**Title and Subtitle**

SCALABILITY, COMPLEXITY AND RELIABILITY IN QUANTUM INFORMATION PROCESSING

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**Abstract**

This program was a theory/experiment collaboration focusing on the fundamental need for scalability in the development of quantum information processing. Theory and experiment were connected and interleaved at several levels. The theory objectives were 1) to develop new theoretical tools to enable the implementation of reliable quantum information processing in scalable systems and 2) to characterize the relations between quantum algorithms and architectures, between fault tolerance and architectures, between quantum and classical complexity classes, and to develop secure primitives for quantum cryptography. The experimental objectives were to develop scalable quantum component technology based on gas phase systems using atoms and light fields. Specific experimental goals were scalable implementation of universal quantum logic in optical lattices and achievement of deterministic control in atom/cavity systems.

**Subject Terms**

Scalable quantum information processing, quantum computing, quantum algorithms, optical lattices, gas phase systems
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1.0 Introduction and Overview

1.1 Project Objectives

Our program was a theory/experiment collaboration focused on the fundamental need for scalability in the development of quantum information processing. Theory and experiment were connected and interleaved at several levels. Our theory objectives were two-fold. The first objective was to develop new theoretical tools to enable implementation of reliable quantum information processing in scalable systems. This included improving the crucial threshold and computational overhead for fault-tolerant quantum computation to a practically viable range, as well as formulating novel universal models for quantum computation more suitable for physical realization. The second objective was to study exponential speedups by quantum algorithms, as well as the relationship between quantum and classical complexity. Our experimental objectives were to develop scalable quantum component technology based on gas phase systems, using atoms and light fields. Specific experimental goals included scalable implementation of universal quantum logic in optical lattices and achieving deterministic control in atom/cavity systems. These experimental studies were accompanied by theoretical design of efficient schemes for quantum logic, state initialization, and error correction of trapped atom quantum computation, as well as by theoretical and numerical simulations based on detailed physical modeling of the experimental systems to assess the reliability of quantum state preparation and logic operations.

1.2 Theory Overview

There are four fundamental goals of theoretical work in quantum computation, and they provide the most appropriate categories in which to describe the accomplishments of our project. We categorize the theoretical study of specific physical implementations as an additional fifth goal, which will be described together with the experimental work in the following subsection.

The first category is the study of the power and limits of quantum algorithms. Our understanding of exponential speedups by quantum algorithms is best described in the framework of the hidden subgroup problem (HSP), which includes Shor’s algorithms for factoring and the discrete logarithm. In [1], we gave a polynomial time algorithm for solving Pell’s equation. This algorithm extends the hidden subgroup framework to abelian groups which are not finitely generated. An extension of the basic algorithm breaks the Buchmann-Williams cryptosystem, which was proposed as an alternative to the RSA cryptosystem. Five years ago at the start of this project, arguably the most promising direction for exponential speedups by quantum algorithms was the non-abelian hidden subgroup problems, which included the graph isomorphism problem and finding short lattice vectors. In [2], we showed that the generalization of the standard method --- random coset state preparation followed by fourier sampling --- required exponential time for sufficiently non-abelian groups including the symmetric group, at least when the fourier transforms are carried out in a random basis. There has been intense follow up work since our result, which has recently established that the result holds in an arbitrary basis, all but ruling out efficient quantum algorithms using this approach. However,
negative results about algorithms can lead to positive results in cryptography. In [3], we proposed an efficient new cryptosystem based on the quantum intractability of finding short vectors in a lattice. An important feature of the proposed cryptosystem is its improved efficiency, whose proof of security relies on a quantum reduction to the lattice problem. In our project we also explored different, non-HSP based approaches to designing quantum algorithms. The most dramatic such proposal was made by Farhi et al., who gave a general framework for solving optimization problems by adiabatic evolution. Most intriguing was their claim in a paper published in Science, that simulations of quantum adiabatic optimization on small random instances of (NP-complete variants of) 3SAT appeared to indicate that the algorithm runs in polynomial time. In [4], we showed that exponential black-box lower bounds for NP-complete problems do not apply to the particular form of adiabatic optimization that Farhi et. al. had formulated. This optimistic situation did not last, since we later showed that adiabatic optimization requires exponential time on certain instances of 3SAT [5]. Our understanding about the power of adiabatic optimization was further clarified by showing an exponential lower bound for “physically realistic” local 2SAT, a special case of 2SAT (which can be solved in polynomial time on a classical computer), where each literal is involved in just two clauses [6]. Another approach we explored for designing quantum algorithms is based on quantum walks. In [7], we showed that discrete time quantum walks can be used to construct a quantum search algorithm. In [8], we showed how to use discrete quantum walks to give a new quantum algorithm for element distinctness. The algorithm is optimal, running in $O(N^{2/3})$ time on an $N$ element array, contrasted with $O(N)$ time required by classical algorithms. This was the first successful algorithmic application of discrete time quantum walks, and has since lead to a number of polynomial speedups by quantum algorithms.

The second goal is the study of quantum complexity theory to clarify the power of quantum computation. In [9], we used a novel technique to establish a lower bound for the Quantum Collision Problem: $\Omega(n^{1/5})$ for the number of quantum queries required to decide whether a function is one-to-one or two-to-one. This central problem had been open for a number of years, with no non-constant lower bound known. A poly-log time algorithm would imply an efficient polynomial time algorithm for graph isomorphism. In [10], we showed that the 2-local Hamiltonian problem is Quantum Merlin-Arthur (QMA) complete. In [11], we showed that adiabatic quantum computation is a universal model for quantum computation, a result we will elaborate upon below under quantum models. Finally, [12, 13] we provided a novel connection between quantum and classical complexity theory, by using quantum techniques to resolve an open question in classical complexity theory. These papers give a lower bound on the minimal size of locally decodable codes, using a technique based on quantum random access codes.

The third goal is the formulation of new models for quantum computation – models that might be more convenient to implement in the laboratory. In [11], we showed that a suitable reformulation of the adiabatic computation paradigm --- adiabatic state generation --- is universal for quantum computation. A remarkable consequence of this result is that for any quantum circuit, there are two local hamiltonians $H_i$ and $H_f$ such that starting in the ground state of $H_i$ and gradually turning the dial from $H_i$ to $H_f$ is sufficient
to simulate the quantum circuit. This is clearly an attractive scheme from the viewpoint of the physical realization of quantum computers and has inspired a lot of follow up work. In [14, 15, 16], we not only formalized the paradigm of encoded universality within a Lie algebraic analysis to derive specific conditions under which physical interactions can provide universality, but demonstrated that this is possible with the isotropic, anisotropic, and asymmetric exchange interactions. In a series of papers, [17, 18, 19], we developed and employed a geometric approach to generation of 2-qubit operations that allowed systematic analysis of optimal constructions for 2-qubit universal circuits, i.e., circuits capable of realizing any arbitrary 2-qubit operation. We showed that the optimal controlled-unitary gate is CNOT, and together with DCNOT shares the ability to realize any arbitrary 2-qubit operation with only three applications. We subsequently discovered the existence of a new gate, the B gate, which remarkably requires only two applications. Analysis of the ability to generate these gates from physical Hamiltonians showed that one or other of the two gates may be more efficient, depending on the physical realization.

The fourth goal is the design of efficient schemes for fault-tolerant quantum computation. At the start of this project five years ago, although it had been proved that in principle quantum computation can be carried out fault-tolerantly, the tolerated decoherence rates were miniscule, and the computational overheads astronomical. In [20], we gave a procedure for fault-tolerance based on an improved concatenated encoded ancilla preparation scheme. Simulations show that this procedure significantly improves the error threshold for fault tolerant quantum computation, to a new value of about 1% and significantly reduces ancilla overhead. This was one of the first results in a renaissance in quantum fault-tolerance, that have since resulted in thresholds as high as 5%, albeit at a cost of increasing the computational overhead. In a different direction, in [21], we developed a quantum search-based scheme for correcting systematic control errors, with a new class of composite pulse sequences that allow for arbitrary reduction of the control error. Finally, the evaluation of quantum fault-tolerant schemes requires simulation of quantum circuits carrying out quantum error-correction. In [22], we developed a fast classical algorithm for simulating quantum circuits composed of CNOT, Hadamard, and phase gates (Gottesman-Knill theorem) and implemented it for thousands of qubits in the freely-available CNOT-Hadamard-Phase (CHP) package.

1.3 Experimental Overview

On the experimental side, we are developing scalable quantum component technology based on gas phase systems using atoms and light fields. We are developing tools for initialization and robust manipulation of cold atoms in optical lattices for quantum computation, and we are working towards manipulation of cold atoms in high finesse optical cavities in order to produce numbered photon states and undertake reliable quantum logic operations. The similarity in the technology used in our quantum computation and photon generation schemes presents the long term possibility of integrating the two systems, e.g., so that quantum computers will be able to communicate via photons, or conversely, quantum computers can be used as repeater stages in the transmission of quantum photon states. Technological uniformity makes it easier to develop unified theoretical models of the experiments. The simplicity and scalability of atom based approaches to quantum computing is beneficial to theory, allowing the latter
to address important novel questions relating to the architecture of quantum computation in lattices and networks.

Our experimental efforts were funded through the Quantum Information Science and Technology (QUIST) contract only starting in the third year, and support for the experiments was terminated after the fourth year. This inconsistent support somewhat hampered our experimental work. Nevertheless, significant progress was made, albeit somewhat short of our goals at the outset of the program. We have taken important steps toward realizing quantum information protocols with neutral atoms, and towards demonstrating new ways to control and detect atoms in both optical lattices and high finesse cavities. These new techniques have been implemented in systems that are specially designed for quantum computing and information.

1.4 Optical Lattice Experiments

We have built an apparatus for trapping neutral atoms in a 3D optical lattice. The system is designed to trap hundreds of single atoms at lattice sites, each of which can act as a qubit, to allow single atoms to be addressed in order to execute single qubit gates, and to entangle atoms in pairs to execute two-qubit gates. Relatively small modifications can expand the available number of qubits into the thousands. Decoherence times are intrinsically very long in this system and the performance of the gates can be calculated, and are expected to have high fidelity. To date, we have demonstrated the trapping of single atoms at half the lattice sites in a 500 site volume. We have also shown that we can image atoms in this lattice without causing any site hopping.

During the project lifetime we have built a vacuum chamber with optical access that can accommodate the following experimental steps:
(a) Configure a 3D site addressable optical lattice;
(b) Laser cool ~1000 individual atoms to vibrational ground states; and
(c) Image selectable planes of lattice-trapped atoms.
This paves the way to the next generation of experiments which consist of
(d) Selectively addressing atoms with light and microwaves, to move atoms around and to execute single qubit gates; and
(e) Selectively entangling atom pairs to execute two-qubit gates.

1.5 Trapped atoms/cavity QED Experiments

At the onset of this project, we identified that advancing the state of the art in cavity quantum electrodynamics (CQED) was important for the development of quantum information processors. On one hand, the capabilities offered by CQED, as spelled out by theory and include quantum interfacing between stationary and flying qubits, photonic quantum networking between distant qubits, and a tool for processing quantum information encoded in optical fields, were powerful additions to the arsenal of tools needed for implementing quantum computing. On the other hand, analyzing the specific upcoming needs of quantum computing architectures had identified the looming bottleneck arising from the need to transfer quantum information between distant regions of a complex lattice of encoded logical qubits. CQED, with the promise of transmitting
quantum information through light, rather than by transport or a series of gate operations between material qubits, offered a way out of that dilemma.

Further, we identified that one limitation to existing CQED schemes was their indeterminism, and, relatedly, their non-scalability. That is, in systems which were in place at the beginning of our project, single atoms (potential material qubits) were introduced into high-finesse optical cavities only with small probability on each run of the experiment, being trapped within the cavity only for short and indeterminate times, and located in random and unknown positions inside of the optical cavity. It was hard to imagine an elaborate network of such haphazard devices operating as a quantum network and serving a useful function.

With support from the QUIST program, we undertook an experiment which would address this indeterminism by preparing atomic samples using well-established techniques of ultracold atomic physics and delivering these atoms to specific locations in a high-finesse optical cavity. There, one could utilize the entire ensemble for some applications in quantum optics and quantum information processing. Alternately, single atoms could be selected from this reservoir of ultracold atoms and utilized for CQED device operations.

During the project lifetime, we assembled a system capable of deterministic cavity QED operations for single-atom/single-photon entanglement and quantum information processing that has the following characteristics:
(a) Utilizes laser-cooling, evaporative-cooling, and magnetic trapping to produce ultracold atomic gases confined to small spatial regions;
(b) Translates cooled atoms to a high-finesse optical cavity within an integrated Ultra High Vacuum (UHV) chamber;
(c) Enables single-atom-level counting using CQED;
(d) Differentiates between “active” atoms to be used for quantum information processing, and “passive” atoms used as an atom number reservoir.

This development now sets the stage for several important advances in quantum information science. The storage of a long-lived reservoir of ultracold atoms in a high finesse cavity may allow for the activation of single atoms in the cavity at the moment they are needed in a quantum information protocol, making cavity QED device operation more robust. Alternately, the entire atomic ensemble may serve as the medium for storing quantum information, e.g. in the form of spin squeezing, for which the cavity provides a strong enhancement on the degree of squeezing (many-body entanglement) which can be attained. Various new studies of spin squeezing or other aspects of quantum information and quantum measurement may be pursued using the system we have developed.

1.6 Theoretical Work on Implementations

A key element of our theory/experiment collaboration was the integration of theoretical and experimental activities aimed at realization of robust scalable components with neutral atoms and light fields. Two sets of projects were pursued here:
(a) design of efficient schemes for state initialization and error correction of quantum computation with neutral atoms in optical lattices, as well as detailed assessment of
decoherence mechanisms, control options and gate fidelities for quantum operations; and (b) theoretical investigation of the possibilities afforded by application of CQED to atomic ensembles, addressing in particular how strong-coupling CQED can be utilized to generate correlated or entangled many-body states, and how characteristics of these states may be measured.

Significant progress was achieved in both of these directions. In [23], we developed an efficient and experimentally realizable scheme to compact an optical lattice with sparse random occupation of sites, to a smaller lattice with perfect occupation of one atom per site. As an interesting fundamental aside, this scheme provides the first implementation of the original Maxwell demon that reversibly reduces the entropy of a system to zero [24]. In [25], we developed an error correction protocol for qubit loss corresponding to loss of an atom, as well as other leakage errors out of the qubit space. This protocol relies on a quantum non-demolition measurement, realized with a CQED system, and is feasible as an extension of our current experimental setup. Gate fidelities were studied with a wave packet simulation package that we developed for analysis and control of experimentally variable parameters (QSIMS [26]) and detailed studies made of decoherence mechanisms (e.g., photon scattering, off resonant transitions of non-target atoms and addressing beam errors) analyzing the scaling of these with lattice size, N. For realistic parameters we find that in order to carry out quantum operations within the conventional fault tolerance threshold, these decoherence mechanisms limit the lattice size to $10^5 – 10^6$ atoms. In a more exploratory theoretical study, we investigated the feasibility of implementing a model for topological quantum computation using an extended Hubbard model Hamiltonian proposed by M. Freedman et al. in 2003 with our system of neutral atoms trapped in optical lattices. Realization of such a topological system would provide natural fault tolerance to local errors, and is thus highly desirable. The challenge is however to find a physical system that can realize what initially appear to be non-physical interactions unlike those normally encountered in the solid state. We have explored realizations with neutral atoms as well as a more promising scheme employing polar molecules that allows for much stronger and more flexibly tunable effective interactions than neutral atoms.

Our joint theory/experiment investigations into the use of CQED with atomic ensembles to general novel many-body states started with the question of whether a cavity can be used to convert states of a many-body atomic system into an equivalent quantum-optical state. In [27], we provided a first answer to this question by showing that N atoms trapped in a high-finesse cavity could be induced to generate an N-photon Fock state on demand. This generalizes the previously known and recently realized single-photon generation from single atoms in optical cavities. Because of the far greater complexity of the many-body states, our new result is also encouraging for pursuing the more general and challenging question as to whether any many-body atomic state can be converted into an equivalent quantum-optical state, with implications for quantum communication. We also considered some of the practicalities of this scheme. In particular, the number of atoms in the cavity would ideally be measured prior to generating the optical Fock state. In [28], we showed how CQED effects could be used to make such a measurement, via measurement and analysis of transmission/reflection
characteristics of the cavity when this is filled with N atoms, for both weakly and strongly confined atoms. More recently we have developed a measure of the effective size of macroscopic quantum superpositions and applied it to superposition states of N cold atoms trapped in double well potentials [29]. Such superposition states, termed Schrodinger cat states, are extreme examples of entangled many-body states that have applications in high precision measurements and also constitute a valuable testing ground for the ability to make the large scale entangled states required for quantum computation. These theoretical analyses, while focused on relatively simple forms of many-body states, indicate that high fidelity conversion of quantum information stored in atomic states to optical states is possible and offers a rich field for future investigation.
2.0 Evaluation of problems and difficulties encountered during the research.

2.1 Theory

Improving thresholds and overheads for fault-tolerant quantum computation proved to be easier than expected. On the other hand, we found that there appear to be very serious bottlenecks to the design of new quantum algorithms. One surprise that emerged here was the discovery that quantum techniques can be used to resolve purely classical problems. Our extensive visitor program was very beneficial in making progress on the theoretical side. This could be systematized more in future grants and contracts for even greater impact.

2.2 Optical Lattice Experiments

Achieving the required interferometric stability and optical access were harder than expected, but we achieved them both. Early on, we redesigned the experiment to use a blue instead of a red-detuned lattice. Given this change, we remain happy with the design of the experiment. The problems we have encountered have been of a technical nature, and all have been tractable. This seems likely to continue to be the case.

2.3 Trapped Atoms/Cavity QED Experiments

The tough requirements for magnetically trapping and positioning a cloud within a small optical cavity were not identified in the original experiment designs and required exploratory work before a suitable approach was identified. Fortunately, the product of this time-consuming exploration, a millimeter-scale magnetic trap, yielded the unexpected spin-off of discovering a means for trapping ultracold atoms in a circular waveguide. This novel trapping technique is being pursued at present using funding from a different Defense Advanced Research Projects Agency (DARPA) program Guided BOSE-Einstein Condensate Interferometry (gBECi) for the purpose of precise gyroscope. The integration into the experiment of a high-finesse optical cavity, and the development of the requisite systems for controlling and probing this cavity, proceeded as expected. As for the potential for scaling neutral atom/cavity quantum electrodynamics (QED) devices for quantum information processing, we now consider microfabricated atom chips (rather than the millimeter-scale bulk system we have developed) to be the most suitable platform. Toward this end, we have successfully developed two technological approaches which allow high finesse cavities to be integrated onto atom chips. With support from the Air Force Office of Scientific Research (AFOSR) and leveraging some of the infrastructure developed in the QuIST program, we are now developing what will be the first multi-cavity setup for neutral-atom cavity QED.
3.0 Outstanding Challenges Identified During the Research

3.1 Theory
(a) Reducing overheads for achieving fault tolerance is critical to realizing quantum computers. Given the developments over the last 2-3 years it now appears that there is a significant possibility of accomplishing this.
(b) Designing new algorithms with exponential speedups. Recent results have ruled out several approaches, making this problem harder.
(c) The difficulty in designing quantum algorithms represents an opportunity for cryptography. We need to use our new insights into the limits of quantum algorithms to design classical cryptosystems that are impervious to quantum attack.
(d) Whether naturally fault tolerant qubits, such as with topological quantum computation schemes can be realized in either gas phase or solid state systems is still an open question.

3.2 Optical Lattice Experiments
We have shown that the performance of all elements needed for quantum computation in an addressable optical lattice can be reliably calculated and that with sufficient control over the environment and many laser beams, there will be a high ratio of decoherence to gate time. Thus in the long term, realizing the many quantum operations needed for a quantum computation appears realistic with this approach. The challenge remains to attain the requisite level of control over all elements together. Our detailed theoretical modeling and our experiments to date suggest that no technical difficulty is insurmountable. It appears to be only a matter of time and financial investment.

3.3 Trapped Atoms/Cavity QED Experiments
Theoretical challenges remain in understanding how best to create and make use of increasingly sophisticated many-body quantum states interfacing coherently with optical fields through cavity QED. Our research made some progress in addressing this challenge, e.g. by working through a scheme for generating Fock states of the light using properly prepared atomic samples in the strong-coupling regime of cavity QED. Now, our cavity QED experiment, which is unique in that entire atomic ensembles are stored within a high finesse cavity in the single-atom strong-coupling regime, allows for this theoretical possibility to be explored experimentally. Technically, the major challenge of scalability remains, with our present experiment and those of others only confirming the difficulty of building high-quality, cavity-QED-based, optical interconnects for quantum information. However, we point to present work in our group on integrating multiple high-finesse cavities with “atom-chips” as a promising step toward greater scalability of cavity QED tasks.


4.0 Research Accomplishments

We list here in itemized form the main accomplishments of the project. Details of selected accomplishments are given in chapters 5 (theory), 6 (optical lattice experiments) and 7 (trapped atoms/cavity QED experiments).

- Found a new quantum algorithm for solving Pell’s equation. This algorithm breaks the Buchmann and Williams cryptosystem.
- Designed an efficient new cryptosystem based on the quantum intractability of finding short vectors in a lattice.
- Found a new quantum algorithm for element distinctness in an unsorted $N$ element array, using a new algorithmic technique based on quantum random walks. The algorithm is $O(N^{2/3})$ in time, contrasted with $O(N)$ time required by classical algorithms. The new algorithm is optimal.
- Proved equivalence of adiabatic quantum computation with the quantum circuit model up to polynomial factors, i.e., is universal.
- Showed adiabatic quantum computing gives a quadratic speedup for general search and that exponential query lower bounds do not apply to the quantum adiabatic algorithm for 3SAT. Provided a class of 3SAT formulae for which adiabatic quantum optimization takes exponential time.
- New lower bound for the Quantum Collision Problem: $\Omega(n^{1/5})$ for the number of quantum queries required to decide whether a function is one-to-one or two-to-one, with bounded probability.
- Showed an exponential lower bound for the single register standard method for the non-abelian hidden subgroup problem over the symmetric group.
- Proved that the 2-local Hamiltonian problem is QMA complete.
- Determined the first example of exponential separation between quantum and randomized one-way communication complexity.
- Determined lower bound on the minimal size of locally decodable codes, a classical problem, by means of quantum concepts.
- Developed a geometric theory of non-local two-qubit operations via a geometrically defined set of local invariants. Found that exactly one half of all non-local two-qubit gates are perfect entanglers, that arbitrary two-qubit gates can be very efficiently simulated with a combination of at most three two-qubit operations and four local operations, and demonstrated optimal generation of gates starting from arbitrary one and two-qubit physical Hamiltonians. Applications have been made to solid state implementations and also to optimal quantum circuits.
- Formalized the paradigm of encoded universality within a Lie algebraic analysis to derive the specific conditions under which physical interactions can provide universality. Demonstrated that realization is possible with isotropic, anisotropic, and asymmetric exchange interactions.
- Derived an improved concatenated encoded ancilla preparation scheme. Simulations show that this procedure significantly improves the error threshold for fault tolerant quantum computation, to a new value of about 1% and significantly reduces ancilla overhead.
• Developed a quantum search-based scheme for correcting systematic control errors, with a new class of composite pulse sequences that allow for arbitrary reduction of the control error.
• Developed a fast classical algorithm for simulating quantum circuits composed of CNOT, Hadamard, and phase gates (Gottesman-Knill theorem) and implemented it for thousands of qubits in the freely-available CNOT-Hadamard-Phase (CHP) package. This package forms the backbone of simulation tools subsequently developed by the quantum architecture team of I. Chuang et. al.
• Developed a measure of effective size of macroscopic quantum superpositions in strongly interacting quantum systems. The measure is based on what measurements can be performed to distinguish the different branches of the superposition and allows comparison of effective ‘cat size’ in very different physical systems.
• Developed and implemented a freely available quantum simulation software (QSIMS) package for analysis of quantum logic operations on neutral atoms trapped in optical lattices.
• Designed and modeled a lattice based neutral atom quantum computer with error correction.
• Developed a model for physical implementation of topological quantum computation with atoms and polar molecules trapped in 2D optical lattice.
• Constructed a site addressable 3D optical lattice apparatus.
• Loaded a stable 3D optical lattice with cold neutral atoms and took images with individual site resolution.
• Developed a rapidly-switched, single-pass, dispenser-based source for loading atoms to laser cooling experiment.
• Constructed an integrated ultracold atom/cavity quantum electrodynamics experiment using novel magnetic trapping technology. Demonstrated the transport and trapping of ultracold atomic gases within a stabilized optical cavity suitable for strong-coupling cavity QED operations.
• Developed scheme for generation of N-photon Fock states, counting of N atoms in high finesse cavity.
• Constructed the first circular waveguide for Bose-Einstein condensed atoms. Discovered a novel dispersion management technique incorporating concepts from accelerator physics, and implemented Wigner-function tomography using bichromatic superradiant pump-probe spectroscopy.
5.0 Theory: Details of key accomplishments

• **Found a new quantum algorithm for solving Pell’s equation. This algorithm breaks the Buchmann and Williams cryptosystem.**

The new algorithm extends the hidden subgroup framework to abelian groups which are not finitely generated. An extension of the basic algorithm breaks the Buchmann and Williams cryptosystem which was proposed as an alternative to the RSA cryptosystem [1].

• **Designed an efficient new cryptosystem based on the quantum intractability of finding short vectors in a lattice.**

We showed via a quantum reduction that solving noisy equations mod p is as hard as finding short vectors in a lattice. This greatly improves upon the efficiency of cryptosystems based on the hardness of the shortest lattice vector problem. The new cryptosystem can be efficiently implemented classically, but is hard to break provided that finding short vectors is intractable for a quantum computer [3].

• **Found a new quantum algorithm for element distinctness in an unsorted N element array, using a new algorithmic technique based on quantum random walks. The algorithm is \(O(N^{2/3})\) in time, contrasted with \(O(N)\) time required by classical algorithms. The new algorithm is optimal.**

We developed a new quantum algorithm for element distinctness. Element distinctness is a well-known problem in computer science. It can be informally described as a problem of finding two equal elements in an unsorted array. We give a new \(O(N^{2/3})\) time quantum algorithm for element distinctness on N elements. This contrasts with \(O(N)\) time required by classical algorithms for this problem. The new algorithm is optimal, since it matches a lower bound of Shi. The algorithm uses a new algorithmic technique based on quantum walks [8].

• **Proved equivalence of adiabatic quantum computation with the quantum circuit model up to polynomial factors, i.e., is universal.**

We have made a connection between the local Hamiltonian problem and adiabatic quantum computation. Building on previous results we can now show that universal quantum computation is equivalent, up to polynomial factors, to adiabatic computation with a local Hamiltonian, such that all of the subsystems can be laid out in a two-dimensional grid and interact only with their nearest neighbors. Extending this, we then showed explicitly that it is possible to construct a 2D lattice model with 6-level systems on a grid such that with only local interactions one can adiabatically simulate every quantum circuit [11].
Showed adiabatic quantum computing gives a quadratic speedup for general search and that exponential query lower bounds do not apply to the quantum adiabatic algorithm for 3SAT. Provided a class of 3SAT formulae for which adiabatic quantum optimization takes exponential time.

We have investigated the power of the recently proposed paradigm of adiabatic quantum computing, arriving at several critical results. First, we have shown that adiabatic quantum computing does give a quadratic speedup for general search, thus demonstrating the value of this new paradigm for the design of quantum algorithms. We have also shown that exponential query lower bounds (which show that any quantum algorithm for general search must take exponential time) do not apply to the quantum adiabatic algorithm for 3SAT. We have demonstrated that the dynamics of this algorithm are nevertheless governed by very local features of the energy landscape, and proven that even for very simple landscapes the eigenvalue gap is exponentially small, implying that an exponentially large delay is required [4].

A major open question left by this initial work was whether the quantum adiabatic algorithm is efficient for physically realistic Hamiltonians - which correspond to instances of 3SAT where each variable occurs in only a constant number of clauses. We further resolved this question by showing that an exponential delay is required even for 2SAT instances where each variable occurs in 2 clauses. On the other hand, we have shown that for certain cost functions, the adiabatic algorithm can tunnel through local optima in polynomial time, even though sub-exponential time local search algorithms get stuck. We also showed that the spectral gap for the adiabatic quantum optimization algorithm proposed by Farhi et.al. remains exponentially small for 2SAT formulae with only two occurrences of each literal [38]. This resolves an open question of Mosca and Vazirani, and shows that even for some “physical” hamiltonians, the adiabatic optimization algorithm does not offer an exponential speedup.

New lower bound for the Quantum Collision Problem: \( \Omega(n^{1/5}) \) for the number of quantum queries required to decide whether a function is one-to-one or two-to-one, with bounded probability.

We have found a new lower bound for the Quantum Collision Problem, namely \( \Omega(n^{1/5}) \) for the number of quantum queries required to decide whether a function is one-to-one or two-to-one, with bounded probability. This result implies the existence of cryptographic hash functions that would be immune to quantum cryptoanalysis. This result, which has been an open problem for five years, also implies existence of an oracle such that statistical zero knowledge is not contained in quantum polynomial time. [9, 48]

Showed an exponential lower bound for the single register standard method for the non-abelian hidden subgroup problem over the symmetric group.

The status of the non-abelian hidden subgroup problem is one of the most fundamental open problems in quantum algorithms. In particular, the graph automorphism problem
may be formulated as a hidden subgroup problem over the symmetric group $S_n$. It is natural to generalize the standard method for the abelian hidden subgroup to non-abelian groups. Fourier transforms over non-abelian groups are defined in terms of the irreducible complex representations of the group. There are efficient quantum circuits for computing these transforms for some groups of interest such as the symmetric group. However, since the dimension of these irreducible representations is in general greater than one, the Fourier transform is not unique, and is defined only up to a unitary change of basis for each irreducible. The Fourier sampling step in the standard method now yields the name of an irreducible representation $\rho$, together with the indices $i,j$ of the entry within that irreducible. The main question, then, is whether the statistics of a sample from the Fourier transform of a coset state reveal sufficient information about the hidden subgroup to allow for efficient reconstruction. In [2], we showed that with respect to a random choice of basis, the Fourier sampling statistics reveal, in general, an exponentially small amount of information about the hidden subgroup. The results hold not only for the symmetric group, but also for any sufficiently non-abelian group. We also conjectured that if the hidden subgroup is uniformly random among subgroups of order 2, then the exponential lower bound for quantum Fourier sampling based algorithms should hold for arbitrary basis. There has been intense follow up work that recently not only established this conjecture, but also proved that entangled measurements on polynomially many registers are necessary for any efficient quantum algorithm.

In [49], we give a characterization of many subgroups of the symmetric group that can be distinguished from the identity group. Our results show that for a large set of subgroups quantum Fourier sampling has no advantage whatsoever over classical exhaustive search.

In [50], we explore the question of designing effective joint measurements on polynomially many registers, each containing a random coset state. We have shown that the optimal joint measurement has a special structure: perform a fourier transform on each register, and then measure the character (name of the irreducible representation) for each register. The registers are now in a superposition of the entries of that particular irreducible representation. Now perform some joint measurement on these registers. This is a possible step towards the design of efficient measurements for non-abelian HSP.

- **Proved that the 2-local Hamiltonian problem is QMA complete.**

We have developed a new perturbation theory technique, which has allowed us to show that the two-local Hamiltonian problem is QMA-complete. The complexity class QMA is the quantum analog of the classical class NP. The complexity of the 2-local problem has long been outstanding. Our technique also allowed us to show, that adiabatic computation with 2-local Hamiltonians is equivalent to standard computation. Moreover, we believe that we have developed a powerful tool for the analysis of Hamiltonians, which many are now using. We hope that it will allow simplification of existing quantum computing architectures and the development of new ones. [10, 51]
• Determined the first example of exponential separation between quantum and randomized one-way communication complexity.

We have answered a main open question in the field of quantum communication by providing the first exponential separation between quantum and randomized one-way communication complexity. Previously, no asymptotic gap (even polynomial) was known in this model [52].

• Determined lower bound on the minimal size of locally decodable codes, a classical problem, by means of quantum concepts.

We obtained a result which solves an open question that had been worked on by Goldreich, Schulman, and Trevisan on the minimal size of locally decodable codes—codes which have the property that the j-th bit of the message can be reconstructed by querying only a few random bits of the noisy code word. The most intriguing aspect of this paper is that even though this question is purely classical, our lower bound argument makes essential use of quantum concepts such as querying in superposition, density matrices, and the von Neumann entropy of an ensemble. The paper opens up the possibility of the use of quantum techniques for tackling problems in classical complexity theory and information processing [12, 53].

• Developed a geometric theory of non-local two-qubit operations via a geometrically defined set of local invariants. Found that exactly one half of all non-local two-qubit gates are perfect entanglers, that arbitrary two-qubit gates can be very efficiently simulated with a combination of at most three two-qubit operations and four local operations, and demonstrated optimal generation of gates starting from arbitrary one and two-qubit physical Hamiltonians. Applications have been made to solid state implementations. Also optimal quantum circuits.

We developed a geometric theory of non-local two-qubit operations that allows the quantification of maximally entangling gates via a geometrically defined set of local invariants. The theory shows that exactly one half of all non-local two-qubit gates are perfect entanglers, and also shows that arbitrary two-qubit gates can be very efficiently simulated with a combination of at most three two-qubit operations and four local operations [17]. Using this theory we obtained several new results concerning the optimal construction of two-qubit universal quantum circuits. First, we have established the minimal number of applications needed for an arbitrary controlled-unitary gate to construct a universal quantum circuit [18]. We showed that the optimal such gate is CNOT, which requires only three applications, and we gave an explicit analytic construction for a universal circuit based on this. We also gave an analytic construction of a universal quantum circuit that requires only three applications of the double CNOT (DCNOT), which is not a controlled unitary gate. Subsequently, we discovered a new gate, the B gate, which requires only two applications for construction of a universal quantum circuit, and found an analytic circuit for this [19]. This new gate is remarkable because it is more efficient than the standard paradigm of the CNOT gate in several
respects. It requires fewer applications to make a two-qubit universal quantum circuit, and consequently will be a better gate on which to base the construction of quantum compilers. In addition, it is directly accessible from the physical Hamiltonian in at least one important scalable implementation, namely superconducting qubits, in the range of feasible physical parameters where the CNOT gate requires at least two on/off switches of the interaction. We also compared ease of generation of the B and CNOT gates from physical Hamiltonians, showing that in some cases the B gate may also be physically more accessible than CNOT in that it requires fewer switchings of the interactions for its generation.

*Formalized the paradigm of encoded universality within a Lie algebraic analysis to derive the specific conditions under physical interactions can provide universality. Demonstrated that realization is possible with isotropic, anisotropic, and asymmetric exchange interactions.*

We formalized our previously proposed paradigm of encoded universality within a Lie algebraic analysis to derive specific conditions under which physical interactions can provide universality [54]. We demonstrated that this is possible with the anisotropic exchange interaction (the XY interaction), in addition to the isotropic (Heisenberg) exchange interaction. The minimal encoding for universality from the XY interaction alone is into qutrits [54]. We have found very efficient discrete gate sequences for universal computation with the XY interaction [14]. We have generalized our results for the universality of the exchange interaction to a generalized form of exchange that contains asymmetric exchange tensors and also cross-product terms such as would derive from symmetry breaking effects like spin-orbit coupling. Using a general relation between commutators and operator conjugation that we have expanded, we have found explicit discrete gate sequences for universal computation with this generalized exchange alone [15]. These results are significant for spin-coupled solid state quantum computation.

We undertook an extensive simulation analysis of the performance of exchange-only quantum computation [55, 56, 57]. We calculated the algorithmic fidelity for a three qubit Deutsch-Jozsa algorithm, using the three qubit encoding for exchange-only quantum computation with the isotropic Heisenberg exchange and the gate sequences determined for this by us in 2000. The sensitivity of the scheme to dephasing and spin flip errors was investigated using a quantum Monte Carlo wave function simulation approach. We found that algorithmic fidelities greater than 95% may already be achieved with current experimental parameters, and that the overall performance is more a function of the total time of simulation rather than of the physical complexity of the gates [55]. We also made a numerical determination of a discrete universal gate-set for exchange-only quantum computation with an encoding of four qubits, the smallest such encoding that provides a non-trivial decoherence free subspace. The numerical procedure was based on searches that minimized a fitness function designed to optimize both the fit to local gate invariants and leakage out of the subspace [56].
• Derived an improved concatenated encoded ancilla preparation scheme. Simulations show that this procedure significantly improves the error threshold for fault tolerant quantum computation, to a new value of about 1% and significantly reduces ancilla overhead.

We have derived an improved concatenated encoded ancilla preparation scheme. Simulations show that this procedure significantly improves the error threshold for quantum computation to a new value of about one in a hundred [20].

• Developed a quantum search-based scheme for correcting systematic control errors, with new class of composite pulse sequences that allow for arbitrary reduction of the control error.

We give a quantum search-based scheme for correcting systematic control errors. Such errors occur, for example, when there is a small and constant over-rotation error while applying a rotation about angle theta in the Bloch sphere. We have discovered a new class of composite pulse sequences, inspired by quantum search (amplitude amplification) and dynamical decoupling. The new sequences concatenate nicely, unlike previous techniques, allowing for arbitrary reduction of the control error and giving a threshold result for this noise model. The new sequences also can correct for a larger class of errors than previous techniques [21].

• Developed a fast classical algorithm for simulating quantum circuits composed of CNOT, Hadamard, and phase gates (Gottesman-Knill theorem) and implemented it for thousands of qubits in the freely-available package CHP (CNOT-Hadamard-Phase). This package forms the backbone of simulation tools subsequently developed by the quantum architecture team of I. Chuang et. al.

We showed how to turn the Gottesman-Knill theorem into practical tools for designing and simulating quantum computers. In particular, we developed a faster classical algorithm for simulating quantum circuits composed of CNOT, Hadamard, and phase gates, by removing the need for Gaussian elimination at the cost of a factor-2 increase in the number of bits needed to represent a state. This helped remove the mystery around the Gottesman-Knill theorem by showing that the problem of simulating stabilizer circuits is Parity L-complete; therefore, these circuits are almost certainly not even universal for classical computation. We further proved that any stabilizer circuit has an equivalent circuit with only $O(n^2 / \log(n))$ gates. This result matches the Shannon lower bound and has possible applications to the design of quantum fault-tolerance circuits. We have implemented the improved algorithm in a freely-available program called CHP (CNOT-Hadamard-Phase), which can handle thousands of qubits easily [22]. This program forms the backbone of the simulation tools developed by the quantum architecture team of I. Chuang, et. al.

• Developed a measure of effective size of macroscopic quantum superpositions in strongly interacting quantum systems. The measure is based on what measurements can
be performed to distinguish the different branches of the superposition and allows comparison of effective ‘cat size’ in very different physical systems.

Macroscopic superposition states, termed Schrödinger cat states, are extreme examples of entangled many-body states that have applications in high precision measurements and also constitute a valuable testing ground for the ability to make the large scale entangled states required for quantum computation. Recent experiments claiming formation of cat states in near macroscopic systems raise the question of how the sizes of general quantum superposition states in an interacting system are to be quantified. We proposed a measure of size for cat-like states that is based on what measurements can be performed to probe and distinguish the different branches of the cat state [29]. The measure allows comparison of the effective size for superposition states in very different physical systems. It can be applied to general superposition states and reproduces known results for near-ideal cat states. Comparison with a prior measure based on analysis of coherence between branches indicates that significantly smaller effective cat sizes result from this measurement-based approach. Application to a system of interacting bosons in a double-well trapping potential shows that the effective cat size is strongly dependent on the relative magnitude of the barrier height and interparticle interaction.

- Developed and implemented a freely-available quantum simulation package QSIMS for analysis of quantum logic operations on neutral atoms trapped in optical lattices.

We have developed a quantum simulation software package, entitled “qsim” for performing wave packet simulations of atomic motion and internal state dynamics relevant to quantum logic operations on neutral atoms trapped in optical lattices. A public alpha version is now available at the following site: http://sourceforge.net/projects/qsim/.

- Designed and modeled lattice based neutral atom quantum computer with error correction.

We have developed an efficient and experimentally feasible scheme for compression of sparsely populated optical lattice with large inner-site separation to a perfect lattice. This compression scheme converts a partially filled optical lattice (CO$_2$) to complete filling. The scheme is efficient, i.e., scales polynomially with the number of atoms or lattice sites, and uses feasible atom-laser interactions. Its key elements are flipping of the sensitivity of atoms to the magnetic component of the trapping field and continuous variation of this trapping potential. The scheme can be implemented with existing experimental technologies [23]. We subsequently developed an interpretation of this optical lattice compacting (compression) scheme as a physically realizable example of a Maxwell demon with a memory. This appears to be the first implementation of the original Maxwell demon that reversibly reduces the entropy of a system to zero [24].

We developed error correction protocols for loss of neutral atoms in optical lattices that provide a mapping of these non-standard errors onto standard error models of quantum
error correction [25]. The scheme relies on the ability to selectively image individual sites in the large wavelength optical lattice being built in our experimental effort. The protocol in its simplest version uses four-qubit Grassl-Beth-Pelizari erasure code and consists of two main steps. The first one is reduction of a qubit (atom) loss into a standard quantum error. This is accomplished using a sequence of a quantum non-demolition measurement (QNDM), implemented using a cavity-QED system, and a qubit source, implemented by an adiabatic optical tweezer, conditioned on the QNDM result. The second part of the protocol corrects the residual quantum error using one projective qubit measurement and a sequence of 3 two-qubit and two single qubit operations. No ancillary qubits are required. The scheme is experimentally feasible with current technology of atomic and optical physics either using neutral atoms in optical lattices or trapped ions. The protocol is general and can also be adapted for correction of qubit loss in quantum computation and communication schemes based on photons. We further adapted this scheme to eliminate arbitrary leakage error for neutral atom qubits in optical lattices, with the following procedure. A cavity QED system allows discrimination between field insensitive (qubit) states and field sensitive magnetic hyperfine levels. A state flip operation between two consecutive measurements can further distinguish field sensitive states having comparable AC Stark shifts. A complete correction of non-standard quantum errors not restricted to qubit subspace is now possible. Two distinct implementations of projective measurement were also proposed as part of this, one based on fluorescence and implementable in parallel, the second scheme using cavity QED to perform the projective measurement on a qubit state after unitary transformation into the atomic field sensitive states.

We developed an analysis of decoherence during single-qubit operations on trapped atoms in optical lattices due to vibrational excitation of atoms resulting from switching on and off the site addressing optical field. The simulation model, based on adaptation of the Chebyshev polynomial algorithm to solution of the Schroedinger equation with a time varying potential, allows evaluation of the heating over a broad range of parameters including both adiabatic and sudden approximation limits. Calculations revealed the dependence of heating on the addressing field parameters and were then utilized to control and minimize the vibrational excitation of atoms during single-qubit operations [23]. The results suggest that heating can be minimized by using timing-based control techniques. Simulation of the entire single-qubit gate has subsequently been carried out.

A second heating mechanism arises in implementation of an entangling Rydberg gate between neutral atoms. The change of the atomic internal states leads to variations in trapping potential that triggers undesired evolution of atomic wavepacket between laser pulses which reduces the gate fidelity. We have developed and implemented a realistic simulation model for the Rydberg gate between two cesium atoms and evaluated its fidelity [57].

• Developed model for physical implementation of topological quantum computation with atoms and polar molecules trapped in 2D optical lattice.
We found that using neutral atoms interacting via tunneling between auxiliary sites in principle implements the required interactions for an Abelian version of this model, but the feasibility is somewhat hampered by the weakness of the effective interactions. A scheme employing polar molecules allows for much stronger and more flexibly tunable effective interactions than implementations using neutral molecules and is currently being developed further.
6.0 Optical Lattice Experiments: Details of Key Accomplishments

Our optical lattice is made from three pairs of 60 μm waist λ=845.5 nm laser beams, 7 nm blue-detuned from the D2 line in Cs (see Figure 1). Two beam pairs propagate in the horizontal plane and one pair propagates in a vertical plane. The two beams in each pair are θ=5.5° away from copropagating along one of the horizontal lattice axes. All beam pairs are linearly polarized perpendicular to their plane of propagation. To prevent mutual interference, the pairs are shifted in frequency relative to each other by tens of MHz [30]. The result is a square 3D lattice with a spacing \( L=\frac{\lambda}{2\sin\theta} \approx 4.5 \) μm, and effectively linear polarization everywhere [30]. The effective linear polarization is critical to the success of our imaging while laser cooling.

![Figure 1: Schematic of the beam configuration for the 3D far-detuned optical lattice with 4.5um site spacing.](image)

The experiment takes place in a custom made high optical quality fused silica cell (5 cm × 6 cm × 7 cm, with 5 mm walls). The cell affords high numerical aperture (N.A.) access from three directions, along with space for lattice beams and a variety of cooling and optical pumping beams. Cs atoms are pulsed from a heated cartridge, and loaded into a magneto-optic trap (MOT). The loading takes between 3 and 10 seconds, depending on experimental choices. The MOT magnetic field is then shut off and the optical lattice is turned on, which initially leaves ~4 atoms per lattice site. Atoms are rapidly lost in pairs due to photoassociation, so that initially even-occupied sites become empty and initially odd-occupied sites end up with a single atom [31]. To improve cooling and reduce stray light scattering, the MOT beams are then replaced by a set of smaller (100 μm waist) optical molasses beams, 85 MHz below the F=4 to F′=5 resonance.

The steady state laser–cooled atom temperature is \( T \sim 6.5 \) μK, while the depth of the optical lattice near the central sites is \( U_0 = k_B \times 150 \) μK. The probability of an atom having enough energy to hop to an adjacent well is \( P_h \sim \exp\left(-\frac{U_0}{k_B T}\right) \). Each atom’s energy is scrambled in a characteristic polarization gradient laser cooling time, \( \tau_c \sim 50 \) μs,
so the expected average time between site hops is $T_h \sim P_h \tau_c$, which far exceeds the timescale of the experiment (it is about two months).

We filter the trapping light with a narrowband interference filter and collect scattered laser cooling light with an objective outside the cell. It has a 0.55 N.A., 18 mm working distance, and is diffraction limited for infinite conjugate ratio. A 58 cm focal length infinite conjugate ratio lens 1.5 m away, forms a 32 times magnified image on a cooled front-illuminated CCD camera. The depth of field of the imaging system is 2.7 μm, so only one plane is in focus at a time. We can reliably determine if there is an atom at a given site in ~20 ms. The image plane can then be changed using a piezo-electric transducer to change the axial objective position. The first row of Figure 2 shows successive pictures of non-adjacent lattice planes, while the second row shows subsequent images of the same planes. Each small bright spot is a single atom. We observe the lattices from along the negative z-axis. The haze in each photo is from atoms trapped in non-imaged lattice planes. The site occupation in the central areas of each image is unchanged over the 3 second window, while at the much shallower depth lattice edges some hopping has occurred. The out of focus contribution of the atoms in the central plane can in many cases be discerned in the adjacent planes, and vice versa. Although site hopping can be observed at the shallower edges of the lattice, it is clear that taking the pictures does not affect the locations of atoms at the central 500 lattice sites. Figure 2 shows only three imaged planes, but we can image the occupied part of the lattice in its entirety.

![Figure 2: Photographs taken of three adjacent lattice planes, at 3 second time intervals.](image)

With $T \sim 6.5 \, \mu{K}$ the rms extent of each atom is 0.3 μm, which makes the time between spontaneous emission events due to the blue detuned lattice light hundreds of milliseconds. We will next 3D Raman sideband cool atoms to their vibrational ground state [32] and reduce the lattice depth, leaving the atom size at 100 nm. Since the atoms will then see much less lattice light, this will increase the time between spontaneous emissions to ~2 minutes, comparable to the lifetime of the atoms due to background gas collisions. Combined with the vacancy filling operations that we have invented as part of this project [24, 23] this will produce a manifestly zero entropy state of 250 atoms, with very long decoherence times. Trapped ions have been the most successful qubits to date,
with steady progress having realized six qubit entangle systems [33]. Neutral atoms in these highly scalable traps are now poised to compete in this field.
**7.0 Trapped Atoms/Cavity QED Experiments: Details of key accomplishments**

Our experimental work began with development of the overall experimental infrastructure of vacuum systems, laser systems, electronics, imaging, computer control, etc. The technical approach which we chose to pursue involves producing ultracold atoms in a magneto-optical trap (MOT) several centimeters away from the high-finesse cavity, moving these atoms to the cavity location, transferring them into a more tightly confining trap, and then using them for quantum information studies. A schematic of the approach is shown on Figure 3: (I) Atoms are loaded into the MOT and trapped in a magnetic trap, (II) transferred 1.75 inches toward the mm-scale trap and cavity, and (III) evaporated to a phase space density of around $10^{-5}$. (IV) The cloud is then transferred to the Ioffe-Pritchard trap. The arrangement of curvature (red), antibias (blue) and gradient (yellow) coils are indicated to scale, with the gap between the antibias coils being 1 mm. (V) Atoms are then evaporated to quantum degeneracy, producing BECs of about $10^6$ atoms. These will be tightly confined within a high-finesse, strong-coupling cavity.

![Figure 3: Sketch of experimental sequence for preparation of trapped cold atomic gases.](image)

We explored a scheme for simplifying some steps in the preparation of ultracold atomic samples, namely the use of a single-pass atomic beam produced by a rubidium dispenser. After assessing this system [34], we went back to a more complex but robust approach to introducing atoms to our experiment. We then tested a new approach to magnetic trapping that we had devised for this work. That is, we noted that the requirements for magnetically trapping atoms to sufficiently small volumes inside an optical microresonator were difficult to achieve with existing magnetic trapping structures. This new trap, the “millitrap” was successfully demonstrated, as reported in [35].

At this stage, our funding under the DARPA QUIST program ended and we decided to forsake the pursuit of CQED for a very productive detour. The extremely tight confinement offered by our millitrap allowed us to invent a means for confining ultracold atoms in a ring-shaped magnetic trap. This trap may be suitable for high-precision atomic gyroscopy and also for other precision measurement applications. Following the first demonstration of this trapping technique [36], we studied aspects of atomic motion in the ring trap with precedents in high-energy accelerator physics [37], and also

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*24*
developed a new measurement technique to measuring coherence properties of atoms in the ultracold-atom storage ring [38].

Finally, during the past year, we have returned to our original path and introduced ultracold trapped gases to the interior of a high-finesse optical cavity. The systems for stabilizing the cavity resonance and for probing the atom-filled cavity were also developed during this year. The present status of our work is described below.

7.1 Novel Atom Source

We have developed a simple system for loading a MOT which does not require the infrastructure of typical sources for ultra-cold atom experiments [34]. This technique utilizes alkali metal dispensers, or “getters,” which are current-driven, resistively-heated sources of atomic vapor. Such devices have been employed in other experiments [39, 40], but only at rather high pressures. In our design, summarized in Figure 4, strict UHV conditions are maintained by use of a cold “shroud” which traps those rubidium atoms not captured by the MOT. The experiment consists of two parts: (a) The getter is brought within 1.2 inches of the MOT center and produces a hot rubidium flux used to load the MOT. Rubidium atoms not directed at or not loaded by the MOT are trapped by the cold plate and nozzle. (b) Pulsing the getter yields large MOT populations (crosses), while cooling the shroud eliminates the background rubidium vapor as determined by the low MOT populations produced without pulsing the getter (circles). We have found the fast atom flux emitted directly from the getter to be an inefficient loading source for a MOT. However, a secondary, lower temperature source, formed by short-lived desorption off the shroud, was found to yield efficient loading. A next generation design would employ this secondary beam in a more controlled manner.

![Figure 4: Novel atom source incorporating a getter and cold shroud system.](image)

7.2 Millimeter-scale magnetic trapping

In spite of promising alternatives [41], successful optical CQED experiments have used exclusively Fabry-Perot cavities. Magnetically trapping atoms inside these high-finesse resonators presents a significant design challenge. Standard inch-scale magnetic traps cannot provide sufficient field curvatures necessary for confining atoms inside a strongly-coupled cavity. While surface magnetic traps produce ample curvature (~10^6 G/cm²), their fields are limited to within 100 μm of the chip surface, complicating their integration with existing Fabry-Perot cavities.
Nevertheless, in recent work, funded by a “seedling grant” from DARPA and AFOSR, we have indeed accomplished the integration of high-finesse Fabry-Perot resonators with atom chips. We are presently building an experiment to use this newly developed technology to study quantum measurement and control of single atoms and guided atomic beams. For this, we are leveraging infrastructure established through the DARPA QUIST program.

We have thus turned to a novel magnetic trap design capable of trapping atoms within a ~100 $\mu$m long optical cavity. The mirrors we are using constrain us to place electromagnetic coils several millimeters away from the cavity center. At this distance, currents on the order of 100 Amperes in a small cross-sectional area are required to confine atoms sufficiently. We have developed a technique to harness such high current densities (200 Amperes/mm$^2$) in a magnetic trap which produces curvatures of $\sim 5 \times 10^3$ G/cm$^2$ under steady state operation. The trap uses anodized aluminum conductors as multi-turn coils operating at liquid nitrogen temperatures. The trap generates 20 W of heat at normal operation; this heat is dissipated \textit{in vacuo} by contact with a nitrogen-cooled heat sink.

We used this trap to produce Bose-Einstein condensates of several million atoms. The strong, nearly isotropic trapping fields restrict the diameter of the condensate to be less than 40 $\mu$m, much smaller than the ~150 $\mu$m planned spacing between mirrors for our Fabry-Perot cavity. Measuring the lifetime of atoms trapped with the millitrap indicates pressures of $\sim 10^{-11}$ Torr or less, confirming the UHV-compatibility of materials used in the millitrap. Technical details are illustrated in Figure 5 below. The primary curvature coils (red), the anti-bias coils (blue), and the gradient coils (yellow) are depicted in this diagram as solid bodies, but are in reality multiple turns of wire with protruding leads. For clarity, the coil leads have been omitted and the nearest gradient coil is shown as transparent.

![Figure 5: Sketch of the mm-scale IP trap.](image-url)
7.3 The ultracold-atom storage ring

Two major uses of atomic Bose-Einstein condensates are the studies of a new type of quantum fluid and the development of coherent atom optics and eventual applications of the atom laser. For the former, theories abound which posit a multiply-connected gaseous condensate in which to consider quantized circulation, soliton formation and propagation, and even analogues of general relativity (“sonic black holes”), although experimentally such systems did not yet exist. For the latter, waveguiding structures have been sought which would conduct coherent atom waves through a large-scale interferometer and allow for precision measurements through atom interferometry.

Using the millitrap which was developed for the DARPA QUIST project, we developed a novel, millimeter-scale magnetic ring trap which addresses both of these needs. Using just a subset of the electromagnets of this trapping device, we demonstrated that Bose-Einstein condensates could be formed within and then sent traveling around a circular closed-loop waveguide [36]. The magnetic trap differs from existing ones, e.g. in having a multiply, rather than simply, connected topology, and in permitting trapped quantum gases to travel indefinitely in an unterminated waveguide. A schematic is shown in Figure 6: (a) Four coaxial circular electromagnets (see Ref. [34] for details) are used to generate both the static (currents as shown) and rotating fields needed for the waveguide. Axes are indicated; gravity points along -z. (b) As shown schematically, the field (arrows) from just the two outer coils (curvature coils, colored red) points axially in the midplane between the coils, with largest fields at the axis. (c) Adding a uniform opposing bias field (using anti-bias coils, colored blue) produces a ring of field zeros (X) in the x-y plane about which weak-field seeking atoms (shaded region) are trapped. (d) Rapidly rotating the field zeros around the trapped atoms produces the TORT. In our new TORT design, only the curvature coils need be at the millimeter scale, as larger scale coils may be advantageously used for bias fields and gradients.

One promising application of the circular waveguide is the Sagnac gyroscope, in which two coherent matter waves are made to travel in opposite directions along a path with a large enclosed area, accumulating a relative phase in a rotating frame, which is then read out interferometrically.
In our work, condensates completing several complete orbits in the ring encompassed a total area which would be sufficient to detect rotation rates as low as $5 \times 10^{-9}$ rad/s in a single shot. In the future, rotation sensors similar to a SQUID may use condensates which completely fill the ring. We are presently improving this technology in a new apparatus to explore Sagnac interferometry with both bosonic and fermionic ultracold gases. Figure 7 shows the launching of a Bose-Einstein condensate into circular motion along such a closed waveguide: (a) Top absorption image, taken after very short (2 ms) time of flight, shows a Bose-Einstein condensate in a portion of the waveguide. From such images taken at variable duration after the launch, the azimuthal (b) width and (c) position is determined for these waveguided atoms. For this launch, the angular (linear) velocity of the propagating coherent matter wave was 57 rad/s (69 mm/s), and the rms longitudinal velocity spread corresponded to an effective temperature of 25 nK. (d) From the velocity of the atoms, one can determine the kinetic energy at different portions of the ring, and thereby measure variations in the magnetic and gravitational potential along the guide. For these settings, a peak-to-peak variation of about 5 μK is observed. In (e) we show “unwrapped” images, i.e. ones converted to cylindrical coordinates, of the propagating matter wave after a variable number of complete revolutions around the ring. The longitudinal velocity spread causes the atomic cloud to fill a substantial portion of the ring after about 7 revolutions (about 750 ms).

Using our ultracold-atom storage ring, we revisited the phenomenon of betatron resonances that is well known from high-energy circular accelerators and storage rings. In such a resonance, small defects in the ring can excite large transverse motion of the particle beam when it is traveling at particular resonant velocities. Figure 8 shows resonant dispersion management in a multimode circular atomic waveguide based on such defects. At launch angular velocities tuned near subharmonics of the transverse trapping frequencies, resonant transfer of longitudinal energy to transverse excitation can cause a dramatic reduction in the longitudinal dispersion of an atomic wave packet. Tuned to one of these resonances (the 5th subharmonic), the atomic cloud is seen to propagate with little dispersion for as many as 13 complete revolutions around the guide.
-- this is to be compared to the non-resonant, normal dispersion case in Figure 7. While these resonances must be avoided in high energy devices, in our case they yield the coldest atomic beam ever produced, with a kinetic temperature in the picokelvin range. Our work indicates a new research direction, drawing together concepts of accelerator physics, atom and molecular interferometry, and optics. We foresee that many concepts in high-energy accelerator physics may find new applications in the ultralow energy domain [37].

Figure 7: Launching of a Bose-Einstein condensate into circular motion along a closed waveguide.

Figure 8: Resonant dispersion management in a multimode circular atomic waveguide.
During the past year, we also completed a paper on characterizing the coherence properties of an ultracold atomic beam guided in a circular waveguide. The success of guided-atom interferometers will depend on the ability to maintain a high degree of coherence during the long propagation of atoms in waveguides, an ability which is not guaranteed and which may be stymied for various reasons, such as heating due to static or time-varying defects in the waveguiding structures, phase diffusion due to non-linear interactions, or the onset of one-dimensional effects, e.g. the loss of phase coherence due to the concentration of low energy phase fluctuations, as the atoms propagate. In light of such concerns, we undertook to measure the coherence of a guided atom beam as it propagated for several revolutions in our millimeter scale ring trap.

The transverse character of the guided atom beam was easily assessed using conventional time-of-flight techniques. These confirmed that as a trapped Bose-Einstein condensate was launched into a waveguide and expanded longitudinally in the guide, it remained in a self-consistent single transverse quantum state which evolved toward the non-interacting ground state in the transverse confining potential.

The harder task was to measure the longitudinal quantum state of the beam. For this purpose, we developed a pump-probe technique, bichromatic superradiant pump-probe spectroscopy (bSPPS) which allows for tomographic measurements of the Wigner function of the propagating atom beam. The bichromaticity was provided naturally by the rotation of the atomic beam in the curved waveguide. The upshot of applying this measurement was that we confirmed that the guided atom beam maintained a long coherence length, even in the presence of a significant coherent velocity chirp across the cloud (which would normally obscure the coherence of the beam if probed using conventional means), and also after multiple complete revolutions in the waveguide [38].

7.4 Optically-trapped ultracold atoms in a high-finesse cavity

In the last year, we have added the high-finesse optical cavity into our apparatus. This Fabry-Perot cavity is constructed using high-reflectivity curved mirrors fabricated by Research Electro-Optics (Boulder, CO). The mirrors are glued to BK7 glass blocks which extend into an open channel in the magnetic trap assembly. Measured properties of this cavity are given in Table 1. The position of the cavity relative to the millitrap assembly is adjusted using a three-dimensional UHV translation stage so that there is no mechanical contact between the two systems.

The cavity spacing is stabilized using a two-stage passive isolation system to suppress high frequency vibrations, while actively controlling lower frequency noise. For this active stabilization, we monitor the cavity spacing using laser light at $\lambda_{\text{FORT}} \approx 850$ nm, exciting one of the cavity resonances. Figure 9 shows the laser system for stabilizing and probing the CQED system. Off-resonant light at $\lambda_{\text{FORT}} = 850$ nm is stabilized to a transfer cavity. The transmitted signal from this light allows the high-finesse cavity mirror spacing to be actively stabilized. Probe light near the atomic D2 line at 780 nm is simultaneously locked to the transfer cavity. Rubidium cell spectroscopy allows the transfer cavity, and thereby the high-finesse cavity, to be precisely stabilized. The transmitted probe light is monitored by a polarimeter which will be used for detecting
Larmor precession of atoms trapped in the cavity. Various frequency shifters provide flexibility in system settings. This light also serves as a far off-resonant optical trap (FORT) when the in-cavity intensity of the light is increased to the point that the AC Stark shift depth satisfies $U > 4 E_{r,FORT}$ with $E_{r,FORT} = \left(\frac{2 \pi}{\lambda_{FORT}}\right)^2 \hbar^2 / 2 m$ being the recoil energy at the FORT wavelength. Our system is configured so that the cavity remains locked both for settings in which the AC Stark shift does ($U > 4 E_{r,FORT}$) and does not ($U < 4 E_{r,FORT}$) trap the atoms.

![Laser system for stabilizing and probing the CQED system.](image)

The FORT wavelength is related to the cavity probe wavelength $\lambda_{probe}$ by means of a narrow-line (100 kHz) external transfer cavity to which both the FORT and probe lasers are locked with suitable frequency shifts. In turn, the probe wavelength can be stabilized by saturation spectroscopy to a Rb spectroscopy cell. At present, the overall stability of this system may be assessed by the effective linewidth $\kappa_{eff} \approx 2 \pi (1.5 \text{ MHz})$ of the in-vacuum cavity which is measured by slowly sweeping $\lambda_{probe}$ across the cavity resonance. This value is slightly larger than the cavity linewidth $\kappa = 2 \pi (0.8 \text{ MHz})$ which is measured by ring-down spectroscopy, indicating that the combination of laser linewidths and cavity jitter still significantly contributes to the apparent cavity stability. We are presently implementing higher bandwidth feedback to the laser diodes in this setup so as to further narrow their laser linewidths and overcome this problem.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirror transmissions</td>
<td>3.7 ppm, 3.7 ppm</td>
</tr>
<tr>
<td>Other losses</td>
<td>7.4 ppm</td>
</tr>
<tr>
<td>Mirror spacing</td>
<td>196 μm</td>
</tr>
<tr>
<td>Free spectral range (at 780 nm)</td>
<td>1.5 nm</td>
</tr>
<tr>
<td>Beam waist w₀ (1/e² radius at 780 nm)</td>
<td>23.4 μm</td>
</tr>
<tr>
<td>Measured finesse F</td>
<td>550,000</td>
</tr>
<tr>
<td>Cavity half-linewidth κ/2π</td>
<td>0.8 MHz</td>
</tr>
<tr>
<td>Atomic half-linewidth γ/2π</td>
<td>3 MHz</td>
</tr>
<tr>
<td>Maximum coupling strength g/2π</td>
<td>16 MHz</td>
</tr>
<tr>
<td>Single atom cooperativity C = g² / 2κγ</td>
<td>52</td>
</tr>
<tr>
<td>Critical photon number γ²/2g²</td>
<td>0.018</td>
</tr>
</tbody>
</table>

Table 1: Cavity parameters indicated by measurement for our in-vacuum high-finesse cavity, which uses 5 cm radius-of-curvature high-reflectivity mirrors. The maximum coupling strength g is calculated for a ⁸⁷Rb atom at the antinode on the cycling resonance line.

We are presently investigating prospects for carrying out experiments on spin squeezing with these cavity-enclosed ensembles. Several of the capabilities needed for such experiments have already been demonstrated, namely the ability to trap optically an ultracold atomic gas within a high-finesse cavity, to prepare the gas in a well defined initial state, and then to detect a precessing atomic spin within the cavity. At present, magnetically trapped gases with 5 x 10⁴ atoms at the Bose-Einstein condensation temperature, or 2 x 10⁵ atoms in a pure condensate, are delivered to the cavity mode volume. There, we have three choices of trap potentials: (1) the Time Orbiting Potential (TOP) trap, with the AC Stark shift from the locking light negligibly influencing their motion; (2) a hybrid trap which confines atoms both magnetically in the TOP trap and also optically in a sufficiently deep FORT; or (3) a purely-optical trap established by the locking laser light.

The system is probed with the cavity resonance detuned by Δ/(2π) = 3 GHz or more below the D2 atomic resonance. At this setting, a single atom maximally coupled to the cavity shifts its resonance frequency (further to the red) by

\[
\delta(m_z, \varepsilon) = -2\pi (81 kHz) \times \left[ \frac{2}{3} - \frac{1}{6 + 2 \varepsilon^2} m_z \right]
\]

Equation 1

Here, m_z is the eigenvalue of the atomic spin along the cavity axis, and ε is the ellipticity of the probe laser beam. The high-finesse cavity, which shows little birefringence in absence of trapped atoms, becomes circularly birefringent if it contains spin-polarized atoms.

To probe the atoms at this setting, we red-detune the probe light from the cavity resonance by variable amount, couple this light into the cavity, and monitor the transmitted optical power continually. Here, the atomic spin is maintained along the cavity axis, and circular polarized light is used. When the probe light is switched on, the resonance shift due to the trapped atoms exceeds the detuning of the probe light from resonance, and thus the cavity transmission is low. As the number of trapped atoms decays, the cavity resonance shifts to the blue and eventually matches the probe-light frequency, at which point an “uptick” in the transmitted power is observed. Further decay
of the trapped atoms shifts the cavity away from resonance, and extinguishes the transmitted light.

One data trace obtained in this manner is shown in Figure 10, with atoms trapped in the purely-optical trap. Panel (a) shows that atoms in the cavity shift the empty cavity resonance to lower frequencies. We probe the cavity with laser light detuned to the red of the empty cavity resonance, and to the blue of the cavity resonance when shifted by the initial number of atoms trapped in the cavity. Over time, the number of trapped atoms drops, and the cavity resonance shifts toward the probe frequency. Panel (b) shows that when the shifted cavity resonance matches the probe frequency, probe light transmitted through the cavity is detected, providing an accurate measurement of the effective number of trapped atoms. Panel (c) shows how the detected effective atom number decays over time.

![Figure 10: Cavity-aided detection of optically-trapped atoms in a high-finesse optical cavity.](image)

During the uptick, the instantaneous cavity resonance frequency is well resolved, implying some knowledge as to the number and state of the atoms in the cavity. In our present experiment, atoms occupy a large number of optical-trap minima; the total length of the cloud along the cavity axis is around 100 μm. In each minimum, the coupling strength to the cavity field differs, so that the average cavity frequency shift per atom is reduced by about $\frac{1}{2}$. The “effective atom number” in the optical trap, defined using the average shift described above, decays over a timescale of as many as 10’s of seconds. At our present settings, the observed linewidth $\kappa_{\text{eff}}$ of the cavity corresponds to a variation in the effective atom number of 37; since the probe-cavity frequency difference is well resolved during the uptick, the uncertainty in the effective atom number is smaller than this.

We find that the decay of atoms from the optical trap is presently dominated by mechanical effects of the cavity probe. This is confirmed by switching the probe off when a transmission uptick is detected, waiting a variable period of time, and then switching the probe back on. For short wait times, the cavity transmission indicates almost no change in the state of atoms in the cavity, while for longer wait times, the diminished transmission indicates some loss had occurred. This triggering capability in our experiment will be helpful for experiments on cavity-aided detection of Larmor precession.

The cavity transmission also displays a strong sensitivity to the spin-state of the atoms within the cavity. To demonstrate this, we probe a cavity which contains atoms still trapped in a TOP magnetic trap, in which the rotating bias field rotates the atomic
spin in a plane containing the cavity axis. The transmission lineshapes reveal this motion, see Figure 11. Here, atoms are held in the rotating magnetic field of a TOP trap and probed via the cavity transmission. Panel (a) shows that the transmission signal is much longer than in the case of optical trapping, indicating a wider effective line width for the cavity transmission. Panel (b) shows that this signal is strongly modulated at the rotation rate of the atomic spins. This demonstrates the strong spin-sensitivity of cavity-based optical measurement of atoms. The transmission signal for magnetically-trapped atoms persists for much longer than for the optically-trapped atoms (20 ms vs. 2 ms). This is explained by the fact that the cavity resonance frequency is swept over a wide frequency range as the atomic spin is rotated, thus allowing for probe transmission for a greater range of atomic numbers. We have confirmed this picture by directly measuring the cavity transmission linewidth for either type of trapping. Furthermore, the transmitted signal is clearly modulated at the 5 kHz rotation frequency of the TOP fields. Here, this modulation occurs due to the forced guiding of atoms by a strong magnetic bias field. In future experiments on Larmor precession, similar signals should appear due to the free precession of the atomic spin(s).

![Figure 11: Cavity-aided detection of dynamic atomic spin.](image)

### 7.5 Theoretical progress on CQED with many-body states

The groups of Whaley and Stamper-Kurn investigated theoretically the possibilities afforded by application of CQED to atomic ensembles, addressing the following questions: How can strong-coupling CQED be utilized to observe many-body atomic systems, e.g. in the sense of various microscopy techniques? How can CQED be utilized to generate correlated or entangled many-body states? And how can many-body systems be utilized to generate novel quantum-optical capabilities through CQED? Several answers to these questions were obtained.

It is known that single atoms stored in high-finesse optical cavities can be used to generate single photons [42, 43, 44] and to map the state of a single atom onto the state of a single optical mode [45]. We have considered whether a cavity could be used generally to convert any state of a many-body atomic system into an equivalent quantum-optical state. As a starting point, we found that N atoms trapped in a high-finesse cavity could be induced to generate an N-photon Fock state on demand. This procedure should succeed with an error rate that scales with the number of atoms. Thus, an N-photon Fock state could be generated with high-fidelity for $N < g^2 / \kappa \gamma \approx 100$ [46].
In the aforementioned scheme, the number of atoms in the cavity would ideally be measured prior to generating the optical Fock state. We considered how CQED effects could be used to make such a measurement. The influence of trapped atoms on the transmission/reflection characteristics of a cavity grows with increasing atomic number; for instance, the splitting between the red- and blue-sideband cavity resonances grows as $2gN^{1/2}$ if all atoms are coupled to the cavity with coupling strength $g$. We analyzed the situation in which atoms are imperfectly trapped within the cavity volume. Analytical results confirmed that atomic confinement in the Lamb-Dicke regime is very helpful to transmission-based atom counting. However, even weakly-confined atoms can be counted reliably up to $N \approx 10$ [47].
8.0 Summary

This research effort achieved collaboration between theoreticians and experimentalists who were able to focus on the fundamental need of scalability in quantum information processing. The results that have been shown demonstrate the connections and interleaving of the theory and experimental work on several levels. The theory research 1) developed new theoretical tools enabling reliable implementation of quantum information processing in scalable systems, 2) characterization of the relationships between quantum algorithms and architectures, between fault tolerance and architectures and between quantum and classical complexity classes. The experimental research developed scalable quantum component technology based on gas phase systems using atoms and light fields and demonstrated deterministic control in atom/cavity systems. While tremendous progress was made over the course of this effort, challenges still remain in the development of scalable quantum information processing systems.
9.0 References


10.0 Appendix

10.1 Publications Resulting From the Work of this Program


S. Aaronson, Book Review of A New Kind of Science by S. Wolfram, Quantum Information & Computation (QIC), September 2002.


S. Aaronson, separate part II combined in reference 65.


10.2 Dissertations Resulting From the Work of this Program


10.3 List of Invited Presentations (over the period of this grant)


J. Kempe, ERATO seminar, Tokyo, Japan, “Encoded Universality”; University of Tokyo, Computer Science Seminar, “Quantum Random Walks”; Japanese-French workshop, Tokyo, “Quantum Random Walks”; all three talks in period December 6–12, 2001

S. Aaronson, Avaya Labs, Tel Aviv, Israel, “Quantum Computing: What’s It Good For?”, January 10, 2002

K. B. Whaley, American Association for the Advancement of Science (AAAS), Boston, Massachusetts, “Decoherence-free Quantum Computation,” February 17, 2002


J. Vala, SQuInT Annual Meeting, University of Colorado at Boulder, Poster, “Encoded Universality with Anisotropic Exchange Interaction,” March 8–10, 2002


J. Vala, JILA Seminar, University of Colorado at Boulder, Contributed Talk, “Encoded Universality,” March 11, 2002


S. Aaronson, STOC’02, Montreal, Quebec, Canada, Contributed Talk, “Quantum Lower Bound for the Collision Problem,” May 21, 2002


K. B. Whaley, Presentation to College of Chemistry Advisory Board: “Quantum Nanoscience at Berkeley”, May 2002


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M. Hsieh, Berkeley in Silicon Valley Faculty Forum and Networking Event, “New Directions in Technology,” San Jose, CA, Poster, “Exact Gate Sequences for Exchange-only Quantum Computation,” June 1, 2002

U. V. Vazirani, Theory Seminar, Hebrew University, Israel, “How Powerful are Adiabatic Quantum Algorithms?”, June 4, 2002


U. V. Vazirani, CS Colloquium, Tel Aviv University, Israel, “New Quantum Algorithms”, June 10, 2002


S. Aaronson, University of Bristol, Bristol, England, “Quantum Computing and Dynamical Quantum Models,” August 15, 2002


S. Aaronson, M.S.R.I. Workshop on Quantum Algorithms and Complexity, Banff Centre, Banff, Canada, “Quantum Lower Bounds You Haven’t Seen Before,” September 24, 2002

I. Kerenidis, M.S.R.I. Workshop on Quantum Algorithms and Complexity, Banff Centre, Banff, Canada, “Exponential Lower Bound for 2-query Locally Decodable Codes via a Quantum Argument,” September 24, 2002


W. van Dam, Workshop on Quantum Information and Quantum Computation, Abdus Salam International Centre for Theoretical Physics, Trieste, Italy, “Quantum algorithms: Fourier transforms and group theory,” October 21, 2002


W. van Dam, Workshop on Quantum Information and Quantum Computation, Abdus Salam International Centre for Theoretical Physics, Trieste, Italy, “Quantum algorithm for the hidden shift problem,” October 23, 2002

J. Kempe, M.S.R.I. Workshop on Quantum Information and Cryptography, Berkeley, CA, “Quantum Random Walks,” November 4-8, 2002


K. R. Brown, SQuInT Annual Meeting, Santa Fe, NM. Poster: “Scalable Ion Trap Quantum Computation on Decoherence Free Subspaces,” February 6–9, 2003
J. Von Korff, SQuInT Annual Meeting, Santa Fe, NM. Poster: “Nash Equilibria in Quantum Games,” February 6–9, 2003


J. Vala, ERATO Conference on Quantum Information Science, Kyoto, Japan; Contributed Talk, “Two-qubit Module Design of a Universal Quantum Compiler,” September 4–6, 2003


I. Kerenidis, Quantum Reading Group, UC Berkeley, Berkeley, California; Invited Talk, “Exponential Separation of quantum and classical one-way communication complexity,” November 2003


J. Kempe, Annual DARPA QuIST PI Annual Review 2003, Fort Lauderdale, Florida; Poster, “Recent Results by the UC Berkeley Theory Group,” November 13, 2003


J. Vala, Quantum Information Science and Technology (Chem, CS, Phys 191) class, University of California, Berkeley, California; Guest Lecture, “Optical Lattice Quantum Computation,” November 25, 2003


O. Regev and D. Aharonov, Bay Area Theory Seminar (BATS), Mountain View, California; Invited Talk, “Lattice Problems in NP Intersect coNP,” January 2004

J. Von Korff, QIP’2004, Waterloo, Ontario, Canada; Poster, “Encoding Permutations into Quantum States,” January, 2004


S. Aaronson, UC Berkeley Quantum Reading Group, Berkeley, California; Invited Talk, “Quantum Advice and the Glorious Return of the Polynomial Method,” January 30, 2004


S. Aaronson, UC Berkeley, Berkeley, California; Contributed Talk, “MARKOVIA, A Quantum Lower Bound Saga Spanning Three Centuries,” February 6, 2004


J. Von Korff, Gordon Research Conference on Quantum Information Science, Ventura, California; Poster, “Encoding Permutations into Quantum States,” February, 2004

J. Vala, Gordon Research Conference on Quantum Information Science, Ventura, California; Poster, “Quantum Error Correction of a Qubit Loss,” February 22–27, 2004


I. Kerenidis, Caltech, Institute for Quantum Information, Pasadena, California; Invited Talk, “Exponential separation of quantum and classical one-way communication complexity,” March 2004; and Invited Talk at IBM Almaden Research Laboratory, San Jose, California, March 2004


S. Aaronson, UC Berkeley Quantum Computing Seminar, Berkeley, California; Invited Talk, “Improved Simulation of Stabilizer Circuits,” March 9, 2004

K. B. Whaley, Retirement Symposium for Mark S. Child, Oxford University, Oxford, United Kingdom; Invited Talk, “The Elusive Path to Quantum Computation,” March 27, 2004

J. Vala, Berkeley Quantum Control Seminar (EE 298-14), Department of Electrical Engineering, University of California, Berkeley, California; Invited Talk, “Control and Decoherence of Molecular Quantum States,” March 31, 2004

S. Aaronson, UC Berkeley Quantum Reading Group, Berkeley, California; Invited Talk, “Is Quantum Mechanics An Island In Theoryspace?” April 2, 2004

S. Aaronson, San Jose State University Computer Science Colloquium, San Jose, California; Invited Talk, “Quantum Computing and Hollywood,” April 15, 2004

S. Aaronson, Dissertation Talk, University of California, Berkeley, California; “Limits on Efficient Computation in the Physical World,” April 26, 2004


J. Kempe, UC Berkeley Mathematics Department Colloquium, University of California, Berkeley, California; Invited Talk, “Quantum Computation and the Symmetric Group,” April 29, 2004

J. Vala, CONtrol of QUantum mEchanical SysTems (CONQUEST) Workshop, University of California, Berkeley, California; Invited Talk, “Control and Decoherence of Quantum Molecular Vibrational Motion,” April 30, 2004

J. Vala, Department of Physics Colloquium, Santa Clara University, Santa Clara, California; Invited Talk, “Scalable Quantum Computation and Simulation with Neutral Atoms,” May 3, 2004


J. Kempe, UC Davis Statistics Colloquium, University of California, Davis, California; Invited Talk, “Quantum Walks—An Approach to Quantum Computing,” May 6, 2004

J. Vala, Yamamoto Group Meeting, Department of Physics, Stanford University, Palo Alto, California; Invited Talk, “Scalable Quantum Computation with Neutral Atoms,” May 14, 2004


S. Aaronson, Lorentz Center, Leiden, Netherlands; Invited Talk, “Quantum States That Pack An Exponential Punch,” May 24, 2004


D. M. Stamper-Kurn, Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California; Invited Talk, “Periodically-dressed Bose-Einstein condensates and other goings-on at Berkeley,” June 2, 2004


I. Kerenidis, Theory Seminar, University of California, Berkeley, California; Dissertation Talk, “The power of quantum encodings,” August 30, 2004


N. Shenvi, Jan von Delft Group Seminar, LMU, Munich, Germany; Invited Talk, “Hyperfine-Induced Electron Spin Decoherence,” September 7, 2004


J. Vala, Berkeley Quantum Information and Computation Seminar, University of California, Berkeley, California, U.S.A.; Invited Talk, “Quantum computation with neutral atoms in an addressable optical lattice,” September 21, 2004

K. B. Whaley, Joint Workshop of Center for Nanoscience (CeNS) and Ludwig Maximilian University of Munich, entitled “Nanoscience—linking disciplines,” San Servolo, Venice, Italy; Invited talk, “Rational design of optimally coherent solid state nanostructures,” September 27-October 1, 2004


D. Aharonov, FOCS 2004, Rome, Italy; Invited Talk, Presentation of “Adiabatic quantum computation is equivalent to standard quantum computation,” by D. Aharonov, W. van Dam, J. Kempe, Z. Landau, S. Lloyd and O. Regev, October 18, 2004


K. B. Whaley, Helsinki University of Technology (HUT), Materials Physics Department, Helsinki, Finland; Invited talk: “Quantum control of gates and circuits from solid state qubits,” November 25, 2004


K. B. Whaley, Technical University of Munich, Chemistry Department Seminar, Munich, Germany; Invited Talk, “Quantum Control of Gates and Circuits from Solid State Qubits,” December 9, 2004


N. A. Shenvi, University of California, Berkeley, Department of Chemistry, Head- Gordon Group Seminar, Berkeley, California, U.S.A.; Invited Talk, “Ground State Energies via the 2-Particle Reduced Density Matrix,” January 21, 2005

J. I. Korsbakken, Berkeley Quantum Information and Computation Seminar, UC Berkeley, California, U.S.A.; Invited Talk, “Topological Quantum Computing in an Extended Hubbard Model - or at least a nice try,” February 8, 2005

K. B. Whaley, University of California, Berkeley, Department of Chemistry, Physical Chemistry Seminar, Berkeley, California, U.S.A.; Invited Talk, “The Elusive Path to Quantum Computation,” February 8, 2005


T. Beals, Gordon Research Conference on Quantum Information Science, Ventura, California, U.S.A.; Poster Presentation, “Quantum computing with addressable optical lattices: simulation and optimization of one-qubit gates,” February 27–March 5, 2005


J. Korsbakken. Pendelvake, annual gathering for students and faculty at the Physics Department, University of Oslo, Norway. Invited talk entitled “Does God still play dice? -- Quantum mechanics, probability and (non-) determinism 50 years after Einstein,” May 2005.


R. Jain. 20th IEEE Conference on Computational Complexity (CCC), San Jose, California. Technical talk entitled “Prior entanglement, message compression and privacy in quantum communication,” June 2005


D. M. Stamper-Kurn, Center for Advanced Studies Thursday Seminar Series, University of New Mexico, Albuquerque, NM. Invited talk entitled “Seeing spinning condensates,” August 2005


D. S. Weiss. CIAR Cold Atom Workshop, Toronto, Canada, October, 2005, “1D Bose gases and optical lattice experiments”


D. S. Weiss. Division of Atomic and Molecular Physics (APS, DAMOP), Lincoln, Nebraska, May, 2005, “Experiments with 1D Bose gases: from Thomas-Fermi to Tonks-Girardeau”


