The goal of the research was to create enhanced practical methods to estimate and simulate hyperspectral images by using recorded data of low spectral resolution images. These data can be used to test algorithms designed for many hyperspectral image analysis tasks, such as target detection, classification and tracking. An advantage
Final Report: Estimation and Simulation of Hyperspectral Images

ABSTRACT

The goal of the research was to create enhanced practical methods to estimate and simulate hyperspectral images by using recorded data of low spectral resolution images. These data can be used to test algorithms designed for many hyperspectral image analysis tasks, such as target detection, classification and tracking. An advantage is that these images are unclassified and can be used in an university environment where international students are often used. The images are also readily defined to allow ground truth to be known so that accuracy of the image analysis tasks can be measured.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Number of Papers published in peer-reviewed journals: 0.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 0.00

(c) Presentations


Number of Presentations: 1.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

(d) Manuscripts

Number of Manuscripts: 0.00
## Graduate Students

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT_SUPPORTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jayakumar Nanjappan</td>
<td>0.50</td>
</tr>
<tr>
<td>Pradeep Dorairaj</td>
<td>0.25</td>
</tr>
<tr>
<td>Karthik Krish</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**FTE Equivalent:** 1.00  
**Total Number:** 3

## Names of Post Doctorates

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT_SUPPORTED</th>
</tr>
</thead>
</table>

**FTE Equivalent:**  
**Total Number:**

## Names of Faculty Supported

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT_SUPPORTED</th>
</tr>
</thead>
</table>

**FTE Equivalent:**  
**Total Number:**

## Names of Under Graduate students supported

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT_SUPPORTED</th>
</tr>
</thead>
</table>

**FTE Equivalent:**  
**Total Number:**

## Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period.

- The number of undergraduates funded by this agreement who graduated during this period: 0.00
- The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields: 0.00
- The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields: 0.00
- Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): 0.00
- Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering: 0.00
- The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense: 0.00
- The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

## Names of Personnel receiving masters degrees

<table>
<thead>
<tr>
<th>NAME</th>
<th></th>
</tr>
</thead>
</table>

**Total Number:**
### Names of personnel receiving PHDs

<table>
<thead>
<tr>
<th>NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number:</td>
</tr>
</tbody>
</table>

### Names of other research staff

<table>
<thead>
<tr>
<th>NAME</th>
<th>PERCENT SUPPORTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTE Equivalent:</td>
<td></td>
</tr>
<tr>
<td>Total Number:</td>
<td></td>
</tr>
</tbody>
</table>

### Sub Contractors (DD882)

### Inventions (DD882)
Final Report
Estimation and Simulation of Hyperspectral Images

H. J. Trussell
Department of Electrical and Computer Engineering
North Carolina State University
Raleigh, NC 27695-7911

November 23, 2009

Objective

The goal of the research was to create enhanced practical methods to estimate and simulate hyperspectral images by using recorded data of low spectral resolution images. These data can be used to test algorithms designed for many hyperspectral image analysis tasks, such as target detection, classification and tracking. An advantage is that these images are unclassified and can be used in a university environment where international students are often used. The images are also readily defined to allow ground truth to be known so that accuracy of the image analysis tasks can be measured.

1 Introduction

This work developed and implemented basic theory for estimation and simulation of realistic hyperspectral images. The dual goals of both estimation and simulation are reasonable since the proposed process for creating simulated hyperspectral images begins with a multiband image of much lower dimensionality, e.g., R,G,B. Since this image provides a starting point for the creation of the hyperspectral image, we can consider the simulated image an estimate of an actual hyperspectral image. The quality of the simulations can be judged not only by how realistic they appear, but how well they match a true hyperspectral image that is used to create the lower dimensional data.

This effort resulted in methods to create hyperspectral images based on common three band (RGB) images. The images that were created appeared indistinguishable to the standard observer in the visible color region. The spectral properties in the IR region are currently undefined and provide a topic for additional research. The images were obtained using methods developed by Vrhel and Trussell [3]. This paper introduced three distinct methods with which spectra could be generated.
The work was extended by this effort to use prototype spectra to help generate spectra that have random characteristics but are statistically similar to those in the class from which the prototype comes. The results from this work include:

1. Implemented spectral generation methods developed in [3]. The methods used were
   (a) copulas, based on probability density transformations,
   (b) projection onto convex sets (POCS), based on defining sets representing statistical properties of the class,
   (c) neural networks, based on training the network using members of the class.

2. Employed the methods to produce random spectra that may be representative of a class of object in the image. This was done primarily by use of a mean spectra that is representative of a class.

3. Evaluated methods with respect to computational efficiency and ease of use. The basic results are that POCS is the most versatile method but is significantly slower than the others. The Neural networks approach had significant problems in attaining the flexibility needed to easily represent different classes of objects.

4. Developed methods to produce textured images with random variations that are perceptually uniform across the image. The bounds on the sets used in POCS can be varied according to the region of the perceptual space used for the problem.

5. Created a small database of classes of hyperspectral data that can be used to characterize objects. Currently, the classes include such common objects as grass, leaves, asphalt, bark (tree trunks) and various colored flowers.

6. Created small hyperspectral images using these methods.

One example of this effort is shown in Figure 1, where we display the original RGB image and seven bands of the hyperspectral image created from this. Note that the deep red band at 700 nm looks quite different from the red band at 600nm. This property of distinguishing objects by their different signatures in many spectral bands makes hyperspectral images most useful in target detection and classification. Creating such images will be useful in the academic and commercial environments. Some of these results have been presented in conferences [1]. Additional submissions are being prepared.

This project is motivated by the potential uses of hyperspectral data for ATR which exists, but is hampered by the fact that most military hyperspectral data is classified. Even if the a majority of graduate student research assistants in the relevant disciplines were not US citizens, the classified data would limit testing at U.S. universities. In addition to simulating a single image, this research will allow us to create a series of images of similar scenes that represent the same physical objects recorded from a different perspective or under different illumination. A final by-product comes from the fact that the algorithms developed to create the simulations can also be used to estimate an underlying hyperspectral image.
Figure 1: (a) original RGB image. (b-h) images for bands at 400nm to 700nm at 50 nm increments.

2 Approach

2.1 Use of Low-resolution Spectral Images

We assume we have the responses of the sensors through the Red, Green, and Blue filters that were used to construct the color image. Suppose, just for example, that we have a spectral resolution of
3nm, and we know the response of each filter from 400-700 nm. Our filters may then be represented by a vector of 100 bands. We refer to those filters as \( f_r, f_g, \) and \( f_b \). Define the input image to also be a 100 element vector, denoted \( i_\lambda \) over the same range, and the filtering process may be written as a vector products,

\[
\begin{align*}
t_r &= i_\lambda^T f_r \\
t_g &= i_\lambda^T f_g \\
t_b &= i_\lambda^T f_b
\end{align*}
\]

For the case of an image that represents an intensity distribution in the visible range, we can assume a scene of reflectances, \( r(\lambda) \) that is illuminated by a radiant source whose energy distribution is defined by \( l(\lambda) \). The intensity image is obtained by representing the illiminant as a diagonal matrix, \( L \), and

\[ i = Lr \]

where we have written the reflectances and intensities as vectors. The equations of can be written

\[ t = F^T Lr \]

where the filters of eq.(1) are the columns of \( F \). For the estimation problem, for a single pixel, we have three equations and 100 unknowns to find \( i \). This as an extremely underdetermined estimation problem, and can be addressed effectively only if we employ significant a priori constraints. These constraints will be many of the same ones used for the generation of spectra for a desired class of object. Thus, we will concentrate here on the constraints and how they are employed in by the generation methods.

### 2.2 Short Review of Approaches

In our first work, we implemented the three methods suggested by [3]: copulas, projection onto convex sets (POCS) and neural networks. We will review those methods here and evaluate their suitability for modeling classes of objects.

#### 2.2.1 Copulas

The approach using copulas is derived from a common method of generating multivariate random variables [4]. The copula arises from Sklar’s Theorem [5], which proves that any joint probability distribution can be written in the form

\[
F(x_1, ..., x_N) = C[F_1(x_1), ..., F_N(x_N)],
\]

where \( C(u_1, ..., u_n) \) is a joint distribution function with uniform marginals (that is \( u_i = F_i(x_i) \) is uniformly distributed). Assuming the derivatives exist, the joint density function is given by

\[
f(x_1, ..., x_N) = f_1(x_1) \times ... \times f_N(x_N) \frac{\partial^{N-1} C}{\partial F_1 \cdots \partial F_N},
\]

where \( c = \frac{\partial^{N} C}{\partial F_1 \cdots \partial F_N} \) and \( f_i(x_i) \) is the density corresponding to the distribution function \( F_i(x_i) \).
The function $C$ is called a copula. The function serves to “couple” the marginals $F_i(x_i)$ into the joint distribution. The function, in essence, introduces correlation between the variables. The most common form for the copula uses a multivariate Gaussian distribution function.

In this case, the process for generating the vectors is straightforward. To begin, a set of zero-mean unity-variance correlated multivariate Gaussian distributed vectors are generated. These vectors are easily created using standard commercial software, e.g., MATLAB.

In the next step, each component of the vector is transformed to a uniform distribution using the Gaussian CDF. In other words, if the Gaussian distributed random vector is denoted by $v$, then the uniformly distributed vector is given by

$$u = \frac{1}{2} \text{erf} \left( \frac{v}{\sqrt{2}} \right),$$

where the operations occur on each of the vector elements. Note that the elements of $u$ will remain correlated since the elements of $v$ were correlated.

Finally, the elements of each of the uniformly distributed random variables $u$ are mapped using the appropriate inverse of the CDF that you desire of your final variable. For example, if it is desired to have the marginal CDF at wavelength $i$ be $F_i(x_i)$, then the random reflectance value at wavelength $i$ is given by

$$[r]_i = F_i^{-1}([u]_i).$$

Again, note that the elements of $r$ will be correlated. The exact characteristics of the resulting correlation is an open research question. For most previous uses it was sufficient to have any simple correlation. In our case, the correlation is used to enforce smoothness in the spectral domain.

**Evaluation:** The copula method is easily modified to produce spectra with a given mean distribution. As mentioned above, the method produces correlated spectra but the exact correlation of the output cannot be specified. The method is quite fast compared to the POCS below. Unfortunately, the copula method cannot be modified to produce spectra with a specific visual color appearance, i.e., tristimulus or chromaticity value.

### 2.2.2 POCS

Set theoretic methods have been applied to color science problems [6, 7] and, of course, for problems in image and signal restoration [2]. Here we used this powerful method to constrain the vectors such that they have the desired properties. We assume that they are properly bounded, have a specific mean and correlation, have a variation distribution similar to a particular data set and are appropriately smooth.

The method of successive projections onto convex sets (POCS) is an iterative algorithm to find a point in $C = \cap_{k=1}^N C_k$. For POCS, the sets must be closed and convex. It is also assumed that the sets have a non-empty intersection that contains a feasible solution. The update $\hat{a}_{k+1}$ at iteration $k + 1$ is found by sequentially projecting onto the $N$ sets. Let $P_c(a)$ denote the projection of $a$
onto the set $C_c$, i.e.

$$P_c(a) = \left\{ \begin{array}{ll}
    a & a \in C_c \\
    \hat{a} & a \notin C_c
\end{array} \right.,$$

(10)

where $\hat{a}$ satisfies

$$\min_{\hat{a}} d(a, \hat{a}) \quad \text{subject to} \quad \hat{a} \in C_c,$$

(11)

for some metric $d(\cdot, \cdot)$. Then, the updated estimate $\hat{a}_{k+1}$ at iteration $k + 1$ is given by

$$\hat{a}_{k+1} = P_N (P_{N-1} (\cdots P_2 (P_1 (\hat{a}_k)))).$$

(12)

The method is guaranteed to converge. The constraints previously mentioned can be used to define appropriate sets. In addition, a common set defined by

$$C_f = \{ i : \| t - F^T i \|^2 \leq \Delta_f \}$$

(13)

is used to enforce the condition that the simulated pixel is consistent with the recorded data. The matrix $F$ represents the sensor responses or CIE color matching functions. The bound, $\Delta_f$, allows for the possibility that there is some noise in the recording of the low-resolution data.

One significant result of our research has been the development of methods to vary the bounds of the color sets, Eq.(13), according to the region of the color space in which we are working. This allows us to produce variations in the output spectra over different classes that are perceptually similar. An example of this effect is shown in Fig. 2. This extension has application beyond the hyperspectral simulation problem by allowing fast computation of perceptual metrics.

To use the POCS method, we generate a set of $M$ spectra of length $N$ stored in a matrix $R$. This matrix should be contained in the intersection of the following convex sets:

$$C_{\text{bound}} = \{ R \mid r_{ij} \leq r_{ij} \leq r_{ij} \mid i = 0, \ldots, (N - 1) \quad j = 0, \ldots, (M - 1) \},$$

(14)

where $r_{ij}$ is the element in the $i$th row and $j$th column of $R$ and $[r_{min}, r_{max}]$ define the reflectance range at wavelength $i$,

$$C_{\text{mean}} = \{ R \mid \| R1 - M\bar{r} \|^2 \leq \delta_{\text{mean}} \}$$

(15)

where $1$ denotes a vector of all ones. This set is used to allow generation of different classes of objects based on their mean. The object class can also be represented a covariance structure. This set can be defined by

$$C_{\text{cov}} = \{ R \mid \| RR^T - M(K_r + \bar{r}\bar{r}^T) \|_F \leq \delta_{\text{cov}} \}$$

(16)

where $\| \cdot \|_F$ denotes the Frobenius norm, $\bar{r}$ is the desired mean of the data, and $K_r$ is the desired covariance matrix. The values of $\delta_{\text{mean}}$ and $\delta_{\text{cov}}$ are be selected based upon a confidence interval of the estimated covariance matrix.

**Evaluation:** The POCS method is easily modified to produce spectra with a given mean distribution and covariance. The POCS method can easily be modified to produce spectra with a specific color appearance by using the set of Eq.(13). The major problem with POCS is its slow speed. However, for cases where images need to be produced infrequently, but used often once they are available, the method is useful.
Figure 2: (a) Original RGB image. (b) RGB version of hyperspectral image that is based on the original but has spectral variations added.

2.2.3 Neural Networks

To use artificial neural networks (ANNs) to simulate the reflectance spectra requires appropriate design of the network and proper training. In this case, the input into the network will be white noise
vectors and the output should be simulated reflectance spectra. The structure of the ANN should be designed based upon \textit{a priori} knowledge of the function that the network is to approximate. It is known that a 2-layer feed forward network with a sigmoidal nonlinearity can be used to approximate any function with a finite number of discontinuities, arbitrarily well, given sufficient neurons in the hidden layer [8, 9, 10]. For this reason, we will concentrate on this architecture.

Given a \( N \) element input vector \( \mathbf{e} \), the \( N \) element output of the 2-layer feed-forward ANN with \( S \) neurons in layer one is expressed as

\[
\mathbf{r} = \mathcal{L} (\mathbf{e}) = \Psi [ \mathbf{V} \Phi (\mathbf{W} \mathbf{e} + \mathbf{b}) + \mathbf{c}] ,
\]

where \( \Phi (\mathbf{x}) = [\phi(x_1), ..., \phi(x_S)]^T \), \( \mathbf{b} \) is an \( S \) element vector of bias terms for the hidden neurons, \( \mathbf{W} \) is an \( S \times N \) matrix, \( \mathbf{V} \) is an \( N \times S \) element vector of bias terms for the output neurons, \( \mathbf{c} \) is a \( N \) element vector, \( \phi \) is the sigmoidal function, which is given by

\[
\phi(x) = \frac{2}{1 + \exp(-2 \times x)} - 1 ,
\]

and the output function is given by \( \Psi (\mathbf{x}) = [\psi(x_1), ..., \psi(x_{171})]^T \),

\[
\psi(x) = \frac{1}{1 + e^{-x}},
\]

which insures the output values will be bounded between zero and one. The parameters \( \mathbf{V}, \mathbf{c}, \mathbf{W}, \) and \( \mathbf{b} \) are optimized through appropriate training.

**Evaluation:** The neural networks method is the least versatile of the three. We modified the method by using radial basis functions but with little improvement. The network would need to be trained for each class. From our experiments, the number of training samples for a class would be almost as many as the number that would be generated after the network is trained. However, if we envision a library of trained networks (or library of network parameters), the process would be quite fast. We did not investigate the possibility of creating such a library.

### 3 Conclusion

Our work produced several extensions to the work of Vrhel and Trussell. A more complete evaluation of the method introduced in that work was done. The creation of hyperspectral images that are realistic pictorially by using low-dimensional spectral images is now possible. There are additional enhancements that are worth pursuing and will be investigated in later work.

### References


