**Understanding the Effect of Atmospheric Turbulence on Optical and Infrared Propagation using Hilbert Phase Analysis**

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Understanding the Effect of Atmospheric Turbulence on Optical and Infrared Propagation using Hilbert Phase Analysis

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Introduction: The refractive index structure parameter $C_n^2$ is a figure of merit used to describe the magnitude of the effect of turbulence in the atmosphere. Understanding the influence of different local climate parameters upon $C_n^2$ is an important step toward developing a prediction model for in-field laser interrogators. The layered structure of the atmosphere and its random behavior present unique challenges to theoretical and experimental studies in propagation research. This article presents a novel technique termed Hilbert Phase Analysis (HPA) to provide insights into the physical interactions. We explore the technique’s use in analyzing the dependence of $C_n^2$ on local climate variables using data recently obtained from campaigns conducted in Puerto Rico.

The Method: HPA is a new technique that consists of Empirical Mode Decomposition (EMD) and the Hilbert Transform (HT). EMD is an instantaneous frequency filter bank that generates a family of empirical eigenmodes called Intrinsic Mode Functions (IMF) from a non-linear, non-stationary time series.1 By transforming the IMFs with HT, we produce physically interpretable instantaneous phase angles, $\phi_i$. These values are akin to Fourier phase angles, except that they apply per sample point. This feature now enables the study of the temporal phase relationship for many measurables in a point-wise manner. This is not possible using standard Fourier techniques. Legacy time series tools such as wavelets provide relatively poor temporal resolution. Figure 8 illustrates how, by using HPA, the phases $\phi_1$ and $\phi_2$ of two interdependent signals can be shown to be related. On the left, Signal 1’s vector rotates clockwise, while that of Signal 2 on the right rotates counter-clockwise. The time-dependent phase angles of these signals are clearly inversely correlated. The relationship can provide insight into how climatic parameters might affect $C_n^2$.

Experiment: A series of campaigns to collect $C_n^2$ and simultaneous climate data are being performed at the University of Puerto Rico at Mayagüez.2 The results described here are from a campaign conducted in February and March 2006. Scintillometers were placed on the roofs of buildings at the main campus of the university. The transmitter and receiver were separated by 90 meters over land. Climate data was acquired at a nearby weather station.

Data Analysis: Figure 9(a) compares meteorological data with $C_n^2$ over time. This data is from a very clear and sunny day. We can see close tracking between the measured solar radiation flux (SR) and the temperature (T), which is expected, since the solar flux controls the heating of the atmosphere. We also observe similar tracking of $C_n^2$ with T, validating that temperature plays a major role in defining optical turbulence. From the relative humidity (RH) record, we see an inverted relationship to T and therefore to $C_n^2$. If we directly cross-correlate $C_n^2$ and a climate variable, the resulting function is very difficult to interpret, since $C_n^2$ is highly variable with time.

FIGURE 8 Illustration of HPT phase angles where the colors represent the changes in time. The arrows show the signal vectors, which change over 20 time steps.
FIGURE 9
(a) Data for March 9, 2006; (b) Cross-correlations of the instantaneous phase angles of the different measurements within ±500 minutes of the zero lag position. Black: The correlation function between solar radiation flux (SR) and temperature (T) phase angles, showing a strong positive correlation. Red: The correlation function between Cn and temperature (T) also showing a strong positive correlation. Blue: The correlation function between Cn and relative humidity (RH), showing a negative correlation.
Applying HPA to this family of measurements provides us with the time-varying phase angles. Since we expect correlations between our raw parameter records, we calculated cross-correlations of the HPA-determined phase angles, as shown in Fig. 9(b). The curve plotted in black is the cross-correlation function of the instantaneous phase angles SR and T; it shows that T faithfully follows SR, with an approximate ten-minute lag. The cross-correlation function of $C_n^2$ and T indicates that the turbulence parameter follows a similar relationship. These results support the conventional theory that temperature strongly influences $C_n^2$, thereby calibrating the HPA technique against known atmospheric behaviors. Finally, the cross-correlation function between $C_n^2$ and RH shows an inverse dependence, which corroborates the first-order approximation of an inverse relationship existing between temperature and relative humidity.

**Conclusions:** HPA provides the ability to study the time-varying phase angles of non-linear signals.

In this research, the technique was tested with optical turbulence data and its simultaneous local climate record taken in a recent campaign in Puerto Rico. The technique demonstrated consistency with conventional physics. That is, the direct influence of temperature on $C_n^2$ was verified. Additionally, the technique reveals an approximate inverse relationship between $C_n^2$ and relative humidity. The experiment is being extended to over-the-water ranges of up to 600 meters between Magueyes Island and the Villa Parguera Hotel in La Parguera, Lajas, Puerto Rico, as shown in Fig. 10.

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**References**


**FIGURE 10**

Propagation path shown for $C_n^2$ experimental studies for 2006–2007 campaigns.