TRANSPORT PHENOMENA IN FLUID DYNAMICS:
MATRIX HEAT EXCHANGERS AND THEIR
APPLICATIONS IN ENERGY SYSTEMS

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Heat exchangers play an important role in maintaining the operating temperatures required for high thermal performance of energy systems. Energy systems such as heat pumps, electric generators, water heating and steam generation, conversion chemical processes, and heat dissipation equipment are parts of the deployed equipment.

The fact that heat transfer intensifies by increasing the surface area of contact between the working fluid streams and the added benefit of near elimination of axial heat transfer gives the Matrix Heat Exchanger (MHE) an edge over many other compact heat exchangers concepts. MHE consists of a stack of alternating perforated plate-spacer pairs which are bonded together to form leak free passages for the fluid streams.

This effort explored the possibility of using MHE as steam generator and established a model to estimate the heat transfer coefficient and the flow friction coefficient in case of two phase flow in MHE.
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SUMMARY

This report summarizes a literature review performed in order to study current work in understanding matrix heat exchangers (MHEs). While efficient, the heat transfer process in a matrix heat exchanger is complex, as there are two heat conduction paths (along the perforated plates, and across the spacer plates), as well as three convection surfaces (upstream of the plates, downstream of the plates, and the inner wall surface area of each perforation). The literature review reveals a long list of derived correlations of Nusselt number, heat transfer coefficient and friction factor in terms of Reynolds number, flow characteristics and geometry for a matrix heat exchanger. In addition to the empirical correlations, a myriad of analytical and numerical relations have been established and verified against the experimental results.
1. INTRODUCTION

For compact heat exchangers to have a high thermal effectiveness, they require the following characteristics: large heat transfer surface area per unit volume, high heat transfer surface coefficients, very small longitudinal heat conduction through the walls, and uniform distribution of flow throughout any cross section of the exchanger.

The known type of heat exchanger that best meets the above requirements is the matrix heat exchanger (MHE). As shown in Figure 1, a matrix heat exchanger is a stack of alternating perforated plate-spacer pairs which are bonded together to form leak free passages, allowing two flow streams to exchange heat. The perforated plates are made of high thermal conductivity materials and are used as an intermediary to transfer heat between two fluid streams. The spacers are made of less thermally conductive material and are used to inhibit axial conduction and enable flow redistribution.

![Figure 1. Matrix Heat Exchanger Schematic](image)

When the fluids pass through the perforated plate, heat is exchanged between the plate and each fluid. The rate of heat exchange is dependent on the surface area in contact with the fluids. The perforated plate surface area consists of the front and back of the plate as well as the inner wall surface area of each perforation. The large surface area of each perforated plate gives the matrix heat exchanger a large surface area to volume ratio, enabling compact exchangers with high heat transfer. Figure 2 shows a cross section of a matrix heat exchanger to indicate the three different convective heat transfer surfaces.

While efficient, the heat transfer process in a matrix heat exchanger is complex, as there are two conduction paths (along the perforated plates, and across the spacer plates), as well as three
convection surfaces (upstream of the plates, downstream of the plates, and the inner wall surface area of each perforation).

Figure 2. Matrix Heat Exchanger Cross Section

The total heat transfer of a matrix heat exchanger is composed of five components:

1) Convective heat transfer between hot fluid streams and perforated plates.
2) Conductive heat transfer along the perforated plates up to the separation wall.
3) Conductive heat transfer across the separation wall.
4) Conductive heat transfer along the perforated plates from the separation wall.
5) Convective heat transfer between perforated plate and cold fluid stream.

All these modes of heat transfer (convection on three different surfaces and conduction in two different directions) are coupled, requiring them to be determined together.

This project surveys the available literature for applications of matrix heat exchangers in steam generation, which will result in the ability to design a more efficient and more compact steam generator.

2. LITERATURE REVIEW

Venkatathrnam, et al. [1] gave a chronological development of the matrix heat exchanger for cryogenic applications covering four decades, from when the matrix heat exchanger was first introduced by McMahon et al. [2] in 1949, until late 1990. They surveyed different methods of fabrication, heat transfer and fluid flow characteristics, and design and simulation procedures. They concluded that better correlations should emerge, and recommended that attempts should be made to predict heat transfer and flow friction performance from fundamental relations on different heat transfer mechanisms, which will result in optimized geometries and operating conditions.
Presented are some of the publications summarized in the above survey, followed by a survey of the literature from 1990 until present. This presentation is focused on what has been established for fabrication, heat transfer and friction coefficient correlations, and the design processes and optimization methods, with a view toward their applications in industry, specifically in steam generation.

2.1. Highlights of Literature Review (From 1949 Until 1990)

Matrix heat exchangers were first introduced by McMahon et al. [2] in 1949, mainly to be fabricated commercially for use in production of liquid oxygen. Their design consisted of a series of perforated aluminum plates separated by thin, die cut neoprene gaskets, which served as channels to separate the fluid streams. The whole package, together with a cast aluminum header at each end, was held together by means of steel tie rods. The neoprene spacers nearly eliminated axial conduction and provided gas tight seals, even at liquid air temperatures. The construction was extremely simple and repairs were considered easy. Performance data was presented for heat transfer and flow resistance for matrix heat exchangers with air streams flowing countercurrently to study the effects of hole diameter, plate thickness, and gasket thickness on the heat transfer coefficient and friction coefficient for a flow with a Reynolds number between 800 and 4300.

Fleming’s [3] heat exchanger design was similar to McMahon’s [2] in that it consisted of stacked, commercially available punched plates with plastic separators. Fleming considered the matrix heat exchanger’s perforated plates as fins, with efficiencies in the range of 0.4 to 0.6. Because of the inaccuracies in using the traditional fin formulas to examine a matrix heat exchanger, a method was needed to study the thermal and flow characteristics and coefficient of heat transfer. To accomplish this, the equivalent fin efficiency was defined as:

\[
\eta_{\text{fin}} = \left( 1 + \frac{hA_L L^2}{3k_e A_c} \right)^{-1}
\]  

(1)

Where,

- \( \eta_{\text{fin}} \) = fin effectiveness
- \( h \) = heat transfer coefficient at fin surface
- \( A_L \) = heat transfer surface area per unit length of fin
- \( A_c \) = cross sectional area of fin perpendicular to heat flow direction
- \( L \) = length of fin in heat flow direction
- \( K_e \) = effective thermal conductivity of fin material

Most authors have derived empirical correlations for heat transfer coefficient (expressed in terms of Nusselt number, Stanton Number, J number, or simply \( h \)), and friction factor, \( f \), versus Reynolds number. The general approach has been to find a relation of the form: \( Nu = C Re^n \), where \( C \) and \( n \) are functions of geometric parameters, and \( Re \) is usually determined based on flow velocity in the perforation, using the perforation diameter as the characteristic dimension.

In an attempt to quantify the effect of the matrix heat exchanger’s geometry on its performance, Anashkin et al. [4] investigated the feasibility of using such heat exchangers in helium
refrigerators of low, medium, and large capacity. Their analysis showed that the hole diameter is one of the most important parameters. They also developed a relation similar to Fleming’s fin efficiency (Eq. 1).

Sparrow et al. [5] examined the flow pattern on the upstream face using a lampblack–oil technique. The lampblack-oil technique involves the use of a lampblack powder and oil mixture. The mixture was thinly spread on the upstream face of the perforated sheet. The shear stresses of the air flow on the plate, resulting from the air being drawn through the perforated sheet, resulted in the oil particles aligning themselves with the direction of flow. They demonstrated that a hexagonally shaped flow tube could be used to represent the approaching flow for each hole. Extrapolating the solution for a single hole over the entire plate results in an average solution for the perforated plate. They were able to derive an average Nusselt number correlation based on the Reynolds number and Prandtl number as follows:

\[ Nu = 0.881Re^{0.476}Pr^{0.333} \text{ for } 2000 \leq Re \leq 20000 \]  

Shevyakova et al. [6] obtained generalized theoretical relations based on experimental studies of the hydraulic resistance, expressed in terms of Euler number, and heat transfer, expressed in terms of the Stanton number, in heat exchangers made of perforated plates with different internal geometries (specifically the porosity, \( p \)).

The relations were:

\[ E_u = 16.34Re^{-0.55} \left( \frac{(1.707-p)^2}{2} \right) \]  

and \[ St = CRe^nPr^{-2/3} \]  

where \[ C = 3.6X10^{-4}[(1-p)p - 0.2]^{-2.07} \]  and \[ n = -4.36X10^{-2}p^{-2.34} \]  

Their relations were suitable for designing heat exchangers made of perforated plates 0.5mm thick with perforated plate porosity between 0.3 and 0.6, plate spacing between 0.4 and 1.6 mm, and arbitrary location of the holes of adjacent plates relative to each other.

2.2. Recent Literature Review (From 1991 Until Present)

Since Venkatarathnam, et al.’s literature review, the fabrication methods for matrix heat exchangers have evolved to take advantage of vacuum brazing and diffusion bonding methods. Nilles et al. [7] listed the advantages of diffusion bonding over vacuum brazing as follows:

- Heat exchanger assembly is simplified
- Bond strength is greater
- No sharp boundary between pieces
- Less chance of plugging the exchanger since there is no liquid-phase brazing materials to contaminate the passages of the exchanger
• No extra materials (braze or solder) to add to the exchanger
• Diffusion bonding can be used for copper/stainless steel pairs up to 800 °C.

For this review, matrix heat exchangers were fabricated using all-metal materials and assembled using diffusion bonding; no review was carried out for different methods of fabrication.

This review was divided into three groups: heat transfer and flow characteristics analysis; modeling, simulation and design optimization; and a survey of two phase flow in heat exchangers to apply a matrix heat exchanger in steam generation. The report concludes with recommendations on the approach to choose, fabricate, design, and characterize a matrix heat exchanger to be used for steam generation.

2.2.1 Heat Transfer and Flow Characteristics Analysis

Venkatarathnam [8] opined that there are two approaches to analyze the performance of matrix heat exchangers. The first approach considers a matrix heat exchanger as a conventional exchanger with the plates modeled as fins. This approach, adopted by early workers [2, 3], was solved analytically. The second approach involves the treatment of a matrix heat exchanger as a discrete set of plate-spacer pairs, with the number of plates as an important parameter, involving numerical solution of the governing equations [9, 10]. Venkatarathnam explained the two approaches and presented analytic closed form solutions where he defined the Number of Transfer Units ($N_{tu}$) as a function of effectiveness. It was shown analytically that the first approach is a special case of the second approach, and that the solutions obtained with the first approach were very close to those obtained with the complex numerical models used in the second approach.

Linghui et al. [11] studied how the length to diameter ratio ($\delta/d$) of the plate’s holes affected the heat transfer coefficient for a perforated plate. They studied ratios varying from 0.333 to 1.1666, holding the diameter constant while thickness was varied. Their experiments used the naphthalene sublimation technique to determine the heat transfer of the plate. In their experiments, they found that after the third division plate, the Sherwood number was relatively constant. They also found that there was little change in the heat transfer coefficients between the ($\delta/d$) ratios of 0.5 and 1.1. Their final equation for the Nusselt number inside the tube was:

$$Nu = 2.058 \text{Re}^{0.487}$$

H.H. Cho, et al. [12] performed laboratory experiments to determine the heat transfer coefficient characteristics of a short hole in a plate. Their experiments were conducted using the naphthalene sublimation technique, and examined hole length to diameter ratios of 0.5 to 1.5 and outer boundary to hole ratios of 1.4 to 4.5. The holes’ diameters were 12.6 mm to 38.1mm, and the Reynolds number varied from 100 to 30,000. The results were broken into three sections: windward face, inside the hole, and leeward face. The windward face was compared to a sink flow. The study found that the heat transfer coefficient increases rapidly as the flow approaches the hole. This is due to the acceleration of the flow, which leads to thinning of the boundary layer. The section inside the hole was also divided into three distinct sections: the
separation/recirculation region at the inlet of the hole, the reattachment to the tube wall region, and the developing region. The flow separates at the edge of the inlet, which leads to low heat transfer coefficients. At the reattachment region, the heat transfer coefficients rapidly increase due to high turbulence. Finally, the heat transfer coefficients decrease gradually with developing pipe flow. This was not found, however, to be the case with thin plates, $\delta/d \geq 0.6$, in which no reattachment takes place and the recirculation zone extends to the leeward side. The leeward side has low heat transfer characteristics due to the low entrainment velocity on the surface compared to the hole exit velocity. The results were further separated into two groups based on Reynolds numbers. At $Re \leq 3200$, in which weak reattachment and long recirculation was expected, the heat transfer coefficient is low. At $Re \geq 4900$, the heat transfer coefficient would be high due to the turbulent transition reattachment zone. For these Reynolds numbers, it was found that the separation length was closely approximated by $Y/d \approx 0.56$, where $Y$ was the distance along the hole axis.

Brunger et al. [13] studied the effectiveness for each of the three zones of heat transfer on a perforated plate: the front of the plate, the inside of the tube, and the back of the plate. In their study, they considered large pitch to diameter ratios (> 6.67). Tests were conducted to determine the degree to which the plates were isothermal. Modeling the plates as isothermal was found to be a reasonable assumption for metallic plates. Also found was that a square arrangement of holes does not perform as well as a triangular arrangement. For each of the heat transfer regions, an equation for effectiveness was given. The authors also stated that under typical operating conditions, about 62% of the ultimate temperature rise of the air was predicted to occur on the front surface, 28% in the hole, and 10% on the back of the plate.

Venkatarathnam et al. [14] introduced an apparatus for testing small cryogenic heat exchangers, which they described: “The apparatus, enclosed in a vacuum vessel, provided a variable cold end temperature and means of direct measurement of ineffectiveness. Temperature approaches at both ends and pressure drop in both channels were also measured.” The working principle of their test apparatus was based on relating the liquid nitrogen boil off rate to the ineffectiveness of the exchanger, where that relation had been derived mathematically.

A widely used method for determining the heat transfer coefficient for compact heat exchangers is to determine the maximum slope of an exchanger’s outlet temperature verses time using the single blow transient test method. The maximum slope is a function of the number of transfer units ($N_{tu}$). The longitudinal heat conduction parameter ($\lambda$) was estimated for simple compact heat exchangers, but the estimation of the longitudinal heat conduction parameter presents difficulties for complex surfaces, such as those in a matrix heat exchanger.

Foad et al. [15] used the above method for testing matrix heat exchangers after some manipulation. An analytical expression was derived to highlight the effect of a finite number of plates on the transient response of a matrix heat exchanger during a single blow transient test. They concluded that the number of plates strongly influences the transient response of perforated plate heat exchanger surfaces. During single blow tests, the maximum slope of the temperature-time response curve was finite, even when the number of transfer units per plate ($N_{tu}$) was infinite, and the slope of the temperature-time response curve went through an inflection only when the number of plates in the test bed exceeded the value given by:
\[ n_{\text{min}} = \frac{2/c_p \cdot 1}{1 - e^{-Ntu}} = e^{-Ntu} \]  

Krishnakumar et al. [16] utilized the single blow transient test method to relate the maximum slope to the longitudinal heat conduction parameter so that the data from a single test could be used to correlate the prediction of \( N_{tu} \) based on the value of the maximum slope, and the parameter of longitudinal heat conduction, \( \lambda \), based on the time when the maximum slope occurred. They verified the data experimentally and published their data in a table format, which is useful in cases where the heat exchanger’s geometry is complex and estimation of the longitudinal heat conduction parameter \( (\lambda) \) is not straightforward. The proposed method was found to be particularly useful in the case of perforated plate matrix heat exchangers and for wire mesh surfaces.

2.2.2. Modeling, Simulation, and Design Optimization

Venkatarathnam, et al. [10] derived the matrix heat exchanger’s governing equations, including the five sources of heat transfer mentioned in the introduction of the present paper, and simplified those equations using assumptions. Because of the discrete structure of the exchanger, the partial differential equations were reduced to sets of algebraic and ordinary differential equations, and a numerical scheme for solving these equations was presented. They showed that an efficient and stable algorithm was required for an iterative solution of this system of equations, and they gave specific steps of that algorithm in their paper. Based on the algorithm and method of solution, they wrote a computer program to predict the performance of a perforated plate matrix heat exchanger with rectangular geometry, gave two illustrated examples of how to apply their findings, and compared their results to results published by various authors.

Nilles et al. [7] obtained a numerical solution of the heat exchange equations for a perforated plate counter flow heat exchanger with two annular flow passages. They also outlined a method to determine the characteristics of the exchanger required for a given application directly from the parameters of the application, the material properties of the perforated plate matrix, and the properties of the working fluid. Topics addressed in their presentation include pressure drop, plate conduction, corrections for longitudinal thermal conductivity in the exchanger, and entrance effects. They confirm their analysis and their design approach with experimental data.

Venkatarathnam [17] developed a minimum volume method for sizing perforated plate matrix heat exchangers based on designing the matrix heat exchanger with a small cross section and a large length to reduce lateral temperature differences and axial heat conduction. Mathematically, the optimization involved minimizing the matrix heat exchanger’s volume subject to constraints, such as specified effectiveness and required pressure drop. He presented closed form expressions for the effectiveness of the exchanger in terms of various heat transfer resistances. Those expressions can be used to determine the effectiveness of a matrix heat exchanger of arbitrary shape. Based on those expressions, a procedure was illustrated with a numerical example.

Kumar et al. [18] introduced optimization of rectangular and circular shaped matrix heat exchangers. They established methods for the optimum sizing of both balanced and unbalanced
flow matrix heat exchangers. With elaborate analysis, they concluded that their methods can be summarized as follows:

For Rectangular Geometry:

- Express width and height of the exchanger in terms of $N_{tu,f,i}$ and $N_{tu,p,i}$ using their correlations.
- Express the width and height in all other expressions in terms of the longitudinal heat conductivity parameters $\lambda$ and $\lambda_p$ using the same correlations.
- Solve the optimization equations simultaneously by the Newton-Raphson technique.

For Circular Geometry:

- Express $R_i$ and $R_o$ of the exchanger in terms of $N_{tu,f,1}$ and $N_{tu,f,2}$ using their correlations.
- Express $R_i$ and $R_o$ in all other expressions in terms of the longitudinal heat conductivity parameters $\lambda$ and $\lambda_p$ using the same correlations.
- Solve the optimization equations simultaneously by the Newton-Raphson technique.

They concluded with the following remarks:

1. The volume of matrix heat exchangers of rectangular (two channels) and circular geometry are quite close at effective $N_{tu}$ less than 15, but at higher values the volume of a matrix heat exchanger of multichannel rectangular geometry will be lower than other configurations.
2. It is preferable to use the inner channel for the low pressure stream and the annular channel for the high pressure stream in the case of a circular matrix heat exchanger.
3. The low pressure stream is more likely to operate in a laminar flow regime, particularly at high effectiveness in an optimum matrix heat exchanger. Conversely, the high pressure stream is more likely to operate in a turbulent flow regime.

Krishnakumar et al. [19] researched utilizing transient testing of perforated plate matrix heat exchangers, and commented that they are well understood and that the effectiveness of a matrix heat exchanger is strongly dependent on the number of plate-spacer pairs used. Contrary to opinions preceding their work, they showed that the single blow method to determine heat transfer coefficients can be used to analyze a matrix heat exchanger if the overall $N_{tu}$ of the test section is less than 4, and the number of plates used is approximately 10 to 20.

They commented that no maximum slope in the time-temperature history of the fluid stream leaving the test section would occur if the overall $N_{tu}$ is less than 2, as is the case of a conventional heat exchanger. They also noted that differing numbers of plates need to be used for different Reynolds number ranges in order to maintain the $N_{tu}$ between 2 and 4. The authors compiled data from open literature after converting correlations of heat transfer coefficients as functions of Reynolds number to conform to their correlation as:
A plate porosity of 0.3246 was chosen for comparison so that a number of correlations could be compared. A plate thickness to diameter ratio of 1.0, which is a typical value in heat exchangers, and a Prandtl number of 0.7 were assumed. The $N_u$ per plate values were plotted over the Reynolds number ranges investigated by the respective authors. The comparison confirmed that a maximum slope will occur only when total $N_u$ is between 2 and 4.

Imke [20] developed a numerical model to simulate micro-channel flow and heat transfer in compact heat exchangers. The model is based on a forced convection, porous body approach, combined with conventional pipe flow closure correlations. The resultant model may be utilized as a tool for designers of heat exchangers to estimate thermal behavior dependent on geometry and operation conditions. The program calculates outlet temperatures, pressure losses and, in the case where boiling occurs, vapor volume fractions. In addition, the thermo-hydraulic conditions inside the heat exchanger can be determined. Results obtained using the model were compared to experiments with cross flow and counter flow heat exchangers for single-phase flow, and with an electrical evaporator for dual-phase (boiling) conditions.

Andrew et al. [21] developed a model to determine the heat transfer convective coefficient of the upstream face; the upstream face and the tube walls; and the upstream face, tube walls, and the leeward face of a perforated plate using computational fluid dynamics (CFD). The plate’s holes were modeled as hexagonally shaped flow patterns, which is similar to Sparrow’s [5] model presented earlier. The data obtained from the CFD model were found to agree within a few percent of Sparrow’s [5] data. This led to their conclusion that the CFD model and solution were valid. A final equation for the Nusselt number for the front side of a perforated plate was presented as:

$$\frac{N_u}{n} = St\left\{4\frac{l}{d} + 2\left(\frac{1-p}{p}\right)\right\}$$

which is similar to equation 2.

The model was applied to the tube surface of the holes and the leeward side of the perforated plate to study their effect on the overall convective heat transfer coefficient of the matrix heat exchanger. This showed that the plate thickness had a substantial influence on the amount of heat transfer occurring within the tube part of a matrix heat exchanger, and that the leeward side of the perforated plate requires more investigation. In conclusion, an equation for the Nusselt number as a function of the Reynolds number was presented, taking into account the convection of the front, the back, and the inside of the perforation hole, considering air as a working fluid ($Pr=0.7$):

$$Nu = 0.397Re^{0.652}$$

Krishnakumar et al. [22] indicated that the convective heat transfer and flow friction characteristics of a perforated plate matrix heat exchanger are strongly dependent on a number of geometric parameters such as: plate porosity ($p$), spacer thickness ($s$), plate thickness ($l$), and
perforation diameter \((d)\). These geometric parameters have been varied experimentally using the single blow transient test method [16] and the results were used to produce generalized correlations for the heat transfer coefficient \((j)\) and flow friction factor \((f)\) for a matrix heat exchanger as follows:

\[
\begin{align*}
    j &= 0.519 Re^{-0.51} p^{-0.31} \left(\frac{s}{l}\right)^{-0.76} \left(\frac{l}{d}\right)^{1.21} & 100 < Re < 370 \\
    j &= 0.206 Re^{-0.33} p^{-0.22} \left(\frac{s}{l}\right)^{-0.75} \left(\frac{l}{d}\right)^{1.32} & 370 < Re < 1000 \\
    f &= 0.101 Re^{-0.44} p^{-1.76} \left(\frac{s}{l}\right)^{-0.21} \left(\frac{l}{d}\right)^{0.59} & 100 < Re < 370 \\
    f &= 0.006 Re^{-0.18} p^{-2.49} \left(\frac{s}{l}\right)^{-0.31} \left(\frac{l}{d}\right)^{0.49} & 370 < Re < 1000
\end{align*}
\]

Equations 12 through 15 were obtained by using the least squares method, with a confidence level of 94.5\% and uncertainty of 4\%, and they were valid for the following range of parameters:

\[
\begin{align*}
    p &= 0.17 \text{ to } 0.26 \quad (16) \\
    s/l &= 0.5 \text{ to } 2.0 \quad (17) \\
    l/d &= 0.438 \text{ to } 1.02 \quad (18)
\end{align*}
\]

It may be observed from the above correlations that the heat transfer coefficient and friction factor decrease with an increase in porosity or a decrease in the ratio of plate thickness to diameter ratio, and increase with the ratio of spacer thickness to plate thickness. The correlations presented will be useful for optimizing the size of a matrix heat exchanger.

Andrew et al. [23] investigated the heat transfer and fluid flow through porous media by both numerical simulation and experiment. For the numerical simulation, two models were created. The first consisted of a two-dimensional numerical model which was solved using the finite difference approach. The two-dimensional model’s flow in the porous media was described by means of the Brinkman-Forchheimer-extended Darcy equation with thermal non-equilibrium boundary conditions. The second model consisted of a computational fluid dynamics porous media model, which was solved using the finite volume approach. Both models assumed constant fluid phase and properties. Pore diameters were held constant for each simulation and two different porosities were investigated. Boundary conditions were applied at the wall, and the temperatures of the fluid and the porous media were determined by coupled energy equations. The effects of the boundary condition, the Reynolds number, porosity, and heat input were examined.

The experimental investigation consisted of a flow channel with a porous media section that was heated from below. The variation in temperature of the fluid in the porous media was measured along the centerline and along the top and bottom walls. The heat source temperature and the fluid inlet and outlet temperatures were also measured. The results of the numerical model compared well with the laboratory experiment, while the computational fluid dynamics model using the thermal equilibrium equation over predicted the heat transfer. Criteria set forth by other authors were also used to validate the thermal non-equilibrium model’s usage when modeling the heat transfer in porous media. The thermal dispersion was also investigated and it was
determined to have a very small contribution, such that its effect can be neglected. Their conclusion was that the thermal non-equilibrium model was the correct model to use when modeling heat transfer in porous media.

### 2.2.3. Two Phase Flow in Heat Exchangers

Since one of the applications of matrix heat exchangers is steam generation, the present literature review also investigated the boiling mechanisms in compact heat exchangers. There are two principal relevant mechanisms of boiling: nucleate and convective boiling. The heat transfer coefficients for nucleate boiling are high for large temperature differences. However, this effect is suppressed in compact heat exchangers due to the so-called high flux surfaces. Convective boiling relies on the normal convective mode of heat transfer with a phase change occurring by evaporation at the liquid-vapor interface. For a mass ratio of vapor to total flow (e.g. quality) between 0 and 0.95, a commonly used assumption is that the two mechanisms operate simultaneously. Numerous authors correlate the heat transfer coefficient for the two phase flow, \( h_{\tau p} \), in compact heat exchangers to include those two mechanisms of boiling.

Most of the published correlations investigate compact heat exchangers, with few of them tailored toward matrix heat exchangers. We chronologically discuss these publications, and establish a final procedure to apply the correlations to the matrix heat exchanger in the case of two phase flow.

Wang et al. [24] indicated that heat transfer in steam-water two-phase flow involves instantaneous local conduction and convection, as well as energy and mass exchanges across the interface. A numerical method was presented to simulate the micro-scale transients of such flows by modeling the constituent fundamental processes and the interfacial configurations. The bubble dynamics and the liquid temperature distribution were calculated. The fourth order Runge-Kutta scheme was used with respect to time and the volume approach was used with respect to space. This technique has potential for industrial applications.

Kandlikar [25] summarized a number of saturated-flow boiling correlations available in the literature. He chose some of the well-known correlations, and explained that, in general, these may be classified into two categories. In the first category, the correlations were developed by experimental investigators using primarily the authors’ own data, with some including a few other data sources with the same fluid. After ascertaining the accuracy of the experiments conducted, those individual correlations may be used by a designer within the same range of parameters. The correlations in the second category were developed on the basis of a large number of data sets involving a number of fluids over a wide range of parameters. Those correlations represent a larger data base and cover a much broader range of operating conditions, and therefore may have more general applicability.

Kandlikar [26] built on his previous simple correlation [25] for predicting saturated flow boiling heat transfer coefficients inside horizontal and vertical tubes. This was based on a model utilizing the contributions due to nucleate boiling and convective boiling mechanisms. It incorporated a fluid dependent parameter, \( F_{fl} \), in the nucleate boiling term of his previous correlation. The proposed correlation, Equation 19, along with the constants defined in his work,
gives a deviation of approximately 16% with water data, and approximately 19% with refrigerant data. It also predicted the correct two phase flow boiling heat transfer coefficient as a function of the steam fraction as verified experimentally with water and R-113.

\[
\frac{h_{TP}}{h_t} = c_1 CO^{c_2}(25 F_{\eta_0})^{c_3} + c_3 BO^{c_4} F_{\eta}
\]  

(19)

The Dittus-Boelter Equation, 20, can be used to calculate the single phase (liquid only) heat transfer coefficient, \(h_t\), for the saturated boiling heat transfer inside tubes:

\[
h_t = 0.023 \Re_{\nu}^{0.8} \Pr_{\nu}^{0.4} (\frac{k_l}{D})
\]  

(20)

The correlation proposed in Equation 19 can be extended to other fluids by evaluating the fluid dependent parameter \(F_{\eta}\) for that fluid from its flow boiling or pool boiling data.

Kandlikar [27] presented a comprehensive review of literature on evaporation in small diameter passages, along with some results obtained by the author for water operating in 1 mm hydraulic diameter multi-channel passages. He concluded that three flow patterns are commonly encountered during flow boiling in minichannels: isolated bubble, confined bubble, or plug/slug, and annular. The literature review of flow patterns in microchannels was insufficient to draw conclusions. Finally, in designing the evaporators with small diameter channels, the length to diameter ratio depends on the heat transfer and pressure drop characteristics: larger pressure drops were generally accepted in evaporators with small diameter channels.

Customary evaporator or condenser characterization provides data concerning the circulating mass flow rate and inlet and outlet conditions. Corberan et al. [28] analyzed these estimations and the corresponding uncertainty based on typical experimental results of the average heat transfer coefficient in the phase change region. Their analysis showed the influence of measurement uncertainties, i.e. temperatures, pressures, and mass flow rates, and operating conditions on the total uncertainty in estimating two phase flow heat transfer coefficients. They applied their study on a brazed plate heat exchanger working as an evaporator at both co-current and counter-current flow arrangements. Their results showed that the operating conditions in which this kind of equipment is typically characterized and used makes the estimation of the two phase flow heat transfer coefficient difficult. The influence of the model used for data reduction was also discussed. Finally, sample values of the two phase flow heat transfer coefficients obtained from the analysis were presented and briefly discussed.

Imke [20] introduced a numerical simulation tool to estimate pressure losses, temperature, and boiling conditions. His code allows for implementation of special micro-channel correlations if necessary. First, two-phase flow simplified one-dimensional simulations were made for an electrically heated water evaporator. The results for heat transfer (temperatures) agreed well with the experimental findings. However, large deviations were found for the pressure loss, which is more sensitive to the manufacturing accuracy of the channel geometry or deposits at the channel walls.

Thome [29] presented a summary of recent research on boiling in microchannels. He addressed the topics of macro scale versus micro scale heat transfer, two phase flow regime, flow boiling
heat transfer results for microchannels, heat transfer mechanisms in microchannels, and flow boiling models for microchannels. It was observed that for microchannels the most dominant flow regime appeared to be the elongated bubble mode that can persist up to vapor qualities as high as 60-70, followed by annular flow. Flow boiling heat transfer coefficients had been shown experimentally to be dependent on heat flux and saturation pressure while only slightly dependent on mass velocity and vapor quality. Hence, those studies had concluded that nucleate boiling controlled evaporation in microchannels. A recent analytical study has shown that transient evaporation of the thin liquid films surrounding elongated bubbles was the dominant heat transfer mechanism, as opposed to nucleate boiling, and was able to predict those trends in the experimental data. Newer experimental studies have further shown that there is a significant effect of mass velocity and vapor quality on heat transfer when covering a broader range of conditions, including a sharp peak at low vapor qualities at high heat fluxes. Furthermore, it was concluded that macro scale channel models are not realistic for predicting flowing boiling coefficients in microchannels as the controlling mechanism is not nucleate boiling nor turbulent convection, but rather transient film evaporation. Also, it was observed that microchannel flows are typically laminar, and not turbulent as assumed by macro scale models. He indicated that a more advanced three zone flow boiling model for evaporation of elongated bubbles in microchannels was under development attempting to qualitatively describe all those trends. He mentioned that numerous fundamental aspects of two phase flow and evaporation in microchannels remained to be better understood.

Yohanis et al. [30] proposed a simplified method of calculating heat flow through a two phase heat exchanger in which one or both heat carrying media are undergoing a phase change. The method is based on the enthalpies of the heat carrying media rather than their temperatures. This enables the determination of the maximum rate of heat flow, providing the thermodynamic properties on both heat carrying media are known. They claimed that there are no requirements to separately simulate each part of the system or introduce boundaries within the heat exchanger if one or both heat carrying media undergo a phase change. This model can be used at the pre-design stage when the parameters of the heat exchangers may not be known, i.e., to carry out an assessment of a complex energy scheme such as a steam power plant.

Garcia-Cascales, et al. [31] studied refrigeration cycles in which plate heat exchangers were used as either evaporators or condensers. It was indicated that the performance of the cycle was analyzed by means of a method introduced in previous papers, [32, 33], which consisted of assessing the calculation method by looking at representative variables such as the evaporation or the condensation temperature, depending on the case evaluated. The assessment was also used to compare several heat transfer coefficients on the refrigerant side. As in their previous work, [33, 34], the models of the cycle components were considered together with heat exchanger models in such a way that the system of equations they provided was solved by means of the Newton-Raphson algorithm. They also compared the calculated and measured values of the evaporation and the condensation temperatures. The experimental results corresponding to the same air to water heat pump studied in other papers and they have been obtained by using refrigerants R-22 and R-290.
3. CONCLUSION

A literature review was performed to evaluate analytical techniques applicable to matrix heat exchangers, especially in regards to their application as steam generators, where heat is provided to generate superheated steam. The process includes changing phase from a liquid state, as a single phase, to a transition state, where both phases exit concurrently and finally to a gaseous state, again as a single phase, where the ratio of steam to liquid (or degree of superheat capacity) can be described using the steam quality factor, $X$. Using the standard design equations for matrix heat exchangers, the models we recommend for each of the three cases are as follows:

- **Case (1):** for heating water ($X = 0$), as a single phase, Equation 10 or Equations 12 to 18 for the suitable Reynolds number range and heat exchanger parameters should be used to calculate the heat transfer coefficient ($h_l$) and the flow friction coefficient.

- **Case (2):** for boiling water ($0 < X \leq 1$) as two-phase flow, substitute Equation 10, or Equations 12 to 18, for the single phase heat transfer coefficient, $h_l$, in Equation 19 to obtain the two-phase flow boiling heat transfer coefficient, $h_{tp}$.

- **Case (3):** for superheated steam ($X > 1$), equations are selected as in case (1), above.

The authors caution that these equations need to be confirmed experimentally.
REFERENCES

# LIST OF SYMBOLS, ABBREVIATIONS AND ACRONYMS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area (m²)</td>
</tr>
<tr>
<td>b</td>
<td>Width of channel separator, Figure 1</td>
</tr>
<tr>
<td>B0</td>
<td>Boiling Number</td>
</tr>
<tr>
<td>C</td>
<td>Constant</td>
</tr>
<tr>
<td>C₁</td>
<td>Constant</td>
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<tr>
<td>C₂</td>
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<td>C₅</td>
<td>Constant</td>
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<tr>
<td>C₀</td>
<td>Convection Number</td>
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<tr>
<td>d</td>
<td>Diameter (m)</td>
</tr>
<tr>
<td>Eₜ</td>
<td>Euler Number</td>
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<tr>
<td>f</td>
<td>Flow friction factor</td>
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<tr>
<td>Fₘ₁</td>
<td>Fluid-dependent Parameter</td>
</tr>
<tr>
<td>Frₖ₀</td>
<td>Froude Number with liquid flow</td>
</tr>
<tr>
<td>h</td>
<td>Heat transfer coefficient (W/m² K)</td>
</tr>
<tr>
<td>H</td>
<td>Plate Height, Figure 1</td>
</tr>
<tr>
<td>j</td>
<td>Colburn factor of heat transfer</td>
</tr>
<tr>
<td>k</td>
<td>Thermal Conductivity (W/m K)</td>
</tr>
<tr>
<td>l</td>
<td>Plate thickness, Figure 1</td>
</tr>
<tr>
<td>L</td>
<td>MHE’s length (m)</td>
</tr>
<tr>
<td>n</td>
<td>Number of holes, or number of iterations, or number of plates, or index</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>Nₜ</td>
<td>Number of heat transfer units</td>
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<td>p</td>
<td>Porosity</td>
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<td>Pitch [m]</td>
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<td>Pr</td>
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<td>W</td>
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<tr>
<td>X</td>
<td>Steam fraction, or quality</td>
</tr>
<tr>
<td>Y</td>
<td>Distance along hole’s direction</td>
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**Greek symbol**

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<tr>
<td>δ</td>
<td>Plate thickness (m), or hole length</td>
</tr>
<tr>
<td>ε</td>
<td>Heat exchanger effectiveness</td>
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<tr>
<td>λ</td>
<td>Thermal conductivity (W/m K), or Longitudinal Heat Conductivity Parameter</td>
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<tr>
<td>η</td>
<td>Efficiency</td>
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**Subscripts**

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<td>Fin</td>
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<td>i</td>
<td>Stream number or inside</td>
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<tr>
<td>l</td>
<td>Liquid</td>
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<tr>
<td>L</td>
<td>Unit length</td>
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<td>o</td>
<td>Outside</td>
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<tr>
<td>min</td>
<td>Minimum</td>
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<tr>
<td>p</td>
<td>Plate</td>
</tr>
<tr>
<td>TP</td>
<td>two phase flow</td>
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