DISCREPANCIES ASSOCIATED WITH THE
DRAG CHARACTERISTICS OF PRIMARY FRAGMENTS

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**Report Documentation Page**

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Standard Form 298 (Rev. 8-98)
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INTRODUCTION

In the calculation of the velocity of primary fragments a number of important parameters are involved. Some of these parameters are standard and seemingly well-defined [1-14]. Included in this group would be the drag coefficient, $C_D$, the fragment density, $\rho_f$, the ambient air density, $\rho_{\text{air}}$, the fragment diameter, $d_f$, and the fragment mass, $m_f$.

In addition to these standard parameters there exist a number of other parameters, which are not completely standard and/or not well-defined. The most important of these is the velocity decay coefficient, $k_v$ [1, 2, 4, 8, 13, 14]*. In three of these references [1, 2, 13] the parameter “caliber density”, $D$, is also introduced. Another parameter is the “presented area”, $A_f$ [1-14], along with the “shape factor or ballistic density”, $k$ [3, 4, 8], and the “L” parameter [3, 4]. In addition, the fragment “form factor” is utilized in a number of references [2, 10-13]. In a recent study [5], a methodology taken from Reference 4 introduces an “L_{1}” parameter, along with a specific value of the shape factor, $k$.

In the development and application of these parameters, both standard and nonstandard, certain apparent discrepancies have developed. The discussion which follows represents a summary of these discrepancies. A complete description of all such parameters and related discrepancies is beyond the scope of this discussion, but such a description is available elsewhere [15].

DRAG COEFFICIENT

The most important of the parameters noted is the drag coefficient, $C_D$, which according to standard fluid dynamic texts [16-20] is defined as

$$C_D \equiv \frac{2 F_D}{\rho_{\text{air}} A_f V_f^2} \text{ (dimensionless)**}$$

where

$F_D$ = drag force (f)

$\rho_{\text{air}}$ = ambient air density (m $\ell^{-3}$)

$A_f$ = presented area ($\ell^2$)

* Numbers in brackets refer to references cited, included at the end of the paper.
** Immediately following each equation, within the parentheses, the dimensions are indicated.
\[ V_f = \text{fragment velocity (} \ell \text{ t}^{-1}) \]

Unfortunately, this definition of the drag coefficient, although widely recognized and accepted, is not the only definition. In one of the early studies dealing with the measurement of primary fragment drag [6] the drag coefficient was defined as

\[
K_D = \frac{\pi}{4} \frac{F_D}{\rho_{\text{air}} A_f V_f^2} \quad \text{(dimensionless)} \tag{2}
\]

In three other early experimental studies [7 -9] the drag coefficient was defined as

\[
C_D^* = \frac{F_D}{\rho_{\text{air}} A_f V_f^2} \quad \text{(dimensionless)} \tag{3}
\]

where the asterisk is used in the current discussion to distinguish the drag coefficient in Eq. (3) from the original coefficient defined by Eq. (1). By inspection, \(K_D\) and \(C_D^*\) are seen to be related as follows:

\[
K_D = \frac{\pi}{4} C_D^* \quad \text{(dimensionless)} \tag{4}
\]

In one other early study [10] the symbol “k” was used to represent drag coefficient. For this case \(k\) can be shown to be equivalent to \(C_D^*\). (This “k” parameter should not be confused with the “k” parameter representing shape factor [3, 4, 8]). For simplicity, the drag coefficient associated with these first five studies [6 -10] will be represented by the symbol \(C_D^*\) in subsequent discussions. It is important to note that based on a comparison of Eqs. (1) and (3),

\[
C_D^* = C_D / 2 \tag{5}
\]

In much of subsequent literature [1, 2, 11-14], \(C_D^*\) is not generally distinguishable from \(C_D\) because the same symbol is used for both, without the asterisk. **Experimental values of the fragment drag coefficient, \(C_D^*\), as originally recorded [6 -10], have continued to be used without regard to the missing factor of 1/2.** In certain references [3, 4, 14] the correct definition of \(C_D\) appears to be used consistently. In another reference [5] the correct definition for drag coefficient is used, but, for purposes of conservatism, \(C_D\) is limited to a value of 0.8, representing a cube, oriented edge-on to the direction of flow [20].

References 1 to 14 are not necessarily a complete list of all documents of interest, dealing with primary fragment drag coefficients, but they are certainly representative of the more
important references. The relationship between these documents, so far as the use of $C_D$ or $C_D^*$, is depicted in Figure 1. As indicated in the figure references 1, 2, and 6 to 13 appear to have made use of some version of $C_D^*$ in total or in part, while references 3 to 5 appear to have made use of $C_D$ only. The fact that both types of $C_D$ are used without distinction in two very important references [1, 2] is most significant.

**VELOCITY DECAY COEFFICIENT**

The situation concerning the definition of drag coefficient, as previously described, has proven to be the source of considerable confusion, especially as related to the definition of the velocity decay coefficient. Based on the equation of motion of a primary fragment, taking into account drag, but neglecting gravity, the velocity decay coefficient, $k_v$, is defined as [4, 14]

$$k_v = \frac{\rho \, C_D \, A_f}{2 \, m_f} \quad (t^{-1})$$  \hspace{1cm} (6)

This, however, is not the only definition of $k_v$. In several references [1, 2, 8, 13] $k_v$ is defined as

$$k_v^* = \frac{\rho \, C_D^* \, A_f}{m_f} \quad (t^{-1})$$  \hspace{1cm} (7)

Inspection of Eq. (7) reveals that the factor $1/2$, as given in Eq. (6), is missing. This omission is not a typographical error but results from the use of $C_D^*$, as defined by Eq. (3), in place of $C_D$. **Because the asterisk is omitted, there is no way for the reader to determine which drag coefficient is involved.** The problem is aggravated by uncertainties associated with the proper definition of the “presented area”.

**PRESENTED AREA**

The presented area, $A_f$, [1-14], represents the projected surface area of the fragment normal to the flight path upon which the fragment drag coefficient is based. In fluid dynamics texts [16-20] this parameter is generally referred to as the “reference area” or “frontal area” or “cross-sectional area”. For primary fragment shapes [1] the “presented area” is clearly defined as

$$A_f = \pi \, d_f^2 / 4 \quad (t^{-2})$$  \hspace{1cm} (8)
where
\[ d_f = \left[ \frac{m_f}{(0.654 \rho_f)} \right]^{1/3} \]  \( (\ell) \)  
(9)

for “standard fragment shape” [1] and
\[ d_f = \left[ \frac{m_f}{(1.2 \rho_f)} \right]^{1/3} \]  \( (\ell) \)
(10)

for “alternate fragment shape” [1]

In other references [2-14] the “presented area” is not defined in terms of the dimensions of the fragment. In one case it is defined simply as a function of fragment mass [7]. In others it is expressed in terms of the form factor [2, 9-13] and fragment mass or the shape factor/ballistic density [3, 4, 8] and fragment mass. In reference 5, the term “presented area” does not appear but its value can be derived by means of the value assigned to the “shape factor” for (what are assumed to be) cubical shaped steel fragments.

Based on the use of the various parameters noted, an alternate “presented area”, \( A_f^* \), can be defined as follows:
\[ A_f^* = F \pi d_f^{2/4} \]  \( (\ell^2) \)
(11)

where the fragment area factor, \( F \),
\[ F \approx 2.0 \]  \( \text{dimensionless} \)
(12)

If this alternate “presented area” is used in either Eq. (6) or (7) to compute velocity decay coefficient, the resulting values of \( k_v \) could increase by a factor of 2.

Clearly, in the experimental measurement of drag coefficients, the manner in which the “presented area” is defined can strongly affect the magnitude of the measured drag coefficients, which can in turn affect the velocity decay coefficient.

OTHER PARAMETERS

The form factor [2, 10-13] represents the ratio of “presented area” to fragment mass, while the “shape factor/ballistic density” [3, 4, 8] represents the ratio of fragment mass to the “presented area” raised to the 3/2 power. Likewise, the “caliber density” is the ratio of fragment mass to the fragment diameter cubed [1, 2, 13]. These three parameters are closely related to one another and, either directly or indirectly, involve the “presented area”. Because of uncertainties in the definition of “presented area”, as already noted, certain questions arise as to the correct values for these three parameters.
The “L” parameter [3, 4] and the “L1” parameter [5] are closely associated with each other, and both are related to the velocity decay coefficient. **As already noted, because of the factor of 1/2 missing from some versions of the velocity decay coefficient, some doubt arises with regard to the correct values for the L and L1 parameters.**

**COMPARISON OF RESULTS PRODUCED BY USE OF DIFFERENT DEFINITIONS**

The uncertainties associated with the values of drag coefficient and “presented area” can lead to significant errors in calculating the value of the velocity decay coefficient, which in turn can result in major inaccuracies in the computed striking velocities and hazardous ranges for primary fragments. These inaccuracies are greatest when either the wrong value for drag coefficient is used in the correct equation for $k_v$, as given by Eq. (6), or the correct value of drag coefficient is used in an incorrect equation for $k_v$, such as given by Eq. (7).

The velocity decay coefficient is used in calculating hazardous range, $R_{HAZ}$, according to the relation,

$$R_{HAZ} = -\ln\left(\frac{U_{HAZ}}{U_0}\right)/k_v \quad (13)$$

where

$$U_{HAZ} = \text{hazardous striking velocity} \quad (\ell \ t^{-1})$$

$$U_0 = \text{initial velocity} \quad (\ell \ t^{-1})$$

If the correct values of $C_D$ are used with Eq. (6) to compute $k_v$ and Eq. (13) to compute $R_{HAZ}$, the results represent the correct values. For standard-shaped steel fragments [1], with a $C_D$ value of 1.2 assumed, the resulting values of $R_{HAZ}$ are given in Figure 2.

For purposes of comparison, if values of $C_D$, which are 50% of the correct value, are used with Eq. (6) to compute $k_v$ and Eq. (13) to compute $R_{HAZ}$, the resulting values for $R_{HAZ}$ will be twice the correct values. The results of using a drag coefficient value of 0.6 are presented in Figure 3. Comparison of Figure 2 and 3 reveals, as expected, that the hazardous ranges as given in Figure 3, computed by the smaller (incorrect) drag coefficient, are twice as great as the correct ranges given in Figure 2.

For further comparison, if the correct value of $C_D$ is used with Eq. (7) to compute $k_v$ and Eq. (13) to compute $R_{HAZ}$, the resulting value for $R_{HAZ}$ will be half the correct value. Such results, with a drag coefficient of 1.2 are presented in Figure 4. Comparison of Figure 4 with Figure 2 reveals, as expected, that the resulting hazardous ranges in Figure 4, are only 50% of the values given in Figure 2.
CONCLUSIONS

The most important conclusion resulting from the preceding discussion is the fact that the definition of drag coefficient for primary fragments in early studies [6-10] was nonstandard and results in drag coefficient values which are one-half the value resulting from the standard definition [16-20]. Because the same symbol, \( C_D \), has generally been used for both versions, the potential exists for a significant amount of confusion in using such values for primary fragment calculations. Current explosion literature [1-4] contains some values based on the original definition, as well as some based on the standard definition.

A second important conclusion is the observation that at least two definitions for the “presented area” of primary fragments currently exist. The first is generally based on the “form factor” [2, 10-13] or “shape factor/caliber density” [3, 4, 8] and expresses “presented area” as a function of fragment mass. This approach is the most commonly used. Unfortunately, one of the most widely accepted reports dealing with primary fragments [1] utilizes a different definition, based on the properties of a “standard fragment shape”. According to the definition, the “presented area” for such a fragment is equal to the minimum projected area of the fragment. Such a definition results in a value for “presented area” which is approximately one-half the value based on the first definition. This result, however, is strictly applicable only to steel fragments.

The definition of the velocity decay coefficient [1, 2, 4, 8, 13, 14] contains both the drag coefficient and the “presented area” of the fragment. Based on which definition is used, for each of the two parameters, values for the velocity decay coefficient may differ by a factor of 2 or even 4. This can produce significant differences in the calculation of hazardous fragment ranges. In some cases these differences will cause the underestimation of primary fragment ranges and velocities. Such a result could raise serious safety issues when explosive safety arcs are involved.

Of the various parameters noted for which some questions exist, concerning proper definitions and correct values, notice should be taken that at least four are included in reference 2. Establishment of a set of clearly defined parameters for primary fragmentation analysis, with numerical values that are founded on good engineering, is essential to the development of accurate tables, mathematical models, and engineering software pertaining to this field of analysis.

REFERENCES CITED

Figure 1. Usage of $C_D$ and $C_D^*$ in Primary Fragmentation Literature [1–13]
Figure 2. Hazardous Range as a Function of Fragment Mass and Velocity Ratio ($U_{HAZ}/U_0$) for $C_D = 1.2$ with Eqs. (6) and (13)
Figure 3. Hazardous Range as a Function of Fragment Mass and Velocity Ratio ($U_{HAZ}/U_0$) for $C_D = 0.6$ with Eqs. (6) and (13)
Figure 4. Hazardous Range as a Function of Fragment Mass and Velocity Ratio ($U_{HAZ}/U_0$) for $C_D = 1.2$ with Eqs. (7) and (13)
DISCREPANCIES ASSOCIATED WITH THE DRAG CHARACTERISTICS OF PRIMARY FRAGMENTS

by
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and
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July 15, 2010
INTRODUCTION

• IMPORTANT PARAMETERS FOR CALCULATION OF PRIMARY FRAGMENT VELOCITY
  • STANDARD PARAMETERS
  • NON-STANDARD PARAMETERS

• STANDARD PARAMETERS [1-14]
  • DRAG COEFFICIENT, \( C_D \)
  • FRAGMENT DENSITY, \( \rho_f \)
  • FRAGMENT DIAMETER, \( d_f \)
  • FRAGMENT MASS, \( M_f \)
  • AMBIENT AIR DENSITY, \( \rho_{air} \)
INTRODUCTION (cont.)

• NON-STANDARD PARAMETERS
  • VELOCITY DECAY COEFFICIENT, $k_v$ [1, 2, 4, 8, 13, 14]
  • CALIBER DENSITY, $D$ [1, 2]
  • SHAPE FACTOR/BALLISTIC DENSITY, $k$ [3, 4, 8]
  • PRESENTED AREA, $A_f$ [1 - 14]
  • “L” PARAMETER [3, 4]
  • FRAGMENT FORM FACTOR [2, 10 - 13]
  • “$L_1$” PARAMETER [5]

• DISCOVERY OF DISCREPANCIES [15]
• STANDARD PARAMETERS
• NON-STANDARD PARAMETERS
DRAG COEFFICIENT

• STANDARD DEFINITION [16 - 20]

\[ C_D \equiv 2 \frac{F_D}{(\rho_{\text{air}} A_f V_f^2)} \] (dimensionless)

WHERE

\[ F_D = \text{DRAG FORCE (f)} \]

\[ \rho_{\text{air}} = \text{AMBIENT AIR DENSITY (m} \text{l}^{-3}) \]

\[ A_f = \text{PRESENTED AREA (l}^2) \]

\[ V_f = \text{FRAGMENT VELOCITY (l t}^{-1}) \]
DRAG COEFFICIENT (cont.)

• ALTERNATE DEFINITIONS
  
  • $K_D = (\pi/4) \frac{F_D}{(\rho_{\text{air}} A_f V_f^2)}$ (dimensionless) [6]
  
  • $C_D^* = \frac{F_D}{(\rho_{\text{air}} A_f V_f^2)}$ (dimensionless) [7-9]  
    (NOTE: ASTERISK ADDED TO DISTINGUISH FROM STANDARD $C_D$)
  
  • $k = \frac{F_D}{(\rho_{\text{air}} A_f V_f^2)}$ (dimensionless) [10]

• RELATIONSHIP BETWEEN DRAG COEFFICIENTS
  
  $C_D = 2 \ C_D^* = 2 \ k = \frac{8}{\pi} \ K_D$ (dimensionless)

• CONFUSION BETWEEN VALUES FOR $C_D$ AND $C_D^*$
FIGURE 1. USAGE OF $C_D$ AND $C_D^*$ IN PRIMARY FRAGMENTATION LITERATURE [1-13]

NOTE: SEE CHARTS #17 AND #18 FOR A LIST OF REFERENCES CITED
VELOCITY DECAY COEFFICIENT

- BASED ON EQUATION OF MOTION OF PRIMARY FRAGMENT
  - TAKING INTO ACCOUNT DRAG
  - NEGLECTING GRAVITY

- NORMAL DEFINITION (WITH $C_D$) [4, 14]
  \[ k_v = \rho_{\text{air}} \frac{C_D A_f}{2 M_f} \quad (l^{-1}) \]

- ALTERNATE DEFINITION (with $C_D^*$) [1, 2, 8, 13]
  \[ k_v^* = \rho_{\text{air}} \frac{C_D^* A_f}{M_f} \quad (l^{-1}) \]
  (NOTE: ASTERISKS ADDED TO DISTINGUISH FROM STANDARD $k_v$ & $C_D$)

- USE OF $C_D^*$ WITH NORMAL DEFINITION OF $k_v$,
  OVERESTIMATES FRAGMENT VELOCITY

- USE OF $C_D$ WITH ALTERNATE DEFINITION OF $k_v$
  UNDERESTIMATES FRAGMENT VELOCITY
PRESENTED AREA

- REPRESENTS PROJECTED SURFACE AREA OF FRAGMENT NORMAL TO FLIGHT PATH

- OTHER NAMES
  - REFERENCE AREA
  - FRONTAL AREA
  - CROSS-SECTIONAL AREA

- FOR PRIMARY FRAGMENT
  - ACCORDING TO ONE REFERENCE [1]

\[ A_f = \pi \frac{d_f^2}{4} \quad (l^2) \]

WHERE

\[ d_f = \begin{cases} 
\left[ \frac{M_f}{(0.654 \rho_f)} \right]^{1/3} \text{ (FOR STANDARD FRAGMENT SHAPE)} \\
\left[ \frac{M_f}{(1.2 \rho_f)} \right]^{1/3} \text{ (FOR ALTERNATE FRAGMENT SHAPE)} 
\end{cases} \]
PRESENTED AREA (cont.)

- IN OTHER REFERENCES [2 - 13] EXPRESSED
- AS A FUNCTION OF FRAGMENT MASS [7]
- IN TERMS OF FORM FACTOR AND FRAGMENT MASS [2, 9-13]
- IN TERMS OF SHAPE FACTOR/BALLISTIC DENSITY AND FRAGMENT MASS [3, 4, 8]
- IN TERMS OF SHAPE FACTOR FOR CUBICAL SHAPED STEEL FRAGMENTS [5]

- RESULTING ALTERNATE PRESENTED AREA

\[ A^* = F \pi \frac{d_f^2}{4} \quad (l^2) \]

WHERE THE FRAGMENT AREA FACTOR

\[ F \approx 2.0 \quad \text{(DIMENSIONLESS)} \]
OTHER PARAMETERS

• FORM FACTOR [2, 10-13]
  \[ f_f \equiv \frac{A_f}{M_f} \quad (l^2 \text{ m}^{-1}) \]

• SHAPE FACTOR/BALLISTIC DENSITY [3, 4, 8]
  \[ k \equiv \frac{M_f}{A_f^{3/2}} \quad (\text{m} \text{ l}^{-3}) \]

• CALIBER DENSITY [1]
  \[ D \equiv \frac{M_f}{d_f^3} \quad (\text{m} \text{ l}^{-3}) \]

• “L” PARAMETER [3, 4]
  \[ L \equiv 2 \left( k^2 M_f \right)^{1/3} / (C_D \rho_{air}) \quad (l) \]

• “L_1” PARAMETER [5]
  \[ L_1 \equiv 2 k^{2/3} / (C_D \rho_{air}) \quad (l \text{ m}^{-1/3}) \]
COMPARISON OF RESULTS PRODUCED BY USE OF DIFFERENT DEFINITIONS

• CALCULATION OF HAZARDOUS RANGE

\[ R_{HAZ} = - \ln \left( \frac{U_{HAZ}}{U_0} \right) / k_v \ (l) \]

WHERE

\[ U_{HAZ} = \text{HAZARDOUS STRIKING VELOCITY} \ (l \ t^{-1}) \]
\[ U_0 = \text{INITIAL VELOCITY} \ (l \ t^{-1}) \]

• CASE #1
  • INPUTS
    • CORRECT VALUE OF \( C_D \) (= 1.2)
    • CORRECT DEFINITION OF \( k_v \)
  • RESULTS – CORRECT HAZARDOUS RANGE (FIGURE 2)
FIGURE 2. HAZARDOUS RANGE AS A FUNCTION OF FRAGMENT MASS AND VELOCITY RATIO ($U_{HAZ}/U_0$) FOR $C_D = 1.2$ WITH EQS. (6) AND (13)
COMPARISON OF RESULTS PRODUCED BY USE OF DIFFERENT DEFINITIONS (cont.)

• CASE #2
  • INPUTS
    • INCORRECT VALUE OF $C_D$ (= 0.6)
    • CORRECT DEFINITION OF $k_v$
  • RESULTS – COMPUTED HAZARDOUS RANGES (FIGURE 3)
    • TOO LARGE
    • TWICE CORRECT VALUE
FIGURE 3. HAZARDOUS RANGE AS A FUNCTION OF FRAGMENT MASS AND VELOCITY RATIO ($U_{HAZ}/U_0$) FOR $C_D = 0.6$ WITH EQS. (6) AND (13)

CASE #2
COMPARISON OF RESULTS PRODUCED BY USE OF DIFFERENT DEFINITIONS (cont.)

• CASE #3
  • INPUTS
    • CORRECT VALUE OF $C_D (= 1.2)$
    • INCORRECT DEFINITION OF $k_v$
  • RESULTS – COMPUTED HAZARDOUS RANGES (FIGURE 4)
    • TOO SMALL
    • 1/2 CORRECT VALUE
FIGURE 4. HAZARDOUS RANGE AS A FUNCTION OF FRAGMENT MASS AND VELOCITY RATIO ($U_{HAZ}/U_0$) FOR $C_D = 1.2$ WITH EQS. (7) AND (13)
CONCLUSIONS

• DRAG COEFFICIENT DISCREPANCY
  • DEFINITION IN EARLY STUDIES
    • NONSTANDARD
    • ONE HALF STANDARD VALUE
    • EXPERIMENTAL VALUES STILL IN USE
  • DEFINITION IN MORE RECENT STUDIES
    • STANDARD
    • CONFUSION DUE TO USE OF SAME SYMBOL

• PRESENTED AREA DISCREPANCY
  • SEVERAL DIFFERENT DEFINITIONS
  • DIFFER BY A FACTOR OF APPROXIMATELY TWO

• VELOCITY DECAY COEFFICIENT DISCREPANCY
  • TWO DIFFERENT DEFINITIONS
  • DIFFER BY A FACTOR OF TWO

• RESULTS OF DISCREPANCIES
  • INACCURATE HAZARDOUS RANGE CALCULATION
  • SAFETY ISSUES
REFERENCES CITED


