Computational Methods for Identification, Optimization and Control of PDE Systems

Final Report on AFOSR Grant FA9550-07-1-0273
for the period
1 April 2007 - 30 November 2009

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This final report contains a summary of the activities supported under the Air Force AFOSR Grant FA9550-07-1-0273 during the period 1 April 2007 through 30 November 2009. The research focused on the construction of high fidelity numerical methods and the development of a rigorous mathematical framework for attacking complex optimization and control problems with partial differential equations (PDEs) as constraints. The research is motivated by applications to two distinct but related application areas: (1) Optimal design and control of ultra-light large space structures which will serve as the platforms for many future space applications such as space-based radar; (2) Optimal design and control of flexible air vehicles and feedback control of fundamental fluid flows. We generated new numerical methods specifically for parameter estimation, shape optimization, optimal control and feedback control of PDE systems of the type that govern ultra-light inflatable space structures, aerodynamic design and fluid flows. We developed a rigorous mathematical framework to analyze convergence of the resulting control, design and optimization algorithms, obtain error estimates and rates of convergence.

Introduction and General Goals of the Research Program

The research program is focused on the development of numerical methods and software specifically for the purpose of solving control, design, and optimization problems where the underlying dynamics are described by partial and partial-functional differential equations. Although simulation tools and the numerical methods that provide the foundations of simulation software must play an important role in any research of this type, the demands placed on numerical methods developed specifically to solve control, design, and optimization problems are often different and more complex in nature. With this in mind, we focused on the following general objectives.

- We conducted research on new numerical methods specifically for parameter estimation, shape optimization, optimal control and feedback control of PDE
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Dr. Fariba Fahroo

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systems of the type that govern ultra-light inflatable space structures and specific fluid flows. These numerical schemes will be sufficiently general so as to be applicable to a broad range of Air Force problems.

- We developed a rigorous mathematical framework to analyze convergence of the resulting control, design and optimization algorithms, obtain error estimates and rates of convergence. This framework is essential to the verification of the mathematical design models and corresponding numerical algorithms.

- We worked develop high fidelity and high level computational tools based on the new numerical methods that are suitable for use by engineers working in the design and optimization of structures and fluids. In addition, we investigated a new computational approach to the design of experiments based on bounded error methods.

**Accomplishments and Final Report on the Grant**

Through the efforts funded under this grant we accomplished several important goals.

- We have shown that continuous sensitivity equations can be used to analyze the impact of un-modeled parameters on closed-loop simulations. Also, we employ the corresponding discrete sensitivity equations to provide information on computational uncertainty with respect to grids and un-modeled parameters. We defined a new “Lyapunov Exponent” in terms of the the system parameters and use this new concept to determine a prediction horizon where qualitative and numerical predictions may be invalid beyond this horizon. We employed this idea to develop insight into the validity of long term simulations. This is particularly important for closed-loop simulations.

- We developed a new computational approach for optimally placing sensor / actuator pairs in order to maximize the controllability and observability of the reduced order approximations of the PDE control systems. We developed a method to study controllability and conditioning of POD models of PDE systems. This work is provided the framework for addressing optimal sensor / actuator location problems.

- We established that the reduced order models must be constructed so that the numerical conditioning of the key control equations (Ricatti, Liapunov, etc.) is as small as possible. We also, discovered new fundamental relationships between the concepts of distance to the nearest uncontrollable system and the conditioning of the approximate Riccati equation for different reduced order models.
• We developed a new reduced basis construction method which allows for separate consideration of baseline and actuated dynamics as opposed to the case where the baseline and actuated dynamics are considered simultaneously. These different bases give rise to control models with varying numerical properties that allow for efficient controller design.

• Developed a new Mesh Independence Principle for PDE Riccati equations and used this method to solve large scale flow control problems.

• Developed well-posed high fidelity model of flexible space structure to support AFRL/VSSV work on ISAT and solved the identification problem for this application.

• We developed a new parameter estimation scheme for the design of experiments when insufficient data is available to allow for statistical verification. The method is based on bounded error techniques, but reduces the computational effort by solving an inverse "data-to-parameter" problem. This new approach to parameter estimation and validation that uses the sensitivity methods also developed under this grant and implicit function theorems to map sets in the model’s output to the boundary of the parameter membership set.

• We began the work to extend our research on optimal sensor/actuator location to problems with dynamic sensor networks. This effort was motivated by the potential use of MAV based sensors to for information gathering and battlefield management. This work allowed for the inclusion of the MAV sensor platform dynamics.

• We developed an infinite dimensional gradient based algorithm to solve the optimal control problem for the optimal sensor platform control problem with dynamic sensor networks.

In addition, during the grant period we have:

• Produced more than 26 scientific papers,

• Made more than 39 presentations at conferences and colloquium,

• Directed more than 8 graduate students and 1 postdoc,

• Worked with more than 14 visitors, representing 5 different countries,

• Made visits to AFRL at Wright-Patterson and Kirtland AFB to work with Air Force personnel.
Personnel Supported

During this period, the following people were supported in part under Grant FA9550-07-1-0273:

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<tr>
<th>Senior Investigators</th>
<th>Postdoctoral Fellows</th>
<th>Graduate Students</th>
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<tr>
<td>John A. Burns</td>
<td>Imran Akhtar</td>
<td>Ermira Cami</td>
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<td>Eugene M. Cliff</td>
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<td>Adam Childers</td>
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<td>Lizette Zietsman</td>
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<td>Weiwei Hu</td>
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<td>Jeff Borggaard</td>
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<td>Betty Paredez-Alvarez</td>
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<td>Miroslav Stoyanov</td>
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<td>Dan Sutton</td>
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Participation and Presentations at Meetings 2007-2008

John Burns


4. Atlantic Coast Symposium on the Mathematical Sciences in Biology and Medicine, Raleigh, NC, April, 2008.


8. International Conference on Shape and Topology Optimization, Graz, Austria, September, 2008.


Gene Cliff


Lizette Zietsman


6. 47th IEEE Conference on Decision and Control, Cancun, Mexico, December.


Publications Supported Under this Grant for 2007-2009


Honors & Awards Received

- Mr. Carlos Rautenberg: The Lee and Regina Steeneck Scholarship Prize for Excellence in Research

AFRL and DOD Points of Contact

- Dr. Siva Banda, Control Sciences Center of Excellence, AFRL/VACA, WPAFB, (937) 255-8676.

- Dr. Thomas Murphey, ISAT IPT Lead, AFRL/VSSV, Kirtland AFB, New Mexico, (505) 846-9969.