I. Introduction

Since the time of World War II the propagation of sound in ship wakes has been subject of considerable interest. A ship’s wake is a mixture of turbulent seawater created by the motion of a surface ship’s hull and air bubbles that are created by the breaking of a ship’s bow and stern waves, and by the cavitation of the ship’s propellers. The signature of a ship’s wake will vary, depending primarily on hull design, speed through the water, and local oceanographic conditions. As the wake ages, it goes from a violent breakup and mixing of bubbles due to turbulent diffusion, to one where the turbulence decays and the bubbles begin to rise slowly toward the surface due to their buoyancy and changes in the buoyancy of the water mass. This time and frequency dependent bubbly mixture has dramatic effects on acoustic signals due to increases in absorption and refraction. As the wake ages, bubbles of different sizes rise at different rates, and the horizontal and vertical distributions of bubble densities give rise to changes in both sound speed and absorption. It is these generated bubble densities that create large acoustic resonance scattering cross sections that are responsible for the acoustic signature of a ship’s wake.

Some of the earliest acoustic measurements on surface ship wakes are documented in “Physics of sound in the Sea,” [1]. This book discusses the acoustic probing of numerous ship wakes and still is an excellent starting point in surface ship wake characterizations. Since then, there have been numerous measurements made of acoustic propagation through bubble clouds. Extensive bibliographies can be found in [ 2-6 ].

In this paper, we will describe a method of measuring the average across the wake excess absorption due to bubbles within the wake. Pulsed cw signals ranging from 30 kHz to 140 kHz, in 10 kHz increments, were transmitted across a surface ship wake. Using the average excess signal absorption at 12 frequencies and, and their resonance frequencies, estimates of bubble number densities as a function of wake age were obtained at 7.4-meter increments behind the wake generating ship. These measurements were continued for about 2 km behind the ship.
**Broadband acoustic transmission measurements in surface ship wakes**

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II. Theory

Measurements of the absorption as a function of frequency gives a method of estimating bubble distributions that allows the determination of the sound speed as a function of frequency that is given by:

\[
\beta(a_r) = \frac{2\pi c_\omega}{\omega} \int_{0}^{\infty} a \delta n(a) da = 0 \quad (a)\n\]

\[
c = c_0 \left( 1 - \frac{2\pi}{\kappa\rho\omega^2} \int \frac{a_0\left(\frac{a_r^2}{a^2} - 1\right)}{\left(\frac{a_r^2}{a^2} - 1\right)^2 + \delta^2} da \right) = 0 \quad (a)\n\]

Where \(a_r\) is the radius of the bubble that resonates at a frequency \(f\), \((\omega = 2\pi f)\), and \(\delta\) is a damping parameter. It is easily seen that the integrand for \(\beta\) has its largest contribution near the resonant value \(a_r\). Since that the numerator is slowly varying over the range of values near resonance, an approximation can be made that relates the absorption to number density (resonant bubble approximation RBA). It has been shown to a good approximation that [7]

\[
n(a) \approx \frac{4.6 \cdot 10^{-6} f^3 \beta(a_r)}{1 + 0.1z} = 0 \quad (a)\n\]

where \(n(a)\) is the bubble density distribution, \(f\) is the frequency in Hz, \(z\) is the depth, and \(\beta\) is the absorption coefficient. Thus by obtaining measured estimates of the frequency dependent absorption coefficient in dB/m, estimates of the bubble number density can be obtained.

II. Instrumentation and Experimental techniques

A. Experimental configuration

The measurements were conducted in an area 10 miles south of Panama City, Florida in the summer of 2004. The water depths in this area were about 35-m. During the measurements, sea conditions were about sea state 1. The wake generating ship (Neptune) was provided by the NAVDIVESALVTRACEN in Panama City, Fl. It measured 131 ft long, 26.6 ft wide, had a draft of 4.95 ft., and displaced 173 light tons.
Neptune is powered by two Hamilton water jet systems, and one forward bow thruster. The ship had a top speed of 18 knots and she is shown in Figure 1.

![Image of Neptune](image1.png)

**Figure 1.** Wake generating ship Neptune.

Using Neptune to generate the wake, a series of across the wake acoustic propagation measurements were taken using a towed acoustic source system and towed array. A broadband, narrow vertical beam, acoustic source was installed on a stable tow body. A second ship, towed this system down one side of the wake generated by Neptune. CW signals were transmitted through the wake to a receiving array being towed by a third ship.

Each measurement sequence began with Neptune moving a constant speed at a distance of 4 km from the two measurement systems. The ships towing the measurement systems proceeded towards Neptune at a speed of 4.5 knots. The measurement ships positioned themselves across from each other, with a source-to-receiver range of approximately 150-m. This distance was monitored using a laser range finder, and did not vary by more the a few meters during each series of measurements. Acoustic transmission loss measurements were taken across Neptune’s wake for a speed of 15 knots. Figure 2, is a sequence of pictures showing Neptune’s wake as it passes between the two measurement ships.

![Image of Neptune wake](image2.png)

**Figure 2.** Photographs of Neptune passing between the two measurement ships
A. Instrumentation

A towed receiving array was designed and assembled using ten general-purpose hydrophones. The hydrophones were mounted in holders placed inside a 10-m long, 9-cm diameter freely flooding tygon tube. The outputs of each hydrophone were wired through the tube to a connector on the end of the array. A separate cable was attached to the connector, and the data from each hydrophone was sent to a data acquisition system located on the array tow ship. The array was secured to the stern of a dismantled Klien side scan sonar body. A depressor wing was attached, to stabilize the array’s tow depth. A 60-m long polypropylene rope was attached to the end of the array to minimize array deformations. The towed array assembly is shown in figure 3.

![Figure 3. Towed array configuration.](image)

The towed array was deployed, behind and off to the starboard side of Island Diver using a 1 ton davit. The draft of Island Diver was about 1.2-m, and during the measurements, the starboard engine was shut down to minimize bubble formations. The array was towed at a minimum depth of 3.6-m putting it well below the draft of the tow ship. Three Honeywell depth sensors were located at the forward, center, and end of the array. These transducers monitored the depth and stability of the array. Pressure data was acquired via a serial RS232 link and recorded along with the received acoustic data. During the data run the vertical array displacements averaged 20-cm.

A pair of 1-3 composite broadband acoustic sources were mounted on a stable tow body system with their maximum response axis (MRA) 3-degrees down from the horizontal. These sources had vertical beam pattern –3dB down points that ranged from 17° at 30 kHz, to 4° at 140 kHz. The horizontal beam –3dB down points ranged from at 50° at 30 kHz to 75° at 140 kHz. The broad horizontal beam patterns enabled the entire receiving array to be ensonified at the source to receiver distance of 150-m. The source system was towed off the port side and at a depth of 3.6 m. The draft of the tow ship was about 2 m and during the data runs, the port engine was also shut down. A photograph of the towed system is shown in Figure 4.
The relative positions of the three ships were monitored using Garman GPS unit. Date, time, latitude & longitude were recorded each second, and used to synchronize the relative ship positions with the acoustic data. A schematic of the source generation and array data acquisition systems are shown in figure 6. Three pulse sequences were transmitted. Each sequence included four 0.5 ms long cw pulses that were separated by 1 ms. The frequencies in the first pulse sequence ranged from 30 to 60kHz, the second ranged from 70 to 100kHz, and the third from 110 to 140kHz. The pulse sequence reputation rate was 1 Hz and each cw pulse was separated by 10 kHz. The distance traveled by the measurement systems between each pulse sequence was approximately 2.5-m. Thus each pulse sequence was repeated every 7.5 m of travel down the wake.

B. Measurements and results

The source–to-receiver transmissions began several minutes before Neptune passed between the two measurement ships. These initial transmission loss measurements were used to obtain signal levels in the bubble free water. Each of the ten-hydrophone channels were band passed filtered, and sampled at 1MHz. A 1024 point PSD was calculated, and the intensity levels at each of the transmitted frequencies estimated. These measurements were repeated as Neptune traveled between the two measurement systems. By comparing the signal energy levels before and after passage of Neptune, direct estimates of the average excess absorption due to changing bubble densities across the turbulent portion of Neptune’s wake were obtained from

\[
TL_{(EXCESS)} = 20\log \left( \frac{\text{Power spectrum levels in blue water}}{\text{power spectrum levels in bubbly water}} \right)
\]

Figure 5 shows a typical series of average excess absorption results across Neptune’s wake, at four of the twelve acoustic frequencies (70, 80, 90, and 100 kHz). The zero point on the range scale is the point at which the measurement systems began acquiring acoustic transmission loss data through the bubbly wake. A scale showing the wake age is also displayed. These plots are 5-point running averages of the data taken by
hydrophone 4 (hydrophone one was the hydrophone closest to the tow ship, and hydrophone 10, the last hydrophone in the array). For these measurements, Neptune’s speed was 15 knots.

Figure 5. Typical across the wake excess absorption measurements as a function of range and frequency for a 15-knot wake.

As acoustic transmissions encounter the intense beginning of the wake there is an increase in the excess absorption. This intense section of the wake extends for about 700 to 800 m behind Neptune, after which there is a steady decrease in the excess absorption as the wake ages and the bubbles rise to the surface or are broken up into smaller bubbles that redisolve. During the measurements, the surface signature of the edge of the turbulent bubbly portion of the wake was clearly visible and the two measurement ships were able to proceed down the outside of the wake until visually, it was difficult detect. At this range, (2500 m), the signal levels were approaching those for bubble free water.

Figure 6 shows the excess absorption in dB/m, as a function of frequency for four hydrophones at four selected ranges of 519, 742, 1113, and 1485 meters. At the closest range of 519 meters, all of the hydrophones show about the same average level of absorption over the frequency range with the maximum excess absorption occurring at a frequency of 110 kHz. At a range, 742 meters, the absorption showed little change from
the levels at the 519 meter range, indicating little change in the bubble size and number densities at these the two ranges. The maximum excess absorption at 110 kHz was approximately -4dB/m at these two ranges.

**Figure 6.** Excess absorption measured at hydrophones 2, 4, 6, and 8 for a 15-knot wake.

At the range of 1113 meters, the excess absorption levels decreased and leveled out at about −2.5 dB/m. This indicates a change in the number of the bubbles that resonate at 110 kHz. At a range of 1485 meters, the levels approached those for bubble free water.

Using the RBA (eqn 3), the bubble number densities were estimated for each frequency and as a function of range. These results are shown in Figure 7. Again there is the large increase in the bubble numbers with the onset of the wake, and the general decline in the numbers as the wake aged. As shown on the absorption curves in figure 6, there is a larger number of resonant bubbles in the 100 to 120 kHz frequency range.
Figure 7. Bubble number density as a function of range and transmitted frequency for a 15 knot wake for hydrophone number 4.

Figure 8 shows the bubble number densities for a 15 knot wake as a function of bubble size at ranges of 519, 742, 1113, and 1485 meters. The maximum bubble number densities ranged from \((5) \times 10^{10}\) at a range of 519 meters to \(10^{10}\) at the longest range of 1485 meters. As expected, some of the smaller bubbles have probably risen to the surface. The number density of the larger bubble sizes is approximately \(10^8\) at the shortest ranges of 519 and 742 meters. But as the wake ages, there is a decrease in the number of larger bubbles. At this long range, it was anticipated that the larger bubble number density would have shown a larger decrease. One possible explanation is that there were a large number of bubbles that were driven deep into the water column below the depth of the towed array and were still rising.
Figure 8. Bubble number densities for ranges of 519, 742, 1113, and 1485 meters for a 15-knot wake.

**III. Summary**

This paper describes a series of experiments designed to obtain estimates of the spatial bubble characteristics within a surface ship wake. By using measurements of the across the wake time dependent transmission loss at a single depth, estimates of the excess absorption due to the bubbles in the wake were obtained as a function of wake age and frequency. Using the theory of absorption in a bubbly medium, estimates of the bubble number densities (n(a)) as function of wake age, and acoustic transmission frequency were then obtained. In the future, the authors will use the data from the towed array to obtain estimates of the signal correlation functions.

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IV References


