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12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

This report results from a contract tasking FORTH/IACM as follows: Accurate predictions of skin friction and thermal loads caused by complex compressible flows require high resolution computations. High order, discontinuous Galerkin (DG) discretizations are used to compute viscous supersonic flows with strong shocks. Emphasis is given to accurate predictions of surface heat transfer. The two key ingredients, which make practical the use of the DG method for these calculations, are implementation of efficient implicit time marching methods, and use of solution adaptive procedures. Implicit time marching is necessary in order to overcome the severe stability limitations encountered with the increase of the order of spatial DG discretizations. On the other hand, solution adaptive schemes ensure high accuracy (p-type refinement) in regions with smooth but complex flow features, such as wall layers detached shear layers and vortices, while at the same time allow resolution of strong discontinuities without oscillations on a finer mesh (h-type refinement) where lower order expansions are used. The development and validation of such an implicit hp adaptive DG method for mixed-type elements is proposed.

15. SUBJECT TERMS

EOARD, High Speed Aerodynamic, Computational Fluid Dynamics (CFD), Numerical Simulation

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Parallelization of the DG method for the Navier-Stokes Equations

Summary

Accurate predictions of skin friction and thermal loads of high speed complex flows, in both simple and nontrivial geometries, require good shock capturing capability and high accuracy in the viscous flow region. High order discontinuous Galerkin (DG) discretizations possess features that make them attractive for accurate computation of complex flows with strong shocks. A key ingredient that would make the DG method more attractive for these computations, is application of p-adaptive procedures that ensure accurate capturing of discontinuities with low order expansions and resolution of smooth complex features, such as vortices and shear layers, with higher order accuracy. A limiting procedure of DG discretizations capable of commuting high speed flows with strong shocks around complex geometries, using a p-adaptive procedure on mixed type (quadrilateral and triangular) meshes was developed and preliminary results were presented in the previous report. In this report further validation of the limiting approach is presented for standard computationally demanding flow cases, such as the Mach reflection and the wind tunnel with a step. Application examples for both quadrilateral, triangular and mixed type meshes are shown.

These large scale computations were made possible by parallelizing the code using domain decomposition and MPI. Viscous terms were added for the DG method using the local DG approach. Incorporation of an implicit time marching scheme, which will make possible high Reynolds number computations, is underway for a single processor. Implementation of implicit time marching with domain decomposition is more involved and requires use of a dual time stepping for time accurate computations. Developments of capabilities to compute viscous flow numerical solutions with implicit time stepping for multiprocessor will be presented in the next report.

Parallelization

The compact form of the DG discretization stencil (information is needed only from the immediate neighbors) permits efficient implementation on parallel processors. The data structure of the code was designed in order to meet the requirements of the DG method on mixed type element meshes and p-adaptive expansions. Moreover, it allows application of the domain decomposition method through MPI without serious additional coding complexity.

Every simulation with domain decompositions initiates with the partitioning of the computational domain using free available software Metis (http://glaros.dtc.umn.edu/gkhome/views/metis). In order to handle mixed type meshes, a graph file representing the mesh is initially created and subsequently partitioning of the graph in to subdomains leads to the final decomposition of the mesh. Every partition (subdomain) includes layers of elements belonging to neighboring partitions. This set of elements is the receiving list for a partition, and the set of elements of the partition sharing the same edges with the receiving list, is the sending list to neighboring partitions.

A TVD Runge–Kutta method was used for time marching. Limiting is applied after every stage of the RK cycle. Finally, an exchange between the partitions of the solution is also performed at the end of each stage. For a parallel p-adaptive DG code, all coefficients of the expansion for the highest order polynomial approximation are exchanged between partitions of the mesh.

The parallel algorithm for every partition is as follows:

1. Transfer the solution (expansion coefficients) at the end of the RK cycle between partitions. Wait for the transfer to complete.
2. Apply the limiter.
3. Transfer of all coefficients of the highest polynomial application. Wait for the transfer to complete.
4. Assign solution coefficients from P0 up to the highest polynomial application.
5. Compute the volume integrals.
6. Compute the line integrals.
7. Compute residual for every element belonging to the partition.
8. Enter the next RK stage.
9. Complete the RK cycle and repeat.

Results

Results from parallel computations of flows with strong shocks on single- and mixed-type meshes are presented next. The supersonic flow over a cylinder is the first example. The single type computational mesh with triangular elements and the computed density field for flow at M=2.0 is are shown in Figs. 1-2. The same flow was computed on a mesh with arbitrary quadrilateral elements (see Figs. 3 and 4). This computation was not possible with the original limiting procedure suggested by Cockburn and Shu [1] and difficult to implement with limiting approaches presented in Ref. 2-6, but it is straightforward with our new limiting approach. Next, an example of a computation on a mixed-type mesh is presented. The mesh and the computed density field is shown in Fig. 5. For this computation, a p-adaptive procedure is applied where the flow near the discontinuity is computed with P1 polynomials as flagged by the limiter while the rest of the smooth flow field is computed with P2 polynomials. A comparison of the computed density along the symmetry line is shown in Fig. 6. The agreement with the quadrilateral mesh solution is excellent.

Computations for two classical examples are presented next. These are the M=3.0 tunnel with a step and the double Mach reflection of a strong shock at Mach=10 (P. Woodward and P. Colella, "The numerical simulation of two-dimensional fluid flow with strong shocks," Journal of Computational Physics, Vol. 54, No. 1, 1984, pp. 115-173). The computational domain for the tunnel with a step discretized with a mixed type mesh and the partition of this mesh to subdomains for parallel processing is shown in Fig. 7. At the corner the mesh was refined in order to diminish the Mach stem that is created from the artificial entropy layers caused by the sharp corner. The computed flow field and the elements flagged for limiting are shown in Fig. 8. The elements flagged for limiting for the double Mach reflection problem computation are shown in Fig. 9 and the computed flow field is shown in Fig. 10. For these computations the elements flagged for limiting are updated continuously during the time accurate computation. Computations with higher order expansions using p-adaptive procedures are underway and they will presented at the AIAA ASM meeting in January 2011.

Conclusions and Outlook

The DG code was parallelized using domain decomposition and MPI. The new limiting procedure for DG discretizations was then applied for standard computationally demanding test problems. The proposed approach applied to quadrilateral, triangular, and mixed-type meshes. The extension of the limiting approach in three dimensions is underway. Numerical solutions of viscous flows with shocks will be computed using a *p*-adaptive procedure. For high Reynolds number flows and long time integration with small size meshes use of an implicit time marching scheme is necessary.

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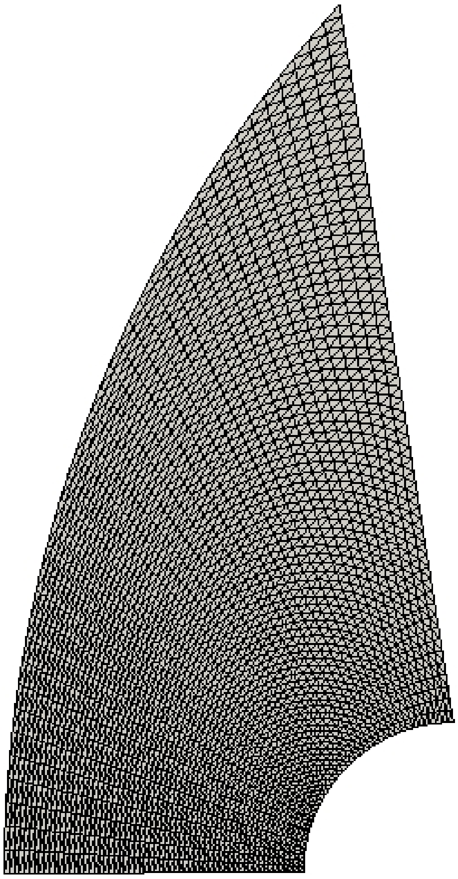


Figure 1. Triangular element mesh over the cylinder

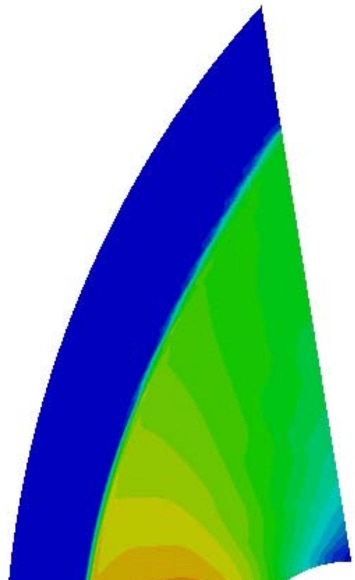


Figure 2. Computed density field at M=2.

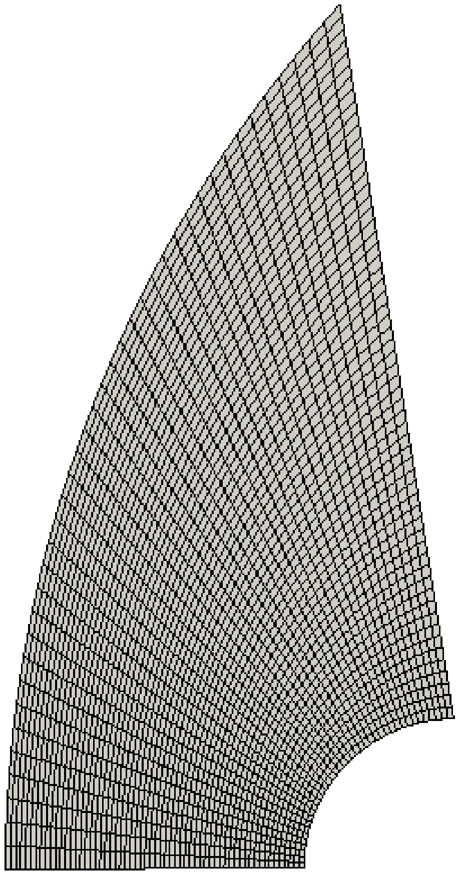


Figure 3. Quadrilateral element mesh over the cylinder

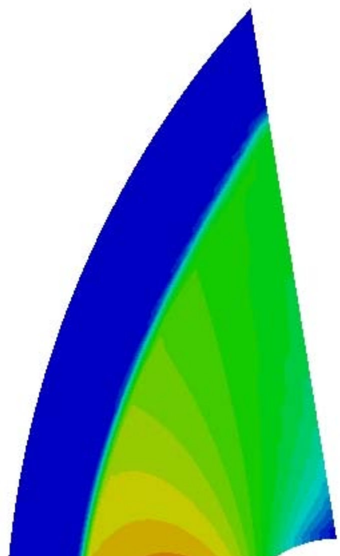


Figure 4. Computed density field at M=2.

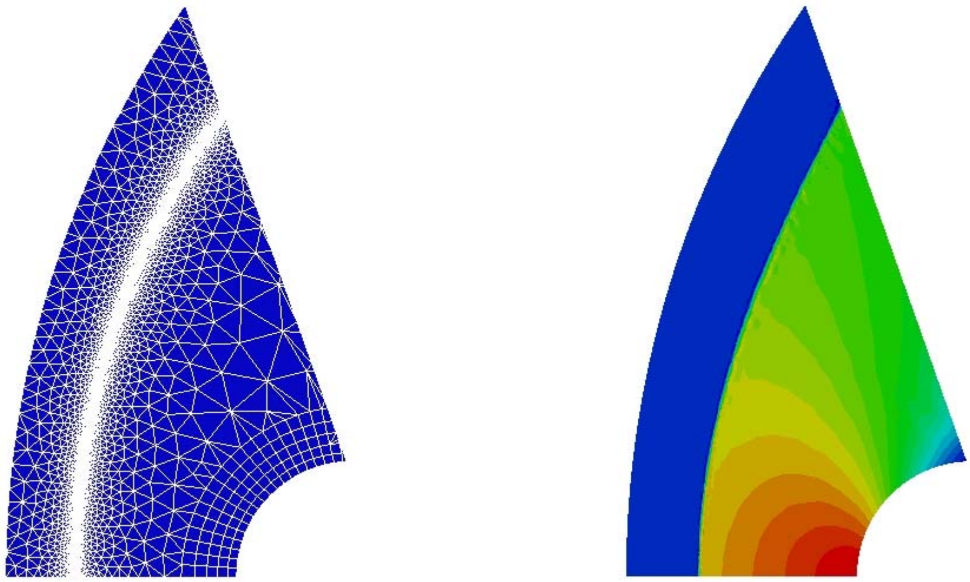


Figure 5. Mixed-type mesh for the computation of supersonic flow.

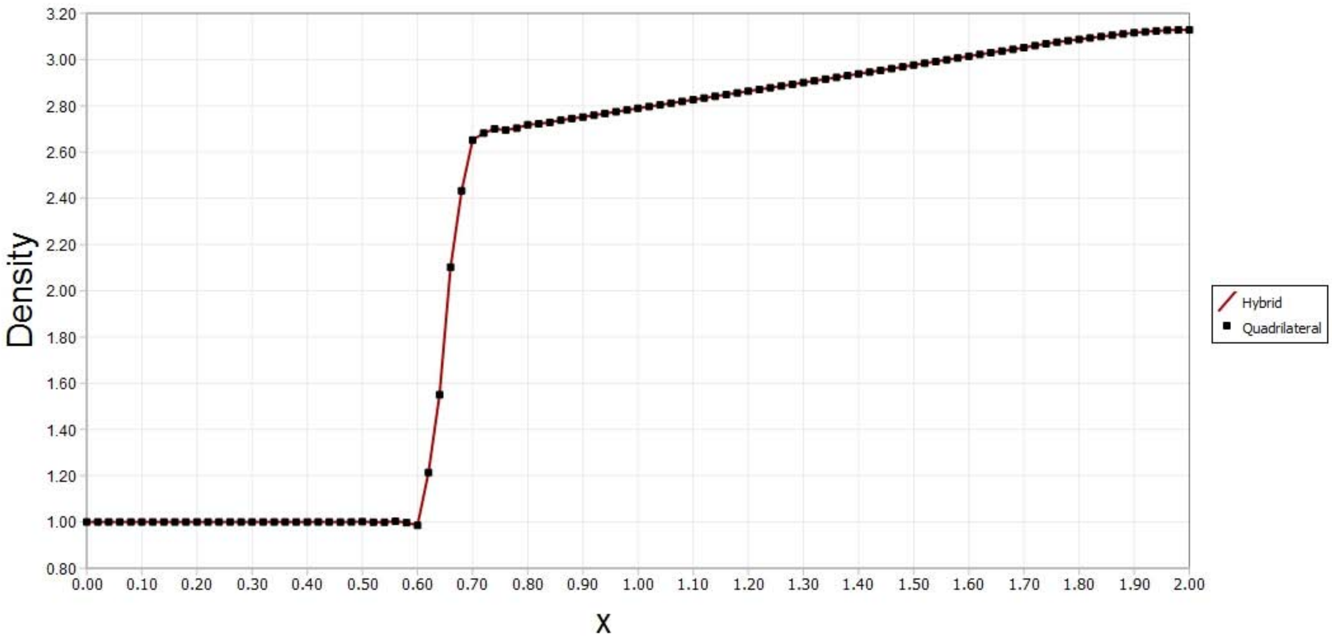


Figure 6 Comparison of the density along the symmetry stagnation line obtained with quadrilateral (P1) and mixed type meshes (p2 adaptive).

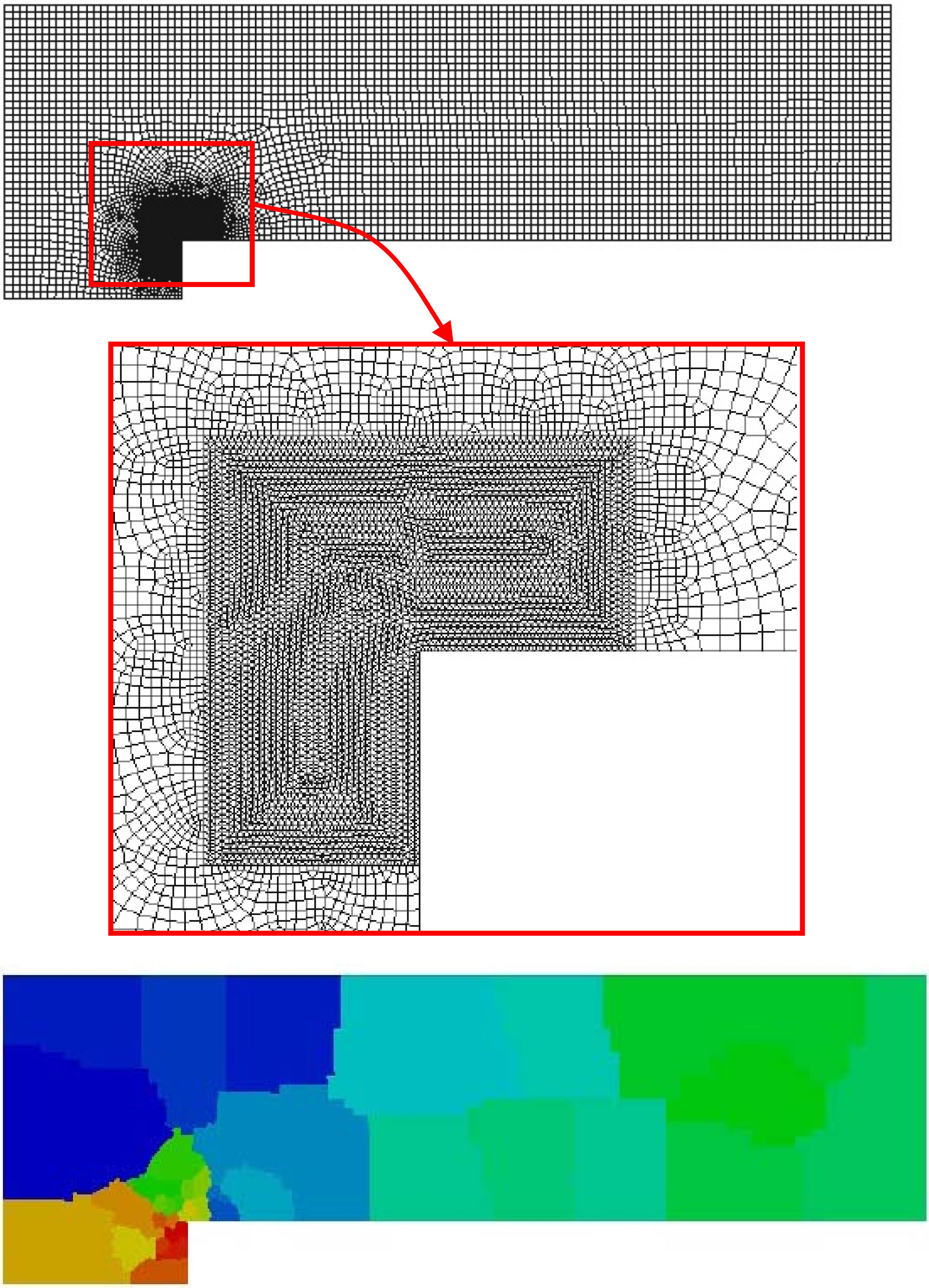
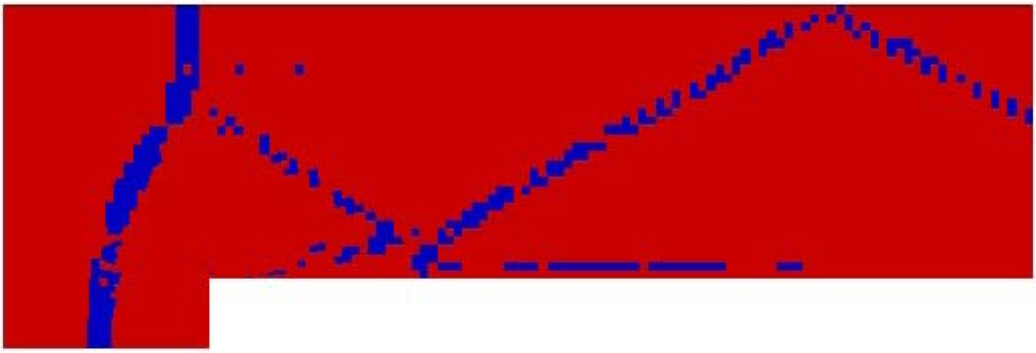
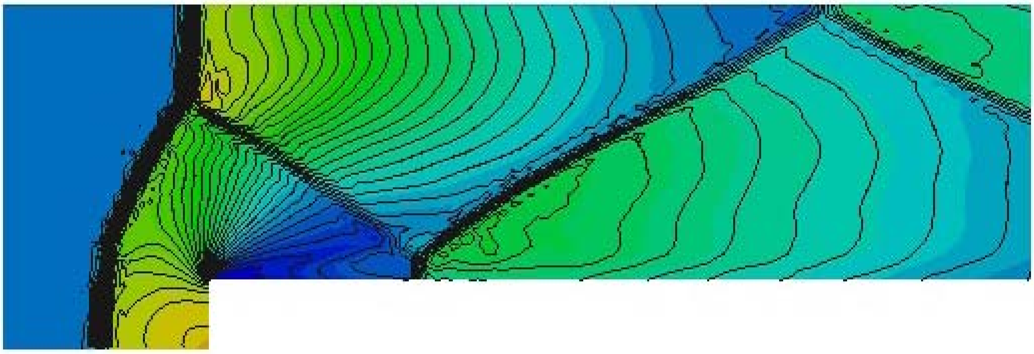


Figure 7. Mixed-type mesh and partition for MPI parallel processing.



(elements flagged for limiting)



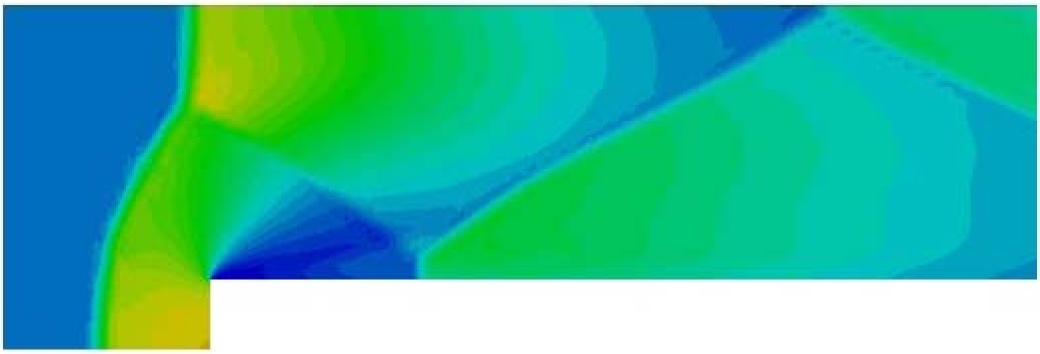


Figure 8. Elements flagged for limiting (top) and computed density and pressure for the flow in a tunnel with a step.

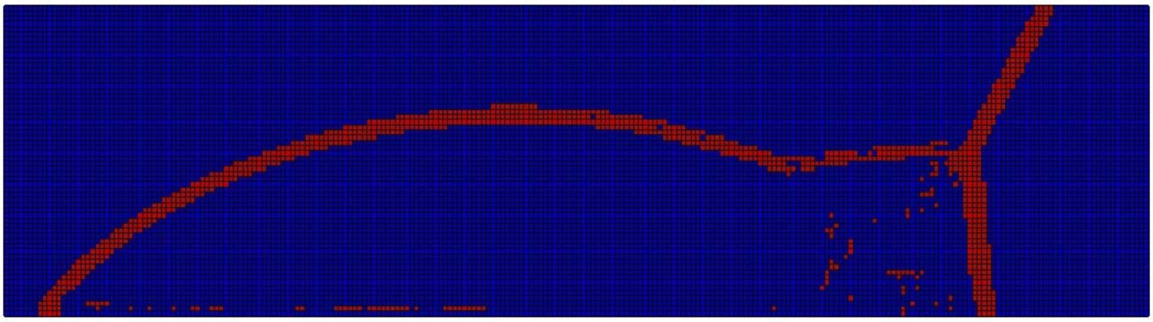


Figure 9. Elements flagged for limiting

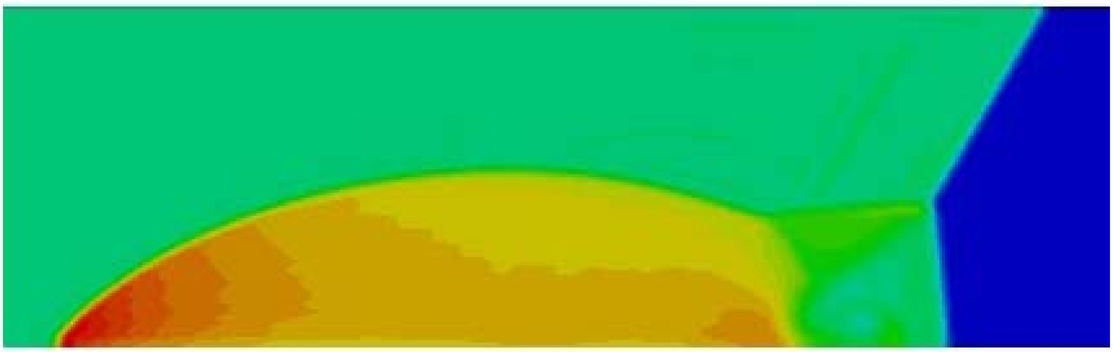


Figure 11. Computed density field.