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**14. ABSTRACT**  
This project deals with the new and challenging problem of implementing large deformation finite element (FE) formulations in flexible multibody system (MBS) algorithms. Successful integration of large deformation FE and MBS algorithms will lead to a new generation of simulation codes that can be used in the analysis, design and performance evaluation of mechanical and aerospace systems. Specifically, the objectives of this project are (1) Different formulations used in the dynamic modeling of flexible MBS will be reviewed and evaluated in order to identify the advantages and drawbacks of each method; (2) The constrained nonlinear equations of motion expressed in terms of different sets of coordinates that can be efficiently used to describe the dynamics of rigid, flexible, and very flexible bodies will be developed; (3) Large deformation Cholesky coordinates will be used to develop a new form of the MBS dynamic equations with optimum sparse matrix structure; (4) Quantitative and qualitative evaluation of the effect of the nonlinear dynamic coupling between the reference motion and the large deformation modes will be performed; (5) The use of the computational methodology developed in this research project in the analysis of mechanical systems will be demonstrated. This

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**Integration of Large Deformation Finite Element Formulations in Flexible Multibody  
System Algorithms**

**Grant # 48772-EG  
(Project Period: July 17, 2006 – July 16, 2010)**

**Final Report**

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## 1. SUMMARY

This project deals with the new and challenging problem of implementing large deformation finite element (FE) formulations in flexible multibody system (MBS) algorithms. Successful integration of large deformation FE and MBS algorithms will lead to a new generation of simulation codes that can be used in the analysis, design, and performance evaluation of mechanical and aerospace systems. Specifically, the objectives of this project are (1) Different formulations used in the dynamic modeling of flexible MBS will be reviewed and evaluated in order to identify the advantages and drawbacks of each method; (2) The constrained nonlinear equations of motion expressed in terms of different sets of coordinates that can be efficiently used to describe the dynamics of rigid, flexible, and very flexible bodies will be developed; (3) Large deformation Cholesky coordinates will be used to develop a new form of the MBS dynamic equations with optimum sparse matrix structure; (4) Quantitative and qualitative evaluation of the effect of the nonlinear dynamic coupling between the reference motion and the large deformation modes will be performed; (5) The use of the computational methodology developed in this research project in the analysis of mechanical systems will be demonstrated. This report summarizes the main accomplishments made in this research project.

## 2. APPROACH

This research project is focused on integrating a new large deformation FE formulation with flexible MBS algorithms in order to allow modeling complex systems that consist of rigid, flexible, and very flexible bodies. The approach used in this project has the following main features:

1. Reference coordinates for the rigid body motion, elastic coordinates for small deformations, and absolute nodal coordinates for large deformations will be used to develop a multi-formulation, multi-scale approach for modeling the nonlinear dynamics of bodies with different degrees of flexibility.
2. The nonlinear kinematic constraint equations that describe mechanical joints and specified motion trajectories and account for the kinematic coupling between different modes of displacements will be formulated in terms of the reference, elastic, and absolute nodal coordinates.
3. The dynamic differential and algebraic equations of very flexible components will be developed using the large deformation ANCF finite elements. ANCF finite elements have desirable features that will facilitate the integration of large deformation FE and MBS algorithms. Fully parameterized ANCF finite elements, in particular, allow for implementing general constitutive material models in a straight forward manner.
4. An optimum sparse matrix structure of the augmented form of the equations of motion will be obtained using the large deformation Cholesky coordinates. The generalized mass matrix associated with the ANCF Cholesky coordinates becomes an identity matrix.
5. New nonlinear finite element ANCF beam, plate and shell models will be developed in this project and used to examine the effect of the coupling between different modes of

6. A non-incremental solution algorithm based on the proposed new formulation will be developed and tested using large deformation mechanical system applications. The applications will include belt drives and rubber chain tracked vehicles. In the applications considered in this research project, the important features of ANCF finite elements will be demonstrated, and the nonlinear dynamic coupling between different modes of displacements will be discussed.

### 3. SIGNIFICANCE OF THE RESEARCH

The subject of MBS dynamics and computer vehicle simulations is one of the main thrust areas of the Structures and Dynamics Program of the Army Research Office. Numerous large, complex mechanical systems used by the Army consist of interconnected multi-body structures. Examples of these systems are heavy machinery, wheeled/tracked ground vehicles, robotic devices, rotorcraft, etc. Substantial weight reductions are sought in order to develop more efficient and reliable systems. The proposed research is directed to respond to this need by developing and implementing new methods that will lead to new generation of MBS simulation tools that can be effectively and efficiently used in the analysis of large deformation applications such as cables used in rescue missions and heavy load handling belts used in power transmission, leaf springs that are common in heavy vehicles and trucks, tires, rotor blades, etc. The new simulation algorithms will be also used to develop new computer models that include significant details that can not be captured using existing modeling techniques and computer codes.

### 4. ACCOMPLISHMENTS

The accomplishments made in this project can be summarized as follows:

1. A review of different approaches used for the integration of large deformation FE and MBS algorithms was presented [1]. The main features of the approaches were outlined in order to explain the main differences. This review was prepared in collaboration with researchers from Georgia Tech and the University of Michigan in order to present a comprehensive overview of all the approaches used in the integration of large deformation FE and MBS algorithms [2].
2. The new large deformation finite element ANCF approach can be used to develop beam and plate models that relax the assumptions of the classical beam and plate theories. This approach allows for the deformation of the cross sections of the beams and plates. The new *ANCF coupled deformation modes* are defined and the significant effect of these modes on the nonlinear dynamics and stability of structures that undergo large deformations was demonstrated [1, 3, 4]]. It was shown that, while these modes can be neglected for stiff structures, the effect of the geometric stiffness associated with these modes can not be neglected for very flexible structures.
3. Several ANCF models for formulating the elastic forces were proposed; and the results obtained using these different models are compared. These different methods, which can

4. The sparse augmented form of the nonlinear dynamic equations of motion which allows for using different formulations to study systems that consist of rigid, flexible and very flexible bodies were presented [1, 2]. This multi-formulation approach allows for developing efficient multi-scale computational algorithms for the solution of the constrained MBS dynamic equations.
5. The ANCF finite elements used in this project allow for straight forward implementation of general nonlinear constitutive models in the case of beam, plate and shell finite elements. This important ANCF feature was demonstrated by using the general large deformation Neo-Hookean and Mooney-Rivlin constitutive models [5]. The results obtained showed that the use of linear Hookean constitutive models can lead in some large deformation applications to singular configurations.
6. A comparative numerical study in collaboration with Georgia Tech was performed [6]. In this study, the results obtained using the large deformation ANCF elements are compared with results obtained using large rotation vector formulation. The eigenvalue solution for a Helicopter rotor is obtained taking into account the effect of the stiffness due to the centrifugal forces. The frequencies obtained using the two different finite element approaches are compared. The results obtained showed a good agreement between the solutions of the two methods.
7. A new numerical integration procedure based on the implicit HHT method was developed for the solution of the differential and algebraic equations of constrained MBS [7, 8]. The proposed method ensures that the kinematic constraint equations are satisfied at the acceleration level. The new procedure was used to solve problems that include very flexible bodies modeled using ANCF finite elements. The performance of the proposed implicit HHT numerical procedure was examined by comparing with the performance of the explicit methods. The results obtained showed that the implicit procedure can be much faster than the explicit Adams method as the stiffness of the flexible bodies increases. This study was performed in collaboration with University of Wisconsin-Madison [7, 8].
8. Accurate modeling of many engineering systems requires the integration of MBS and large deformation FE algorithms that are based on general constitutive models, account for the coupling between the large rotation and deformation, and allow capturing coupled deformation modes that can not be captured using beam formulations implemented in existing computational algorithms and computer codes. In this research project, new three-dimensional nonlinear dynamic rubber chains and belt drives models are developed using ANCF finite elements that allow for a straight forward implementation of general

9. The problem of slope discontinuity in the case of *gradient deficient ANCF finite elements* was addressed [10, 11, 12]. It is shown that slope discontinuities and T-junctions can be modeled in a straight forward manner using fully parameterized and gradient deficient ANCF finite elements. Linear transformations that define the element connectivity can always be obtained and used to preserve ANCF desirable features that include constant mass matrix and zero Coriolis and centrifugal forces in the case of spinning structures. A general method that allows for modeling slope discontinuities and T-junctions using *gradient deficient* ANCF finite elements that do not have a complete set of coordinate lines and gradient vectors was proposed [12]. At the discontinuity nodes, one can always define a complete set of independent coordinate lines that lie on the structure. These coordinate lines can be used to define a complete set of independent gradient vectors at these nodes. Since the proposed method is based on linear coordinate transformations, the method can be implemented in a preprocessor computer program. The application of the proposed general method is demonstrated using ANCF gradient deficient beam element example.
10. In ANCF, the elimination of the relative translations and rotations at a point does not necessarily define a fully clamped joint, particularly in the case of fully parameterized ANCF finite elements that allow for the deformation of the cross section. In this project, the formulations and results of two different sets of clamped end conditions that define two different joints were compared [13]. The first joint, called the *partially clamped joint*, eliminates only the translations and rotations at a point on the cross section. The second joint, called the *fully clamped joint*, eliminates all the translation, rotation and deformation degrees of freedom at a point on the cross section. The kinematic equations that define the partially and fully clamped joints were developed, and the dynamic equations used in the comparative numerical study presented in this paper were shown. As discussed in [13], the fully clamped joint does not allow for the deformation of the cross section at the joint node since the gradient vectors remain orthogonal unit vectors.

11. Because of the ANCF isoparametric property in the cases of beams, plates and shells, ANCF finite elements lend themselves easily to the geometric description of curves and surfaces. ANCF finite elements, therefore, are ideal for what is called *isogeometric analysis* that aims at the *integration of computer aided design and analysis (ICADA)*, which involves the integration of what is now split into the separate fields of *computer aided design (CAD)* and *computer aided analysis (CAA)*. In this project, the relationship between the B-spline and NURBS, which are widely used in the geometric modeling; and the ANCF finite elements was established [14-17]. It was shown that by using the ANCF finite elements, one can in a straight forward manner obtain the control point representation required for the Bezier, B-spline and NURBS geometry. To this end, a coordinate transformation is used to write the ANCF gradient vectors in terms of control points. Unifying the CAD and CAA will require the use of such coordinate transformations and their inverses in order to transform control points to position vector gradients which are required for the formulation of the element transformations in the case of discontinuities as well as the formulation of the strain measures and the stress forces based on general continuum mechanics theory. In particular, fully parameterized ANCF finite elements can be very powerful in describing curve, surface, and volume geometry, and they can be effectively used to describe discontinuities while maintaining the many ANCF desirable features that include a constant mass matrix, zero Coriolis and centrifugal forces, no restriction on the amount of rotation or deformation within the finite element, ability for straight forward implementation of general constitutive equations, and ability to capture coupled deformation modes that can not be captured using existing finite element beam, plate and shell formulations. Because of the relationship between the ANCF finite elements and the B-splines, the development of a *rational absolute nodal coordinate formulation (RANCF)* is feasible.

## **5. TECHNOLOGY TRANSFER**

The publications that result from this project were regularly sent to Dr. David Lamb, and Mr. Randolph Jones. The principal investigator visited TARDEC and met with TARDEC engineers several times during the course of this project.

Currently some federal laboratories, companies and universities are using the simulation tools developed at the University of Illinois at Chicago for modeling dynamics of flexible bodies and vehicle systems. Examples of these organizations are Volpe National Transportation Systems Center, ENSCO Inc., Northern Illinois University, and University of Seville (Spain). Many other

institutions are using the results obtained and formulations developed in this ARO sponsored research project.

## **6. AWARDS AND HONORS RECEIVED**

1. UIC College of Engineering Gold Faculty Research Award, 2006
2. University Distinguished Professor Award
3. Appointed Associate Editor of the IMechE Journal of Multibody Dynamics
4. The principal investigator was appointed to the Editorial Board of the following four journals:
  - 4.1 Journal of the Franklin Institute
  - 4.2 Journal of Open Mechanics
  - 4.3 Advances in Mechanical Engineering
  - 4.4 ASME Journal of Computational and Nonlinear Dynamics (Reappointment)

## **7. GRADUATE STUDENTS INVOLVED DIRECTLT IN THE PROJECT**

1. Luis G. Maqueda (received his Ph.D. in Spring 2008)
2. Bassam Hussein (received his Ph.D. in Spring 2010)
3. F. Marina Gantoi (currently working on her Ph.D.)

## 8. PUBLICATIONS

1. Hussein, B. A., Sugiyama, H., and Shabana, A.A., “Coupled Deformation Modes in the Large Deformation Finite Element Analysis: Problem Definition”, *ASME Journal of Computational and Nonlinear Dynamics*, Vol.2, 2007, pp. 146-154.
2. Shabana, A.A., Bauchau, O. A., and Hulbert, G.M., “Integration of Large Deformation Finite Element and Multibody System Algorithms”, *ASME Journal of Computational and Nonlinear Dynamics*, Vol. 2, 2007, pp. 351 - 359.
3. Hussein, B., Sugiyama, H., and Shabana, A.A., “Absolute Nodal Coordinate Formulation Coupled Deformation Modes”, Proceedings of the 2006 ASME International Mechanical Engineering Congress & Exposition, November 5-10, 2006, Chicago, Illinois.
4. Dmitrochenko, O.N., Hussein, B., and Shabana, A.A., “Coupled Deformation Modes in the Large Deformation Finite Element Analysis: Generalization”, *ASME Journal of Computational and Nonlinear Dynamics*, Vol.4 (2), 2009, pp. 021002-1 – 021002-8.
5. Maqueda, L.G., and Shabana, A.A., “Poisson Modes and General Nonlinear Constitutive Models in the Large Displacement Analysis of Beams”, *Journal of Multibody System Dynamics*, Vol. 18(3), 2007, pp. 375 - 396.
6. Maqueda, L.G., Bauchau, O.A., and Shabana, A.A., “Effect of the Centrifugal Forces on the Finite Element Eigenvalue Solution of a Rotating Blade: A Comparative Study”, *Journal of Multibody System Dynamics*, Vol. 19 (3), 2008, pp. 281 - 302.
7. Hussein, B., Negrut, D., and Shabana, A.A., “Implicit and Explicit Integration in the Solution of the Absolute Nodal Coordinate Differential/Algebraic Equations”, *Nonlinear Dynamics*, Vol. 54(4), 2008, pp.283 - 296.
8. Shabana, A.A., and Hussein, B.A., “A Two-Loop Sparse Matrix Numerical Integration Procedure for the Solution of Differential/Algebraic Equations: Application to Multibody Systems”, *Sound and Vibration*, Vol. 327, 2009, pp. 557-563.
9. Maqueda, L.G., Mohamed, A.A., and Shabana, A.A., “Use of General Nonlinear Material Models in Beam Problems: Application to Belts and Rubber Chains”, *ASME Journal of Computational and Nonlinear Dynamics*, Vol.5, 2010, pp. 021003-1 – 021003-10.
10. Shabana, A.A., and Maqueda, L.G., “Slope Discontinuities in the Finite Element Absolute Nodal Coordinate Formulation: Gradient Deficient Elements”, *Multibody System Dynamics*, Vol. 20, 2008, pp. 239 - 249.
11. Maqueda, L.G., and Shabana, A.A., “Numerical Investigation of the Slope Discontinuities in Large Deformation Finite Element Formulations”, *Nonlinear*

12. Shabana, A.A., "General Method for Modeling Slope Discontinuities and T-Sections Using ANCF Gradient Deficient Finite Elements", *ASME Journal of Computational and Nonlinear Dynamics*, accepted for publication.
13. Hussein, B.A., Weed, D., and Shabana, A.A., "Clamped End Conditions and Cross Section Deformation in the Finite Element Absolute Nodal Coordinate Formulation", *Multibody System Dynamics*, Vol. 21 (4), 2009, pp. 375-393.
14. Sanborn, G.G., and Shabana, A.A., "On the Integration of Computer Aided Design and Analysis Using the Finite Element Absolute Nodal Coordinate Formulation", *Multibody System Dynamics*, Vol. 22, 2009, pp. 181-197.
15. Sanborn, G.G., and Shabana, A.A., "A Rational Finite Element Method Based on the Absolute Nodal Coordinate Formulation", *Nonlinear Dynamics*, Vol. 58(3), 2009, pp. 565-572.
16. Lan, P., and Shabana, A.A., "Integration of B-Spline Geometry and ANCF Finite Element Analysis", *Nonlinear Dynamics*, Vol.61, 2010, pp. 193-206.
17. Lan, P., and Shabana, A.A., "Rational Finite Elements and Flexible Body Dynamics", *ASME Journal of Vibration and Acoustics*, accepted for publication.