High-Resolution, Nonhydrostatic Simulations of Internal Wave Generation in the Luzon Strait using SUNTANS

Oliver B. Fringer
473 Via Ortega, Room 187
Dept. of Civil and Environmental Engineering
Stanford University
Stanford, CA 94305
phone: (650) 725-6878 fax: (650) 725-9720 email: fringer@stanford.edu

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http://suntans.stanford.edu

LONG-TERM GOALS

The long term goal is to develop the capability to predict the internal wave climate in the ocean with the high-resolution, three-dimensional, nonhydrostratic SUNTANS model.

OBJECTIVES

Our primary objective is to understand the mechanisms leading to the generation of internal waves in the South China Sea by performing high-resolution, nonhydrostatic numerical simulations with the SUNTANS model. A secondary objective is to perform simulations to aid with the planning of the field experiments for the IWISE (Internal Waves in Straits Experiment) DRI.

APPROACH

We are employing a combination of theoretical analyses and numerical simulations with the SUNTANS (Fringer et al., 2006) model to perform a detailed, nonhydrostatic study of the mechanisms governing the generation of internal waves in the Luzon Strait, which forms the eastern boundary of the South China Sea (see Figure 1). Our work builds upon simulations we have performed as part of the NLIWI (Nonlinear Wave Initiative) DRI that have been designed to understand the generation and propagation of internal waves in the South China Sea, as well as how the internal waves interact with the continental shelf on the western side of the sea. The simulations are similar to the NLIWI simulations but focus on the Luzon Strait. We plan on performing simulations in smaller-scale domains in order to focus on the generation region, rather than full basin-scale domains, which are better-suited to studying internal wave generation and the deep-basin propagation and interaction with the Dongsha Plateau and the Chinese continental shelf (see Figure 1). SUNTANS domains for the present work are targeted towards simulating high-resolution, nonhydrostatic flow physics through channels and over sills to understand the fundamental physics of internal wave generation. We expect the grid to be refined around the Babuyan and Batan Islands in order to highly resolve the tidal flow around these islands and to resolve possible hydraulic control features. Based on our experience with
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simulating internal waves in the SCS, we expect to be able to use SUNTANS in a predictive manner to aid in the placement of instrumentation for the field component of the IWISE DRI.

![Figure 1: Bathymetric map of the South China Sea, showing the locations of mooring sites L1, B1 and S7 from the WISE/VANS project in 2005, and the transects that are used for the two-dimensional theoretical analysis.](image)

**WORK COMPLETED**

We have developed a linear theory for the generation of internal waves over ridges to determine the phase in the tide at which peaks in the depression waves cross the ridges.

**RESULTS**

The most basic aspect of nonlinear internal waves in the South China Sea that has yet to be explained is the correlation of peaks in westward-propagating internal waves with maximum eastward (ebb) currents at the eastern ridge in the Luzon Strait. Ramp et al. (2004) used linear theory to compute the internal wave phase speeds and trace peaks in the internal wave signatures observed at B1 back to mooring L1, and showed that the tracebacks always crossed mooring L1 during peak eastward tidal flow. The lee-wave release mechanism proposed by Maxworthy (1979) is a plausible generation mechanism because it implies that a hydraulic control develops over the ridge at L1 that prevents waves from propagating westward until the tide becomes subcritical. This mechanism is unlikely because waves appear to be "released" during peak easterly tidal flow at the ridge throughout the spring-neap cycle during which time the peak ebb currents range from strongly supercritical to subcritical, and the lee-wave mechanism is not possible unless the tidal currents are supercritical. Zhao and Alford (2006) proposed an alternate mechanism in which waves are generated over the eastern ridge during peak flood tidal currents (westward), and these waves steepen into trains of nonlinear waves as they propagate into the basin. Although they proposed a different mechanism, Zhao and Alford (2006) also found that the generation phase was independent of the strength of the tidal currents.
The theoretical analysis that we have developed is based on the hypothesis that the generation mechanism is dominated by linear processes, and this is based on the evidence that wave arrival time at a particular mooring in the basin is roughly the same relative to the phase in the particular tide that generated that wave. That is, diurnally-generated A-waves (Ramp et al. 2004) arrive at mooring S7 at the same time every day, while semidiurnally-generated B-waves (Ramp et al. 2004) arrive roughly one hour later each day. This implies that hydraulic effects at the generation site are weak and that nonlinear effects on the propagation speed are also weak. These justify development of a linear model, which is further simplified by assuming that the internal tides are generated and propagate predominantly as first-mode waves, which is based on evidence, both from modeling and observations, that more than 90% of the internal tidal energy in the Luzon Strait is in the first mode (Simmons 2009).

The linear model is derived from the inviscid equations of motion with the Boussinesq approximation for a two-layer fluid with upper-layer depth \(d_1\) and lower-layer depth \(d_2\), with the free-surface defined by \(\eta\) and the interface between the layers defined by \(\zeta\), and with the bottom bathymetry defined by \(h\), as shown in Figure 2.

![Figure 2: Schematic of the two-layer system used to develop the linearized equations.](image)

The equations of motion for the velocities in the upper layer, \(u_1, v_1\), and lower layer, \(u_2, v_2\), along with continuity, are given by
\[
\frac{\partial \eta}{\partial t} = -\frac{\partial}{\partial x} \left[ u_1 (d_1 + \eta - \zeta) \right] - f v_1 + \frac{\partial}{\partial x} \left[ u_2 (d_2 + \zeta - h) \right],
\]
\[
\frac{\partial \zeta}{\partial t} = -\frac{\partial}{\partial x} \left[ u_2 (d_2 + \zeta - h) \right],
\]
\[
\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} - f v_1 = -g \frac{\partial \eta}{\partial x},
\]
\[
\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} - f v_2 = -g \frac{\partial \eta}{\partial x} + \frac{g}{R^2} \frac{\partial \zeta}{\partial x},
\]
\[
\frac{\partial v_1}{\partial t} + f u_1 = 0,
\]
\[
\frac{\partial v_2}{\partial t} + f u_2 = 0,
\]

where \( f \) is the Coriolis parameter and we have assumed quasi two-dimensionality by ignored all gradients of \( v \) and all gradients with respect to \( y \). After linearization and assuming a rigid lid, weak topography (\( \alpha = h/d_2 << 1 \)) and a shallow upper layer (\( \beta = d_1/d_2 << 1 \)), the nondimensional equation governing the behavior of the interface when subjected to a half-cosined-shaped ridge is given by the inhomogeneous Klein-Gordon equation,

\[
\zeta_{tt} + \frac{1}{R^2} \zeta - \frac{1}{1 + \beta} \zeta_{xx} = -Af(x) \sin(k_b x) \cos(t),
\]

where \( R = \omega f \) is the Rossby number and \( \omega \) is the tidal frequency, \( k_b \) is the nondimensional wavenumber of the topography. Here, \( x \) is the nondimensional distance and \( t \) is the nondimensional time, such that, if * represents a dimensional quantity, then \( x = k x^* \) and \( t = \omega t^* \), and \( k = 2\pi/\lambda \) is the internal tidal wavenumber and \( \lambda \) is its wavelength, for \( \beta = 0 \) and \( R = \infty \). In the governing equation for \( z \), \( f(x) = H(x - \pi/k_b)H(-x + \pi/k_b) \) and \( H(x) \) is the heaviside function, and the amplitude is given by

\[
A = \frac{1}{2} \alpha \beta k_b F_0 \left( 1 - \frac{1}{R^2} \right),
\]

where in the present analysis \( F_0 = 1 \). The solution for \( \zeta \) is given by a standing wave in the region \( |x| < \pi k_b \),

\[
\zeta_C = -\frac{\zeta_0}{2 \sin(k_0 \pi/k_b)} \left[ \frac{2k_0}{k_b} \sin(k_b x) \cos(t) + \sin \left( k_0 x + t - \frac{k_0 \pi}{k_b} \right) + \sin \left( k_0 x - t + \frac{k_0 \pi}{k_b} \right) \right]
\]

and freely-propagating waves that radiate away to the left and right of the ridge (located at \( x = 0 \)),

\[
\zeta_L = \zeta_0 \cos(k_0 x + t) \quad (x < -\pi/k_b),
\]
\[
\zeta_R = -\zeta_0 \cos(k_0 x - t) \quad (x > \pi/k_b),
\]

where the amplitude of the waves that radiate away is given by
and the function $G$ is given by

$$G(\gamma) = \frac{\gamma \sin(\pi \gamma)}{1 - \gamma^2}.$$

The nondimensional wavenumber of the waves in the presence of rotation is given by the dispersion relation for Poincare waves

$$c_i^2 k_0^2 = 1 - \frac{1}{R^2},$$

where $c_i^2 = (1+\beta)^{-1}$. This solution shows that the phase in the tide in which troughs in left- or westward-propagating linear waves cross the sill is strong function of $k_0/k_b$, since the function $G$ is not monotonic. For most problems of interest, $k_0/k_b$ is small since the scale of the topography is typically of the order of or smaller than the internal tidal wavelength. However, different values can lead to wave generation at different tidal phases. Figure 3 depicts the analytical result for four different values of $k_0/k_b$, and shows that for $k_0/k_b=0.5, 1.5, and 3.5$ (Figures 3(a), (b) and (d)), left-propagating depressions coincide with the end of the ebb (eastward) tide, while for $k_0/k_b=2.5$ (Figure 3(c)), left-propagating waves coincide with the end of the flood (westward) tide. Therefore, because the value of $k_0/k_b$ affects the sign of $G$, the ratio of the topographic scale to the wavelength of the internal tide determines the apparent phase in the tide at which depressions are generated. Short topography relative to the internal tidal wavelength yields westward-propagating waves that originate at the end of the ebb tide, while longer waves can alternate between the end of the ebb and the end of the flood tide.

These results are contrary to the observations because they indicate that waves are generated during slack rather than peak tides. We hypothesize that the linear theory will show that a two-ridge system can lead to left-propagating waves that are generated during peak ebb tides over the eastern ridge, and that nonlinear effects are not significant in altering this behavior. This is the subject of ongoing work.
IMPACT/APPLICATIONS

Understanding internal tides and waves is important for developing parameterizations of their effects in larger-scale ocean models.

REFERENCES


PUBLICATIONS


HONORS/AWARDS/PRIZES

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