

# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Washington Headquarters Service, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington, DC 20503.

**PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

<b>1. REPORT DATE (DD-MM-YYYY)</b> JULY 2010		<b>2. REPORT TYPE</b> Journal Article Postprint		<b>3. DATES COVERED (From - To)</b> May 2008 – February 2010	
<b>4. TITLE AND SUBTITLE</b>  A GAME THEORETIC FRAMEWORK FOR POWER CONTROL IN WIRELESS SENSOR NETWORKS				<b>5a. CONTRACT NUMBER</b> In House	
				<b>5b. GRANT NUMBER</b> N/A	
				<b>5c. PROGRAM ELEMENT NUMBER</b> 62702F	
<b>6. AUTHOR(S)</b>  Shamik Sengupta, Mainak Chatterjee, and Kevin Kwiat				<b>5d. PROJECT NUMBER</b> 23G4	
				<b>5e. TASK NUMBER</b> IH	
				<b>5f. WORK UNIT NUMBER</b> 01	
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  AFRL/RIGG                                      University of Central Florida 525 Brooks Road                              Electrical Engineering and Computer Science Rome, NY 13441-4505                              Orlando, FL 32816				<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  N/A	
<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>  AFRL/RIGG 525 Brooks Road Rome NY 13441-4505				<b>10. SPONSOR/MONITOR'S ACRONYM(S)</b> N/A	
				<b>11. SPONSORING/MONITORING AGENCY REPORT NUMBER</b> AFRL-RI-RS-TP-2010-27	
<b>12. DISTRIBUTION AVAILABILITY STATEMENT</b> APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED. PA #: WPAFB 08-4920 Date Cleared: July 30, 2008					
<b>13. SUPPLEMENTARY NOTES</b> © 2010 IEEE. Published in the IEEE Transactions On Computers, Vol. 59, No. 2, pages 231-242; February 2010. One or more of the authors is a U.S. Government employee working within the scope of their Government job; therefore, the U.S. Government is joint owner of the work and has the right to copy, distribute, and use the work. All other rights are reserved by the copyright owner.					
<b>14. ABSTRACT</b>  In infrastructure-less sensor networks, efficient usage of energy is very critical because of the limited energy available to the sensor nodes. Among various phenomena that consume energy, radio communication is by far the most demanding one. One of the effective ways to limit unnecessary energy loss is to control the power at which the nodes transmit signals. In this paper, game theory is applied to solve the power control problem in a CDMA-based distributed sensor network.					
<b>15. SUBJECT TERMS</b> Wireless Sensor Networks, Game Theory, Distributed Power Control, Energy Efficiency					
<b>16. SECURITY CLASSIFICATION OF:</b>			<b>17. LIMITATION OF ABSTRACT</b>  UU	<b>18. NUMBER OF PAGES</b>  13	<b>19a. NAME OF RESPONSIBLE PERSON</b> Kevin A. Kwiat
<b>a. REPORT</b> U	<b>b. ABSTRACT</b> U	<b>c. THIS PAGE</b> U			<b>19b. TELEPHONE NUMBER (Include area code)</b> N/A

# A Game Theoretic Framework for Power Control in Wireless Sensor Networks

Shamik Sengupta, Mainak Chatterjee, and Kevin A. Kwiat

**Abstract**—In infrastructure-less sensor networks, efficient usage of energy is very critical because of the limited energy available to the sensor nodes. Among various phenomena that consume energy, radio communication is by far the most demanding one. One of the effective ways to limit unnecessary energy loss is to control the power at which the nodes transmit signals. In this paper, we apply game theory to solve the power control problem in a CDMA-based distributed sensor network. We formulate a noncooperative game under incomplete information and study the existence of Nash equilibrium. With the help of this equilibrium, we devise a distributed algorithm for optimal power control and prove that the system is power stable only if the nodes comply with certain transmit power thresholds. We show that even in a noncooperative scenario, it is in the best interest of the nodes to comply with these thresholds. The power level at which a node should transmit, to maximize its utility, is evaluated. Moreover, we compare the utilities when the nodes are allowed to transmit with discrete and continuous power levels; the performance with discrete levels is upper bounded by the continuous case. We define a distortion metric that gives a quantitative measure of the goodness of having finite power levels and also find those levels that minimize the distortion. Numerical results demonstrate that the proposed algorithm achieves the best possible payoff/utility for the sensor nodes even by consuming less power.

**Index Terms**—Wireless sensor network, game theory, distributed power control, energy efficiency.

## 1 INTRODUCTION

THE advancements in wireless communication technologies coupled with the techniques for miniaturization of electronic devices have enabled the development of low-cost, low-power, multifunctional sensor networks. The sensor nodes in these networks are equipped with sensing mechanisms that gather and process information. These nodes are also capable of communicating untethered over short distances [1]. Oftentimes, sensor networks are deployed at locations that do not allow human intervention due to difficulty in accessing such areas; hence, refurbishing energy via replacing battery is infeasible. As a result, these networks are deployed only once with finite amount of energy available to every sensor node. As energy is depleted for sensing, computing, and communication activity, the algorithms and protocols that are used must be as energy efficient as possible. Since the transmission of data signals consumes the most energy, transmission at the *optimal* transmit power level is very crucial. This is because a node will always try to transmit at high power levels just to make sure that the packets are delivered with a high-success probability. Though this is a good short-term

strategy, it proves to be counterproductive in the long run as energy is depleted faster. Also, transmitting at higher power levels will increase the interference to other nodes, which in turn will increase their power levels to combat the interference. This will create a “cascade” effect, where the nodes will continue to increase their power levels in response to the increased interference. Of course, transmission at lower power levels will compromise the quality of communication, and the desired quality of service might not be met. Hence, smart power control algorithms must be employed that find the optimal transmit power level for a node for a given set of local conditions. Some distributed iterative power control algorithms have been proposed for cellular networks; these algorithms investigate to find the power vector for all the nodes that minimizes the total power with good convergence [2], [3].

It is intuitive that the actions of a node, in response to other nodes’ actions, would be focused on maximizing their “profit.” The nodes run a very simple cost-evaluation function, and the appropriate response is motivated by what the nodes desire—usually determined by some *utility function*. Nodes adapt their behavior by learning their utility for each potential action through feedback. The feedback is just the profit or loss as defined by the overall objective function of the network. In this way, nodes dynamically react to changing network conditions, energy budgets, and external stimuli. Since all rational nodes will seek to optimize their utility, it makes more sense to program the utilities of the nodes in line with the system-wide goals such as energy conservation, data fidelity, or latency for data delivery.

In this respect, it is important that concepts from game theory are used to guide the design process of the nodes that work in a distributed manner. Ideas and fundamental results from game theory have been used for solving resource

• S. Sengupta is with the Department of Mathematics and Computer Science, John Jay College of Criminal Justice of the CUNY, New York, NY 10019. E-mail: ssengupta@jjay.cuny.edu.

• M. Chatterjee is with the School of Electrical Engineering and Computer Science, University of Central Florida, Orlando, FL 32816. E-mail: mainak@eecs.ucf.edu.

• K.A. Kwiat is with Information Directorate, Air Force Research Laboratory, Rome, NY 13441. E-mail: Kevin.Kwiat@rl.af.mil.

Manuscript received 22 Feb. 2008; revised 14 July 2008; accepted 29 May 2009; published online 10 June 2009

Recommended for acceptance by A. Zomaya.

For information on obtaining reprints of this article, please send e-mail to: tc@computer.org, and reference IEEECS Log Number TC-2008-02-0085.

Digital Object Identifier no. 10.1109/TC.2009.82.

management problems in many computational systems, such as network bandwidth allocation, distributed database query optimization, and allocating resources in distributed systems such as clusters, grids, and peer-to-peer networks ([4], [5], [6], [7], [8], [9], [10], [11], and references therein). In a game theoretic framework, the nodes buy, sell, and consume goods in response to the prices that are exhibited in a virtual market. A node attempts to maximize its “profit” for taking a series of actions. Whether or not a node receives a profit is defined by the success of the action; for example, whether a packet is successfully received. The node’s goal is to maximize its profit, subject to constraints on resource usage, such as energy or bandwidth. By modeling the objective function (e.g., optimizing energy usage) of the network as a profit, the nodes can be stimulated to work in a cooperative manner. The essence of this research is the application of game theory to achieve efficient energy usage through optimal selection of the transmit power level.

In this paper, we take a game theoretic approach to regulate the transmit power levels of the nodes in a distributed manner and investigate if any optimality is achievable. We focus on the problem of optimal power control in wireless sensor networks with the aim of *maximizing the net utilities* (defined later) for the rational sensor nodes. Due to the distributiveness of the network, the nodes do not have complete information about the strategies taken by other nodes, and thus the games are categorized as incomplete-information games. Also, cooperative behaviors (such as transmission power control, cooperation for increasing system capacity, reducing interference for each other, honestly revealing private information) though highly desired, might not be achievable. One may argue that the sensor nodes usually belong to the same authority, and hence they can be programmed to negotiate strategies that is most advantageous for the entire network. However, this claim may not be applicable to the power control problem in sensor networks as strategies for transmission power and negotiation for self-coexistence must be done in real time and distributed manner [12], [13].

We adopt a noncooperative game model where each node tries to maximize its net utility. Net utility is computed by considering the benefit received and the cost incurred for packet transmissions. As we do not use any information about the separate transmitting power level strategies taken by other nodes, control signals are greatly reduced, thereby helping nodes in conserving energy. We also study practical systems where the nodes are allowed only discrete power levels. We compare such restricted cases with systems that allow continuously variable power levels and show that these continuous power levels provide a bound on the performance. In summary, the contributions of this paper are as follows:

- We formulate a noncooperative game under incomplete information for the distributed sensor nodes. We define the benefit received and the cost incurred, and hence the net utility for successful packet transmission.
- We investigate the existence of Nash equilibrium [14]. We do so for two different cases—with fixed channel and varying channel conditions. We observe

that there exists a transmission power threshold and channel quality threshold that the nodes must comply with in order to achieve Nash equilibrium. We also observe that with repeated games in effect, sensor nodes follow the transmission strategies to achieve Nash equilibrium even without presence of any third party enforcement.

- Next, we consider a system that would allow only finite number of discrete power levels. A metric called distortion factor is defined to investigate the performance of such system and compare it with systems that would allow any continuous power levels. We also propose a technique to find the power levels that would minimize the distortion.
- We present numerical results to verify the performance of the proposed games. The results show that if the nodes comply with the transmit thresholds, net utility is maximized. Also with the proposed mechanism of finding discrete power levels, distortion factor is reduced.

The rest of the paper is organized as follows: In Section 2, we discuss the basics of game theory and show how this theory has been applied to ad hoc and sensor networks in the past. The distribution function for the number of interfering sensor nodes is derived in Section 3. We formulate the noncooperative game under incomplete information and establish the utility functions in Section 4. We study the existence of Nash equilibrium in Section 5 and calculate the thresholds for transmission power and channel conditions based on the equilibrium obtained. In Section 6, we evaluate the desired power level for transmission so that expected power efficiency and thus utility can be maximized. The effect of having discrete allowable power levels and how to obtain those levels are discussed in Section 7. In Section 8, we present the numerical results. Conclusions are drawn in the last section.

## 2 GAME THEORY FOR AD HOC/SENSOR NETWORKS

Game theory has been successfully used in ad hoc and sensor networks for designing mechanisms to induce desirable equilibria by offering incentives to the forwarding nodes [15], [16], [17] and also punishing nodes for misbehaving [18]. Recently, there has been a growing interest in applying game theoretic techniques to solve problems where there are agents/nodes that might not have the motive or incentive to cooperate. Such noncooperation is very likely since the rational agents will not work (e.g., forward packets) for others unless, and until, convinced that such cooperation will eventually be helpful for themselves.

In [13], Niyato et al. investigated energy harvesting technologies required for autonomous sensor networks using a noncooperative game theoretic technique. Nash equilibrium was proposed as the solution of this game to obtain the optimal probabilities of the two states, viz., sleep and wake up, that were used for energy conservation. Their solutions revealed that sensor nodes selfishly try to conserve energy at the expense of high packet blocking probability. Xidong et al. applied game theoretic dynamic power management (DPM) policy for distributed wireless

sensor network using repeated stage games [19]. A game theoretic energy balance routing (GTEBR) algorithm has been proposed to avoid uneven energy consumption in wireless sensor networks in [20]. The framework of static game with incomplete information was modeled and existence of Nash equilibrium was also proved. Energy efficient self-organization protocol for wireless sensor networks with reduced coordination was studied by Olariu et al. in [21]. Maskery and Krishnamurthy described decentralized, game theoretic adaptive mechanisms, which can be deployed to manage sensor activities with low-coordination overhead [22]. Evolutionary game model was discussed with dynamically adaptive regret matching. In [23], Chang and Tassiulas investigated the energy efficiency problem in wireless sensor networks as the maximum network lifetime routing problem. They proposed to adjust the transmit power levels to just reach the intended next hop receiver such that the energy consumption rate per unit information transmission can be reduced.

As far as ad hoc networks are concerned, Buttyan and Hubaux [16] proposed the concept of virtual currency (called “nuglets”) which is a method to reward nodes participating in forwarding packets in a mobile ad hoc network. The Terminodes project [24] has proposed a method that encourages cooperation in ad hoc networks that is based on the principles laid out in [25]. It has been well established that incorporating pricing schemes (in terms of reward and penalty) can stimulate a cooperative environment, which benefits both the network and the nodes. A traffic-pricing-based approach was proposed in [26]. The compensation of the traffic forwarding depends not only on the energy consumption of the transmission but also on the congestion level of the relaying node. As far as *selfishness* in ad hoc networks is concerned, Srinivasan et al. [18] explored the selfish behavior of nodes and showed the effect on the evolution of the topology. In [27], two techniques were presented that deal with selfish nodes in an ad hoc network, where a watchdog identifies the misbehaving nodes. Also, there are nodes that rate the routes and help routing protocols avoid routes containing misbehaving nodes. In [15], the “confidant” protocol was proposed that not only detects misbehavior and routes traffic around the misbehaving nodes but also isolates them from the network. The “core” protocol proposed in [28] was based on profiling the nodes, i.e., each node maintains a reputation table for the other nodes. The reputation value is updated based on the node’s own observations and the information provided by the other nodes. In [29], a power-allocation and signal shaping game is presented for multiple-antenna “ad hoc” networks with the help of iterative power-control and signal-shaping algorithms. Multiple uncoordinated transmit-receive nodes in wireless ad hoc network simultaneously attempt to communicate and undergo *learning* and *training* phase in this algorithm. The authors deal with distributed power allocation in ad hoc networks, and the objective is the competitive maximization of the information throughput perceived in the network. In [30], Michiardi and Molva analyzed whether it is beneficial for a node to join a network under certain assumptions. Through a utility function (that captures the node’s payoff

and its resource consumption), a node decides whether to cooperate or defect.

Though game theory has been used to study various aspects of ad hoc and sensor networks, there is none that tries to find the optimal transmission power levels when the nodes are allowed both continuous and discrete power levels. The problem arises due to the difficulty in characterizing the information that each sensor node has about the others. Hence, seeking the desired operating point in the incomplete-information scenario becomes a challenge. Though there are several game theory power control approaches for cellular networks (see [31] and references therein), those centralized algorithms cannot be directly applied to sensor networks. In this paper, we attempt to develop a game theoretic framework that helps the nodes decide on the optimal power levels for a specified objective given by the utility function.

### 3 INTERFERENCE FOR RANDOMLY DISTRIBUTED NODES

We consider the problem of communication between neighboring nodes in a network that consists of sensor nodes scattered randomly over an area. Given that the sensor nodes have limited energy, buffer space, and other resources, contention-based protocols may not be a suitable option. Here, as an alternative, we use code division multiplexing, where distinct codes (signatures) can be allocated to different nodes with possible code reuse between spatially separated nodes. Note that we do not necessarily consider signature codes with perfectly zero cross-correlation (such as the Walsh-Hadamard code sets) because of 1) the restriction in the number of available orthogonal codes and 2) the loss of orthogonality in practice due to physical layer asynchronicity and/or multipath signal propagation. In general, due to nonzero cross-correlation between node signatures, we understand that there is an upper limit in the number of simultaneously active nodes in the vicinity of a receiver (i.e., within the interference range of a receiver) so that the received SINR stays above a minimum operational threshold. To obtain the node distribution, we use the following assumptions and definitions:

- All nodes have an omnidirectional transmit and receive antenna of the same gain.
- Receiving and interference ranges for each sensor node depend on the transmission power of the sender and the other sensor nodes in vicinity.
- The *receiving distance*,  $r_R$ , is defined as the maximum distance from which a receiving node can correctly recover a transmitted signal.
- The *interference distance*,  $r_I$ , is defined as the maximum distance from which a receiving node can sense a carrier.
- The signal power level at each receiver is controlled by the corresponding transmitter and is equal to the lowest possible operational threshold. Since the internodal distance varies randomly, the required transmit power is different for different transmitter-receiver pairs.

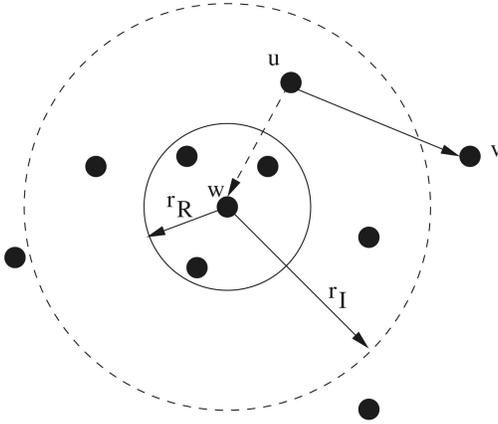


Fig. 1. Interference at node  $w$  from a local neighbor node  $u$ .

Fig. 1 shows node  $w$  as the receiver under consideration. Node  $u$ , while transmitting to node  $v$ , acts as an interferer to node  $w$ . Note that the reverse need not necessarily be true since the transmission power of node  $u$  and node  $w$  can be different. We now proceed to obtain an expression for the average number of *potential* interfering neighbors around a receiver. A rigorous treatment for the distribution of interference power can be found in [32].

### 3.1 Node Distribution

The interference area is  $a_I = \pi r_I^2$ . We consider that there are  $M$  nodes that are uniformly randomly scattered over a region of area  $A$ . If  $M \gg 1$  and  $a_I \ll A$ , we can approximate the distribution of nodes using a Poisson distribution with node density as  $\rho = \frac{M}{A}$ . Theoretically, for randomly scattered nodes, the maximum number of interferers can extend up to infinity. For all practical purposes, we can consider the maximum number of interferers to be the expected value plus a multiple of standard deviation,  $\sqrt{\sigma}$ . For example, if we were to account for 99.6 percent of all the nodes, then we have to consider  $3 \times \sqrt{\sigma}$  in addition to the mean [33]. The expected (mean) number of nodes in area  $a_I$  is  $\rho a_I$ . For a Poisson distribution, the variance ( $\sigma$ ) is equal to  $\rho a_I$ . Thus, the maximum number of interferers for this case can be given by

$$N = \rho a_I + 3\sqrt{\rho a_I}. \quad (1)$$

Note, we considered Poisson distribution for the layout of the nodes. This is primarily because of its mathematical tractability. We could have used any other distribution for the nodes. However, this would have no bearing on the analysis that follows.

## 4 NONCOOPERATIVE GAME UNDER INCOMPLETE INFORMATION

With the average number of interferers for a node known, we formulate a game for such a distributed sensor network and then try to devise the game strategies to find if any steady state equilibrium exists for this game model.

### 4.1 Game Formulation

We assume a set of homogeneous nodes in our sensor network playing repeated game. The information from

previous rounds are used to devise strategies in future rounds. We focus our attention on a particular node with potentially as many as  $N$  neighbors within the interference range. Due to homogeneity of the nodes, the actions allowed by the nodes are the same, i.e., all the nodes can transmit with any power level to make its transmission successful. Also, the nodes have no information if the other nodes are transmitting, hence leading to an incomplete information scenario [34]. If the nodes transmit with an arbitrary high power level, it will increase the interference level of the other nodes. The neighboring nodes in turn will transmit at higher power to overcome the effect of high interference. Soon, this will lead to a noncooperative situation. To control this noncooperative behavior, we try to devise an equilibrium game strategy which will impose constraints on the nodes to act in cooperative manner even in a noncooperative network.

We assume the existence of some strategy sets  $S_1, S_2, \dots, S_{N+1}$  for the nodes  $1, 2, \dots, (N+1)$ . These sets consist of all possible power levels ranging from the minimum transmit power  $s_{min}$  to maximum transmit power  $s_{max}$ . Note that  $s_{min}$  can be zero.  $s_{min} = 0$  implies that a node decides not to transmit at that game iteration. However, the question is: Will there be any finite value of  $s_{max}$  that the nodes will follow and will still be able to maximize their benefits? In other words, does there exist any transmission power upper bound obeying which nodes can reach Nash equilibrium and can extend their network lifetime? We investigate our dynamic repeated game keeping this in mind.

In this game, if node 1 chooses its power level  $s_1 \in S_1$ , node 2 chooses its power level  $s_2 \in S_2$ , and so on, we can describe such a set of strategies chosen by all  $N+1$  (a node with its  $N$  neighbors) nodes as one ordered  $N+1$ -tuple,

$$\mathbf{s} = \{s_1, s_2, \dots, s_{N+1}\}. \quad (2)$$

This vector of individual strategies is called a strategy profile (or sometimes a strategy combination). For every different combination of individual choices of strategies, we would get a different strategy profile  $\mathbf{s}$ .

The set of all such strategy profiles is called the space of strategy profiles  $\mathbf{S}'$ . It is simply the Cartesian product of the power vectors  $S_i$  for each node. We write it as

$$\mathbf{S}' = S_1 \times S_2 \times \dots \times S_{N+1}. \quad (3)$$

We consider that the strategy profile of all the nodes are identical, i.e., all the nodes can transmit with a power level between  $s_{min}$  and  $s_{max}$ . Since all the nodes are identical, we assume that the set of allowable transmit powers is applicable to all the nodes. Hence,  $S_i = S_j = S$ , where  $i$  and  $j$  denote any two nodes, and  $S$  is the fixed strategy profile of any node. Then, (3) reduces to

$$\mathbf{S}' = S \times S \times \dots (N+1) \text{ times}. \quad (4)$$

### 4.2 Utility

The game is played by having all the nodes simultaneously pick their individual strategies. This set of choices results in some strategy profile  $\mathbf{s} \in \mathbf{S}'$ , which we call the outcome of the game. Each node has a set of preferences over these outcomes  $\mathbf{s} \in \mathbf{S}'$ . We assume that each node's preferences

over  $S$  can be represented by von Neumann-Morgenstern utility function [35].

At the end of an action, each node  $i \in I$  receives a utility value  $u_i(\mathbf{s}) = u_i(s_i, \mathbf{s}_{-i})$ .  $\mathbf{s}_{-i}$  is the strategy profile of all the nodes but for the  $i$ th node. Note that the utility each node receives depends not only on the strategy it picked, but also on the strategies which all the other nodes picked. In other words, the utility to any one node depends on the entire strategy profile. The individual utilities for all the  $N + 1$  nodes for a particular strategy profile  $\mathbf{s}$  define a utility vector  $\mathbf{u}(\mathbf{s})$  for that strategy profile  $\mathbf{u}(\mathbf{s}) \equiv (u_1(\mathbf{s}), \dots, u_{N+1}(\mathbf{s}))$ .

With respect to the problem at hand, in every game round, a node will choose to transmit or not to transmit, or to increase or decrease its power, and correspondingly will choose a power level if it decides to transmit. Theoretically, a node should choose its own power level depending on the power levels chosen by all other nodes; but in a distributed sensor network, it is not possible for a node to know about the transmitting strategies of other nodes.

Based on the information available to a node, i.e., its own power level, channel condition, and expected SINR of neighboring receiver nodes (which is obtained through periodic acknowledgment received), we formulate the utility expression for a transmitter sensor node. With the notations already defined and to emphasize that the  $i$ th node has control over its own power level  $s_i$  only, we define the utility (if a node is transmitting) as [36]

$$u_i(s_i, \mathbf{s}_{-i}) = \frac{br}{F s_i} f(\gamma_j), \quad (5)$$

where node  $i$  is transmitting to a node  $j$ , and  $b$  is the number of information bits in a packet of size  $F$  bits.  $r$  is the transmission rate in bits/sec using strategy  $s_i$ .  $f(\gamma_j)$  is the efficiency function which increases with expected SINR of the receiving node. We define the efficiency function,  $f(\gamma_j) = (1 - 2P_e)^F$ , where  $P_e$  is the bit error rate (BER). The bit error rate,  $P_e$ , depends on the channel state and interference from other nodes, i.e., in other words,  $P_e$  is a function of SINR. Though the instantaneous knowledge of SINR will help a node decide on its strategy, this information is not known a priori; however, an expected SINR can be used which the sensor nodes compute based on past observations. Correspondingly,  $P_e$  can only be estimated; for example, with a noncoherent FSK modulation scheme,  $P_e = 0.5e^{-\frac{\gamma_j}{2}}$ , where  $\gamma_j$  denotes the expected SINR of node  $j$ . We also assume that the utility value obtained by a node when it decides not to transmit is 0.

### 4.3 Net Utility

With the utility of a node defined, let us consider the cost/penalty incurred by a node. We assume that the each sensor node tries to maximize its own utility by adjusting its own power optimally as given by utility function. The utility function from a sensor node's perspective takes into account the interference it gets from other nodes; however, it ignores the fact that this node imposes on itself in terms of drainage of energy. Pricing (or regulating cost) has been shown to be effective in regulating this externality, as it encourages the nodes to use resources more efficiently. We use pricing (cost) as a negative incentive signal to model

the usage-based cost that a sensor node must pay for using the resource. Hence, we consider a cost component that accounts for the energy consumed/drawn by the sensor nodes with usage of resources (transmission power). Therefore, we define a metric: net utility, which is the utility achieved minus the cost incurred. This justifies the rational (self-optimizing) behavior of a sensor node even in the distributed scenario. As a result, a node transmitting at a high power will increase the probability of success, but at the same time, it will incur the cost of transmission, providing the possibility to decrease the net utility. Thus, each of the rational sensor nodes' objective function would be to optimize the transmission power such that it can sustain maximized net utility. Formal definition for the cost component is presented in the next section.

## 5 TRANSMISSION THRESHOLDS FOR NASH EQUILIBRIUM

With the strategy set and net utility defined, a game is played by all the players/nodes simultaneously picking their individual strategies. This set of choices results in some strategy profile  $\mathbf{s} \in S$ . We typically assume that all the players are rational and pick their strategy in such a way so as to maximize their utilities. If there is a set of strategies with the property that no player can benefit by changing his strategy unilaterally while the other players keep their strategies unchanged, then that set of strategies and the corresponding utilities constitute the *Nash equilibrium*. In other words, when all players correctly forecast their opponents' strategies and play their *best responses* to these forecasts, the resulting strategy profile is a Nash equilibrium; this exists if and only if there exists a fixed point of a particular best-response correspondence. In this section, we study the existence of Nash equilibrium for two different scenarios—fixed channel conditions and varying channel conditions.

### 5.1 Nash Equilibrium with Fixed Channel Condition

When channel condition is fixed, to gain better utility, nodes try to transmit at a high power which eventually drains their batteries. If the strategy of the  $i$ th node is to transmit at signal power  $s_i \in S$ , the cost incurred is a function of  $s_i$ , which we denote by  $A(s_i)$ .  $s_i$  is a random variable denoting transmitting signal power of  $i$ th node. Motivated by the physical significance of price (cost),  $A(s_i)$  should increase monotonically with the transmit power  $s_i$  (necessary condition) and should also be convex (sufficient condition). The reason for this is as follows: Consider two nodes 1 and 2, transmitting at powers  $s_1$  and  $s_2$ , respectively, such that  $s_1 \ll s_2$ . Let both nodes increase their transmit powers by  $\delta$  units. The impact on the total lifetime of node 1 would then be much lesser than that of node 2. Therefore, the cost (penalty) experienced by node 2 for transmitting an additional power of  $\delta$  units is more than that experienced by node 1. Thus, the rate of change of the cost with respect to  $s_i$ ,  $\frac{dA}{ds_i}$ , is nondecreasing. Hence,  $A(s_i)$  is a convex function of  $s_i$ . Note that a linear pricing is a special case of a convex pricing. Thus, the net utility,  $u_i^{net}$ , of  $i$ th node can be written as

$$u_i^{net} = \begin{cases} u_i(s_i, \mathbf{s}_{-i}) - A(s_i), & \text{if transmitting,} \\ 0, & \text{if not transmitting.} \end{cases} \quad (6)$$

Let us now analyze the existence of Nash equilibrium. If a node is allowed to transmit at the calculated power, however high, then it is obvious that

$$\int_0^{\infty} f_{s_i}(x)dx = 1, \quad (7)$$

i.e., the node transmits with probability 1.  $f_{s_i}(x)$  is the probability density function of  $s_i$ . However, for all practical purposes, a node cannot transmit at arbitrarily high power and must decide on a maximum threshold power  $P_t$ . The imposition of the threshold implies that the node will not transmit if its calculated transmit power is above the threshold  $P_t$ . In other words, exceeding this threshold will introduce nonbeneficial net utility for the node. Due to this restriction, the probability that a node transmits (or does not transmit) will be dependent on the threshold  $P_t$ ; smaller the threshold, smaller the probability of transmission. A node transmits at a power level  $s_i$  such that  $0 < s_i \leq P_t$ . Then, the probability that a node is transmitting can be given by

$$\int_0^{P_t} f_{s_i}(x)dx = p(P_t). \quad (8)$$

This definition of  $p(P_t)$  assumes that a node chooses *not* to transmit if the interference and noise level of the receiver is above a certain threshold.

Now, a node can be in two modes: either it is transmitting or it is not transmitting (idle or receiving). Then, we can assume that the probability with which a node transmits is  $p(P_t)$  and is idle or receiving with probability  $1 - p(P_t)$ .

To prove the existence of Nash equilibrium, we assume that a maximum power threshold exists and all the nodes act rationally and maintain the maximum power threshold  $P_t$ . The probability that any  $l$  nodes out of  $N$  nodes are active is given by

$$p_l = \binom{N}{l} (p(P_t))^l (1 - p(P_t))^{N-l}. \quad (9)$$

The expected net utility of  $i$ th node (if the node is transmitting) is given by

$$E[u_i^{net}] = \sum_{l=0}^N (u_i(s_i, \mathbf{s}_{-i}) - A(s_i)) p_l. \quad (10)$$

As  $\sum_{l=0}^N C_l^N (p(P_t))^l (1 - p(P_t))^{N-l} = 1$ , (10) can be rewritten as

$$E[u_i^{net}] = \sum_{l=0}^N u_i(s_i, \mathbf{s}_{-i}) p_l - A(s_i). \quad (11)$$

If we define  $U_i(P_t)$  as

$$U_i(P_t) = \sum_{l=0}^N u_i(s_i, \mathbf{s}_{-i}) p_l, \quad (12)$$

then the expected net utility obtained by  $i$ th node is given by

$$E[u_i^{net}] = U_i(P_t) - A(s_i). \quad (13)$$

If the node is transmitting, then the expected net utility is given by (13). If the node is not transmitting, then by definition (6) the expected net utility is 0. Thus, the *achievable gain* (net utility considering both modes:

transmitting with  $0 < s_i \leq P_t$ , and not transmitting) obtained by node  $i$  is

$$\begin{aligned} G_i(P_t) &= \int_0^{P_t} [U_i(P_t) - A(x)] f_{s_i}(x) dx \\ &= U_i(P_t) p(P_t) - \int_0^{P_t} A(x) f_{s_i}(x) dx. \end{aligned} \quad (14)$$

For the sake of convenience, let us denote

$$\int_0^{P_t} A(x) f_{s_i}(x) dx = B(P_t).$$

Then, (14) can be written as

$$G_i(P_t) = U_i(P_t) p(P_t) - B(P_t). \quad (15)$$

As far as the Nash equilibrium point is concerned, the expected net utility for transmitting and for being silent should be equal at the threshold, i.e.,  $s_i = P_t$ . Therefore, the solution to the equation

$$U_i(P_t) - A(P_t) = 0 \quad (16)$$

is the required threshold for the power level. Next, we show that if nodes follow this threshold  $P_t$ , then even without the knowledge of other nodes' power levels, the system can attain Nash equilibrium, i.e., all nodes will reach a stable state where the *gain* of an individual node cannot be increased further by unilaterally changing the strategy of that node.

Let us assume  $T_1$  be the solution to (16). Then, the average achievable gain of  $i$ th node obtained from the system is given by

$$\begin{aligned} G_i(T_1) &= \int_0^{T_1} [U_i(T_1) - A(x)] f_{s_i}(x) dx \\ &= U_i(T_1) p(T_1) - B(T_1). \end{aligned} \quad (17)$$

But suppose that a node unilaterally changes its strategy and changes the threshold value to  $T_2$ . Then, the average achievable gain obtained by this particular node is given by

$$\begin{aligned} G_i(T_2) &= \int_0^{T_2} [U_i(T_1) - A(x)] f_{s_i}(x) dx \\ &= U_i(T_1) p(T_2) - B(T_2). \end{aligned} \quad (18)$$

The difference,  $(G_i(T_1) - G_i(T_2))$ , can be then given by the following expression:

$$[U_i(T_1) p(T_1) - B(T_1)] - [U_i(T_1) p(T_2) - B(T_2)]. \quad (19)$$

We use (16) to find the value of  $U_i(T_1)$ . Substituting the value of  $U_i(T_1)$  in the above equation, we get

$$G_i(T_1) - G_i(T_2) = A(T_1) [p(T_1) - p(T_2)] - [B(T_1) - B(T_2)]. \quad (20)$$

Two cases might arise depending on the relative values of  $T_1$  and  $T_2$ .

**Case 1:**  $T_1 > T_2$

In this case, (20) can be written as

$$G_i(T_1) - G_i(T_2) = \int_{T_2}^{T_1} [A(T_1) - A(x)] f_{s_i}(x) dx. \quad (21)$$

Since  $A(s_i)$  is an increasing function of power level  $s_i$ ,  $A(T_1) - A(x) > 0$  for  $x < T_1$ . Therefore, for  $T_1 > T_2$ ,

$$G_i(T_1) - G_i(T_2) > 0. \quad (22)$$

**Case 2:**  $T_1 < T_2$

In this case, (20) can be written as

$$G_i(T_1) - G_i(T_2) = - \left[ \int_{T_1}^{T_2} [A(T_1) - A(x)] f_{s_i}(x) dx \right]. \quad (23)$$

Applying the same logic as in Case 1, we find  $A(T_1) - A(x) < 0$  for  $T_1 < x \leq T_2$ , which gives

$$G_i(T_1) - G_i(T_2) > 0. \quad (24)$$

Thus, for both the cases, we find that  $G_i(T_1) > G_i(T_2)$ . This shows that a node's average achievable gain cannot be increased further by changing its strategy unilaterally. Therefore, Nash equilibrium exists only if the nodes agree to abide by the threshold power level.

## 5.2 Nash Equilibrium with Varying Channel Conditions

When channel conditions vary, it might be advantageous to transmit packets when the instantaneous channel is better than a certain threshold. This will ensure that packets are transmitted and received successfully with a higher probability. Thus, it is intuitive that the cost/penalty to the nodes will depend on the channel conditions in addition to the cost component  $A(s_i)$ ; better the channel conditions, lower the cost. Thus, we define the additional cost as inversely proportional to the varying channel condition  $C$ . Let this cost be  $\xi\left(\frac{1}{C}\right)$ , which is a decreasing function with  $C$ . Then, the net utility of  $i$ th node can be written as

$$v_i^{net} = \begin{cases} u_i^{net} - \xi\left(\frac{1}{C}\right), & \text{if transmitting,} \\ 0, & \text{if not transmitting.} \end{cases} \quad (25)$$

To prove the existence of Nash equilibrium, we hypothesize that a node should transmit, only if its channel condition is better than a given threshold. Let this threshold be  $C_t$ . Therefore, the probability of a node transmitting is

$$\int_{C_t}^{\infty} f_C(x) dx = p'(C_t). \quad (26)$$

Let  $f_C(x)$  be the probability density function of  $C_t \leq C < \infty$ . Then, proceeding in a similar manner as done in Section 5.1, the expected net utility of node  $i$  is given by

$$E[v_i^{net}] = U'_i(C_t) - \xi\left(\frac{1}{C}\right). \quad (27)$$

The gain obtained by the node  $i$  is then

$$\begin{aligned} G'_i(C_t) &= \int_{C_t}^{\infty} \left[ U'_i(C_t) - \xi\left(\frac{1}{x}\right) \right] f_C(x) dx \\ &= U'_i(C_t) p'(C_t) - B'(C_t), \end{aligned} \quad (28)$$

where  $B'(C_t) = \int_{C_t}^{\infty} \xi\left(\frac{1}{x}\right) f_C(x) dx$ .

Now we will show that if the nodes act rationally and transmit only when the channel condition is better than  $C_t$ ,

then Nash equilibrium can be reached. As before, the solution to

$$U'_i(C_t) - \xi\left(\frac{1}{C_t}\right) = 0$$

gives the value of the threshold.

Let  $C_1$  be the solution. Then, the average achievable gain of  $i$ th node is given by

$$\begin{aligned} G'_i(C_1) &= \int_{C_1}^{\infty} \left[ U'_i(C_1) - \xi\left(\frac{1}{x}\right) \right] f_C(x) dx \\ &= U'_i(C_1) p'(C_1) - B'(C_1). \end{aligned} \quad (29)$$

Suppose, a node unilaterally changes its strategy and decides the threshold to be  $C_2$ . Then, the average achievable gain for that node will be

$$\begin{aligned} G'_i(C_2) &= \int_{C_2}^{\infty} \left[ U'_i(C_1) - \xi\left(\frac{1}{x}\right) \right] f_C(x) dx \\ &= U'_i(C_1) p'(C_2) - B'(C_2). \end{aligned} \quad (30)$$

The difference,  $(G'_i(C_1) - G'_i(C_2))$ , in the gain is given by

$$\begin{aligned} &[U'_i(C_1) p'(C_1) - B'(C_1)] - [U'_i(C_1) p'(C_2) - B'(C_2)] \\ &= U'_i(C_1) [p'(C_1) - p'(C_2)] - [B'(C_1) - B'(C_2)] \\ &= \xi\left(\frac{1}{C_1}\right) [p'(C_1) - p'(C_2)] - [B'(C_1) - B'(C_2)]. \end{aligned}$$

Again, two cases might arise depending on the relative values of  $C_1$  and  $C_2$ .

**Case 1:**  $C_1 > C_2$

$$G'_i(C_1) - G'_i(C_2) = - \left[ \int_{C_2}^{C_1} \left[ \xi\left(\frac{1}{C_1}\right) - \xi\left(\frac{1}{x}\right) \right] f_C(x) dx \right] > 0. \quad (31)$$

**Case 2:**  $C_1 < C_2$

$$G'_i(C_1) - G'_i(C_2) = \left[ \int_{C_1}^{C_2} \left[ \xi\left(\frac{1}{C_1}\right) - \xi\left(\frac{1}{x}\right) \right] f_C(x) dx \right] > 0. \quad (32)$$

Thus, we find that a node cannot increase its gain by unilaterally changing its strategy.

## 6 DETECTING TRANSMISSION POWER

In the previous section, we have evaluated the maximum power level and minimum channel condition that a node must comply with in order to achieve Nash equilibrium. However, the optimal transmit power needs to be also evaluated which will depend on the SINR, which in turn depends on the strategies adopted by the other nodes. We have two questions in hand that need to be addressed: 1) for unreliable link, what is the probability of successful transmission, and 2) with this probability of successful transmission, what is the expected power consumption?

First, we try to find the probability of successful transmission. We assume that node  $i$  is transmitting to node  $j$ . Node  $j$  not only hears from node  $i$  but also from other neighboring nodes if they are transmitting; these

signals appear as interference. Now, if  $\gamma_j$  is the SINR perceived by node  $j$ , then the bit error probability for the link ( $i \rightarrow j$ ) is given by some inverse function of  $\gamma_j$ . For example, with noncoherent FSK modulation scheme,  $P_e = 0.5e^{-\frac{\gamma_j}{2}}$ , or with DPSK modulation scheme,  $P_e = 0.5e^{-\gamma_j}$ , where  $P_e$  is the bit error probability of the link ( $i \rightarrow j$ ). The probability of successful transmission of a packet containing  $F$  bits from node  $i$  to node  $j$  can be given by

$$p_s = (1 - P_e)^F. \quad (33)$$

For simplicity, the bit corruption is assumed to be independently and identically distributed. It is clear from the above definition that with increased  $\gamma_j$ , the bit error probability decreases, which in turn increases the probability of successful transmission and vice versa.

With the probability of successful transmission defined, we need to find the desired transmit power level for a link over which the packets are to be transmitted. Before doing so, let us try to find the expected power consumption. We consider a scenario where a node is allowed to retransmit a packet if a transmission is unsuccessful, and it continues to retransmit until the transmission is successful. Let the power level chosen by the transmitter node be  $P$ , and there are  $(n - 1)$  unsuccessful transmission followed by successful transmission. Then, the expected power consumption by the transmitter node can be given by

$$\begin{aligned} E[\text{Power Consumption}]_P &= \sum_{n=1}^{\infty} n(1 - p_s)^{n-1} \times p_s \times P \\ &= \frac{P}{p_s}. \end{aligned} \quad (34)$$

With the power consumption given in (34), we define the expected power efficiency for power level  $P$  as an inverse function of the expected power consumption. Then, the optimal transmit power is the power level, which will maximize the expected power efficiency.

## 7 DISCRETE POWER LEVELS

So far, we have assumed that the power levels chosen by the nodes are continuous, i.e., the nodes can choose any value between the maximum and the minimum power levels. However, most practical (real) systems allow a finite set of predefined power levels. For example, Cisco WLAN cards can be configured with six power levels [37].

Since a practical system would allow finite number of power levels, it is important that the values are chosen carefully. This problem is similar to a scalar quantization problem. In *scalar quantization*, a one-dimensional space is partitioned into multiple nonoverlapping regions and all points in a region map to a representative point of that region. More precisely, an  $L$ -point scalar quantizer,  $\mathcal{Q}$ , is a mapping function such that  $\mathcal{Q} : \mathcal{R} \rightarrow \mathcal{S}$ , where  $\mathcal{R}$  is the real number line and

$$\mathcal{S} \equiv \{M_1, M_2, M_3, \dots, M_L\} \subset \mathcal{R}.$$

The output set  $\mathcal{S}$  has  $L$  power levels, where  $M_i$  ( $1 \leq i \leq L$ ) are the power levels. The goodness of  $\mathcal{S}$  is usually measured by a *distortion* metric which is defined as the nonnegative cost  $d(x, \hat{x})$  associated with quantizing any input  $x$  with a reproduction  $\hat{x}$ . The choice of  $\mathcal{S}$  is said to be *optimal* if it minimizes the average distortion which quantifies the performance of the system.

### 7.1 Distortion Factor

We define distortion factor,  $D$ , as the difference between the best possible net utility obtainable with continuous power level and the best possible net utility obtained with  $L$  discrete power levels. Given the transmission powers in both continuous and discrete cases, respectively, as  $P_C$  and  $P_D$ , the distortion factor for the  $i$ th node is represented by

$$D = u_i(P_C, \mathbf{s}_{-i}) - u_i(P_D, \mathbf{s}_{-i}), \quad (35)$$

where  $\mathbf{s}_{-i}$  represents the strategy profile of rest of the nodes. With increase in number of power levels  $L$ , the distortion can be reduced.

### 7.2 Choice of Power Levels for Fixed $L$

It is well understood that having larger number of transmit power levels allows the nodes to better utilize their (energy) resources and also regulates the SINR of the system with a finer precision. But the question is: given the number of power levels ( $L$ ), what values of transmit power will best span the range from minimum to maximum? One naive way is to have the  $L$  values uniformly spaced between the minimum and the maximum. A more rational approach is to determine the  $L$  values from the probability density function of the interference as observed by a node. We propose to use this interference distribution as a guideline for determining the  $L$  power levels. It can be noted that the eventual goal is to find such  $L$  transmit power levels that will minimize the distortion for that  $L$ .

Theoretically, interference ranges from 0 to  $\infty$ . We divide the interference pdf into  $L$  regions such that the probability of occurrence of every region is equal. That is, for the  $L$  regions, the partitions  $\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_{L-1}$  are such that

$$\int_0^{\mathcal{X}_1} f_{p_I}(p_I) dp_I = \int_{\mathcal{X}_1}^{\mathcal{X}_2} f_{p_I}(p_I) dp_I = \dots = \int_{\mathcal{X}_{L-1}}^{\infty} f_{p_I}(p_I) dp_I, \quad (36)$$

where  $f_{p_I}(p_I)$  is the interference pdf. Or in other words, the area of each of the  $L$  regions is equal. Now we need to find a power level for each of these regions that would best represent that region. For distortion minimization, the power level in every region must bisect that region equally. Thus, for the  $i$ th region ( $1 \leq i \leq L$ ), the transmit power,  $M_i$ , is obtained by solving

$$\int_{\mathcal{X}_{i-1}}^{M_i} f_{p_I}(p_I) dp_I = \int_{M_i}^{\mathcal{X}_i} f_{p_I}(p_I) dp_I. \quad (37)$$

## 8 NUMERICAL RESULTS

We consider that the sensor nodes can transmit uniformly in the range  $\{s_{min}, s_{max}\}$ . We assume that the SINR received by the nodes is uniformly distributed between  $\{SINR_{min}, SINR_{max}\}$ . For our calculation, we assume

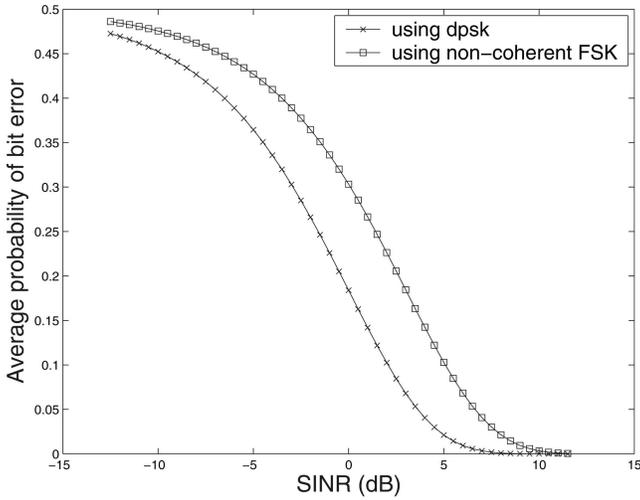


Fig. 2. Average bit error rate.

$s_{min} = 0$  and  $s_{max} = 100$  mW. SINR is assumed to range from  $-12.5$  dB to  $11.5$  dB.

Figs. 2 and 3 show the average bit error rate and probability of successful transmissions, respectively, for different values of SINR (in dB) perceived by node  $j$  from all its neighboring nodes. We show the results for two different modulation schemes: DPSK and noncoherent PSK. As expected, with improvement in channel condition, i.e., with increase in SINR, the probability of successful transmissions increases.

Figs. 4 and 5 present the maximum power efficiency for both schemes. More precisely, from the graphs, we find that if SINR is low and transmitting power  $P$  is high, where  $s_{min} < P \leq s_{max}$ , then the power efficiency is almost equal to zero. This proves our previous claim, which is, during bad channel conditions or below a certain threshold channel condition (when the SINR of the intended receiver node is very low), a node should not transmit. This only increases its power consumption and thus expected power consumption is no longer minimized. On the contrary, when the SINR is

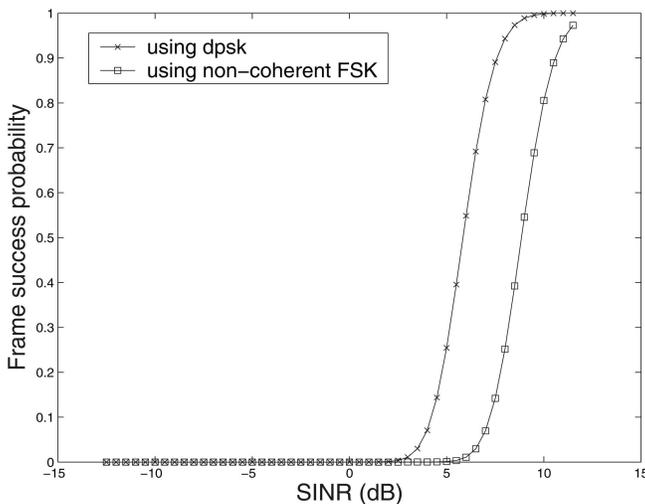


Fig. 3. Successful frame transmission probability.

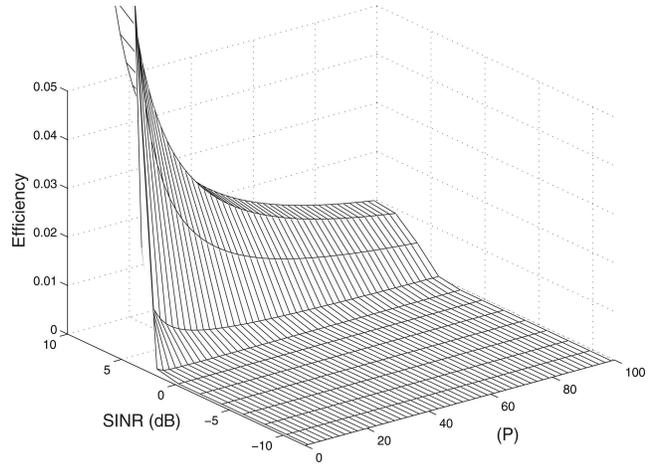


Fig. 4. Maximum efficiency for DPSK.

high, a node should transmit with low power to maximize its power efficiency. In this case, increasing transmitting power unnecessarily will decrease the power efficiency below its maximum. The plots also reveal the existence of an upper bound on transmit power as was obtained in Section 5.1; a condition to reach Nash equilibrium.

Let us now consider the cost component  $A(s_i)$  that results due to transmitting at power  $s_i$ . To get the cost, we must consider  $A(s_i)$  as some function of  $s_i$ . Since the exact relationship between  $s_i$  and  $A(s_i)$  is not known, we consider three types of functions: linear, quadratic, and exponential. Therefore,

$$A(s_i) = \begin{cases} \kappa \times s_i, & \text{for linear,} \\ \kappa \times s_i^2, & \text{for quadratic,} \\ \kappa \times e^{s_i}, & \text{for exponential,} \end{cases} \quad (38)$$

where  $\kappa$  is a scaling factor. Though trivial, we show  $A(s_i)$  in Fig. 6.

Fig. 7 shows the variation of the net utility with increasing transmitting power. It is intuitive that there will be an optimal value of  $s_i$ , beyond which the net utility will only decrease. This figure serves as a guideline for

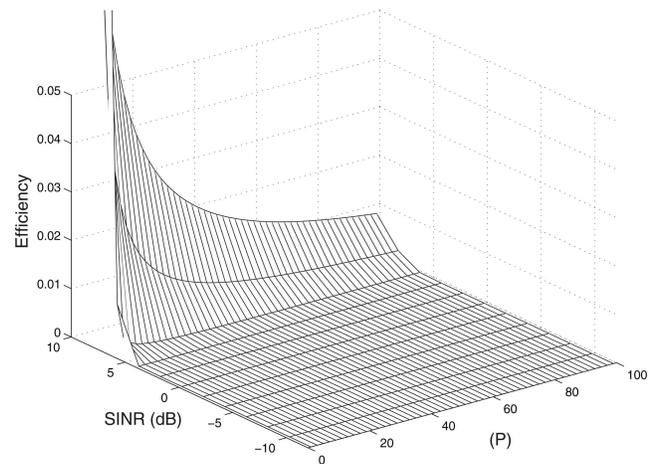


Fig. 5. Maximum efficiency for noncoherent PSK.

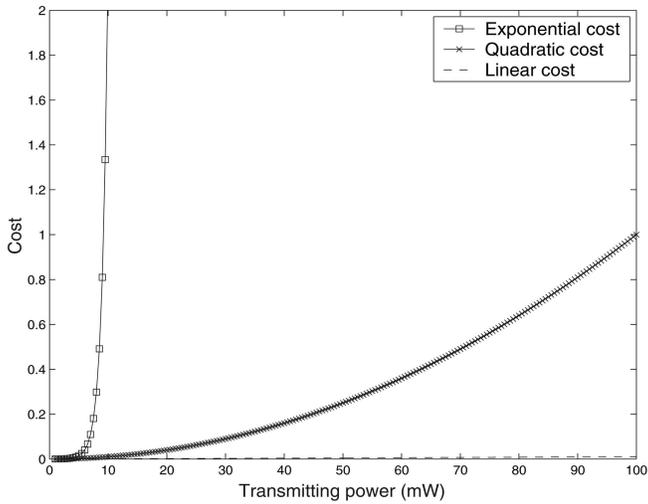


Fig. 6. Different cost functions.

calculating the desired transmitting power to maximize net utility for a node  $i$  transmitting to node  $j$ , given the strategies taken by all other nodes. For finding the best response to the strategies adopted by other nodes, we assume a subset of nodes to be active that are operating with fixed strategies.

For our calculation, we varied transmitting power from 1 to 100 mW. We find that there exist points for each of the cost functions considered (i.e., linear, quadratic, and exponential), which give the maximum net utility given the strategies taken by all other nodes as fixed. This desired transmitting power level gives the best response for the node. If a node unilaterally changes its strategy and does not transmit with this transmitting power level, then the node will not get its best response and will not be able to reach Nash equilibrium even if a Nash equilibrium exists for this model.

Fig. 8 plots the net utility against the transmission power for a fixed received power. We compare continuous power level with two sets of discrete power levels; one set has six and the other has 20 power levels. The power levels are

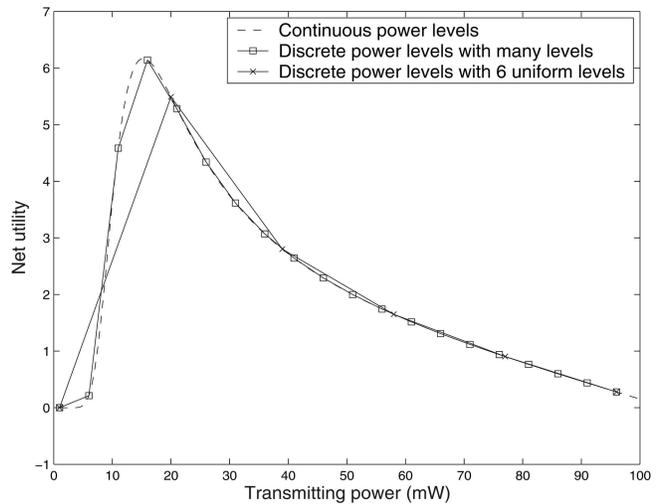


Fig. 8. Net utility for continuous and (6, 20) discrete power levels.

uniformly spaced between the maximum and the minimum. As expected, with more number of allowed power levels, the maximum net utility gets closer to that as obtained by continuous power levels.

Fig. 9 shows the effect of having nonuniform power levels. We choose 1, 5, 20, 30, 50, and 100 mW as the power levels. These values are the power levels specified by the Cisco Aironet cards and are of not any particular significance to this research. We see that even with nonuniform power levels as per Cisco regulations, the interference in the receiving node is such that maximum net utility is not obtained.

Fig. 10 shows the distortion factor with number of discrete power levels. Here, we compare our proposed mechanism of finding discrete power levels based on interference distribution with uniform spaced discrete power levels. The result shows that the distortion factor is reduced with increase in number of power levels. Moreover, the distortion obtained is reduced if the knowledge of the interference is used instead of having uniform equal-spaced power levels. Equation (37) was solved through

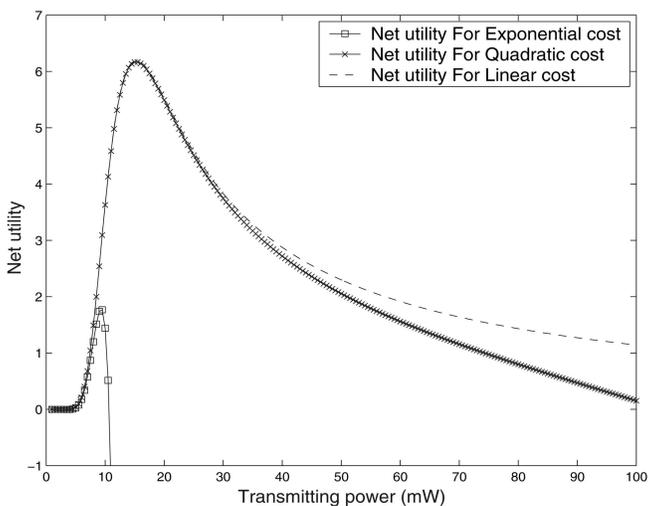


Fig. 7. Net utility.

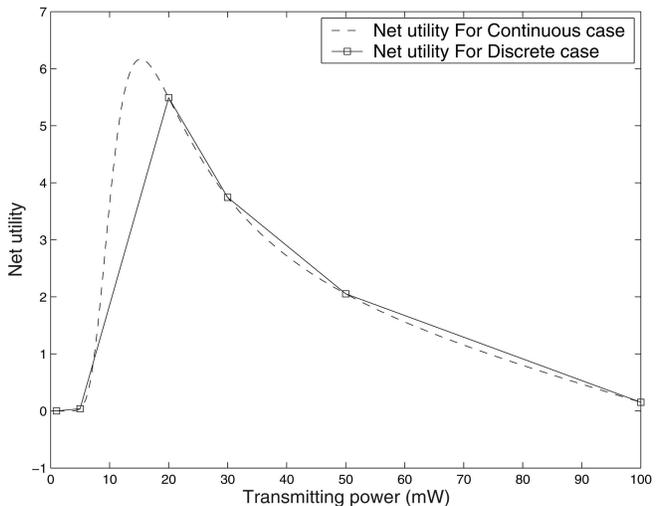


Fig. 9. Net utility for nonuniform power levels.

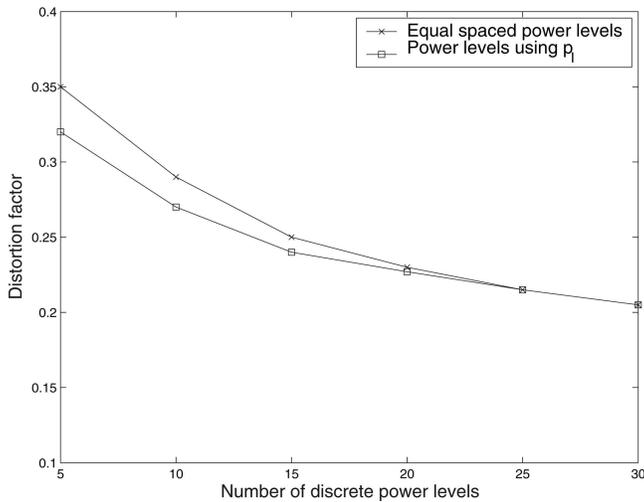


Fig. 10. Distortion factor versus number of discrete power levels.

numerical analysis for the values of  $M_i$ . This result can be used to find the desired number of power levels if the allowed distortion level is known.

## 9 CONCLUSIONS

In this paper, we presented a game-theoretic approach to solve the power control problem encountered in sensor networks. We used noncooperative games with incomplete information and studied the behavior and existence of Nash equilibrium. We found that Nash equilibrium exists if we assume a minimum and maximum threshold for channel condition and power level, respectively. We suggest that a node should only transmit when its channel condition is better than the minimum threshold and its transmission power level is below the threshold power level. We evaluated the desired power level at which the nodes should transmit to maximize their utilities under any given condition. We also analyzed the case where nodes are allowed discrete power levels as in most practical systems and compared their performances with the continuous power levels.

## ACKNOWLEDGMENTS

This research was sponsored by the US Air Force Office of Scientific Research (AFOSR) under the federal grant no. FA9550-07-1-0023.

## REFERENCES

- [1] I. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A Survey on Sensor Networks," *IEEE Comm. Magazine*, vol. 40, no. 8, pp. 102-114, Aug. 2002.
- [2] A. Sampath, P.S. Kumar, and J. Holtzman, "Power Control and Resource Management for a Multimedia CDMA Wireless System," *Proc. IEEE Int'l Symp. Personal, Indoor and Mobile Radio Communications (PIMRC)*, vol. 1, pp. 21-25, Sept. 1995.
- [3] R. Yates, "A Framework for Uplink Power Control in Cellular Radio Systems," *IEEE J. Selected Areas in Comm.*, vol. 13, no. 7, pp. 1341-1348, Sept. 1995.
- [4] S. Clearwater, *Market-Based Control: A Paradigm for Distributed Resource Allocation*. World Scientific, 1996.
- [5] F. Kelly, A. Maulloo, and D. Tan, "Rate Control in Communications Networks: Shadow Prices, Proportional Fairness and Stability," *J. Operations Research Soc.*, vol. 49, pp. 237-252, 1998.

- [6] P. Key and D. McAuley, "Differential QoS and Pricing in Networks: Where Flow Control Meets Game Theory," *IEE Proc. Software*, vol. 146, no. 1, pp. 39-43, Feb. 1999.
- [7] H. Lin, M. Chatterjee, S. Das, and K. Basu, "ARC: An Integrated Admission and Rate Control Framework for CDMA Data Networks Based on Non-Cooperative Games," *Proc. Ninth Ann. Int'l Conf. Mobile Computing and Networking*, pp. 326-338, 2003.
- [8] R. Maheswaran and T. Basar, "Decentralized Network Resource Allocation as a Repeated Noncooperative Market Game," *Proc. 40th IEEE Conf. Decision and Control*, vol. 5, pp. 4565-4570, 2001.
- [9] M. Stonebraker, R. Devine, M. Kornacker, W. Litwin, A. Pfeffer, A. Sah, and C. Staelin, "An Economic Paradigm for Query Processing and Data Migration in Mariposa" *Proc. Third Int'l Conf. Parallel and Distributed Information Systems*, pp. 58-67, Sept. 1994.
- [10] R. Subrata, A. Zomaya, and B. Landfeldt, "Game-Theoretic Approach for Load Balancing in Computational Grids," *IEEE Trans. Parallel and Distributed Systems*, vol. 19, no. 1, pp. 66-76, Jan. 2008.
- [11] S. Sengupta and M. Chatterjee, "An Economic Framework for Dynamic Spectrum Access and Service Pricing," *IEEE/ACM Trans. Networking*, vol. 17, no. 4, pp. 1200-1213, Aug. 2009.
- [12] M. Kubisch, H. Karl, A. Wolisz, L. Zhong, and J. Rabaey, "Distributed Algorithms for Transmission Power Control in Wireless Sensor Networks," *Proc. IEEE Wireless Comm. and Networking Conf.*, vol. 1, pp. 558-563, 2003.
- [13] D. Niyato, E. Hossain, M. Rashid, and V. Bhargava, "Wireless Sensor Networks with Energy Harvesting Technologies: A Game-Theoretic Approach to Optimal Energy Management," *IEEE Wireless Comm.*, vol. 14, no. 4, pp. 90-96, Aug. 2007.
- [14] J. Nash, "Equilibrium Points in N-Person Games," *Proc. Nat'l Academy of Sciences*, vol. 36, pp. 48-49, 1950.
- [15] S. Buchegger and J. Le Boudec, "Performance Analysis of the CONFIDANT Protocol," *Proc. Third ACM Int'l Symp. Mobile Ad Hoc Networking & Computing*, pp. 226-236, 2002.
- [16] L. Buttyan and J.P. Hubaux, "Nuglets: A Virtual Currency to Stimulate Cooperation in Selforganized Mobile Ad-Hoc Networks," Technical Report DSC/2001/001, Swiss Fed. Inst. of Technology, Jan. 2001.
- [17] W. Wang, M. Chatterjee, and K. Kwiat, "Enforcing Cooperation in Ad Hoc Networks with Unreliable Channel," *Proc. Fifth IEEE Int'l Conf. Mobile Ad-Hoc and Sensor Systems (MASS)*, pp. 456-462, 2008.
- [18] V. Srinivasan, P. Nuggehalli, C. Chiasserini, and R. Rao, "Cooperation in Wireless Ad Hoc Networks," *Proc. IEEE INFOCOM*, vol. 2, pp. 808-817, Apr. 2003.
- [19] Z. Xidong, C. Yueming, and Z. Heng, "A Game-Theoretic Dynamic Power Management Policy on Wireless Sensor Network" *Proc. Int'l Conf. Comm. Technology*, pp. 1-4, Nov. 2006.
- [20] Z. Jia, M. Chundi, and H. Jianbin, "Game Theoretic Energy Balance Routing in Wireless Sensor Networks" *Proc. Chinese Control Conf.*, pp. 420-424, July 2007.
- [21] S. Olariu, Q. Xu, and A. Zomaya, "An Energy-Efficient Self-Organization Protocol for Wireless Sensor Networks" *Proc. Intelligent Sensors, Sensor Networks, and Information Processing Conf.*, pp. 55-60, Dec. 2004.
- [22] M. Maskery and V. Krishnamurthy, "Decentralized Adaptation in Sensor Networks: Analysis and Application of Regret-Based Algorithms" *Proc. 46th IEEE Conf. Decision and Control*, pp. 951-956, Dec. 2007.
- [23] J. Chang and L. Tassiulas, "Maximum Lifetime Routing in Wireless Sensor Networks," *IEEE/ACM Trans. Networking*, vol. 12, no. 4, pp. 609-619, Aug. 2004.
- [24] <http://www.terminodes.org>, 2009.
- [25] L. Blazevic, L. Buttyan, S. Capkun, S. Giordano, J. Hubaux, and J. Le Boudec, "Self-Organization in Mobile Ad-Hoc Networks: The Approach of Terminodes," *IEEE Comm. Magazine*, vol. 39, no. 6, pp. 166-174, June 2001.
- [26] J. Crowcroft, R. Gibbens, F. Kelly, and S. Ostring, "Modelling Incentives for Collaboration in Mobile Ad Hoc Networks," *Proc. Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt '03)*, 2003.
- [27] S. Marti, T.J. Giuli, K. Lai, and M. Baker, "Mitigating Routing Misbehavior in Mobile Ad Hoc Networks," *Proc. Sixth Ann. Int'l Conf. Mobile Computing and Networking*, pp. 255-265, 2000.
- [28] P. Michiardi and R. Molva, "Core: A Collaborative Reputation Mechanism to Enforce Node Cooperation in Mobile Ad Hoc Networks," *Proc. Comm. and Multimedia Security Conf.*, 2002.

- [29] E. Baccarelli, M. Biagi, and C. Pelizzoni, "A Distributed Spatial Signal-Shaping for the Competitively Optimal Throughput-Maximization of MIMO 'Ad-Hoc' Networks," *IEEE Trans. Vehicular Technology*, vol. 55, no. 6, pp. 1862-1876, Nov. 2006.
- [30] P. Michiardi and R. Molva, "A Game Theoretical Approach to Evaluate Cooperation Enforcement Mechanisms in Mobile Ad Hoc Networks," *Proc. Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt '03)*, 2003.
- [31] F. Meshkati, H. Poor, S. Schwartz, and N. Mandayam, "An Energy-Efficient Approach to Power Control and Receiver Design in Wireless Data Networks," *IEEE Trans. Comm.*, vol. 53, no. 11, pp. 1885-1894, Nov. 2005.
- [32] S. De, C. Qiao, D. Pados, M. Chatterjee, and S. Philip, "An Integrated Cross-Layer Study of Wireless CDMA Sensor Networks," *IEEE J. Selected Areas in Comm. (JSAC)*, Special Issue on Quality of Service Delivery in Variable Topology Networks, vol. 22, no. 7, pp. 1271-1285, Sept. 2004.
- [33] A. Garcia, *Probability and Random Processes for Electrical Engineering*. Addison-Wesley, 1989.
- [34] D. Fudenberg and J. Tirole, *Game Theory*. MIT Press, 1991.
- [35] J. Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*. Princeton Univ. Press, 1944.
- [36] Y. Xing and R. Chandramouli, "Distributed Discrete Power Control for Bursty Transmissions over Wireless Data Networks," *Proc. IEEE Int'l Conf. Comm. (ICC)*, vol. 1, pp. 139-143, 2004.
- [37] "Cisco Systems Inc. Data Sheet for Aironet," 2004.



**Mainak Chatterjee** received the BSc degree in physics (Hons.) from the University of Calcutta in 1994, the ME degree in electrical communication engineering from the Indian Institute of Science, Bangalore, in 1998, and the PhD degree from the Department of Computer Science and Engineering, University of Texas at Arlington in 2002. He is currently an associate professor in the School of Electrical Engineering and Computer Science, University of Central Florida, Orlando. His research interests include economic issues in wireless networks, applied game theory, cognitive radio networks, and mobile video delivery. He serves on the executive and technical program committee of several international conferences.



**Kevin A. Kwiat** received the BS degree in computer science and the BA degree in mathematics from Utica College of Syracuse University, and the MS degree in computer engineering and the PhD degree in computer engineering from Syracuse University. He has been a civilian employee with the US Air Force Research Laboratory in Rome, New York, for more than 26 years. He holds three patents. In addition to his duties with the Air Force, he is an adjunct professor of computer science at the State University of New York at Utica/Rome, an adjunct instructor of computer engineering at Syracuse University, and a research associate professor with the University at Buffalo. He completed assignments as an adjunct professor at Utica College of Syracuse University, a lecturer at Hamilton College, a visiting scientist at Cornell University, and as a visiting researcher while on sabbatical at the University of Edinburgh. His main research interest is dependable computer design.

► **For more information on this or any other computing topic, please visit our Digital Library at [www.computer.org/publications/dlib](http://www.computer.org/publications/dlib).**



**Shamik Sengupta** received the BE degree (First class Hons.) in computer science from Jadavpur University, India, in 2002 and the PhD degree from the School of Electrical Engineering and Computer Science, University of Central Florida, Orlando, in 2007. He has also worked as a postdoctorate researcher in the Department of ECE, Stevens Institute of Technology, Hoboken, New Jersey. Currently, he is an assistant professor in the Department of Mathematics and Computer Science, John Jay College of Criminal Justice of the City University of New York from Fall 2009. His research interests include game theory, security in wireless networking, dynamic spectrum access, cognitive radio, and wireless sensor networking. He was the cochair of First IEEE International Workshop on Cognitive Radio and Networks (CRNETS) 2008 and also serves on the organizing and technical program committee of several IEEE conferences. He is the recipient of the IEEE Globecom 2008 Best Paper Award.