lambda'_{ij}\hat{\lambda}\right)}{\hat{\theta}}\right\}$$

for $i = 1,\ldots,N$ and $j = 1,\ldots,n_i$, where Λ_{ij} is an $r \times 1$ vector of covariate values corresponding to the j^{th} response nested within the i^{th} group.

3.1.3 Model Inference

Suppose one is interested in testing the hypotheses $H_0:\ \lambda_h = 0$ versus $H_1:\ \lambda_h \neq 0$. Since $\hat{\lambda}$ is a maximum likelihood estimate for λ, we can derive an approximate test. If the variance components are known, it is easily shown that

$$Var\left(\hat{\lambda}\right) = \mathbf{D} = \left[\sum_{i=1}^{N} \Lambda_i' \mathbf{V}_i^{-1} \Lambda_i\right]^{-1}$$

and evaluating $\mathbf{V}_i$ using $\hat{\phi}$ (i.e., maximum likelihood estimate for ϕ), we have the following test statistic

$$z_0 = \frac{\hat{\lambda}_h}{\sqrt{\hat{D}_{hh}}}$$

which is asymptotically standard normal under H_0. Thus, for a given level of α (i.e., type I error rate), an *approximate* test involves computing $|z_0|$ and comparing to the upper $\alpha/2$ quantile of the standard normal distribution.

The model developed in this section can be used as a stand-alone analysis tool in situations where the researcher is interested performing correlational studies (via observational data) or causation studies (via a designed experiment) between individual and/or group level outcomes (or responses, constructs, etc.) and one or more characteristics of groups, group members, interacting groups, or the settings within which groups function.

The next subsection develops an alternative approach to modeling the relationship between group constructs and important variables. This approach is more probabilistic in nature, and is especially useful when interest lies in mapping a group's behavioral state to a set of possible actions. It exploits the use of historical information and subject matter expert opinion that might be available on the conditional distribution of important environmental variables, given the observed attitudes and behaviors of groups under study.

3.2 Markov Affinity Model with Bayesian Updates (MAMBU)

In this subsection we discuss the Markov Affinity Model with Bayesian Updates (MAMBU). MAMBU is our initial efforts to modeling changes in 'between-group' affinities within the NOEM framework, although its use extends beyond that of NOEM. It is anticipated that, for prediction purposes, a statistical model will be employed in helping the researcher to determine which environmental factors most highly correlate with some group construct of interest, e.g., affinity, alliance, prejudice, trust, etc. The objective here is to determine a smaller subset of factors (from some larger pool) that are useful in explaining variations in group constructs. To study these constructs, one might examine the influence of both group-level variables (e.g., unemployment rate, crime rate, violent death rate) and individual-level variables (e.g., age, sex, occupation) on these constructs using the analysis approach discussed in the previous subsection. However, this problem is simplified within the NOEM framework since there are no individual-level variables; rather, only group-level variables exist. As a result, standard regression methods can be used to populate MAMBU so long as all variables included in the study are aggregated at the group-level. However, NOEM's emphasis on only one level of analysis (i.e., group-level) may be a severe limitation. This is because both individual- and group-level processes occur in group settings (Kenny & Voie, 1985), (Moritz & Watson, 1998), thus NOEM may not permit the precise modeling of group dynamics since it does not retain information at the individual level.

The goal of MAMBU is to assign a probability distribution to the behavioral state space of a group towards some construct of interest (e.g., attitude towards immigrants, support for indigenous government, etc) as a function of changes in relevant environmental variables (e.g., unemployment rate, rate of grievances redressed, etc.). For example, suppose that a regional populace is under study, and interest lies in determining how the population's affinity towards the indigenous government changes as a result of changes in important variables (which are perceived by the population to be controlled by the indigenous government). In what follows, the technical development of MAMBU is discussed.

Let $A_{(ij)} = \left[S_{(ij)1},\ldots,S_{(ij)n_i}\right]$ denote the behavioral state space of group *i* towards group *j*, where $S_{(ij)l}$ denotes the l^{th} behavioral state of group *i*. Let $X_t = \left[X_{1t},\ldots,X_{kt}\right]$ denote a *k*-dimensional vector representing the state of the environment at time *t*. We desire a probability distribution over the state space $A_{(ij)}$ for each $i, j = 1,\ldots,m$; however, conditioned on the observed state of the environment at time *t*, or X_t. Thus, we have m groups in the environment, *k* environmental variables, and n_i behavioral states corresponding to group *i*. Let $p_{(ij),t}$ denote the probability distribution assigned to $A_{(ij)}$ at time *t*, and let $\Omega_{(ij)}$ denote a transition probability matrix corresponding to the behavioral state space $A_{(ij)}$. Given an initial state probability distribution over $A_{(ij)}$, say $p_{(ij),0}$, the *prior* probability distribution over $A_{(ij)}$ at time *t* is computed from

$$\left[P_t(S_{(ij)1}),\ldots,P_t(S_{(ij)n_i})\right] = p_{(ij),t} = p_{(ij),t-1}\Omega_{(ij)}$$

for t=1,2,…, suggesting our *prior* knowledge with respect to how a group transitions between its behavioral states over time is adequately modeled by a discrete-time Markov chain. Suppose that at time t we observe the state of the environment X_t, then the *posterior* (i.e., updated) probability distribution over $A_{(ij)}$ at time t is computed by

$$\tilde{p}_{(ij),t} = \left[P_t(S_{(ij)1} \mid X_t), \ldots, P_t(S_{(ij)n_i} \mid X_t) \right]$$

where by combining Bayes' Theorem and the law of total probability we have

$$P_t(S_{(ij)\ell} \mid X_t) = \frac{P(X_t \mid S_{(ij)\ell}) P_t(S_{(ij)\ell})}{\sum_{r=1}^{n_i} P(X_t \mid S_{(ij)r}) P_t(S_{(ij)r})}$$

Note that if the state of the environment remains constant over time (i.e., $X_t = X$ for all t), the probability distribution assigned to the behavioral state space of the group will eventually reach a steady state. This follows directly from the properties of discrete-time Markov chains e.g., see (Kulkarni, 2000). However, once the group is perturbed by changing environmental conditions (assuming the behavioral state of the group is affected by these changes), the probability distribution assigned to the state space again becomes transient.

The computation of $P(X_t \mid S_{(ij)\ell})$ can be messy and complicated, depending on the type of measurements contained in X_t, as well as whether or not covariance exists between the different variables in X_t. If the elements of X_t are all continuous variables, one approach is to let

$$P_t(X_t \mid S_{(ij)\ell}) = \frac{\exp\left\{ -0.5\left(X_t - \mu_{(ij)\ell}\right)' \Sigma_{(ij)\ell}^{-1} \left(X_t - \mu_{(ij)\ell}\right) \right\}}{(2\pi)^{k/2} \left| \Sigma_{(ij)\ell} \right|^{1/2}}$$

which is the multivariate normal density function. Note that $\mu_{(ij)\ell}$ and $\Sigma_{(ij)\ell}^{-1}$ denote the mean vector and covariance matrix, respectively, of the environmental state vector X_t, conditional on the behavioral state $S_{(ij)\ell}$.

There are other forms of data besides continuous data. For example, there are also counts and proportions. Suppose the elements in X_t consist of q counts (e.g., number of violent incidents in a region, number of civilian casualties, etc.), then a reasonable approximation of $P(X_t \mid S_{(ij)\ell})$ is given by the product of Poisson mass functions, or

$$P(X_t \mid S_{(ij)\ell}) = \prod_{u=1}^{q} \frac{\lambda_{(ij)u\ell}^{X_{ut}} \exp\left\{-\lambda_{(ij)u\ell}\right\}}{X_{ut}!}$$

where $\lambda_{(ij)u\ell}$ denotes the mean count rate of the u^{th} variable conditioned on the ℓ^{th} behavioral state contained in $A_{(ij)}$. Suppose that X_t consists of *q* proportions, i.e., a number between 0 and 1. Then an approximation to $P(X_t \mid S_{(ij)\ell})$ can be obtained from the product of beta density functions

$$P(X_t \mid S_{(ij)\ell}) = \prod_{u=1}^{q} \frac{\Gamma\left(\alpha_{(ij)\ell} + \beta_{(ij)\ell}\right)}{\Gamma\left(\alpha_{(ij)\ell}\right)\Gamma\left(\beta_{(ij)\ell}\right)} X_{ut}^{\alpha_{(ij)\ell}-1}\left(1-X_{ut}\right)^{\beta_{(ij)\ell}-1}$$

where $\alpha_{(ij)\ell} > 0$ and $\beta_{(ij)\ell} > 0$ denote parameters corresponding to the shape and scale of the probability distribution of X_t, and, Γ denotes the Gamma function defined generally as

$$\Gamma(z) = \int_0^\infty t^{z-1} \exp(-t)\,dt$$

Note that if the measures in X_t consists of a mixture of continuous, count and proportions data, then a reasonable approximation to $P(X_t \mid S_{(ij)\ell})$ can be obtained by taking the product of the marginal density functions corresponding to each measure. For example, let X_t denote an *a*-dimensional vector of continuous variables, Z_t denote a *b*-dimensional vector of count variables, and U_t a *c*-dimensional vector of proportion variables. Then an approximation to $P(X_t \mid S_{(ij)\ell})$ can be obtained from

$$P\left(X_t \mid S_{(ij)\ell}\right) = \frac{\exp\left\{-0.5\left(X_t - \mu_{(ij)\ell}\right)'\Sigma_{(ij)\ell}^{-1}\left(X_t - \mu_{(ij)\ell}\right)\right\}}{(2\pi)^{a/2}\,|\Sigma_{(ij)\ell}|^{1/2}} \times \prod_{u=1}^{b} \frac{\lambda_{(ij)u\ell}^{Z_{ut}} \exp\left\{-\lambda_{(ij)u\ell}\right\}}{Z_{ut}!}$$
$$\times \prod_{u=1}^{c} \frac{\Gamma\left(\alpha_{(ij)\ell} + \beta_{(ij)\ell}\right)}{\Gamma\left(\alpha_{(ij)\ell}\right)\Gamma\left(\beta_{(ij)\ell}\right)} U_{ut}^{\alpha_{(ij)\ell}-1}\left(1-U_{ut}\right)^{\beta_{(ij)\ell}-1}$$

Of course, in order to compute these probabilities, we must have some knowledge of the parameters (i.e., μ, Σ, λ, α, and β) for each behavioral state space considered. This knowledge is presumed to come from historical data (or subject matter expert knowledge when empirical data is lacking). Standard approaches to estimation and inference (i.e., method of maximum likelihood, two-sample *t*-tests, etc.) can be used to estimate and draw inference on the unknown parameters from observed data. Standard statistical approaches are widely available in a variety of software packages.

To obtain an overall measure of group affinity (or whatever construct is being modeled) at any time *t*, one can compute an expectation across the distribution assigned to the behavioral state space. That is, suppose that the cardinality of group *i*'s state space is odd-numbered, then at each time *t*, group *i*'s affinity toward group *j* is computed as

$$\xi_{(ij),t} = \tilde{p}_{(ij),t} v$$

where $\tilde{p}_{(ij),t}$ is the *posterior* probability distribution over the behavioral state space $A_{(ij)}$ at time *t*, and

$$v = \left[-1 \quad -\left(\frac{\left\lfloor \frac{n_i}{2} \right\rfloor - 1}{\left\lfloor \frac{n_i}{2} \right\rfloor} \right) \quad -\left(\frac{\left\lfloor \frac{n_i}{2} \right\rfloor - 2}{\left\lfloor \frac{n_i}{2} \right\rfloor} \right) \quad \cdots \quad 0 \quad \cdots \quad -\left(\frac{\left\lfloor \frac{n_i}{2} \right\rfloor - 2}{\left\lfloor \frac{n_i}{2} \right\rfloor} \right) \quad -\left(\frac{\left\lfloor \frac{n_i}{2} \right\rfloor - 1}{\left\lfloor \frac{n_i}{2} \right\rfloor} \right) \quad 1 \right]$$

where $\lfloor \ \rfloor$ denotes the "floor" function. Note that $\xi_{(ij),t}$ is an expected value and is contained in the set [-1, 1].

For example, suppose that we again consider a regional populace, and its affinity toward the regional government. Suppose that we consider three behavioral states for the regional populace: 1) Unsupportive, 2) Neither Supportive nor Unsupportive, and 3) Supportive of the regional government. Then an affinity score close to -1 would suggest that the population is in the 'Unsupportive' state. On the other hand, an affinity score close to 1 would suggest that the population is in the 'Unsupportive' state.

As a final addition to the model, it is important to consider the fact that individuals (and hence, groups) retain memory of past events. Thus, MAMBU accounts for this by geometrically weighting the posterior probability distributions assigned to the behavioral state space as they age with time. As the weights decrease geometrically, so does the contribution of the posterior distributions for which the weights are assigned. The posterior probability distribution over the behavior state space at time *t* is then computed from

$$\hat{p}_{(ij),t} = \psi_i \tilde{p}_{(ij),t} + (1 - \psi_i)\hat{p}_{(ij),t-1}$$

where $\psi_i \in (0, 1]$ for all *i* and denotes the weighting coefficient corresponding to group *i*. Larger values for ψ_i indicate that group *i* applies more weight to current environmental conditions, and less weight to past events or conditions. On the other hand, smaller values for ψ_i suggest that group *i* places less weight on current conditions and more on past events or conditions. The value for ψ_i is likely to be set by subject matter experts knowledgeable about group *i*.

4.0 RESULTS AND DISCUSSION

In this section we demonstrate application of the proposed models developed in the previous section and provide some additional discussion. We use simulated data sets in order to control 'ground truth'. We also discuss the use of these models within the NOEM framework.

4.1 Application of Statistical Model

To illustrate the proposed statistical model, consider the following. Suppose that a correlation (as opposed to causation) analysis is to be conducted using observational data with the objective of determining whether group unemployment rate and individual skill level affects attitudes towards immigrants. Note here that 'unemployment rate' is a group aggregate measure, while skill level is an individual characteristic. Thus, we are dealing with cross-level factors in this example. Suppose that response data consists of five-level Likert-item responses to some statement construct regarding immigrants. Suppose further that 8 groups are randomly selected from a larger population of groups available for study, and within each group, 40 individuals are randomly selected for observation. For each group selected, suppose that the unemployment rate for that group was recorded (low or high), as well as the skill level (low or high) of each individual surveyed within the groups. Suppose that each individual was asked to respond to the following statement construct on a five-point Likert-item: "Immigrants are a threat to my economic security." Note that the five-point Likert-item has response options "Strongly Disagree", "Disagree", "Neither Agree or Disagree", "Agree", and "Strongly Agree". The simulated data sets are shown in histogram format in Figure 1.

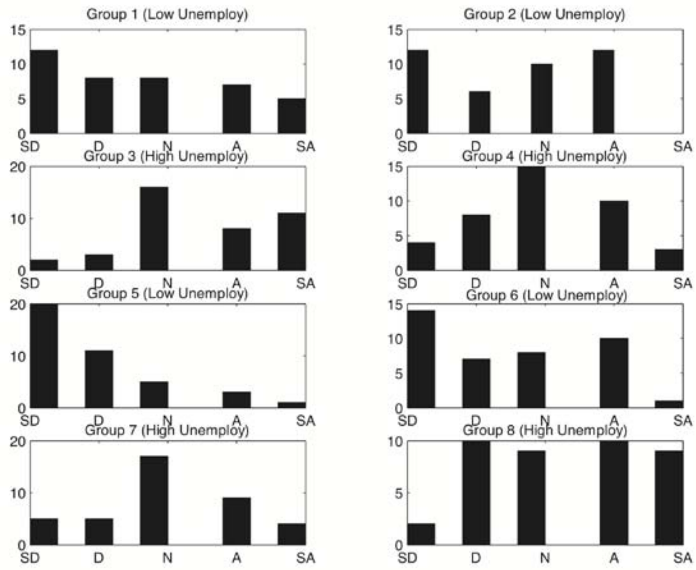

Figure 1: Likert-item Responses from Eight Groups, each with 40 Individuals

Each histogram shown in Figure 1 represents simulated responses obtained from members of the same group. The true simulated effect for *unemployment rate* was 0.8725. Additionally, the true simulated effects due to *skill level* and the *unemployment rate* $\times$ *skill level* interaction were -0.1025 and 0.4550, respectively. A random effect was also simulated to induce additional *unexplainable variation* at the group level. Using this data, we demonstrate our proposed analysis approach. The top plot in Figure 2 shows a plot of the log-likelihood function (given earlier) over a range of values for the transformation parameter, θ. Notice that the recommended transformation on the response is the square-root transformation since $\hat{\theta} = 1/2$.

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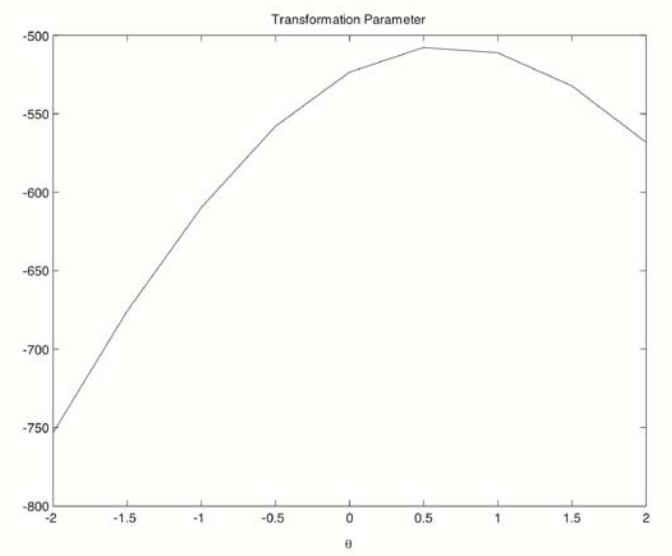

Figure 2: Plot of Log-likelihood Function versus Transformation Parameter

The numerical results of the analysis are shown in Table 1, including estimated coefficients, standard errors, and p-values. Notice that if unemployment rate is high, then Table 1 suggests that the proportion of individuals who agree with the statement construct "Immigrants are a threat to my economic security" increases. Similarly, Table 1 suggests that an increase in individual skill level is often accompanied by an increase in disagreement with the above statement construct. Lastly, we see that the interaction effect between the two variables is also deemed statistically significant. Of course, these results are expected since the data were simulated and the true simulated effects are known. Note that in Table 1 the standard error (S.E.) of the estimated coefficient for *unemployment rate* is larger than the standard errors of the estimated coefficients for *skill level* and the *unemployment rate* $\times$ *skill level* interaction. This is an expected result since we only have eight observations at the *group-level*, compared to 320 observations at the *individual level*. Since we have a small number of samples at the group level, the *t*-distribution with 6 degrees of freedom was used as the reference distribution for *unemployment rate*, while the standard normal distribution was used as the reference distribution for factors at the individual level. Note also that the estimates for the variance components are $\hat{\sigma}_b^2 = 0.015$ and $\hat{\sigma}_\varepsilon^2 = 0.5594$, suggesting most of the unexplainable variability is due to within-group differences. At this point, the researcher might seek other factors at the *individual* level in attempts to mitigate some of the unexplainable variability contained at this level.

Table 1: Effect Estimates and Estimated Standard Errors of Effect Estimates

Source	Coeff	S.E.	p-value
Unemployment Rate	0.2812	0.0601	0.0017
Skill Level	-0.0946	0.0422	0.0125
Unemploy x Skill Level	0.0713	0.0422	0.0455

Since the analysis was conducted in the transformed units, we need to transform the model back into the original units for prediction purposes. Doing so yields the following prediction equation, where z_i and x_{ij} denote observed or known values for group i's *unemployment rate* and individual *j*'s *skill level* (where individual *j* is a member of group *i*):

$$\hat{y}_{ij} = E\left(y_{ij} \mid z_i, x_{ij}\right) = \exp\left\{\frac{\log_e\left(1 + 0.5\left(1.25 + 0.28z_i - 0.09x_{ij} + 0.07z_i x_{ij}\right)\right)}{0.5}\right\}$$

where $\hat{y}_{ij}$ denotes the expected response on a five-point Likert-item to the statement construct "Immigrants are a threat to my economic security," as a function of group unemployment rate and individual skill level. It is important to note that using the proposed approach, inference can be drawn on the entire population of groups, as opposed to only the groups considered in the study, since it is assumed that each group considered in the study was randomly selected from a larger population of groups existing in the OE. Suppose that the entire population of groups had been sampled, then in such a case there is no random effect due to groups, and thus, one can use standard multiple linear regression methods to conduct the analysis.

Suppose that we used standard methods to conduct the analysis instead of the proposed strategy. In this particular case, one can apply the Box-Cox transformations (Box & Cox, 1964) to find the appropriate power transform on the response. For this example, applying the Box-Cox transformations yields $\hat{\theta} = 1/2$, so that the recommended transformation is again the square-root transformation. The numerical results are given in Table 2. Notice that when using standard regression in the presence of a random group effect, the factor *unemployment rate* appears to be highly significant, while the remaining factors at the individual-level go undetected (say, at the 0.05 level of significance). This is a major disadvantage of using standard statistical methods on grouped responses. In particular, the type II error associated with effects studied at the group level increases, and the power to detect factor effects studied at the individual level decreases.

Table 2: Effect Estimates and Estimated Standard Errors of Effect Estimates using Standard Multiple Linear Regression

Source	Coeff	S.E.	p-value
Unemployment Rate	0.2813	0.0679	< 0.0000
Skill Level	-0.0951	0.0679	0.0809
Unemploy x Skill Level	0.0699	0.0679	0.1518

The model proposed in this section is a powerful statistical technique for determining factors that can explain the variability in one or more group constructs of interest, to include attitudes and behaviors towards other (human) groups. Since groups share commonalities on a variety of dimensions, e.g., goals, ethnicity, culture, beliefs, values, etc., this complicates the resulting statistical analysis, particularly when the researcher is interested in assessing the significance of both group-level and individual-level factors. The consequences of analyzing grouped data using standard ANOVA or regression methods can be severe, often resulting in misleading results.

With respect to the NOEM, the proposed model can be used in establishing empirical relationships between factors and groups modeled within the NOEM. Using the proposed model, one can study the ‘forces’ acting on a population/group of interest that shapes its attitude and behavior. Once a fitted model is obtained, it can be implemented into the NOEM in the form of a meta-model. To demonstrate, consider the hypothetical example given above where group attitudes towards immigrants were studied as a function of unemployment rate and individual skill level. Since individual-level variables are not modeled within the NOEM framework, our focus might be on predicting group attitudes and behavior towards another group (e.g., immigrants) as a function of group unemployment rate. However, the above fitted model still can be used as a prediction equation by working with partitions of group *i* on the basis of skill level. That is, group *i* can be partitioned into two groups, one have a high skill set and another having a low skill set. Attitudes and behavior of groups with low skill sets and high skill sets can then be predicted as functions of group unemployment rates. For groups with high skill sets, the prediction equation becomes

$$\hat{y}_i = \exp\left\{\frac{\log_e\left(1.58 + 0.175 z_i\right)}{0.5}\right\}$$

and similarly for groups with low skill sets

$$\hat{y}_i = \exp\left\{\frac{\log_e\left(1.67 + 0.105 z_i\right)}{0.5}\right\}$$

For example, consider individuals in group *i* having a high skill set, then, based on the above analysis, if group unemployment is low (i.e., -1), we expect the average response on the Likert-item to lie somewhere between "Strongly Disagree" and "Disagree". On the other hand, if group unemployment is high, we expect the average response on the Likert-item to lie somewhere between "Neither Agree or Disagree" and "Agree". We find similar interpretations for members of group *i* having a low skill set.

4.2 Applications of MAMBU

In each of the examples that follow, we describe hypothetical examples involving a regional populace and a governing force. We should note that although the following examples involve describing the affinity that a regional populace has towards another group, MAMBU is certainly not limited to the modeling of regional populations and their attitudes towards a governing force. In fact, we believe that MAMBU has application in a number of different areas. For example, MAMBU might be implemented within a combat simulation model in order to model the process of assessing battle damage at the group or individual level. Additionally, it might find application in 'what if' studies involving the implementation of new policies.

4.2.1 Example with Continuous Environmental Variables

Suppose that we are interested in modeling changes in the 'affinity' that a regional populace has toward the governing force over time. Let us consider the following behavioral state space

$$A_{(12)} = [\text{Unsupportive, Neither Supportive or Unsupportive, Supportive}]$$

Suppose that we consider two continuous environmental variables:

1. Units of potable water supplied to the region at time t (X_{1t})
2. Units of electric power supplied to the region at time t (X_{2t})

Suppose that historical observations exist on X_1 and X_2 over some finite time horizon where the regional populace was known to be 'unsupportive', 'neither supportive or unsupportive' and 'supportive' towards the regional government, perhaps determined by analysis of public opinion polls over time. Figures 3-5 show historical (i.e., simulated for the purpose of this example) values for X_1 and X_2 assumed to have been jointly observed over a total of 100 time units, and under the perception that the regional population was 'Unsupportive", "Neither Supportive or Unsupportive", and "Supportive" of the regional government, respectively.

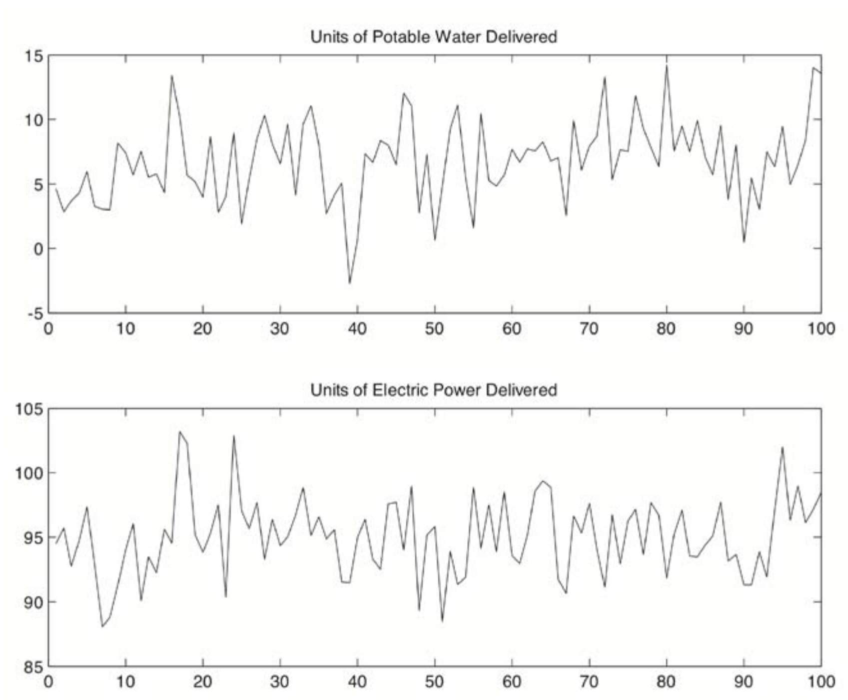

Figure 3: Jointly Observed Values of X1 and X2 when Population is Perceived to be 'Unsupportive' of the Governing Force

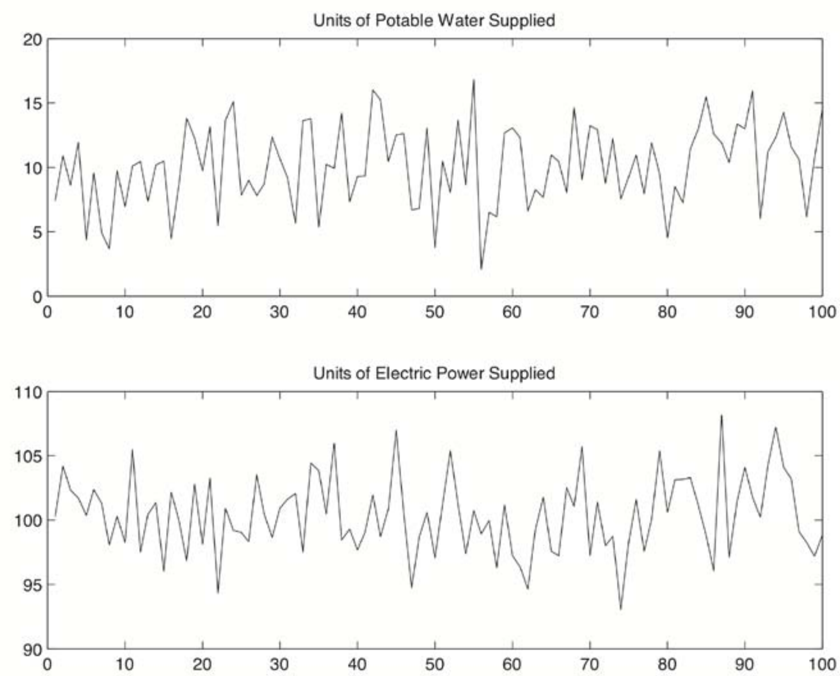

Figure 4: Jointly Observed Values of X1 and X2 when Population is Perceived to be 'Neither Supportive of Unsupportive' of Governing Forces

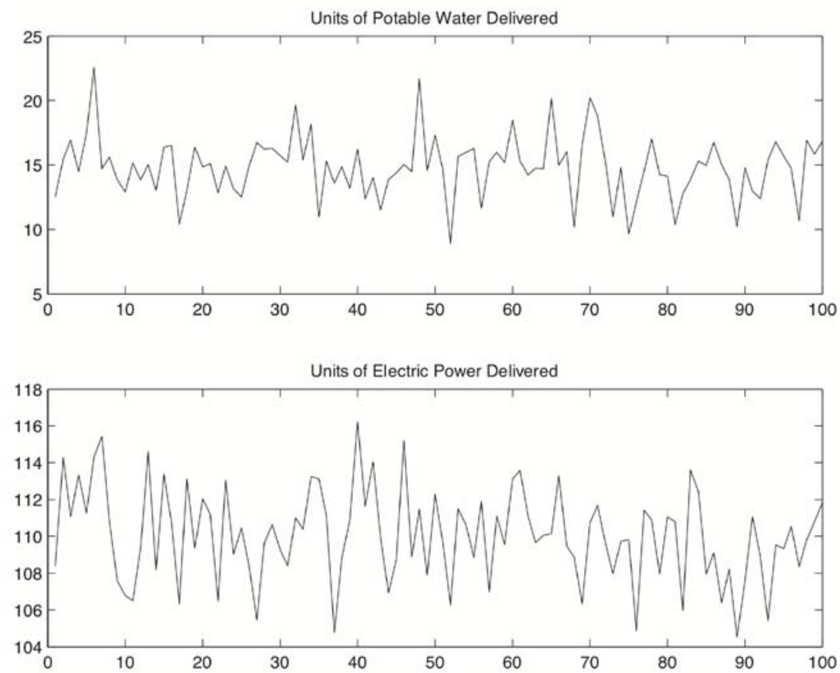

Figure 5: Jointly Observed Values of X1 and X2 when Population is Perceived to be 'Supportive' of Governing Forces

Using the data shown in Figures 3-5, we compute maximum likelihood estimates (under multivariate normal theory model assumptions) of the mean vectors and covariance matrices corresponding to each of the behavioral states using standard statistical software. These are given by:

$$\hat{\mu}_{(12),\text{'Unsupp'}} = [6.7857,\ 95.0406]' \quad \text{and} \quad \hat{\Sigma}_{(12),\text{'Unsupp'}} = \begin{bmatrix} 9.9931 & 2.1967 \\ 2.1967 & 9.0597 \end{bmatrix}$$

$$\hat{\mu}_{(12),\text{'Neither'}} = [10.0968,\ 100.368]' \quad \text{and} \quad \hat{\Sigma}_{(12),\text{'Neither'}} = \begin{bmatrix} 10.1014 & 1.5380 \\ 1.5380 & 8.8726 \end{bmatrix}$$

$$\hat{\mu}_{(12),\text{'Supp'}} = [14.841,\ 110.054]' \quad \text{and} \quad \hat{\Sigma}_{(12),\text{'Supp'}} = \begin{bmatrix} 5.8472 & 2.0341 \\ 2.0341 & 6.4130 \end{bmatrix}$$

In order to assess whether or not there is a statistical difference between, e.g., the mean units of potable water delivered when the populace was observed to be ‘unsupportive’ and the mean units of potable water delivered when the populace was observed to be ‘neither supportive or unsupportive’, one can use standard statistical methods for making multiple comparisons given in, e.g., (Wu & Hamada, 2000) or (Montgomery, 2005).

For example, suppose we want to determine if there is a difference between the mean units of potable water delivered when the populace was assumed to be in the ‘unsupportive’ behavioral state, and the mean units of potable water delivered when the populace was assumed to be in the ‘neither supportive or unsupportive’ state. Using standard methods for multiple comparisons involves computing the *t*-statistics

$$t_{ij} = \frac{\bar{x}_i - \bar{x}_j}{\sqrt{s_p^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}$$

for all $i, j\ (i \neq j)$, where $\bar{x}_i$ and $\bar{x}_j$ denote sample averages and s_i^2 and s_j^2 denote sample variances corresponding to the i^{th} and j^{th} behavioral states, respectively, and

$$s_p^2 = \frac{(n_i - 1)s_i^2 + (n_j - 1)s_j^2}{n_i + n_j - 2}$$

denotes the *pooled estimator* of the *unexplainable* error σ^2 (i.e., the error source not explainable by the populace's behavioral state). Note also that n_i and n_j denote the number of samples corresponding to behavioral states *i* and *j*, respectively. The test is conducted by comparing $|t_{ij}|$ to critical values of the *t* distribution with $n_i + n_j - 2$ degrees of freedom. To better control the overall type I error, one can alternatively use Bonferonni or Tukey critical values, e.g., see (Wu & Hamada, 2000), (Montgomery, 2005).

Note that in order to pool the variances, it is required that the variance of X be homogeneous across the different behavioral states. For the example presented here, although the assumption of constant variance appears to be valid across the 'unsupportive' and 'neither supportive or unsupportive' states (for both variables X_1 and X_2), the variances of the observations observed on the variables when the populace was perceived to be in the 'supportive' state appear to be smaller. If the variances cannot be assumed constant across behavioral states, then an *approximate* test involves computing

$$t_{ij}^* = \frac{\bar{x}_i - \bar{x}_j}{\sqrt{\left(\frac{s_i^2}{n_i} + \frac{s_j^2}{n_j}\right)}}$$

and comparing $|t_{ij}^*|$ to the *t* distribution with degrees of freedom

$$\upsilon = \frac{\left(\frac{s_i^2}{n_i} + \frac{s_j^2}{n_j}\right)^2}{\frac{\left(s_i^2/n_i\right)^2}{n_i + 1} + \frac{\left(s_j^2/n_j\right)^2}{n_j + 1}} - 2$$

For this example, we have a total of 3 comparisons to make: t_{12}, t_{13}, and t_{23} Since we have 100 observations on each variable for each behavioral state, the test statistic t_{12} (for each variable) is compared to the *t* distribution with 198 degrees of freedom, while the test statistics t_{13} and t_{23} (for each variable) are compared to the *t*-distribution with approximately $\upsilon = 188$ degrees of freedom. Note that since the degrees of freedom for each test is large in this case, one can approximate the critical values of the test rather precisely using the standard normal distribution. Doing so we find that, for the variable X_1, we have $t_{12} = -7.41$, $t_{13} = -20.23$ and $t_{23} = -11.89$, and at the $\alpha = 0.05$ level we see that $|t_{ij}| > z_{0.025} = 1.96$ for all *i* and *j* ($i \neq j$).

Repeating this procedure for the variable X_2 we obtain similar results. The results of the multiple comparison analysis provide justification for inclusion of the environmental factors (i.e., *potable water* and *electric power*) in the model. It provides empirical evidence that the levels of the variables of interest are in fact different, depending on the behavioral state of the group.

If we examine the parameter estimates, note that it only takes a loss of approximately 3 units of potable water and 5 units of electric power, on average, before the population transitions from the 'neither supportive or unsupportive' to 'unsupportive' states. However, it takes upwards of 5 additional units of potable water and 10 additional units of electric power, on average, before the population transitions from the 'neither supportive or unsupportive' to 'supportive' states. Also, as noted previously, the variances of X_1 and X_2 are much smaller when the population is in the 'supportive' state. The interpretation of this non-constant variance is that the population has less tolerance to variations in the amounts of potable water and electric power supplied by the government. That is, for the population to maintain its support for the government, it expects to receive roughly 15 units of potable water and 110 units of electric power on a *consistent* basis.

To completely populate MAMBU, we also require a transition probability matrix for the group under consideration, as well as an initial state probability distribution vector. The transition matrix for this example was chosen to be

$$\Omega_{(12)} = \begin{bmatrix} 0.95 & 0.04 & 0.01 \\ 0.25 & 0.60 & 0.15 \\ 0.15 & 0.25 & 0.60 \end{bmatrix}$$

and the initial state probability vector is $p_{(12),0} = [1,\ 0,\ 0]$, suggesting that initially, the regional population is unsupportive of the government. Note that the transition probability matrix represents our prior knowledge with respect to the likelihood of the populace transitioning between behavioral states. For example, given that the population is currently in the 'unsupportive' state, what is the probability that the population will transition into the 'supportive' state at the next point in time? For this example, $\Omega_{(12)}$ was selected on the basis of the steady-state distribution determined from the transition probability matrix, which is

$$p = [0.8113,\ 0.1225,\ 0.0662]$$

suggesting that at any given time, roughly 81% of the population will lie in the 'unsupportive' state. It should be noted that the settings for the elements in $\Omega_{(12)}$ are not crucial since they represent our prior knowledge. If no prior information is available, a reasonable setting for $\Omega_{(12)}$ is

$$\Omega_{(12)} = \begin{bmatrix} 0.34 & 0.33 & 0.33 \\ 0.34 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \end{bmatrix}$$

so that the steady-state probability vector is approximately uniform across the behavioral state space. Additionally, if the initial-state probability distribution vector is unknown, one can use $p_{(12),0} = [0.34,\ 0.33,\ 0.33]$, which is again approximately uniform. A uniform distribution across the behavioral state space implies that the populace can be in any of the three states with equal probability. Lastly, with respect to the geometric weighting, we assumed $\psi = 0.25$, implying that the population places 75% of their weight on past events and only 25% on current events.

Figure 6 shows the output of running MAMBU over a length of 365 time periods. The values of X_1 and X_2 were simulated from a multivariate normal distribution with $E(X_1) = 10,\ E(X_2) = 100$ and

$$\Sigma = \begin{bmatrix} 5 & 2.5 \\ 2.5 & 5 \end{bmatrix}$$

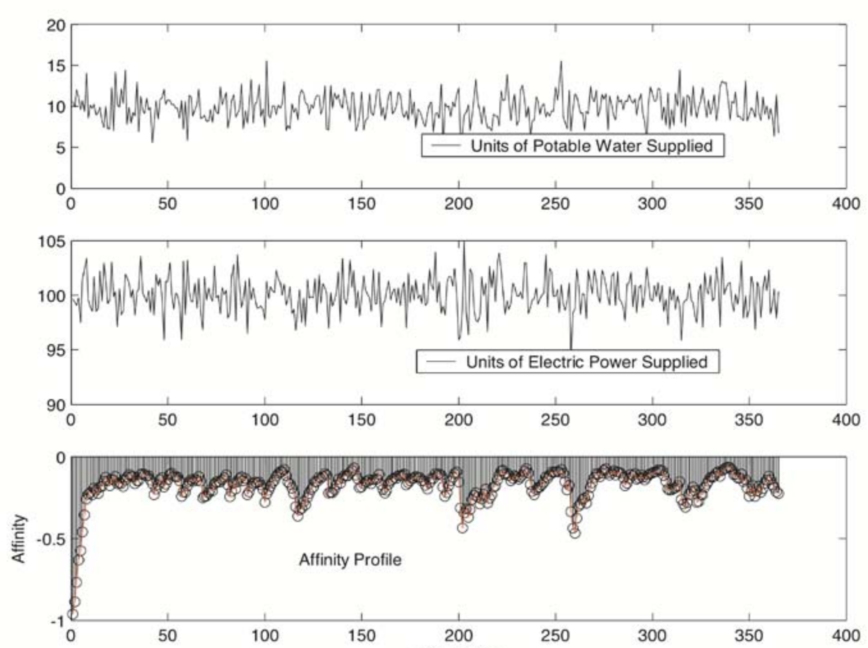

Figure 6: Affinity Profile of Regional Populace Over Time. Note that E(X1)=10 and E(X2)=100

Notice that in Figure 6 that by delivering only an average of 10 units of potable water and 100 units of electric power, the affinity of the population towards the government is never positive. This is because the population applies more weight to electric power than to potable water, as determined by the estimated variance-covariance matrices of X_1 and X_2 conditioned on the behavioral state spaces. If the average units of potable water remain constant at 10 units, then an average of about 103 units of electric power will bring the population to a 'neutral' state. This is shown in Figure 7.

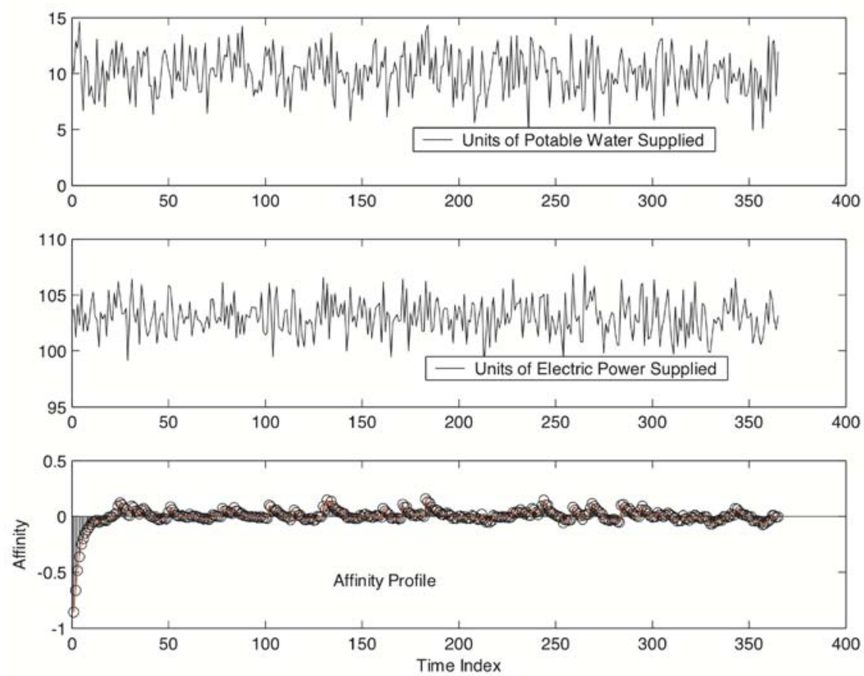

Figure 7: Affinity Profile of Regional Populace Over Time. Note that E(X1)=10 and E(X2)=103

Suppose that the average units of potable water delivered remains at 10 and the average units of electric power delivered increases over time. Then Figure 8 shows increases in affinity as a result of improved environmental conditions (i.e., increased electrical power to the region). Similar results are shown in Figure 9, where although a decrease in average potable water is observed, a large increase in the average units of electricity delivered causes a large increase in the population affinity towards the government.

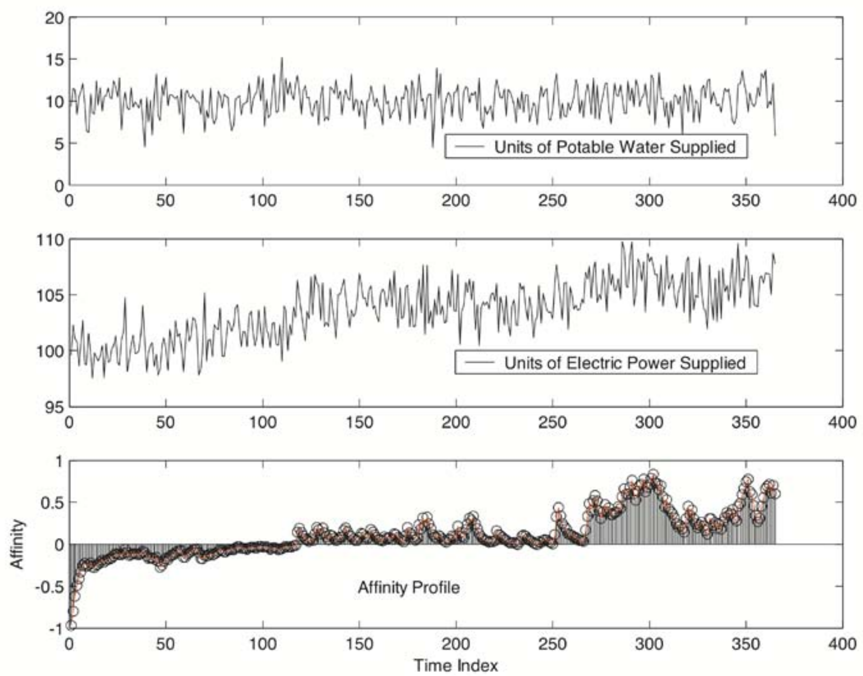

Figure 8: Affinity Profile of Regional Populace Over Time. Note that E(X1)=10 and E(X2) is Increasing Over Time

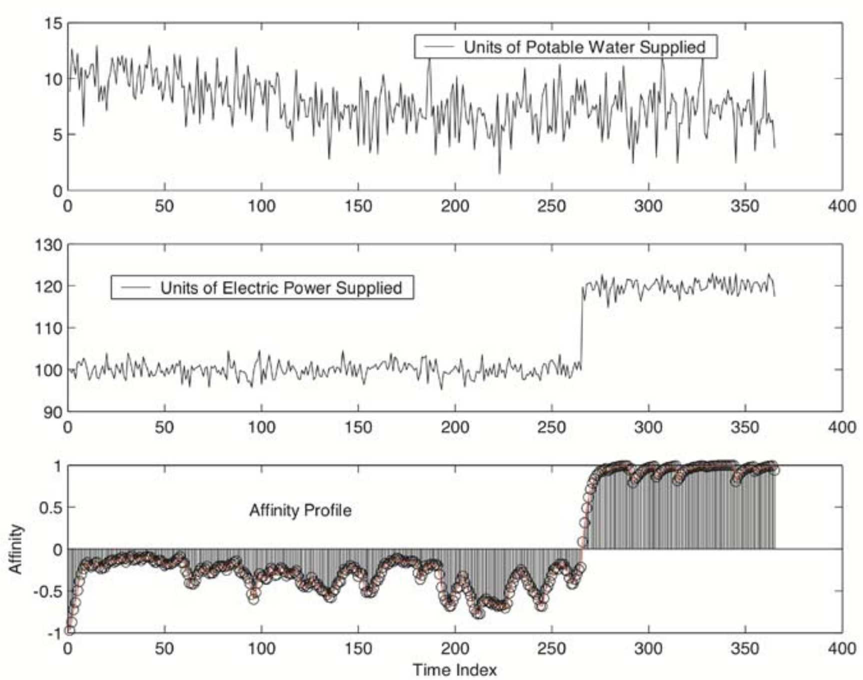

Figure 9: Affinity Profile of Regional Populace Over Time. Note that both X1 and X2 Exhibit Change Points

This example illustrates the utility of MAMBU for implementation into the NOEM. It permits the capture of changes in affinity due to changes in the state of the environment at any given time. As environmental conditions are updated in NOEM, so are the affinities between groups modeled within NOEM via MAMBU. Further, MAMBU is empirically grounded and fairly easy to populate so long as empirical data exists. If empirical data does not exist, then it is recommended that MAMBU be populated by subject matter experts.

4.2.2 Example with Count Environmental Variables

For this example, suppose we are interested in the affinity between a regional populace in Iraq and United States (US) Forces in the region. For this example, the state of the environment at time *t* is measured by the following:

1. Number of violent incidents (X_{1t})
2. Number of civilian casualties (X_{2t})
3. Number of enemy casualties (X_{3t})

Suppose that if the number of violent incidents or the number of civilian casualties increases, the regional populace's stance toward the US Forces will approach hostility. Further, if the number of enemy casualties decreases, so will the regional population's support for the US Forces. On the other hand, decreases in the number of violent incidents and the number of civilian casualties will gain some support for US Forces in the region. Additionally, an increase in the number of enemy casualties will also gain support for US Forces. Of course, all of these effects should be verified by subject matter experts, or validated through empirical studies (perhaps using the statistical model developed earlier or by the multiple comparisons procedure used in the previous example).

Let us define the behavioral state space of the regional population by

$$A_{(12)} = [\text{Hostile, Neutral, Friendly}]$$

and let the transition probability matrix be given as that in the example above, or

$$\Omega_{(12)} = \begin{bmatrix} 0.95 & 0.04 & 0.01 \\ 0.25 & 0.60 & 0.15 \\ 0.15 & 0.25 & 0.60 \end{bmatrix}$$

For this example, suppose it was determined by subject matter experts that, given the regional populace is ‘hostile’ towards US Forces, one can expect 15 violent incidents per day in this region, on average. Further, given the regional population holds a ‘neutral’ stance towards US Forces, one can expect 5 violent incidents per day, on average. Similarly, conditioned on the “friendly’ behavioral state, one can expect 1 violent incident per day, on average. Proceeding in this fashion, suppose that subject matter expert input yielded the following:

$$\lambda_{(12),'\text{Hostile}'} = [15,\ 12,\ 0.05]$$

$$\lambda_{(12),'\text{Neutral}'} = [5,\ 3,\ 10]$$

$$\lambda_{(12),'\text{Friendly}'} = [1,\ 1,\ 20]$$

In this example, the X’s were simulated from Poisson(λ) distributions. Figure 10 shows the changes in the environmental variables and the populace affinities towards US Forces over time. Note that

1. At time 66, the mean number of violent incidents and mean number of civilian casualties increase, while the mean number of enemy casualties decreases.
2. At time 126, the mean number of violent incidents and mean number of civilian casualties decrease, while the mean number of enemy casualties increasers.
3. At time 258, the mean number of violent incidents and mean number of civilian casualties decrease, while the mean number of enemy casualties increases.

Notice in Figure 10 that negative affinities are produced between times 66 and 125, due to a high level of violent incidents and civilian casualties, and a low level of enemy casualties. Further, positive affinities arte produced between times 258 and 365, due to low levels of violent incidents and civilian casualties, and a high level of enemy casualties.

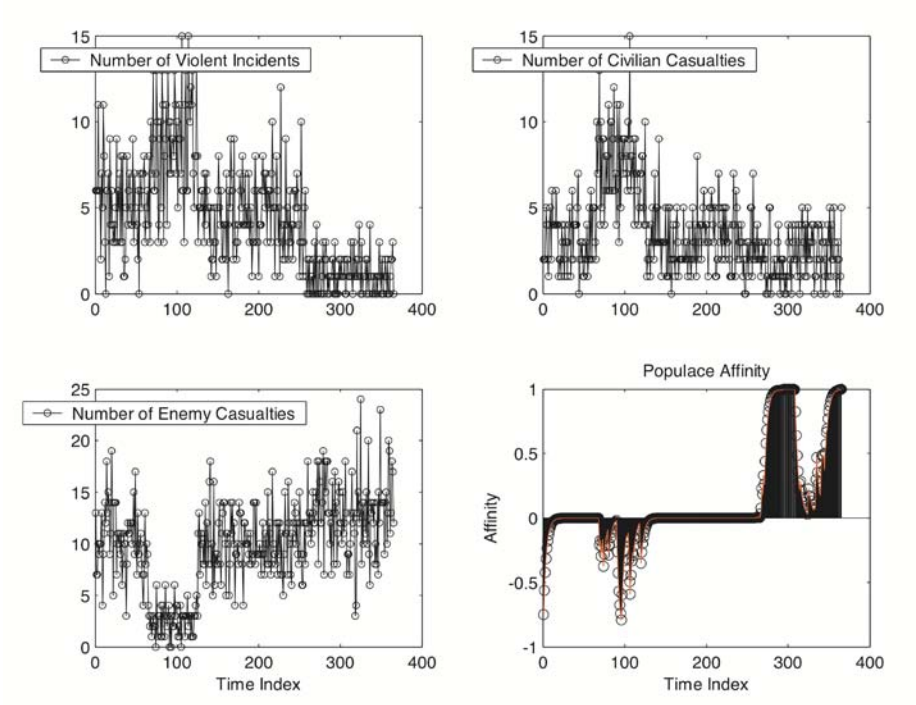

Figure 10: Affinity Profile of Regional Populace Over Time as Function of Number of Three Count Variables

For this example, we set $\psi = 0.25$ in the geometric weighting scheme. As in the previous example, the implication here is that the regional populace places about 25% of their weight on current environmental conditions, and 75% on recent or past events.

4.2.3 Example with Mixed Environmental Variables

For this example, we will assume the same scenario as the example above (i.e., Regional populace and US Forces), except that the state of the environment at time *t* is measured by the following:

1. Potable water supplied (X_{1t})
2. Electric power supplied (X_{2t})
3. Number of violent incidents (Z_t)

which is a mixture of continuous and discrete environmental state variables. Suppose that the following was determined by subject matter expert input:

$$\mu_{(12),'\text{Hostile}'} = [5,\ 95] \text{ and } \lambda_{(12),'\text{Hostile}'} = 15$$

$$\mu_{(12),'\text{Neutral}'} = [10,\ 100] \text{ and } \lambda_{(12),'\text{Neutral}'} = 5$$

$$\mu_{(12),'\text{Friendly}'} = [15,\ 105] \text{ and } \lambda_{(12),'\text{F}\mathit{riendly}'} = 0.01$$

and $\Sigma_{(12),'\text{Hostile}'} = \Sigma_{(12),'\text{Neutral}'} = \Sigma_{(12),'\text{Friendly}'} = \mathbf{I}_{2\times 2}$ (corresponding to the two continuous variables). For example, the mean vector $\mu_{(12),'\text{Hostile}'}$ tells us that the regional population is historically known to turn hostile towards US Forces if potable water level drops below 5 units/day, electric power drops below 95 units/day, and the number of violent incidents rises to 15/day. Similarly, with respect to $\mu_{(12),'\text{Neutral}'}$, in order to contain the regional population in a neutral behavioral state, an average of 10 units of potable water, 100 units of electric power, and 5 violent incidents are required. A similar interpretation can be made for $\mu_{(12),'\text{Friendly}'}$. The specification of Σ suggests that equal importance is given to both continuous variables.

For this example, the X_t's were simulated from multivariate normal distributions, while the Z_t's were simulated from Poisson distributions. Using the same transition probability matrix and initial behavioral state probability distribution as in the previous examples, we find affinities over time as computed by MAMBU in Figure 11. Note that:

1. At time 66, the mean number of violent incidents increases.
2. At time 126, the average units of potable water supplied increases, while the mean number of violent incidents decreases.
3. At time 258, the average units of electric power supplied decreases.

Notice that the increase in the mean number of violent incidents between times 66 and 125 results in a decrease in the affinity during this time. Also note that the decrease in electric power supplied at time 258 also results in a decrease in affinity.

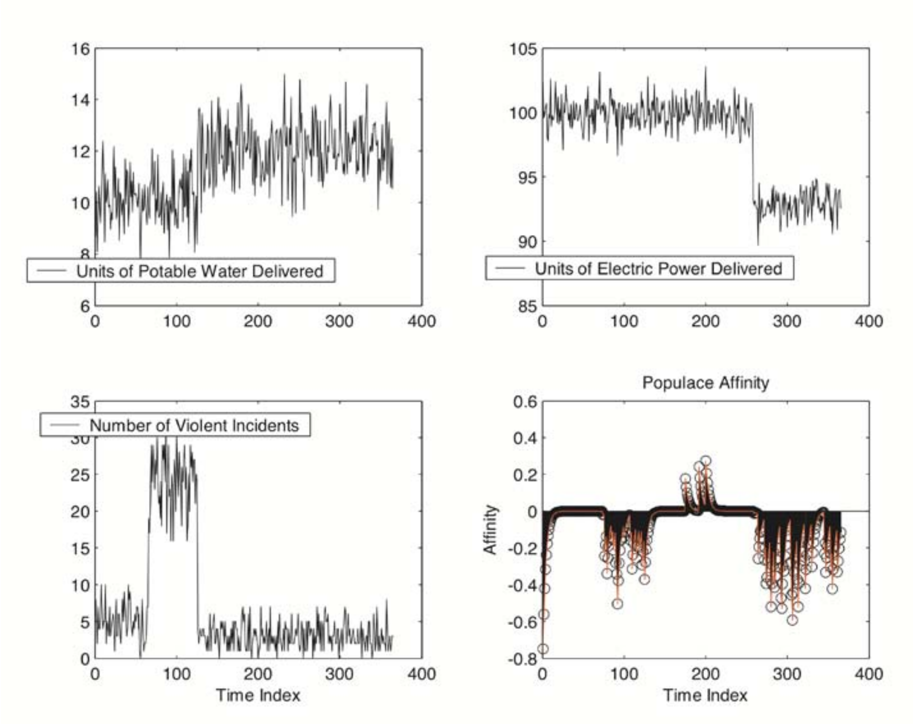

Figure 11: Affinity Profile of Regional Population Over Time as Function of a Mixture of Environmental Variable Types

The examples discussed in this subsection show how historical data and/or subject matter expert knowledge can be exploited to predict affinities between groups contained in the OE over time. It also demonstrates how MAMBU can be used as a basis for a behavioral model within the NOEM framework.

5.0 CONCLUSIONS

In this effort, two novel approaches were developed for predicting group responses or constructs (e.g., attitudes and behavior) as a function of changes in relevant factors. The first is purely statistical in nature and involves a hierarchical linear modeling approach. This approach considers the fact that individuals are nested within groups, which induces non-independence between individuals belonging to the same group. Independence of observations is a crucial assumption when using standard methods of analysis, such as ANOVA and multiple linear regression. As such, the use of standard statistical methods in these cases can severely mislead the analyst into concluding that group-level factors are significant when in fact they are not, and individual-level factors are insignificant when in fact they have significant explanatory power. We also proposed a novel data-based approach for determining an appropriate power transformation on the response variable so that the distribution of the response variable better agrees with the underlying model assumptions. We provided an example using simulated data sets and demonstrated how the proposed modeling technique can be used to develop meta-models for instantiation within the NOEM. The proposed approach can be used with either observational data to facilitate correlation studies, or experimental data to facilitate causation studies.

The second approach is probabilistic in nature and involves the assignment of a probability distribution to the behavioral state space of a group, and then updates this distribution using a Bayesian approach as new information becomes available. The method is most useful when knowledge of the conditional distributions of the environmental variables given the behavioral state of the groups under study is estimable, either via historical observations or subject matter expert opinion. When historical observations are available on each of the independent variables for each behavioral state, standard statistical methods for performing multiple comparisons can be used to determine if there are differences between the means of the variables observed for a given behavioral state. This largely simplifies the analysis required, relative to the hierarchical linear modeling approach discussed above.

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LIST OF ACRONYMS

AFRL	Air Force Research Laboratory
ANOVA	Analysis of Variance
MAMBU	Markov Affinity Model with Bayesian Updates
NOEM	National Operational Environmental Model
OE	Operating Environment
S.E.	Standard Error
TACS	Technology for Agile Combat Support
US	United States
711 HPW/RHXS	Sensemaking & Organizational Effectiveness Branch