# Epitrochoid Power-law Nozzle Concept for Reducing Launch Architecture Propulsion Costs

**Authors:**
- Eric J. Paulson (AFRL/RZST)
- Ryan P. Starkey (University of Colorado)

**Performing Organization:**
Air Force Research Laboratory (AFMC)
AFRL/RZST
4 Draco Drive
Edwards AFB CA 93524-7160

**Abstract:**
A description is presented of a new and original 3-dimensional nozzle class incorporating modular thrust cells, the Epitrochoid Power-law (EP) Nozzle. A parametric design has been defined that uses epitrochoid planar curves for the nozzle cross-section in conjunction with a power law relationship between the primary radius and the nozzle’s axial length, resulting in a radially lobed nozzle configuration. Attributes are described which potentially make the nozzle attractive from a cost and performance perspective in comparison to more conventional nozzle types. The current design approach is described here in detail. Ideal performance of the nozzle is defined. A design reference mission has been selected for nozzle optimization studies and is discussed. Potential research directions for future numerical and physical experiments are discussed.

**Security Classification:**
- **Unclassified**
- **Unclassified**
- **Unclassified**

**Limitation of Abstract:**
SAR

**Number of Pages:**
15

**Name of Responsible Person:**
Mr. Roy Hilton

**Telephone Number:**
N/A

---

**Publication Information:**
For presentation at the JANNAF SMBS/PEDCS/RNTS/SEPS Joint Propulsion Conference, Orlando, FL, 06-10 Dec 2010.
EPITROCHOID POWER-LAW NOZZLE CONCEPT FOR REDUCING LAUNCH ARCHITECTURE

PROPELLION COSTS

Eric J. Paulson
Air Force Research Laboratory
Edwards AFB, CA 93524

Ryan P. Starkey
University of Colorado
Boulder, CO 80309

ABSTRACT

A description is presented of a new and original 3-dimensional nozzle class incorporating modular thrust cells, the Epitrochoid Power-law (EP) Nozzle. A parametric design has been defined that uses epitrochoid planar curves for the nozzle cross-section in conjunction with a power law relationship between the primary radius and the nozzle's axial length, resulting in a radially lobed nozzle configuration. Attributes are described which potentially make the nozzle attractive from a cost and performance perspective in comparison to more conventional nozzle types. The current design approach is described here in detail. Ideal performance of the nozzle is defined. A design reference mission has been selected for nozzle optimization studies and is described. Potential research directions for future numerical and physical experiments are discussed.

NOMENCLATURE

\[A = \text{nozzle cross-sectional area}\]
\[c = \text{power-law coefficient}\]
\[d = \text{minor circle radius fraction}\]
\[\frac{dA}{dx} = \text{derivative of nozzle cross-sectional area with respect to } x\]
\[i = \text{tangential index}\]
\[I_{sp} = \text{specific impulse}\]
\[k = \text{epitrochoid ratio}\]
\[n = \text{power-law exponent}\]
\[\phi = \text{non-physical tangential variable in epitrochoid parametric definition}\]
\[\theta = \text{physical tangential angular measure in cylindrical coordinate system } (r,\theta,x)\]

INTRODUCTION

New propulsion technologies face significant barriers to development and transition into flight systems. These barriers include high costs for development and testing for these high energy-density systems, and infrequent opportunities for flight testing. When one considers the high cost of access to space, it is understandable that launch customers and providers are risk averse when it comes to testing new flight critical technologies such as boost propulsion. One could argue that it is surprising that any propulsion technology has an opportunity to reach the space flight testing phase of development.

By historical standards, modern launch demand in the U.S. is relatively stable at a fairly low rate. This is indicated by comparing one U.S. propulsion company's liquid rocket engine (LRE) annual production rates during a 40-year period at the end of the 20th century to the worldwide annual launch rates at the start of the 21st century, shown in Fig. 1 and 2, respectively.
Therefore the development environment for propulsion is one constrained by a high technology product with high non-recurring research and development costs, relatively high unit production costs, and low production numbers. An examination of historical strategies utilized during the development of new launch capabilities yields four general development categories.

1. Leverage existing Launch Vehicle (LV) systems by developing variants of existing systems. This avoids some development costs and often reuses existing production facilities, avoiding some capital investments in production facilities. Example include:
   a. Titan I → Titan II → Titan III → Titan IV
   c. Delta IV medium → Delta IV heavy

2. Leverage entire propulsion systems from an existing or retired flight system for a new launcher. This potentially eliminates most propulsion technology development/testing costs and production changes, unless the existing production facility requires modifications to support the higher production rate to support multiple vehicles.
   a. Atlas Centaur's RL-10 → Titan III Centaur
   b. Repurposing retired Minuteman and Peacekeeper ICBM stages → Minotaur family
3. Evolve part of an existing propulsion system for a new launcher. This may eliminate a portion of propulsion technology development/testing costs and may minimize the required changes to production facilities.
   a. Atlas ICBM booster engine→Thor IRBM→Delta LV
   b. Falcon 1's Merlin 1C booster engine→Falcon 9's upper stage Merlin 1C vacuum engine

4. Develop a new propulsion system to incorporate technology which significantly lowers the production cost of the new launch system. This generally requires a period of expensive technology development and testing, as well as investment in new production facilities. These significant non-recurring costs must be recouped by the developer over a sufficiently long system production run for new launcher/engine.
   a. Falcon 1 booster Merlin 1A engine
   b. Falcon 1 upper stage Kestrel engine
   c. Delta IV booster RS-68 engine

In the context of transitioning new technology, modern examples from approaches 2 and 3 are discussed in more detail. First consider the RD-170 family of engines. The NPO-Energomash RD-170 was originally developed as a reusable oxidizer-rich staged-combustion engine for the four liquid boosters of the Energia launch vehicle designed to launch the Soviet Buran space shuttle. In parallel with the Buran development, a variant was developed as the RD-171 for the boost propulsion of the expendable Zenit heavy lift cargo vehicle. The kerosene-LOX engine is characterized by the uniquely Soviet/Russian development using four thrust chambers fed by two oxidizer-rich preburners and a single turbopump assembly. In the 1990's, Pratt and Whitney and Energomash reached an agreement forming RD AMROSS in order to market Energomash engines in the U.S., and to develop a derivative engine, the RD-180. The RD-180 was effectively one-half of an RD-171, using two of the thrust chambers, one preburner, and a newly-developed turbopump assembly. The RD-180 was selected as the boost engine for the Atlas V vehicle then being developed by Lockheed-Martin under the Air Force Evolved Expendable Launch Vehicle (EELV) program. After the breakup of the Soviet Union, Russia used this successful experience to develop a new variant of the engine for a Russian analog to the EELV program, the Angara family of cargo launch vehicles. The Angara family uses the new engine as its boost propulsion. The RD-191 variant splits the family in half once again, using a single thrust chamber from the RD-180/RD-171, together with a newly designed preburner and turbopump assembly. This extensive reuse of the thrust chamber design across three current engines leverages the initial investment in their development, testing and production facility construction. Additionally the higher utilization rate more efficiently uses the production facility, notionally lowering the recurring cost of the chamber component yet again. The use of the preburner design in two of the three engine variants has a similar economic payoff. The RD-170 family is shown in Fig. 3.

![Figure 3. RD-170 Family of Engines (Unknown)](image)

The example considered from development category two is that of the Falcon 1 and the Falcon 9. SpaceX's LOX-kerosene gas-generator cycle ablative nozzle Merlin 1A was intended initially to power the
boosters of the Falcon 1, Falcon 5, and Falcon 9 launch vehicles. After flying on the initial Falcon 1 vehicles, the Falcon 5 launch vehicle was cancelled, and a higher performance regeneratively-cooled variant of the engine, the Merlin 1C Block I, was introduced as boost propulsion for the Falcon 1 vehicle. A block upgrade of the Merlin 1C engine (an example of a category 3 development strategy) resulted in the Merlin 1C Block II, with a thrust level of 125 klb at sea-level. The Block II version of the Merlin 1C is the boost propulsion for both the Falcon 1e (enhanced) launcher and the Falcon 9 launcher, which made its initial demonstration flight to low earth orbit (LEO) in June of 2010. The Falcon 9, shown in Fig. 4, uses nine Merlin 1Cs on the first stage with no significant modifications to the engine. This increases the number of engines being built at the SpaceX production facility, increasing the utilization of that facility and spreading the developmental costs for the engine over two launch vehicle families. Indeed, the potential exists for spreading those costs to a 3rd vehicle, should the Falcon 9 heavy concept reach production, using three Falcon 9 boosters burning in parallel similar to the Delta IV heavy vehicle.

Figure 4. Merlin 1C Engines on SpaceX Falcon 9 (SpaceX)

In the previously cited examples, the strategies used were either using a common engine for multiple LVs or reusing a thrust chamber design in a new thrust-class engine design. Can an alternative to those approaches be postulated, and if so, what would such an alternative design look like? Consider again the RD-170 family where the thrust chamber's design was reused, and the turbopump and preburner (in the case of the RD-191 design) required new developments for each additional new member of the engine family. The turbopump of a high performance rocket engine is a highly complex component which requires significant and expensive development and testing efforts, and comprises a large fraction of an engine's total development and manufacturing costs. An engine architecture that leverages investments in development, testing, and production facilities of a specific turbopump could theoretically significantly reduce the unit cost for a new engine, and therefore the overall unit cost for a new launch vehicle. And if an engine design enables usage for a range of vehicle size classes, the future technology transition opportunities would be maximized. A new class of three-dimensional radially lobed nozzles for integrating modular thrusters is proposed here for such an engine architecture, and a specific design concept is described, the Epitrochoid Power-law Nozzle.

RESULTS AND DISCUSSION

Several vehicle concepts have been proposed in the past which incorporate banks of modular thrust cells into linear or axisymmetric aerospike nozzle configurations, such as that shown in Fig. 5. Some of these concepts have resulted in the developments of experimental engines. Although investigated extensively, the technology hasn't yet been integrated into an operational flight system. NASA's X-33 SSTO demonstration program failed to reach vehicle flight testing although its linear aerospike engine, the XRS-2200, was tested at NASA's Stennis Space Center. For a flight system, the physical coupling between full-scale flight vehicle aerodynamics and aerospike propulsion performance is difficult to predict accurately from cold-flow or subscale experiments, and utilizing results from computational fluid dynamics requires validation with full scale data.
The concept of incorporating the exhaust from multiple thrust cells into a single duct has received considerably less attention, other than in the context of air-augmented rocket or rocket-based combined cycle configurations. One analytical effort was funded by NASA-Ames and the Air Force Rocket Propulsion Laboratory (AFRPL) during the mid-80s, and was managed by Mr. Don Hart. This effort focused mainly on developing an efficient computational scheme to compute the flows of a circular arrangement of rocket nozzles exhausting into a cylindrical duct of constant radius. More recently, in 2005 a patent was awarded to Sackheim et al. for an axisymmetric throttleable non-gimbaled engine. This latter concept, shown in Fig. 6, used multiple combustion chambers (four are shown in this instance) exhausting combustion gases through individual throats into either a single diverging axisymmetric bell nozzle, or alternately around a central expansion body similar to an aerospike nozzle. The individually throttleable combustion chambers allow thrust vector control via differential throttling of the combustion chambers. This is similar in concept to the design for the thrust vector control for the X-33. A related TVC concept was used in the Liquid Injection Thrust Vector Control (LITVC) subsystem flown in the Titan III and Titan IVA solid rocket motor nozzles. The Titan SRM implementation used a throttled injection of a secondary working fluid at multiple positions around the SRM nozzle's interior periphery to generate non-axisymmetric flow in the nozzle. This generated the desired unequal moments on the Titan Launch vehicle.
One issue encountered in implementing differential throttling with the X-33 linear aerospike was the tendency of the unthrottled exhaust streams to "fill the gap" left by the throttled exhaust, resulting in less effective actual TVC performance than predicted by theory. This issue can be expected to occur similarly in the Sackheim concept, since it utilizes a single axisymmetric diverging nozzle section. A potential mitigation approach would be to exhaust the multiple hot gas streams into a radially lobed nozzle.

**EPITROCHOID POWER-LAW NOZZLE DEFINITION**

A radially lobed family of planar roulette curves, known as epitrochoids, was studied by Dürer in 1525 and by Huygens in 1679. The curves are formed by tracing the path of a point attached to a secondary circle of radius $R_2$ as it rolls around the circumference of a fixed primary circle of radius $R_1$. A closed curve with radial lobes is generated when the ratio of $R_1/R_2$ is an integer value, with the number of lobes equal to $R_1/R_2$. The point being traced lies along a radial line extending from the center to the circumference of the secondary circle. In Fig. 7 the red solid line is the epitrochoid curve, the blue dashed line shows the fixed primary circle of radius $R_1$, and the black solid line shows the initial position of the outer "generating" circle of radius $R_2$. The blue solid line is the radial line which rotates along with the generating circle. The parameters defining the family of parametric epitrochoid curves in the $y$-$z$ plane are:

- $R_1$: the radius of the fixed primary circle
- $R_2$: the radius of the secondary generating circle
- $k$: the ratio $R_1/R_2$
- $d$: normalized location of generating point along generating circle's radius which traces out the epitrochoid curve

![Figure 7. Epitrochoid Planar Curve](image)

Integer values of $k$ and $0 \leq d \leq 1$ are the epitrochoid's domain of interest for lobed nozzles. The effects on the epitrochoid of varying the values of $k$ and $d$ can be seen in Fig. 8 and 9, respectively. Equations (1) & (2) show the parametric form of the planar epitrochoid in $y$-$z$ Cartesian coordinates, using a nonphysical tangential parametric variable $\varphi$, with $0 \leq \varphi \leq 2\pi$. For the purposes of clarity, the points on the lobed curve of maximum radius for any given $x$ will be labeled the lobe peaks and occur at tangential...
values of $\varphi = \frac{(2i-1)\pi}{k}$ for integers $i = 1, \ldots, k$. Similarly, the points of minimum radius on the epitrochoid will be labeled the lobe cusps, and these occur at tangential values of $\varphi = \frac{2\pi i}{k}$ for $i = 1, \ldots, k$.

$$y(\varphi) = \frac{R_1}{k} * (k + 1) * \cos(\varphi) - \frac{R_1}{k} * d * \cos((k + 1) * \varphi)$$

$$z(\varphi) = \frac{R_1}{k} * (k + 1) * \sin(\varphi) - \frac{R_1}{k} * d * \sin((k + 1) * \varphi)$$

Figure 8. Effect of $k$ Parameter
In order to extend the planar curve into a third dimension corresponding to a nozzle's longitudinal axis (x-axis), a dependency between the epitrochoid's primary radius and x is required. A power-law relationship was implemented with the addition of two more parameters, n & c, in the form \( R_l(x) = c \cdot x^n \). This formulation enables generating a wide range of nozzle shapes including axisymmetric conical and parabolic nozzles. The resulting new parametric forms are shown in Eqs. (3) & (4).

\[
y(x, \phi) = \frac{c}{k} \cdot x^n \cdot \left\{ (k+1) \cdot \cos(\phi) - d \cdot \cos[(k+1) \cdot \phi] \right\} \\
z(x, \phi) = \frac{c}{k} \cdot x^n \cdot \left\{ (k+1) \cdot \sin(\phi) - d \cdot \sin[(k+1) \cdot \phi] \right\}
\]

This defines a semi-infinite epitrochoid power-law body surface extending from \( x=0 \) into the positive x direction (for positive c.) By adding the constraint variables \( x_{\text{min}} \) and \( x_{\text{max}} \), the resultant surface forms a diverging radially lobed nozzle surface suitable for accelerating supersonic gas flow. A FORTRAN code was created to generate arbitrary surface meshes of these EP nozzle shapes. An example of the output mesh is shown in Fig. 10.
Several physically relevant parameters can then be derived after defining the nozzle surface in an orthogonal cylindrical coordinate system \((r, \theta, x)\), expressed in Eqs. (5) & (6). The local slope with respect to \(x\) at the peaks and cusps of the nozzle will be parameters useful for the design of the transition from the individual thruster exits into the lobed nozzle inlet, and are defined by Eqs. (7) & (8).

\[
\begin{align*}
  r(x, \varphi) &= \frac{c}{k} x^n \sqrt{(k+1)^2 + d^2 - 2d(k+1) \cos(k \varphi)} \\
  \theta(\varphi) &= \tan^{-1} \left[ \frac{(k+1) \sin(\varphi) - d \sin[(k+1) \varphi]}{(k+1) \cos(\varphi) - d \cos[(k+1) \varphi]} \right] \\
  \frac{dr}{dx}_{\text{peak}} &= \left( \frac{n c (k+1+d)}{k} \right) x^{n-1} \\
  \frac{dr}{dx}_{\text{cusp}} &= \left( \frac{n c (k+1-d)}{k} \right) x^{n-1}
\end{align*}
\]

Solving the double integral \(A = \iint r \, dr \, d\theta\) using the application of the Chain Rule for \(\theta = f(\varphi)\) in Eqn. 8, results in an analytical expression for the cross-sectional area for the epitrochoid nozzle at arbitrary \(x\) as shown in Eqn. 9. Similarly, an expression for \(dA/dx\) can then be derived, as shown in Eqn 10.

\[
\begin{align*}
  \text{Area}_{\text{cross-sectional}} &= \frac{\pi c^2}{k^2} x^{2n} \left[ 1 + 2k + k^2 + (k+1) d^2 \right] \\
  dA/dx &= \left( \frac{2n \pi c^2}{k^2} \right) x^{2n-1} \left[ 1 + 2k + k^2 + (k+1) d^2 \right]
\end{align*}
\]

The surface area of the parametric EP nozzle is derived via the integral listed in Eqn. 11. By integrating over the range \(x_{\text{min}}\) to \(x_{\text{max}}\) (again utilizing the Chain Rule for \(\theta\) and \(\varphi\)), and using lobe symmetry to integrate from lobe cusp to peak for \(0 \leq \varphi \leq 2\pi/k\), the double integral simplifies to the definite integral in Eq. 12 which must be solved numerically. The terms \(A_{1-4}\) in Eq. 12 are defined by Eq.13a-13d.
\[ \text{Area}_{\text{surface}} = \int \int r \, d\theta \, dx \]  
\[ \text{Area}_{\text{surface}} = \frac{c \times x^{n+1} \times (x_{\text{max}} - x_{\text{min}})}{n+1} \left( 2 \right) \]

Where:

\[ A_1 = \frac{1}{1 + \left[ \frac{(k+1) \times \sin(\varphi) - d \times \sin([k+1] \times \varphi)}{(k+1) \times \cos(\varphi) - d \times \cos([k+1] \times \varphi)} \right]^2} \]  
\[ A_2 = \left\{ (k+1) \times \cos(\varphi) - d \times \cos([k+1] \times \varphi) \right\} \]
\[ A_3 = \left\{ -(k+1) \times \sin(\varphi) + d \times \sin([k+1] \times \varphi) \right\} \]
\[ A_4 = \left\{ (k+1) \times \cos(\varphi) - d \times \cos([k+1] \times \varphi) \right\}^2 \]

**CHARACTERISTICS OF AN EPITROCHOID POWER-LAW NOZZLE**

When considering the characteristics of a notional LRE that would integrate modular thrust cells and an epitrochoid nozzle, several seem worth discussing here. Some of these characteristics are planned to be investigated by the authors during the next phase of research.

Integrating the separate thrusters into a single duct raises several interesting questions. Should the individual thruster end at a throat as in the Sackheim concept, or should it end after a short diverging section to accelerate the gas flow supersonically before entering a plenum section? It seems likely that the latter approach is more desirable. A supersonic thruster exhaust should be more effective than a transonic exhaust in preventing flow instabilities in one thruster's exhaust from propagating into the subsonic region of a nearby thruster's stream.

In theory, the nozzle should be more effective for the use of LITVC or differential throttling as a thrust vector control approach. The lobes of the nozzle would tend to retain an individual thruster's exhaust and allow less “smearing” of the supersonic gas stream into the region of the throttled thruster exhaust. This will require numerical/physical experiments to quantify the degree to which a benefit is derived from the lobed shape.

The nozzle cross-sectional shape for small values of k≤8 lends itself to an arrangement of single center thruster surrounded by a circular array of thrusters aligned with the lobe peaks. As k increases, the nozzle cross-sectional area is dominated by the area of the primary circle, and may require multiple thrusters in a center ring to efficiently fill the center area, if the engine uses a homogeneous array of a single thruster design. Additionally, it is likely that the nozzle performance would be enhanced by canting the peripheral ring of thrusters inward. These variations will be a focus of the optimization study during the next phase of research.
One option that a center-plus-peripheral thruster configuration enables is the potential to provide staged thrust levels. By securing several thrusters symmetrically, the remaining area in the EP section of the nozzle becomes available to accelerate the remaining stream, effectively increasing the area ratio of the full nozzle for the same chamber pressure in the individual thruster(s) combustion chamber. This property of the proposed design stimulated the most interest. The magnitude of any losses during staged thrust operation is a key characteristic that will be investigated in the next phase of the authors' work. This characteristic most directly determined the direction for the selected design reference mission, which is discussed in the next section.

**DESIGN REFERENCE MISSION**

In conjunction with the maturation of the EP nozzle design concept, a launch vehicle design reference architecture was postulated which could effectively utilize the perceived benefits of a lobed nozzle. The most significant potential payoff appears to be operation at staged thrust levels with increased performance. This is unique, and immediately suggested to the authors a payoff in a SSTO vehicle application. In previous launch vehicle studies, the necessity to throttle a SSTO vehicle near orbit in order to limit undue acceleration on the payload and vehicle structure has been a significant tradeoff for SSTO vehicle designs. The SSTO vehicle experiences this effect to a more significant degree than multistage vehicles, due to the necessarily lower structural mass fraction at the end of ascent for a SSTO compared to a multistage vehicle. An examination of the turn-down ratio of several Two-Stage-To-Orbit (TSTO) vehicles was performed to select an initial range for the appropriate turn-down ratio for a SSTO vehicle. The historical TSTO turn-down ratios are plotted in Fig. 11, which shows a cluster of stages with ratios from 4-10.

**Figure 11. Historical TSTO Thrust Turn-Down Ratio**

SSTO vehicles have been analyzed almost exclusively in the context of Reusable Launch Vehicles (RLVs). There has been very little discussion in recent literature of SSTO Expendable Launch Vehicle (ELV) designs. So it appears to be an area worth investigating more fully in this research,
especially in the light of a constrained launch market. An ELV enables achieving a higher propellant mass fraction for a vehicle, since no reentry or long duration in-space support systems are required. In addition, development, production, and operations costs may nominally be lowers since there are fewer stages requiring acquisition and assembly. The propellant mass fractions of the 1st stages for selected historical launch vehicles are shown in Fig. 12.

![Figure 12. Historical TSTO Stage Mass Fractions](image)

One issue that could hamper utilization of an expendable SSTO launch architecture is the question of how to provide in-space propulsion for a wide range of payload destinations. Modern staged ELV upper stages such as the Centaur are excellent orbit transfer systems for moving the payload from a Low Earth Orbit (LEO) parking orbit to a geosynchronous transfer orbit (GTO) or from a GTO into the destination geosynchronous orbit. However, given the recent/current development of multiple unmanned vehicle systems for ISS cargo resupply such as the European ATV, the Japanese HTV, and the commercial Cygnus and Dragon systems in the U.S., and the resurgence of the concept of propellant depots in space, it seems reasonable to postulate the future availability of reusable orbit transfer vehicles (OTV) as part of a new launch architecture using the SSTO to provide a simple delivery to a short term parking orbit in LEO.

Lastly, the thrust and performance requirements for the design reference mission have been initially identified by using the rocket equation and a nominal design payload class of 3,000 lbs, along with the reasonable assumption that the yet to be defined parking LEO requires the SSTO vehicle to deliver an ideal velocity change of 30,000 ft/sec. The resulting vehicle Gross Lift-Off Weights (GLOW) for several different values of vehicle propellant mass fraction and mission-effective Isp are shown in Fig. 13. Plotted on the same chart for comparison are the values of mission average Isp vs. lift-off weight for the 1st stages of several ELVs that have flown. The flight vehicle 1st stage weights do not include the weights of any upper stages or payloads, and the 1st stage mission averaged Isp is calculated by weighting the sea-level and vacuum Isp by 20% and 80% respectively.
SUMMARY AND CONCLUSIONS

A new LRE concept combining modular thrust cells and an original type of lobed nozzle has been developed, the Epitrochoid Power-law nozzle. A methodology for the parametric design of the EP nozzle has been developed, and formulas for useful dependent parameters for the nozzle shape have been derived, including the local slopes at the cusp and peak of the nozzle lobes and the variation of nozzle area in the longitudinal direction. An access-to-space application which could benefit from the theoretical characteristics of the new LRE has been developed and a design reference mission described.

Some thought has been given to tasks required to fully examine the proposed LRE concept. The tasks identified for future work follow, in roughly chronological order. The authors plan to complete tasks 1-4 and present those results in the near future.

1) Complete the creation of a parametric geometry which maps an efficient supersonic gas dynamic transition from radial arrangement of discrete homogeneous modular thrust cell nozzle exits into a pure EP inlet, avoiding discontinuity in dA/dx.

2) Develop three-dimensional steady state full-flow solutions of the Euler and N-S equations for a range of combustor-to-EP nozzle exit configurations.

3) Develop three-dimensional steady state partial-flow solutions of the Euler and N-S equations for a range of combustor-to-EP nozzle exit configurations when the circumferential thrusters are shut down and the center thruster is left on later during an SSTO ascent trajectory.
4) Optimize the parametric modular thruster/EP nozzle rocket engine assembly for the current design reference mission.

5) Develop an efficient structural concept which minimizes the weight penalty of deviating from the highly efficient load carrying circular cross sections of current bell and conical nozzles.

6) Numerically evaluate the efficiency of thruster differential throttling as an engine assembly thrust vector control method.

7) Numerically evaluate potential for heterogeneous arrays of thrusters, i.e. modular tripropellant engine, and how optimization of such a configuration differs from a homogeneous configuration.

8) Conduct physical experiments to validate previous numerical results using cold flow and, if performance warrants, hot fire testing.

9) Finally, if proof of concept evaluations succeed a more detailed examination of the potential cost benefits of an actual LRE architecture based on this modular thruster/EP nozzle.

ACKNOWLEDGMENTS

Mr. Paulson thanks his colleagues at AFRl/RZS for their input during the development of the EPN concept, in particular Messrs. Jason Mossman and Alan Sutton, and Dr. Phillip Kessel.

REFERENCES