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THE HEATING OF THERMAL ELECTRONS IN FAST COLLISIONLESS SHOCKS: THE INTEGRAL ROLE OF COSMIC RAYS

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ABSTRACT

Understanding the heating of electrons to quasi-thermal energies at collisionless shocks has broad implications for plasma astrophysics. It directly impacts the interpretation of X-ray spectra from shocks, is important for understanding how energy is partitioned between the thermal and cosmic-ray populations, and provides insight into the structure of the shock itself. In previous work by Ghavamian et al. we presented observational evidence for an inverse-square relation between the electron-to-proton temperature ratio and the shock speed at the outer blast waves of supernova remnants in partially neutral interstellar gas. There we outlined how lower hybrid waves generated in the cosmic-ray precursor could produce such a relationship by heating the electrons to a common temperature independent of both shock speed and the strength of the ambient magnetic field. Here we explore the mechanism of lower hybrid wave heating of electrons in more detail. Specifically, we examine the growth rate of the lower hybrid waves for both the kinetic (resonant) and reactive cases. We find that only the kinetic case exhibits a growing mode. At low Alfvén Mach numbers the growth of lower hybrid waves can be faster than the magnetic field amplification by modified Alfvén waves.

Subject headings: cosmic rays — shock waves — supernova remnants

1. INTRODUCTION

The main accelerators of cosmic rays (CRs) are widely believed to be high–Mach number shocks in collisionless plasma, here loosely defined as plasma where charged particles interact predominantly through plasma waves rather than by Coulomb collisions (for a thorough conceptual and historical review, see Malkov & Drury 2001). However, a consensus is emerging that CRs are not simply a by-product of collisionless shocks, but in fact play an integral role in the shock structure, dynamics, and energetics. For example, sound waves in a CR pressure gradient can smooth out the shock jump in CR-modified shocks (Drury & Falle 1986). More recent analytic and numerical work has shown that modified Alfvén waves in the CR precursor may amplify the magnetic field to many times its ambient value by generating perpendicular magnetic field from an initially quasi-parallel geometry (Lucek & Bell 2000; Bell & Lucek 2001; Bell 2004, 2005). Observational support for dramatic magnetic field amplification ahead of shocks exists in the form of extremely thin X-ray synchrotron rims of supernova remnants (SNRs) such as Cassiopeia A (Vink & Laming 2003), SN 1006 (Long et al. 2003; Yamazaki et al. 2004), and Tycho's SNR (Warren et al. 2005; Cassam-Chenaï et al. 2007). As noted by Cassam-Chenaï et al. (2007), there are two possible interpretations for the narrow width; however, both require a dramatically amplified magnetic field ahead of the shock. Either the rims are thin because the high magnetic field causes rapid synchrotron cooling of the X-ray-emitting electrons or the scale of the rims represents the scale of magnetic field de-amplification behind the shock.

In this paper we explore another area where CRs may influence the properties of the shock, namely, through the heating of quasi-thermal electrons. For shocks in collisionless plasma the heating of electrons must occur through the damping of waves generated by the other more massive charged particles that dominate the energetics. Given the wide array of possible plasma instabilities at collisionless shocks, an observational relationship for electron temperature, T_e, at the shock front was required to limit theoretical discussions. An inverse relationship between the initial ratio of electron to proton temperatures immediately behind the shock, $(T_e/T_p)_0$, and the shock velocity, v_s, has been reported in a series of observational papers on SNR shocks (Ghavamian et al. 2001, 2002, 2003, 2007; Rakowski et al. 2003; Rakowski 2005) and has also been noted among the higher Alfvén Mach number events in a sample of solar wind shocks (Schwartz et al. 1988). In Ghavamian et al. (2007), we focused on shocks propagating into partially neutral gas. Here the collisional excitation of broad and narrow Balmer line emission at the shock front can be used to diagnose $(T_e/T_p)_0$. In Ghavamian et al. (2001, 2002, 2007) we described the method of simultaneously constraining v_s and $(T_e/T_p)_0$ via measurement of the width of the broad Balmer line and the ratio of broad to narrow Balmer line flux (see also Heng & McCray 2007; Heng et al. 2007). Our results are consistent with an inverse-square relationship, $(T_e/T_p)_0 \propto 1/v_s^2$, for shock speeds above ~400 km s^{-1} (Ghavamian et al. 2007). Given that $T_p \propto v_s^2$ at the shock front by the Rankine-Hugoniot jump conditions, the inverse relationship between equilibration and shock speed implies that the electron temperature itself is nearly *constant*, ~0.3 keV, independent of shock speed.

The insensitivity of electron temperature to shock velocity suggests a heating mechanism within the extended diffusive CR precursor ahead of the shock. In this case the electron heating

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14. ABSTRACT
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would be more reflective of the generic properties of CR acceleration and diffusion than tied to the specific attributes of the shock. In contrast, prior work on heating by shock-reflected ions that are confined to within a gyroradius of the (quasi-perpendicular) shock (Cargill & Papadopoulos 1988) suggested that $(T_e/T_p)_0$ would remain constant with shock velocity. In Ghavamian et al. (2007) we suggested that lower hybrid waves in the CR precursor of a perpendicular shock might be a plausible electron heating mechanism.

Lower hybrid waves are electrostatic ion waves directed nearly perpendicular to the magnetic field with a frequency equal to the geometric mean of the electron and ion gyrofrequencies. Electrons that would otherwise screen the ion oscillation are pinned to magnetic field lines. In addition, the group velocity parallel to the magnetic field greatly exceeds the group velocity perpendicular to the field ($\omega/k_\parallel \gg \omega/k_\perp$). Therefore, the wave can simultaneously resonate with ions moving across the field lines and electrons moving along the field lines, facilitating collisionless energy exchange between them. Based on simple arguments about the width of the CR precursor and the electron diffusion along the field lines, we showed that electron heating from lower hybrid waves in the CR precursor would be independent of both the shock speed and the magnetic field. Here, we explore this mechanism in more physical detail.

In § 2 we calculate the growth rate of lower hybrid waves, first examining the kinetic (resonant) case then the reactive (nonresonant) case. The treatment here is mathematically similar to the work on modified Alfvén waves by Achterberg (1983) and Bell (2004, 2005) involving the CR contribution to the plasma dielectric tensor. The analysis also draws on the work of Laming (2001a, 2001b) on lower hybrid waves from shock-reflected ions. We compare these growth rates with those for magnetic field amplification to assess the conditions under which electron heating might occur. In § 3 we discuss the structure of the CR shock precursor in more detail. We pay particular attention to the magnetic field geometry, since the excitation of lower hybrid waves requires a quasi-perpendicular shock. We show schematically how magnetic field amplification and lower hybrid wave heating might coexist in the shock precursor for either parallel or perpendicular initial geometries. We also review some other ideas for electron heating and make some quantitative predictions from our model for various shock parameters. Included in the appendices are a discussion of CR diffusion coefficients and a derivation of the resonant growth rate for electromagnetic waves.

2. COSMIC-RAY GROWTH RATE OF LOWER HYBRID WAVES

Lower hybrid waves ahead of collisionless shocks have particularly interesting properties. They can have a group velocity away from the shock equal to the shock velocity itself (McClements et al. 1997). This can in principle allow the waves to grow to large amplitudes, even if their intrinsic growth rate is small. To determine if lower hybrid waves can heat the electrons to the $\sim$0.3 keV temperature observed, we must first calculate the growth rate of this instability to see if it will have sufficient power to overcome the damping effect of the electrons as well as to compete with other instabilities in the precursor. We calculate this growth rate in both kinetic and reactive limits, i.e., either considering the CRs with energies in resonance with the lower hybrid wave frequency or the integrated contribution of the entire distribution, respectively (see, e.g., Melrose 1986). Related kinetic and reactive cases were calculated in Laming (2001a, 2001b) but only for the case of shock-reflected, nonrelativistic ions gyrating around the magnetic field, represented as a particle beam. Here we begin the discussion with the resonant case.

2.1. *Kinetic Growth Rate*

We model the normalized CR distribution function diffusing upstream as

$$f(p) = \frac{n'_{\rm CR}}{4\sqrt{2}(\pi\kappa)^{3/2}p_t^3}\frac{(2\kappa-3)\Gamma(\kappa)}{\Gamma(\kappa-1/2)} \times \left[1+\frac{(p_x-mv_s)^2+p_y^2+p_z^2}{2\kappa p_t^2}\right]^{-\kappa}, \qquad (1)$$

where the coordinate system is aligned so that the shock speed v_s lies in the x-direction. Here, p_t is defined as the "thermal" momentum, which we take to be $(3/4)mv_s$, and $n'_{\rm CR}$ denotes the density of suprathermal particles with distribution function $f(p)$. The functional form above, known as a "kappa" distribution, is often seen for particle distribution functions associated with shocks in the solar wind and may be derived as equilibrium distributions for a system of particles and waves under certain conditions (see, e.g., Laming & Lepri 2007), in contrast to a system of particles which only gives a Maxwellian distribution of width p_t. The kappa distribution resembles a Maxwellian for $p < p_t$ and, in fact, for $\kappa \to \infty$ is exactly a Maxwellian. At higher particle momenta it tends smoothly to a distribution $f \propto p^{-2\kappa}$. Below, we shall take $\kappa = 2$ to model the well-known $f(p) \propto p^{-4}$ CR distribution predicted by diffusive shock acceleration in shocks with a compression ratio of 4. In connecting the CRs to the lower energy particles in this way, we are somewhat blurring the distinction between "CRs" and other suprathermal particles reflected from the shock. Hence, we denote the combined density of these particles as $n'_{\rm CR}$ to distinguish it from density of true CRs, $n_{\rm CR}$, which will appear in expressions derived by other authors. Note that all these particles are distinct from the ambient thermal plasma upstream of the shock, which here is considered to be a Maxwellian with much lower temperature than p_t used in equation (1) for the upstream suprathermals. When discussing the kinetic instability we focus on particles that obey a diffusion equation ahead of the shock, due to their interaction with turbulence, rather than gyrate around field lines. We qualify this distinction more carefully below in our discussion of the reactive instability.

The appropriate dispersion relation for equation (1) can be found from the cold plasma dielectric tensor. For electrostatic waves at frequencies close to the lower hybrid wave frequency we have (Laming 2001a, 2001b)

$$K_{\rm L} = 1 + \frac{\omega_{pe}^2}{\Omega_e^2}\sin^2\theta - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2}\cos^2\theta + \frac{4\pi q^2}{k^2}\int\frac{\boldsymbol{k}\cdot\partial f/\partial\boldsymbol{p}}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}}d^3\boldsymbol{p} = 0, \qquad (2)$$

where θ denotes the angle between the wavenumber of the perturbation and the preshock magnetic field; $\omega_{px} = (4\pi q^2 n_x/m_x)^{(1/2)}$ is the plasma frequency of a given species x (electrons, ions, CRs, etc.) with charge q, density n_x, and mass m_x; $\Omega_x = qB/(m\gamma_{\rm L}c)$ is the cyclotron frequency of species x (with $\gamma_{\rm L}$ being the Lorentz factor); and unadorned ω is the lower hybrid wave frequency which is the geometric mean of the electron and proton cyclotron frequencies. Using the Landau prescription for evaluating the integral at the resonant pole and taking only the imaginary parts of the dielectric tensor equation, we find the growth rate for the lower hybrid waves,

$$\gamma = \frac{2\pi^2 q^2}{k^2}\frac{\omega^2}{\omega_{pi}^2+\omega_{pe}^2\cos^2\theta}\int \delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\boldsymbol{k}\cdot\frac{\partial f}{\partial \boldsymbol{p}}d^3\boldsymbol{p}. \quad (3)$$

To compute γ, we take $n'_{\rm CR} \propto \exp(-xv_s/D)$, where D is the CR diffusion coefficient and is in principle dependent on the CR momentum (making $l = D/v_s$ the characteristic diffusive length scale). With this substitution we begin the evaluation of the last integral in equation (3),

$$\int_0^\infty 2\pi p_\perp f\, dp_\perp = \frac{n'_{\rm CR}}{4\sqrt{2}(\pi\kappa)^{3/2}p_t^3}\frac{(2\kappa-3)\Gamma(\kappa)}{\Gamma(\kappa-1/2)}\frac{2\pi\kappa}{\kappa-1} \times p_t^2\left[1+\frac{(p_x-mv_s)^2}{2\kappa p_t^2}\right]^{1-\kappa} e^{-xv_s/D}, \quad (4)$$

where we have separated out the components of $\boldsymbol{p}$ perpendicular to the shock (and $\boldsymbol{k}$) from p_x. Substituting back into equation (3) yields a growth rate

$$\gamma = \left(\frac{\pi}{\kappa}\right)^{3/2}\frac{q^2}{p_t k}\frac{\omega^3 n'_{\rm CR}}{\omega_{pi}^2+\omega_{pe}^2\cos^2\theta}\frac{(2\kappa-3)\Gamma(\kappa)}{\sqrt{2}\Gamma(\kappa-1/2)}\int\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v}) \times\left\{-\left[1+\frac{(p_x-mv_s)^2}{2\kappa p_t^2}\right]^{-\kappa}\frac{(p_x-mv_s)}{p_t^2} + \frac{\kappa}{\kappa-1}\frac{xv_s}{D^2}\frac{\partial D}{\partial p_x}\left[1+\frac{(p_x-mv_s)^2}{2\kappa p_t^2}\right]^{1-\kappa}\right\}e^{-xv_s/D}dp_x. \quad (5)$$

For waves to stay in contact with the shock, $\omega/k \simeq -2v_s$ (Laming 2001a) in the cold plasma electrostatic limit, i.e., reflected ions returning to the shock excite the waves. This remains generally true when these approximations are relaxed (Laming 2001b), so the δ-function picks out $p_x = -2mv_s$ (CRs returning to the shock). Hence,

$$\gamma = 4\left(\frac{\pi}{\kappa}\right)^{3/2}\frac{q^2\omega n'_{\rm CR}v_s^2 m}{p_t\left(\omega_{pi}^2+\omega_{pe}^2\cos^2\theta\right)}\frac{(2\kappa-3)\Gamma(\kappa)}{\sqrt{2}\Gamma(\kappa-1/2)}\left(1+\frac{9m^2v_s^2}{2\kappa p_t^2}\right)^{-\kappa} \times\left[\frac{3mv_s}{p_t^2}+\frac{\kappa x v_s}{(\kappa-1)D^2}\frac{\partial D}{\partial p_x}\left(1+\frac{9m^2v_s^2}{2\kappa p_t^2}\right)\right]e^{-xv_s/D}. \quad (6)$$

Assuming $\partial D/\partial p_x = 0$ (see Appendix A on CR diffusion coefficients) and $\kappa = 2$, only the first term within the square brackets remains. Evaluating the Γ functions and substituting $p_t = (3/4)mv_s$, we arrive at the following expression for the kinetic growth rate of lower hybrid waves,

$$\gamma = \frac{32}{225}\frac{\omega'^2_{pi}\omega}{\omega_{pi}^2+\omega_{pe}^2\cos^2\theta}e^{-xv_s/D}, \quad (7)$$

where ω'_{pi} denotes the plasma frequency for $n'_{\rm CR}$. Substituting in the frequency definitions we note that approximately $\gamma \propto (n'_{\rm CR}/n_i)\Omega_i$. Before proceeding, we pause to compare this growth rate with those for magnetic field amplification. In the case where Alfvén waves are resonantly excited, the growth rate is (Melrose 1986; Pelletier et al. 2006)

$$\gamma_{B,\rm res} = \frac{3\pi}{16}\frac{\Omega_i}{v_{\rm A}}\frac{n_{\rm CR}}{n_i}\left(\frac{\cos\theta}{|\cos\theta|}v_s\cos\phi - \frac{4}{3}v_{\rm A} - \frac{\pi}{4}v_s\sin\phi\right)k_{||}r_g, \quad (8)$$

where $v_{\rm A}$ is the Alfvén speed, n_i is the density of ions in the background plasma, and r_g is the gyroradius of CRs. This expression differs from that in the cited references in the factor $\cos\phi$ and the term in $\sin\phi$, where ϕ is the angle between the shock velocity and the magnetic field. At perpendicular shocks, the growth rate of resonant Alfvén waves can be neglected, but at parallel shocks, may be larger than that for lower hybrid waves, depending on the ratio $n'_{\rm CR}/n_{\rm CR}$. However as we shall argue below, all shocks subject to magnetic field amplification become perpendicular, and this is the geometry where lower hybrid waves are most effectively excited, so we neglect $\gamma_{B,\rm res}$ from here onward. Bell (2004) discovered a nonresonant growth rate for Alfvén waves, with approximate growth rate

$$\gamma_{B,\rm nonres} = \sqrt{\frac{n_{\rm CR}}{n_i}k_{||}v_s\Omega_i - k_{||}^2 v_{\rm A}^2}, \quad (9)$$

which has a maximum value of $M_{\rm A}\Omega_i n_{\rm CR}/2n_i$. According to Bell (2005), this instability operates for arbitrary orientations of $\boldsymbol{B}$, $\boldsymbol{v}_s$, and $\boldsymbol{k}$, indicating that it will also amplify magnetic field at perpendicular shocks. Its growth rate is strongest for $\boldsymbol{k}||\boldsymbol{B}$ and zero for $\boldsymbol{k}\perp\boldsymbol{B}$. Equating the maximum value of $\gamma_{B,\rm nonres}$ with the lower hybrid wave growth rate calculated above, we find the critical Alfvénic Mach number $M_{\rm A} \simeq 12n'_{\rm CR}/n_{\rm CR}$, such that for higher $M_{\rm A}$, CRs preferentially amplify magnetic field, and for lower $M_{\rm A}$, they generate lower hybrid waves. The numerical value depends on the ratio $n'_{\rm CR}/n_{\rm CR}$. In § 2.2 we argue that these two densities should not be the same and that $n'_{\rm CR} > n_{\rm CR}$, following from a consideration of the reactive growth rate for lower hybrid waves.

2.2. *Reactive Growth Rate*

The reactive case involves the integrated contribution to the growth rate from the entire CR distribution. Thus, we examine successive orders in an expansion of $f(p)$ to see if they produce any growing modes. Although no instability is found in this process, we do uncover potentially interesting constraints on the properties of $f(p)$.

We consider again the last term in equation (2), the CR contribution to the dielectric tensor (e.g., Melrose 1986), which includes a factor that reduced to unity for the resonant case,

$$K_{\rm L}^{\rm CR} = \frac{4\pi q^2}{k^2}\int\frac{\boldsymbol{k}\cdot\partial f/\partial\boldsymbol{p}}{\omega-\boldsymbol{k}\cdot\boldsymbol{v}}\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}d^3\boldsymbol{p}. \quad (10)$$

For the case of a beam of CR particles localized around $\boldsymbol{v}_s$, one recovers the usual beam reactive instability (Laming 2001b, eq. [A6]). However, as we demonstrate below, for a more physical quasi-isotropic CR distribution no instability is recovered. We consider the case where CRs drift with velocity $\boldsymbol{v}_s$ and waves are generated with $\boldsymbol{k}\parallel\boldsymbol{v}_s$. The CR distribution function from Appendix A, expanded in terms of the cosine of the angle between $\boldsymbol{k}$ and $\boldsymbol{v}$ or $\boldsymbol{k}$ and $\boldsymbol{v}_s$, $\cos\alpha = \mu$, is

$$f = f_0 + \mu\frac{\partial f}{\partial\mu} + \ldots = f_0\left(1+\mu\frac{3v_s}{v}+\ldots\right), \quad (11)$$

such that $\int \mu v f \, d\Omega = f_0 v_s$ as before. Neglecting terms of order v_s^2/v^2, the CR contribution to the dielectric tensor becomes

$$K_{\rm L}^{\rm CR} \approx \frac{4\pi q^2}{k^2}\int_0^\infty 2\pi\int_{-1}^{1} \frac{vk^2}{\omega}\frac{(\mu^2+3\mu^3 v_s/v)}{\omega - kv\mu}\, d\mu \frac{\partial f_0}{\partial p}p^2\, dp. \tag{12}$$

Expanding the numerator into terms divisible by $(\omega/kv - \mu)$ and evaluating the integral over μ, we obtain

$$K_{\rm L}^{\rm CR} \approx \frac{8\pi^2 q^2}{\omega}\int_0^\infty \left[\frac{-2v_s}{kv} - \left(\frac{3v_s\omega}{kv}+v\right)\frac{2\omega}{k^2v^2} - \left(\frac{3v_s\omega}{kv}+v\right)\frac{\omega^2}{k^3v^3}\ln\left|\frac{\omega-kv}{\omega+kv}\right|\right]\frac{\partial f_0}{\partial p}p^2\, dp. \tag{13}$$

We then evaluate this integral in the two limiting cases away from the pole, $\omega \gg kv$ and $\omega \ll kv$. For $\omega \gg kv$,

$$\ln\left|\frac{\omega-kv}{\omega+kv}\right| \simeq \frac{-2kv}{\omega} - \frac{2}{3}\left(\frac{kv}{\omega}\right)^3 - \ldots, \tag{14}$$

leading to

$$K_{\rm L}^{\rm CR} \approx \frac{8\pi^2 q^2}{\omega}\int_0^\infty \left[\frac{-2v_s}{kv} - \left(\frac{3v_s\omega}{kv}+v\right)\frac{2\omega}{k^2v^2} + \left(\frac{3v_s\omega}{kv}+v\right)\left(\frac{2\omega}{k^2v^2}+\frac{2}{3\omega}\right)\right]\frac{\partial f_0}{\partial p}p^2\, dp. \tag{15}$$

All terms in brackets cancel save for one, giving

$$\begin{aligned} K_{\rm L}^{\rm CR} &\approx \frac{8\pi^2 q^2}{\omega}\int_0^\infty \frac{2v}{3\omega}\frac{\partial f_0}{\partial p}p^2\, dp \\ &= \frac{16\pi^2 q^2}{3\omega^2}\left(\left[vp^2 f_0\right]_0^\infty - \int_0^\infty 3f_0 p^2\frac{dp}{\gamma m}\right). \end{aligned} \tag{16}$$

The first term goes to zero so long as $f_0(\infty) \to 0$ faster than p^{-3}, and the second term is $\propto n_{\rm CR}/(\gamma m)$, leaving

$$K_{\rm L}^{\rm CR} \approx -\omega_{p\,\rm CR}^2/\omega^2, \tag{17}$$

$$\omega^2\left(1+\frac{\omega_{pe}^2}{\Omega_e^2}\sin^2\theta\right) - \omega_{pi}^2 - \omega_{pe}^2\cos^2\theta - \omega_{p\rm CR}^2 = 0. \tag{18}$$

This simply modifies the $1/\omega^2$ term in the dielectric tensor, changing the frequency of the solution but not creating any complex roots; hence, no instability is generated.

Likewise, in the case where $\omega \ll kv$,

$$\ln\left|\frac{\omega-kv}{\omega+kv}\right| \simeq -\frac{2\omega}{kv} - \ldots, \tag{19}$$

leading to

$$K_{\rm L}^{\rm CR} \approx \frac{8\pi^2 q^2}{\omega}\int_0^\infty \left[\frac{-2v_s}{kv} - \left(\frac{3v_s\omega}{kv}+v\right)\frac{2\omega}{k^2v^2} + \left(\frac{3v_s\omega}{kv}+v\right)\frac{2\omega^3}{k^4v^4} + \ldots\right]\frac{\partial f_0}{\partial p}p^2\, dp. \tag{20}$$

All the terms but the first are negligible in this limit; hence,

$$K_{\rm L}^{\rm CR} \approx \frac{16\pi^2 q^2 v_s}{\omega k}\int_0^\infty \frac{1}{v}\frac{\partial f_0}{\partial p}p^2\, dp. \tag{21}$$

This leads to a full dispersion relation that can be written as

$$\omega^2\left(1+\frac{\omega_{pe}^2}{\Omega_e^2}\sin^2\theta\right) - \omega\frac{4\pi q^2 v_s}{k}4\pi \times \int_0^\infty \frac{1}{v}\frac{\partial f_0}{\partial p}p^2\, dp - \left(\omega_{pi}^2 + \omega_{pe}^2\cos^2\theta\right) = 0, \tag{22}$$

which also lacks complex roots, regardless of the actual evaluation of the integral.

Higher order terms in the expansion of $f = f_0(1+\mu(3v_s/v) + \mu^2(3v_s/v)^2/2 + \mu^3(3v_s/v)^3/6 + \ldots)$ give rise to higher order terms in ω. For $\omega \gg kv$, the dispersion relation equation (18) becomes to next highest order

$$\omega^3\left(1+\frac{\omega_{pe}^2}{\Omega_e^2}\sin^2\theta\right) - \left(\omega_{pi}^2 + \omega_{pe}^2\cos^2\theta + \omega_{p\rm CR}^2 - \frac{72}{5}\pi^2 \times q^2 v_s^2\int \frac{1}{v}\frac{\partial f_0}{\partial p}p^2\, dp\right)\omega + \frac{72}{7}\pi^2 q^2 k v_s^3\int \frac{1}{v}\frac{\partial f_0}{\partial p}p^2\, dp = 0, \tag{23}$$

which is stable since the terms $\propto \int (1/v)(\partial f_{\rm CR}/\partial p)p^2\, dp$ are of order $\sim \omega_{p\,\rm CR}^2 v_s^2/v^2 \ll \omega_{p\,\rm CR}^2$. When $\omega \ll kv$, the dispersion relation equation (22) takes on the next highest order terms

$$-\frac{24\pi^2 q^2 v_s^2}{k^4}\int \frac{1}{v^5}\frac{\partial f_0}{\partial p}p^2\, dp\left(3\omega^4 + \omega^3 k v_s\right) - \frac{24\pi^2 q^2 v_s^2}{k^2}\int \frac{1}{v^3}\frac{\partial f_0}{\partial p}p^2\, dp\left(\omega^2 + \frac{3}{5}\omega k v_s\right) \tag{24}$$

to become a quartic equation. This has four real solutions so long as $\int (1/v^5)(\partial f_0/\partial p)p^2\, dp > 0$. In fact if $\int (1/v^5)(\partial f_0/\partial p)p^2\, dp < 0$, the addition of higher order terms in the expansion of the CR distribution function would dramatically alter the character of the solutions, a situation that must be considered unphysical. We require $\int (1/v^5)(\partial f_0/\partial p)p^2\, dp > 0$, which means that at low momenta, $\partial f_0/\partial p > 0$, and the CR distribution cannot be monotonically decreasing from $v = p = 0$. Our use of the kappa distribution in § 2.1 may therefore be questioned. However the resonance at $p_x = -2mv_s$ places it well into the region of the distribution where the gradient is negative, and so modifications to the low-momentum behavior would have very little effect on our result. However, this observation does imply that the distribution of particles obeying a diffusion equation ahead of the shock is unlikely to extend down to zero momentum. Some natural break must exist between the quasi-thermal population gyrating around field lines and the CRs diffusing in turbulence. The foregoing work also neglects the CR-induced current in the background plasma. The inclusion of such effects leads to the modification $\omega_{pi}^2 \to \omega_{pi}^2 - \omega_{p\rm CR}^2$ and has no effect on reactive instabilities.

3. DISCUSSION

3.1. *Electron Heating or Magnetic Field Amplification?*

We have calculated the growth rate for waves that damp by heating electrons, in a CR shock precursor using similar approximations and techniques to those employed by Bell (2004).

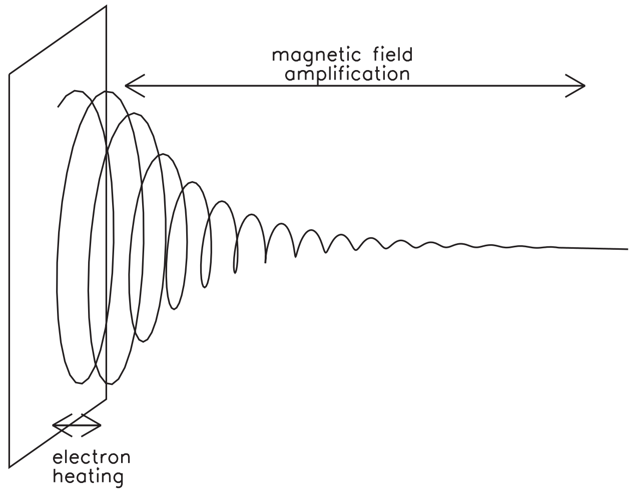

FIG. 1.—Schematic illustrations of the amplification of magnetic field by the nonresonant modified Alfvén waves in the shock precursor in the parallel orientation of the ambient field with respect to the shock normal. The evolution of a single field line in an exponential purely growing mode is shown. As the field is amplified, the shock becomes quasi-perpendicular and the effective $M_{\rm A}$ decreases, eventually to the point where the lower hybrid wave growth takes over, allowing a short region of electron heating.

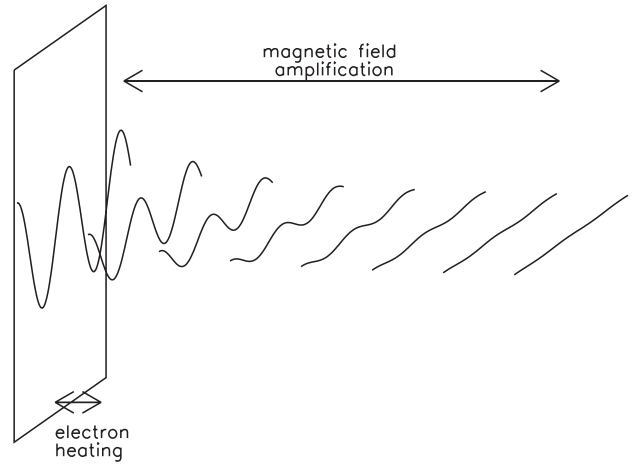

FIG. 2.—Same as Fig. 1, but for an initially perpendicular shock. The evolution of a purely growing mode is illustrated. The magnetic field amplification is less strong than in the quasi-parallel case, and the shock geometry remains quasi-perpendicular.

Both the lower hybrid wave heating of electrons and the growth of magnetic field through modified Alfvén waves redistribute energy within the CR precursor. An important question is which of these is more effective, i.e., which grows faster? Above, we derived a critical Alfvén Mach number, $M_{\rm A} \simeq 12n'_{\rm CR}/n_{\rm CR}$, which divides the regime of magnetic field amplification from that of lower hybrid wave growth. Following from the treatment of the reactive instability above, we estimate $n_{\rm CR} = \int_{p_{\rm inj}}^{\infty} f_{\rm CR} 4\pi p^2\, dp \simeq (6/\pi) n'_{\rm CR} v_s / v_{\rm inj}$, where $f_{\rm CR}$ is given by equation (1) and $p_{\rm inj}$ is the injection momentum where particles may begin to participate in a diffusive shock acceleration process. The approximate result $n'_{\rm CR}/n_{\rm CR} \simeq v_{\rm inj}/2v_s$ gives $M_{\rm A} \simeq 6v_{\rm inj}/v_s$ as the critical Alfvén Mach number.

The next step in determining the critical Alfvén Mach number is to find an appropriate $v_{\rm inj}$ for the injection of seed particles into the CR acceleration process. Zank et al. (2006) argue that quasi-perpendicular shocks have similar injection requirements to quasi-parallel shocks ($v_{\rm inj} \simeq 2v_s$), but that highly perpendicular shocks require much higher injection energies. In the case of nonresonant magnetic field generation, we also consider the case of a highly perpendicular shock, since the generated magnetic field will be perpendicular and much stronger than the initial magnetic field. Zank et al. (2006) give the injection velocity as

$$v_{\rm inj} = 3v_s \left[\frac{1}{(r-1)^2} + \frac{D_{\rm Bohm}^2}{D_\perp^2} \right]^{1/2}, \qquad (25)$$

where r is the shock compression ratio, and $D_{\rm Bohm}$ and $D_\perp$ are the CR diffusion coefficients in the Bohm limit and in the perpendicular direction, respectively. Reville et al. (2008) give $D_{\rm Bohm}/D_\perp \simeq 3$ for CRs where $kr_g \sim 1$, and so $v_{\rm inj} \simeq 10v_s$. Thus, the Alfvén Mach number at which lower hybrid wave growth takes over from magnetic field amplification should be about 12–60, unless the magnetic field saturates at a lower value (i.e., higher $M_{\rm A}$) before this is reached.

The growth of lower hybrid waves is most efficient at a quasi-perpendicular shock, whereas the growth of magnetic field through modified Alfvén waves is strongest at a quasi-parallel shock (Bell 2005). This apparent contradiction is actually easily resolved. At an initially quasi-parallel shock, Bell (2005), Reville et al. (2008), and Zirakashvili et al. (2008) show that a highly helical magnetic field develops. The distortion of an initially parallel field line is shown schematically in Figure 1, showing the evolution of the shock from quasi-parallel to quasi-perpendicular. A similar schematic in Figure 2 shows the evolution of an initially quasi-perpendicular shock, where magnetic field is amplified orthogonal to the preexisting magnetic field, but where the shock remains quasi-perpendicular. In both cases a perpendicular field is generated, thus allowing lower hybrid wave growth and electron heating in a region close to the shock as indicated.

Another potential problem is the cavities seen in simulations of the growth of modified Alfvén waves (e.g., Bell 2005). The helical field from an initially quasi-parallel geometry naturally creates a filamentary structure, dragging the thermal plasma with it, while CRs tend to accumulate in the low-density cavities. This is problematic for our mechanism that requires spatial coincidence between CRs, magnetic field, and thermal plasma. A possible solution is that the growth of lower hybrid waves takes over from the growth of modified Alfvén waves, so that the CR-driven magnetic field never reaches its final saturated state. Bell (2004), Reville et al. (2008), and Zirakashvili et al. (2008) derive a saturation magnetic field by setting $\gamma_{B,{\rm nonres}} = 0$ in equation (9) to give $\delta B \sim jr_g/4\pi$ or $\delta B^2/8\pi \sim n_{\rm CR} m_i v_s v_{\rm inj}/2$. This gives an Alfvén Mach number at saturation of $M_{\rm A}^2 \sim n_i/10n_{\rm CR}$ (assuming $v_{\rm inj} \sim 10v_s$), which for likely parameters $n_i/n_{\rm CR} \sim 10^3$ gives a value of $M_{\rm A}$ of similar magnitude to but possibly lower than that where the electron heating is expected to take over. Bearing in mind that we took the strongest growth rate for magnetic field amplification to estimate where electron heating takes over, it is quite plausible that the amplified magnetic field never reaches saturation. Also, as the initial shock state becomes more quasi-perpendicular, the growth rate slows down, and the circularly polarized Alfvén waves become elliptically polarized, ultimately becoming linearly polarized in the limit of a true perpendicular shock, eliminating the growth of such cavities.

Pelletier et al. (2006) find that the nonresonant instability of Bell (2004, 2005) dominates over the more familiar resonant instability when the shock velocity v_s is greater than a few times

$\epsilon_{\rm CR} c$, where $\epsilon_{\rm CR}$ is the ratio of the CR energy density to the kinetic energy density of the shock. Niemiec et al. (2008) simulated the CR-driven amplification of magnetic field in a parallel shock using particle-in-cell simulations, which can naturally account for the back-reaction of the generated magnetic field on the CR current. In conditions where the nonresonant mode should grow, they find magnetic field amplification only to $\delta B \sim B$. The magnetic field again produces filaments, but they do not find CR accumulation in the filament cavities. They do not find strong growth and argue that saturation occurs because the incoming flow to the shock is decelerated by the CRs, reducing their relative velocity and, hence, the CR current.

For our electron heating model, the precise degree of magnetic field amplification is unimportant so long as the CR diffusion coefficient remains proportional to $1/B$. It is only necessary that the shock be sufficiently quasi-perpendicular to allow CRs to generate lower hybrid waves. A reduced CR current does not necessarily produce an appreciable affect on the kinetic growth rate for lower hybrid waves. So long as the current does not vanish, the initial effect of reducing v_s in equation (6) is to bring more CRs into resonance with the lower hybrid waves. Another estimate of the CR density necessary to heat electrons may come from the long-wavelength limit of the magnetic field amplification, when $\gamma_B = (\boldsymbol{k} \cdot \boldsymbol{B} n_{\rm CR} q v_s / n_i m_i)^{1/2}$, for both parallel and perpendicular cases (Bell 2005). Electron heating then requires $\gamma \sim \Omega_i n_{\rm CR}/n_i > (n_{\rm CR} v_s \cos\phi / n_i v_{\rm inj})^{1/2}\ \Omega_i$, taking $k = \Omega_i / v_{\rm inj}$ (probably an overestimate), yielding $n_{\rm CR}/n_i > \cos\phi/10$. At $\cos\phi \leq (m_e/m_i)^{1/2}$, the values typical for lower hybrid wave propagation, the value for $n_{\rm CR}/n_i$ is low enough (0.001–0.01) to make electron heating by CRs plausible.

3.2. *Other Electron Heating Mechanisms*

Several other researchers have considered the generation of waves in a shock precursor as a means of heating electrons. Ohira & Takahara (2007) and Shimada & Hoshino (2000) have both considered the model of Cargill & Papadopoulos (1988) in more detail, using particle-in-cell codes rather than a hybrid approach. Other references (Dieckmann et al. 2000; McClements et al. 2001; Schmitz et al. 2002) focus more on the electron injection problem for diffusive shock acceleration, rather than the thermal electron temperature, again invoking various wave modes in a reflected-ion precursor. Our principal departure from these works has been to treat similar wave modes upstream of the shock, but excited by CRs undergoing diffusive shock acceleration rather than by quasi-thermal ions reflected from the shock. This allows electron heating to occur over a much more extended upstream region dictated by the CR diffusion coefficient, D, rather than the ion gyroradius. In addition, expressing the thickness of this region as $l \sim D/v_s$ naturally results in electron heating that is essentially independent of shock speed, as argued from observations of Balmer-dominated shocks in Ghavamian et al. (2007). On the other hand, if the thickness of the electron heating region is comparable to the ion gyroradius, then $l \propto v_s$ and $T_e \propto v_s^2$. This results in constant T_e/T_i with v_s, contrary to what is observed.

A number of other authors have investigated the role of the cross shock potential in heating the electrons. Inside the (quasi-perpendicular) shock ramp, the magnetic field may "overshoot," i.e., increase to a value greatly in excess of its asymptotic downstream strength before decreasing again. The electric field arising from the small charge separation associated with this magnetic field gradient, $E \simeq (\partial/\partial x)(B^2)/(8\pi e n_i)$, can decelerate ions and accelerate electrons. Such effects are known to be important at low–Mach number shocks where a laminar approximation holds (e.g., Scudder et al. 1986). At higher Mach numbers, where the shock is turbulent, the importance of such electric fields is less clear. Electron $\boldsymbol{E} \times \boldsymbol{B}$ drift along the shock front will result in periods of energy loss as well as energy gain by the cross shock potential and, hence, no net heating. It has been argued (Gedalin et al. 2008) that in certain cases the shock front may be sufficiently thin (length scales of order c/ω_{pe}) that the electrons are effectively demagnetized. One might expect to see electron heating *increase* with shock velocity (or $M_{\rm A}$) once this condition becomes satisfied. Examination of solar wind shocks suggests that such thin shocks are rare at best and certainly not ubiquitous.

We see no evidence for an increase in electron heating in SNRs up to shock velocities of 6000 km s^{-1} (1E 0102.2−7219; Hughes et al. 2000), and possibly up to 20,000 km s^{-1} (SN 1993J; Fransson et al. 1996). At higher Mach numbers such as those expected in gamma-ray burst afterglows, the convective electron gyroradius may easily reduce to less than the electron inertial length, making the cross shock potential a candidate electron heating mechanism.

Schwartz et al. (1988) have made a survey of a number of solar wind shock crossings observed in situ. They find $T_e/T_i \propto 1/M_{\rm A}$ for $M_{\rm A}$ greater than about 2–3. At lower $M_{\rm A}$, there is a wide scatter in T_e/T_i about $T_e/T_i \sim 1$. At these slower shocks, T_e correlates very well with the change in ion velocity squared, suggesting that both are due to the same mechanism, presumably the cross shock potential. The switch to $T_e/T_i \propto 1/M_{\rm A}$ at $M_{\rm A} \sim 2$–3 is possibly due to the onset of turbulent shock structure at higher Mach numbers.

We can explore the conditions required for the validity of the laminar approximation by adopting the criterion of Tidman & Krall (1971) for the existence of a magnetosonic soliton,

$$\frac{M_S^2}{M_S^2 - 1} < M_{\rm A}^2 < 4M_S^2 \frac{M_S^2 - 4M_S + 3 + 2\ln M_S}{\left(M_S^2 - 1 - 2\ln M_s\right)^2}. \tag{26}$$

The relationship between $M_{\rm A}$ and M_S predicted by this relation is plotted in Figure 3. The criterion above indicates that the laminar approximation breaks down at slightly lower Mach numbers in the solar wind shocks than indicated by the behavior of T_e/T_i in Schwartz et al. (1988). Magnetic field amplification by about an order of magnitude in SNR shocks for the 400 km s^{-1} shocks observed in the Cygnus Loop, where Ghavamian et al. (2007) find complete electron-ion equilibration, would bring $M_{\rm A}$ down to the same range as indicated by Schwartz et al. (1988), possibly suggesting that the cross shock potential is at work for the lower velocity SNR shocks. At the higher velocity shocks in the Schwartz et al. (1988) sample, which are all perpendicular, an empirical relationship $T_e/T_i \propto 1/M_{\rm A}$ emerges. Such a behavior can be consistent with our model if we make the assumption that the CRs accompanying solar wind shocks are nonrelativistic, suprathermal particles. Then the diffusion coefficients take on an extra factor v_s/c, assuming that the CR velocity is proportional to the shock velocity. This extra power of the shock velocity in the diffusion coefficient results in $T_e \propto v_s$ and $T_e/T_i \propto 1/v_s$.

3.3. *Heating versus Damping of Lower Hybrid Waves and the Width of the Precursor*

We have discussed whether growth of lower hybrid waves may compete with CR-induced magnetic field amplification, but have not yet discussed whether this growth rate is sufficient to balance the damping rate of lower hybrid waves by electrons. To answer this question we compare the electron heating rate from diffusive scattering off the lower hybrid waves with the energy input into the lower hybrid waves from the CR turbulence. The

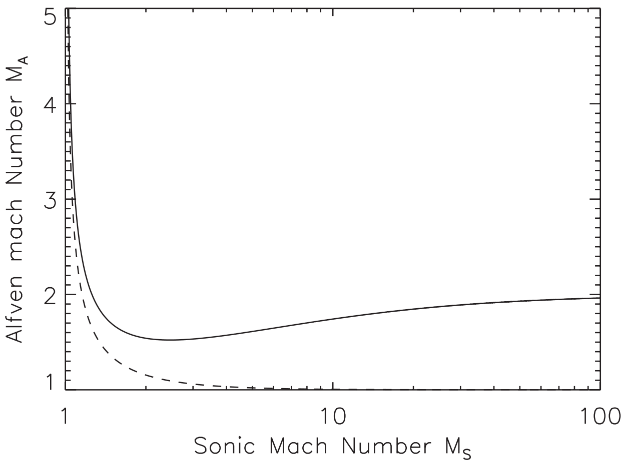

FIG. 3.—Allowed range of M_A as a function of M_S for the existence of a magnetosonic soliton, from Tidman & Krall (1971). The upper limit is given by the solid line, and the lower limit is given by the dashed line.

electron heating rate per unit area of shock is $n_e f_R m_e \kappa_{\parallel\parallel} t v_s/2$, where $f_R \simeq \exp(-\omega^2/2k_\parallel^2 v_{te\parallel}^2) \simeq \exp(-2v_s^2 m_i/v_{te}^2 m_e)$ is the fraction of electrons in resonance with the lower hybrid waves (using $\omega/k \simeq 2v_s$), $\kappa_{\parallel\parallel}$ is the parallel electron velocity diffusion coefficient in lower hybrid turbulence, and t is the period of time spent by an electron in the turbulence. This time $t = l/v_s$, where l is the precursor depth. The electron heating is balanced by energy input to the turbulence by CRs with rate $2\gamma E_{\rm turb} l$. Putting $E_{\rm turb} = (\delta E^2/8\pi)^2 \omega_{pe}^2/\Omega_e^2$ and $\kappa_{\parallel\parallel} = q^2 \delta E^2 k_\parallel^2/4m_e^2 k_\perp^2 \omega$, we deduce a growth rate $\gamma = f_R q^2 B^2/16 m_i m_e \omega c^2 = f_R \omega/16$. The kinetic growth rate derived earlier in equation (7), in units of Ω_i, is proportional to $n_{\rm CR}/n_i$. Thus, as long as this ratio is comparable to or larger than the fraction of electrons that are in resonance with the lower hybrid waves the growth rate outlined above will be sufficient to heat the electrons.

Another constraint on $n_{\rm CR}/n_i$ comes from equation (9). For magnetic field amplification, we require $n_{\rm CR}/n_i > k_\parallel v_{\rm A}^2/(v_s \Omega_i)$. Since $k_\parallel > 1/r_g$, the gyroradius of CRs at injection, $n_{\rm CR}/n_i > v_{\rm A}^2/v_{\rm inj} v_s \sim 1/10M_{\rm A}^2 \sim 10^{-3}$ for $M_{\rm A} \sim 10$ and $v_{\rm inj} \sim 10 v_s$. Taking the maximum growth rate estimated from equation (9), $\gamma = M_{\rm A}\Omega_i n_{\rm CR}/2n_i \sim \Omega_i/20M_{\rm A}$, where $l_i = c/\omega_{pi} = v_{\rm A}/\Omega_i$ is the ion inertial length, we estimate a characteristic length of $v_s/\gamma = 20M_{\rm A}^2 l_i \sim 5 \times 10^{10}$ cm. This requires a CR diffusion coefficient of order 10^{19} cm^2 s^{-1}. This is considerably smaller than the estimate by Bell (2004). Taking a characteristic CR energy of 10^{15} eV, Bell (2004) finds a typical growth time for magnetic field of order 100 yr. This would yield a characteristic length scale for magnetic field amplification of $\sim 10^{18}$ cm for a 3000 km s^{-1} shock, requiring a CR diffusion coefficient of $\sim 3 \times 10^{26}$ cm^2 s^{-1}. We suspect that our simple estimate reflects the growth rate while the shock may be considered quasi-parallel, and that magnetic field amplification slows down considerably once it becomes quasi-perpendicular. Therefore, in taking a characteristic CR energy of 10^{15} eV, Bell (2004) is taking the lowest energy CRs for which the shock may be considered quasi-parallel, and this result may be considered more realistic.

Further, in Ghavamian et al. (2007) we argued that the depth of the CR precursor over which electron heating occurs could not be larger than $\sim 10^8 v_s/n_e$ cm, otherwise neutral hydrogen would not survive to encounter the shock front. We suggest here that lower hybrid waves accelerate the small fraction of electrons that happen to be in resonance, and that these accelerated electrons communicate their energy to the rest of the thermal population by Coulomb collisions, with characteristic timescale $10^{10}(T/10^8\ {\rm K})^{3/2}/n_e$ s. Equating this to $10^8/n_e$ s yields a maximum temperature of $T \sim 10^8(10^{-2})^{2/3} \simeq 5 \times 10^6$ K. This is very close to the temperature found in Ghavamian et al. (2007), 0.3 keV, or 3.5×10^6 K. Put another way, the temperature found in Ghavamian et al. (2007) is consistent with electron heating such that neutral hydrogen can survive to encounter the shock front proper. However, CR precursors at the small end of the range considered above ($\sim 10^{11}$ cm) would not allow any significant electron collisional equilibration to occur. Allowing for compressional heating of the electrons as they go through the shock, a precursor electron temperature of order 10^6 K requires a precursor length of $\sim 10^7 v_s/n_e \sim 10^{15}(v_s/1000\ {\rm km\ s^{-1}})$ cm or a minimum CR diffusion coefficient of $D \sim 10^{23}(v_s/1000\ {\rm km\ s^{-1}})^2$ cm^2 s^{-1}.

The electric field in the lower hybrid waves will be given by the limit derived by Karney (1978),

$$\delta E = B\left(\frac{\Omega_i}{\omega}\right)^{1/3} \frac{\omega}{4k_\perp c} = B\left(\frac{\Omega_i}{\omega}\right)^{1/3} \frac{v_s}{2k_\perp c}. \qquad (27)$$

This is the maximum electric field before ion trapping and heating occurs. Laming & Lepri (2007 and references cited therein) demonstrate that when $\omega/\sqrt{2}k_\parallel v_{te} \ll \omega/\sqrt{2}k_\perp v_{ti}$, ions are heated more effectively than electrons above this threshold. In our case, the ions that are heated will be the lower energy part of the suprathermal ion distribution reflected from the shock, i.e., those below the injection threshold for diffusive shock acceleration in equation (1) or any of its modifications subsequent to the treatment of the reactive lower hybrid wave instability in § 2.2. With the wave electric field given by equation (27), the electron momentum diffusion coefficient in lower hybrid turbulence varies as v_s^2, yielding a constant degree of heating with shock velocity if the time spent in the turbulence varies as $1/v_s^2$, which would be the case if the CRs are obeying a diffusion law.

4. SUMMARY

We have considered in more detail the speculation of Ghavamian et al. (2007) that lower hybrid waves generated in a cosmic-ray precursor could be responsible for the electron heating at collisionless shocks in supernova remnants. We find that there do exist growing modes for the resonant or kinetic case, and that the growth rate in this case may be sufficient both to survive the damping by electrons and to compete with magnetic field amplification by modified Alfvén waves. Below a certain Alfvén Mach number (roughly estimated to be $\sim$12–60) the lower hybrid wave growth rate exceeds that of the modified Alfvén waves. The modified Alfvén wave generation exists for all magnetic field orientations with respect to the shock, but is most effective for quasi-parallel case and always generates new perpendicular field. Lower hybrid waves, on the other hand, require quasi-perpendicular field geometry in order to grow. Thus, a schematic picture emerges in which far ahead of the high-Mach number shock, modified Alfvén waves generate perpendicular field, reducing the effective Mach number closer to the shock front and thus allowing lower hybrid wave growth to occur in a short region before the shock and to heat the resonant electrons. A critical Alfvén Mach number around 15 suggests magnetic field amplification by about an order of magnitude, similar to what a comparison of the surveys of Ghavamian et al. (2007) and Schwartz et al. (1988) would suggest, taking in both cases the shock velocity where $(T_e/T_p)_0 \sim 1$ starts to break down as that where the laminar shock approximation ceases to hold.

We have concentrated on the generation of lower hybrid waves, since for these the group velocity can be equal to the shock velocity itself, meaning that the waves can stay in contact with the shock for long time intervals and in principle grow to large amplitudes. However, other wave modes that heat electrons are certainly possible, and these, such as the Landau damping of kinetic Alfvén waves (e.g., Viñas et al. 2000), do not require perpendicular shocks as lower hybrid waves do. In fact, Bykov & Uvarov (1999) studied the generic case of heating by turbulent modes in the shock precursor and did identify an area of parameter space for which a near-inverse-square relationship between $(T_e/T_p)_0$ and shock velocity could be accommodated. Our model requires that cosmic-ray ions be essentially ubiquitous at SNR shocks, with number densities estimated by various means in § 3. In a wider context, the idea that cosmic rays are responsible for electron heating at fast shocks reinforces the idea that cosmic rays are an intrinsic component of the collisionless shock phenomenon.

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APPENDIX A

COSMIC-RAY DIFFUSION COEFFICIENTS

The parallel spatial cosmic-ray (CR) diffusion coefficient is most easily obtained from its relation to the pitch-angle scattering diffusion coefficient in momentum space. The diffusion coefficient in momentum space is expressed most generally as (Melrose 1986)

$$D_{\lambda\mu} = \sum_{s=-\infty}^{\infty} \int \frac{8\pi^2 q^2}{\hbar} \frac{R_M(\boldsymbol{k})}{\omega_M(\boldsymbol{k})} |\boldsymbol{e} \cdot \boldsymbol{v}(\boldsymbol{k}, \boldsymbol{p}, s)|^2 \delta(\omega_M - s\Omega - k_\parallel v_\parallel) \Delta\lambda\, \Delta\mu\, N_\mu(\boldsymbol{k}) \frac{d^3k}{(2\pi)^3}, \tag{A1}$$

where N_μ is the number density of wave quanta, R_M is the ratio of electric energy to total energy in the wave, such that $R_M \int N_\mu \hbar\, \omega_M\, d^3k/(2\pi)^3 = \delta E^2/8\pi$, $\boldsymbol{e}$ is the wave polarization vector, and $\boldsymbol{v}$ is the CR velocity. For pitch-angle scattering by parallel-propagating Alfvén waves, $\lambda = \alpha$, so

$$\Delta\lambda = \hbar\left(\frac{s\Omega}{v_\perp}\frac{\partial}{\partial p_\perp} + k_\parallel \frac{\partial}{\partial p_\parallel}\right)\lambda = -\frac{\hbar k_\parallel}{p \sin\alpha}. \tag{A2}$$

With $\omega_M = k_\parallel v_{\rm A}$, $R_M = (v_\parallel^2)/(2c^2)$, and $\boldsymbol{e} \cdot \boldsymbol{v} = v_\perp/2$,

$$D_{\alpha\alpha} = \int \frac{8\pi^2 q^2}{\hbar\omega} \frac{v_{\rm A}^2}{2c^2} \frac{v^2 \sin^2\alpha}{4} \delta(\omega_M - s\Omega - k_\parallel v_\parallel) \frac{\hbar^2 k_\parallel^2}{p^2 \sin^2\alpha} \frac{U_M(\boldsymbol{k})}{\hbar\omega} \frac{d^3k}{(2\pi)^3} = \frac{\pi^2 q^2 v}{p^2 c^2 \cos\alpha} \frac{U_M(k_\parallel = \Omega/v_\parallel)}{2\pi}, \tag{A3}$$

where we have put $s = 1$ and taken $\omega_M \ll \Omega$.

We now express $D_\parallel$ in terms of $D_{\alpha\alpha}$ by writing

$$f(p, \alpha) = f_0(p) + f_1(p)\cos\alpha + (1/2)f_2(p)\cos^2\alpha + \ldots \tag{A4}$$

and substituting into the diffusion equation

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} = \frac{1}{\sin\alpha}\frac{\partial}{\partial\alpha}\left(\sin\alpha D_{\alpha\alpha}\frac{\partial f}{\partial\alpha}\right). \tag{A5}$$

Upon integrating the result over $\cos\alpha$ we obtain, with $v_z = v\cos\alpha$,

$$\frac{\partial f_0}{\partial t} + \frac{1}{3}\frac{\partial f_2}{\partial t} + \frac{v}{3}\frac{\partial f_1}{\partial z} = 0. \tag{A6}$$

Multiplying each side by $\cos\alpha$ and then integrating over $\cos\alpha$ yields

$$\frac{2}{3}\frac{\partial f_1}{\partial t} + \frac{2v}{3}\frac{\partial f_0}{\partial z} + \frac{v}{5}\frac{\partial f_2}{\partial z} = -\int_{-1}^{1} \cos\alpha \sin^2\alpha D_{\alpha\alpha} f_2\, d(\cos\alpha). \tag{A7}$$

With $f_0 \gg f_1 \gg f_2$, $f_2 \simeq -(2v/3)(\partial f_0/\partial z)\left[\int_{-1}^{1} \cos\alpha \sin^2\alpha D_{\alpha\alpha}\, d(\cos\alpha)\right]^{-1}$, which when substituted into equation (A6) allows the identification

$$D_\parallel = \frac{2v^2}{9}\left[\int_{-1}^{1} \cos\alpha \sin^2\alpha D_{\alpha\alpha} d(\cos\alpha)\right]^{-1}. \tag{A8}$$

With equation (A3),

$$D_{\parallel} = \frac{p^2 c^2 v}{3\pi q^2 U_M\left(k_{\parallel} = \Omega/v_{\perp}\right)}. \tag{A9}$$

This is a factor of 2π larger than the equivalent expression given by Blandford & Eichler (1987), due to a different definition of U_M. Where $U_M \propto k_{\parallel}^{-\beta}$, $D_{\parallel} \propto p^{2-\beta}$, which evaluates to $D_{\parallel} \propto vp^{1/3}$ or $D_{\parallel} \propto vp^{1/2}$ for Kolmogorov or Kraichnan turbulence, respectively. If $v \sim c$, the dependence of $D_{\parallel}$ on p can usually be neglected.

The perpendicular spatial CR diffusion coefficient has been given in terms of $D_{\parallel}$ by various authors. Based on numerical experiments, Marcowith et al. (2006) give $D_{\perp} = \eta^{2+\epsilon} D_{\parallel}$, where $\eta = \delta B^2/(\delta B^2 + \langle B\rangle^2)$ and the CR distribution function $f(p) \propto p^{-4-\epsilon}$. Shalchi & Kourakis (2007) and Zank et al. (2006) give $D_{\perp} \propto \left(\delta B^2/B_0^2\right)^{2/3} D_{\parallel}^{1/3} (l_{2\mathrm{D}} v)^{2/3}$ from analytic considerations, where $l_{2\mathrm{D}}$ is the two-dimensional bend-over length scale, the inverse of the wavenumber where the inertial range onsets, and consequently has an even smaller dependence on the CR momentum than the parallel diffusion coefficient for relativistic CRs, and has the same dependence in the nonrelativistic case.

APPENDIX B

GROWTH RATE FOR AN ELECTROMAGNETIC INSTABILITY

For completeness, we give here a treatment of the growth rate due to CRs of electromagnetic waves with frequency in the lower hybrid range and show that it is significantly smaller than either the electrostatic instability or the growth of modified Alfvén waves. It is relatively easy to show that the reactive instability of Bell (2004) has higher thresholds and lower growth rates as the frequency of the electromagnetic wave increases first above the proton gyrofrequency and then above the electron gyrofrequency. Here, we concentrate on the kinetic instability that might generate electromagnetic waves in the lower hybrid range, whistlers, adapting the expressions in Bell (2004) and Achterberg (1983),

$$\begin{aligned}\omega^2\left(K^T - 1\right) = {} & \frac{\Omega_i c^2}{\omega v_{\mathrm{A}}^2}\left[\tilde{\omega}_i^2 \mp \frac{k^2 v_{ti}^2}{\Omega_i}\tilde{\omega}_i \mp \frac{\Omega_i}{n_e} k\left(\frac{J_{\mathrm{CR}}}{q} - \frac{\omega}{k} N_{\mathrm{CR}}\right)\right] \\ & + 4\pi q^2 \int \frac{v_{\perp}/2}{\omega - k_{\parallel} v_{\parallel}}\left[\left(\omega - k_{\parallel} v_{\parallel}\right)\frac{\partial f}{\partial p_{\perp}} + k_{\parallel} v_{\perp}\frac{\partial f}{\partial p_{\parallel}}\right] 2\pi p_{\perp}\, dp_{\perp}\, dp_{\parallel} = c^2 k^2 - \omega^2.\end{aligned} \tag{B1}$$

Evaluating the two CR terms,

$$\int\!\!\int v_{\perp}\frac{\partial f}{\partial p_{\perp}} 2\pi p_{\perp}\, dp_{\perp}\, dp_{\parallel} = \int_{-\infty}^{\infty}\left(\left[2\pi p_{\perp} v_{\perp} f\right]_0^{\infty} - \int \frac{4\pi p_{\perp}}{\gamma m} f\, dp_{\perp}\right) dp_{\parallel} = -\frac{2}{\gamma m} n_{\mathrm{CR}}, \tag{B2}$$

$$\begin{aligned}\int\!\!\int \frac{v_{\perp}^2}{\left(\omega - k_{\parallel} v_{\parallel}\right)}\frac{\partial f}{\partial p_{\parallel}} 2\pi p_{\perp}\, dp_{\perp}\, dp_{\parallel} &= \int\!\!\int \frac{-p_{\perp}^2/(\gamma^2 m^2)}{\left(\omega - k_{\parallel} v_{\parallel}\right)}\frac{\left(p_{\parallel} - m v_s\right)/p_t^2}{\left\{1 + \left[\left(p_{\parallel} - m v_s\right)^2 + p_{\perp}^2\right]\Big/\left(2\kappa p_t^2\right)\right\}^{\kappa+1}} 2\pi p_{\perp}\, dp_{\perp}\, dp_{\parallel} \\ &= \int\!\!\int \frac{p_{\perp}\left(p_{\parallel} - m v_s\right)/(\gamma^2 m^2)}{\left(\omega - k_{\parallel} v_{\parallel}\right)}\frac{\partial f}{\partial p_{\perp}} 2\pi p_{\perp}\, dp_{\perp}\, dp_{\parallel},\end{aligned} \tag{B3}$$

explicitly assuming a κ-distribution for CRs in equation (B3). After some algebraic manipulation, an integration by parts, the rewriting of f as a κ-distribution, and making the substitutions $p_{\perp}^2 = P$ and $dP = 2p_{\perp} dp_{\perp}$, the right-hand side of equation (B3) can be written as

$$-\int \frac{4\pi\left(p_{\parallel} - m v_s\right)}{\omega - k_{\parallel} v_{\parallel}} \int \left[\frac{2\kappa p_t^2}{2\kappa p_t^2 + \left(p_{\parallel} - m v_s\right)^2 + P}\right]^{\kappa} \frac{m^2 c^4 + p_{\parallel}^2 c^2}{\left(m^2 c^2 + p_{\parallel}^2 + P\right)^2}\, dP\, dp_{\parallel}. \tag{B4}$$

The integral over dP can be evaluated using a hypergeometric function (Gradshteyn & Ryzhik 1965, eq. [3.197.1]) to give

$$-\int \frac{4\pi\left(p_{\parallel} - m v_s\right)}{\omega - k_{\parallel} v_{\parallel}} \frac{\left(2\kappa p_t^2\right)^{\kappa}}{\left(m^2 + p_{\parallel}^2/c^2\right)} \frac{B(1, 1+\kappa)}{\left[2\kappa p_t^2 + \left(p_{\parallel} - m v_s\right)^2\right]^{\kappa-1}}\, {}_2F_1\left(2, 1; 2+\kappa; 1 - \frac{2\kappa p_t^2 + \left(p_{\parallel} - m v_s\right)^2}{m^2 c^2 + p_{\parallel}^2}\right) dp_{\parallel}, \tag{B5}$$

where $B(1, 1+\kappa)$ is the beta function. Considering only the kinetic case, using the Landau prescription for this integral with the pole at $\omega - k_{\parallel} v_{\parallel}$, the imaginary part (i.e., the portion relevant for the growth rate) is

$$\begin{aligned}\mathrm{Im}\left[\int\!\!\int \frac{v_{\perp}^2}{\left(\omega - k_{\parallel} v_{\parallel}\right)}\frac{\partial f}{\partial p_{\parallel}} 2\pi p_{\perp}\, dp_{\perp}\, dp_{\parallel}\right] = {} & 4i\pi^2\left(\frac{\gamma m \omega}{k_{\parallel}} - m v_s\right)\frac{(2\kappa p_t^2)^{\kappa}}{m^2 + \gamma^2 m^2 \omega^2/\left(k_{\parallel}^2 c^2\right)}\frac{m}{k_{\parallel}} \\ & \times \frac{B(1, 1+\kappa)}{\left[2\kappa p_t^2 + \left(\gamma m \omega/k_{\parallel} - m v_s\right)^2\right]^{\kappa-1}}\, {}_2F_1\left(2, 1; 2+\kappa; 1 - \frac{2\kappa p_t^2 + \left(\gamma m \omega/k_{\parallel} - m v_s\right)^2}{m^2 c^2 + \gamma^2 m^2 \omega^2/k_{\parallel}^2}\right).\end{aligned} \tag{B6}$$

Setting $\omega \to \omega + i\gamma_g$ in the dispersion relation and taking only the imaginary parts, we get

$$0 = \Omega_i\gamma_g + 2\omega\gamma_g \pm \frac{\Omega_i}{\omega^2}\gamma_g\frac{kJ_{\rm CR}}{n_e q} + (2\pi)^3 q^2 k_\parallel\left(\frac{\gamma\omega m}{k_\parallel} - mv_s\right)\frac{\left(2\kappa p_t^2\right)^\kappa}{\left(2\kappa p_t^2 + m^2v_s^2\right)^{\kappa-1}}B(1, 1+\kappa)$$
$$\times\frac{v_{\rm A}^2}{c^2}\frac{1}{m^2}\frac{m}{k_\parallel}\,{}_2F_1\left(2, 1; 2+\kappa; 1 - \frac{2\kappa p_t^2 + m^2v_s^2}{m^2c^2}\right)\frac{n_{\rm CR}}{4\sqrt{2}(\pi\kappa)^{3/2}p_t^3}\frac{(2\kappa-3)\Gamma(\kappa)}{\Gamma(\kappa-1/2)}, \tag{B7}$$

assuming $\gamma\omega/k_\parallel \ll v_s$ and including the normalization of f and the factor of $k_\parallel/2$ that were omitted during the evaluation of the integral. From this we have

$$\gamma_g \simeq -\left(\frac{\pi}{2}\right)^{1/2}\frac{n_{\rm CR}}{n_i}\frac{\Omega_i^2}{2\omega}\frac{\left(\gamma\omega m/k_\parallel - mv_s\right)}{p_t}\frac{2\kappa-3}{\kappa^{1/2}(\kappa+1)}\frac{\Gamma(\kappa)}{\Gamma(\kappa-1/2)}\,{}_2F_1(2, 1; 2+\kappa; 1). \tag{B8}$$

Electromagnetic waves in the lower hybrid frequency range are parallel-propagating whistlers, with

$$\frac{k^2c^2}{\omega^2} \simeq \frac{\omega_{pe}^2}{\omega(\Omega_e - \omega)}, \tag{B9}$$

$$\frac{\partial\omega}{\partial k} \simeq \frac{2\omega/k}{1 + k^2c^2/\omega_{pe}^2}. \tag{B10}$$

We assume $\partial\omega/\partial k \propto \partial\omega/\partial k_\parallel \sim v_s$, and hence, $\omega/k_\parallel \sim v_s/2$ for $k \ll c/\omega_{pe}$. Thus, for $\kappa = 2$

$$\gamma_g \simeq \frac{2n_{\rm CR}}{3n_i}\frac{\Omega_i^2}{\omega}\left(1 - \frac{\gamma}{2}\right). \tag{B11}$$

This is significantly smaller than the growth of lower hybrid waves, which is of order $\Omega_i n_{\rm CR}/n_i$. Further, since whistlers carry energy along magnetic field lines, like Alfvén waves, only for specific shock obliquities will the energy of the waves stay in contact with the shock and allow large wave intensities to build up. Electromagnetic waves with frequency above the electron gyrofrequency (*O*- and *X*-modes) have phase velocities greater than c, and so cannot be excited by kinetic instabilities.

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