MINIMUM TOTAL-SQUARED-CORRELATION QUATERNARY SETS: NEW BOUNDS AND OPTIMAL DESIGNS

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Code-division multiplexing, multiuser communications, quadriphase symbols, quaternary alphabet, sequences, sum capacity, Welch bound.
Minimum Total-Squared-Correlation Quaternary Signature Sets: New Bounds and Optimal Designs

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Abstract—We derive new bounds on the total squared correlation (TSC) of quaternary (quadruphase) signature/sequence sets for all lengths \( L \) and set sizes \( K \). Then, for all \( K, L \), we design minimum-TSC optimal sets that meet the new bounds with equality. Direct numerical comparison with the TSC value of the recently obtained optimal binary sets shows under what \( K, L \) realizations gains are materialized by moving from the binary to the quaternary code-division multiplexing (CDM) alphabet. On the other hand, comparison with the Welch TSC value for real/complex-field sets shows that, arguably, not much is to be gained by raising the alphabet size above four for any \( K, L \). The sum-capacity (as well as the maximum squared correlation and total asymptotic efficiency) of minimum TSC quaternary sets is also evaluated in closed-form and contrasted against the sum capacity of minimum-TSC optimal binary and real/complex sets.

Index Terms—Code-division multiplexing, multiuser communications, quadruphase symbols, quaternary alphabet, sequences, sum capacity, Welch bound.

I. INTRODUCTION

We consider the problem of designing sets of code sequences (signatures) for code-division multiplexing applications from the quaternary alphabet \( \{\pm1, \pm j\} \). A fundamental measure of the cross-correlation properties of a signature set (and subsequently overall code-division system performance) is the total squared correlation (TSC). If \( S = \{s_1, s_2, \ldots, s_K\} \), \( s_k \in \mathbb{C}^L \), \( ||s_k|| = 1 \), \( k = 1, 2, \ldots, K \), is a set of \( K \) normalized (complex, in general) signatures of length (processing gain) \( L \), then the TSC of \( S \) is the sum of the squared magnitudes of all inner products between signatures,

\[
\text{TSC}(S) = \sum_{m=1}^{K} \sum_{n=1}^{K} |s_m^H s_n|^2
\]

where \( H \) denotes the conjugate transpose operation. For real/complex-valued signature sets \( S \in \mathbb{C}^{L \times K} \) or \( S \in \mathbb{R}^{L \times K} \), TSC is bounded from below by [1]-[3]

\[
\text{TSC}(S) \geq \frac{KM}{L}
\]

where \( M = \max\{K, L\} \). The bound in (1) is called the “Welch bound” and the signature sets that satisfy (1) with equality are called Welch-bound-equality (WBE) sets. While for real/complex-valued signature sets the Welch bound is always achievable [4]-[12], this is not the case in general for finite-alphabet signatures. Recently, new bounds were derived for the TSC of binary (alphabet \( \{\pm1\} \)) signature sets for all lengths \( L \) and set sizes \( K \) together with optimal set designs for (almost) all \( K \) and \( L \) [13]-[15]. The sum capacity, total asymptotic efficiency, and maximum squared correlation of minimum-TSC optimal sets were found in [16]-[17]. Minimum-TSC and other digital sequence sets are studied and utilized in [18]-[21].

The gap in TSC between minimum-TSC binary signature sets and Welch-bound-equality real/complex sets can be reduced if we utilize alphabets of larger size. Certainly there is a tradeoff between system performance and transceiver complexity that is associated with our selection of the alphabet size. The quaternary (or quadruphase or 4-phase) alphabet appears as a good candidate since system complexity increase, which is attributed primarily to the addition of a quadrature signal/carrier into the system, may be negligible. Most code-division multiplexing (CDM) systems already employ quadrature signaling, thus utilizing quaternary signature sets would not incur significant additional cost over binary signature sets.

In this paper, we consider a quaternary alphabet and investigate under what \( K, L \) realizations gains can be materialized by moving from the binary to the quaternary code-division multiplexing alphabet. In particular, we first derive new bounds on the TSC of any quaternary signature matrix \( S = [s_1, s_2, \ldots, s_K] \in \mathbb{C}^{L \times K} \) for all possible \( K \) and \( L \) values. Then, via quaternary Hadamard matrix transformations, we design minimum-TSC optimal quaternary signature sets that achieve the new bounds. Finally, we derive analytic expressions for the maximum squared correlation (MSC), total asymptotic efficiency (TAE), and sum capacity \( C_{\text{sum}} \) of the minimum-TSC quaternary sets. In particular, we show that minimum-TSC quaternary sets exhibit the following properties: (i) if \( K \leq L \), MSC(S) is also minimum; (ii) if \( K \leq L \), TAE(S) is single-valued when \( L \equiv 0(\text{mod} \ 2) \) and multi-valued when \( L \equiv 1(\text{mod} \ 2) \); (iii) \( C_{\text{sum}} \) is single-valued when \( \max\{L, K\} \equiv 0(\text{mod} \ 2) \) and multi-valued when \( \max\{L, K\} \equiv 1(\text{mod} \ 2) \). We derive the exact value of MSC, TAE, and \( C_{\text{sum}} \) when these metrics are single-valued. When TAE and/or \( C_{\text{sum}} \) are multi-valued, we establish lower and upper bounds and prove their tightness; the exact
value of $C_{\text{sum}}$ and/or TAE depends on the particular design of the minimum-TSC signature set. A direct conclusion from this study is that minimum-TSC optimal quaternary sets are not necessarily $C_{\text{sum}}$ and/or TAE-optimal, which is also the case for binary antipodal signature sets [16] (we recall that all these metrics are equivalent for real/complex-valued sets [2], [7], [19], [22], [24]). We show, however, that a proposed design of minimum-TSC quaternary signature sets can also minimize MSC, maximize TAE, and maximize $C_{\text{sum}}$, simultaneously.

The rest of the paper is organized as follows. In Section II we present the new bounds and optimal designs. In Sections III, IV, and V we evaluate the maximum squared correlation, total asymptotic efficiency, and sum capacity, respectively, of minimum-TSC quaternary sets. A few conclusions are drawn in Section VI.

II. NEW BOUNDS ON THE TSC OF QUATERNARY SIGNATURE SETS AND OPTIMAL DESIGNS

We recall that the Karystinos-Pados bounds on the TSC of a binary signature set can be given compactly by the following expression [13]-[15]

$$
\text{TSC}(SB) \geq \frac{KM}{L} + \left\{ \begin{array}{ll} \frac{M}{\sqrt{L}} & M \equiv 0 \pmod{4} \\ \frac{4\sqrt{L}}{M} \left[ \left\lfloor \frac{M}{4} \right\rfloor^2 - m \right] & M \equiv 1 \pmod{2} \\ \frac{4\sqrt{L}}{M} \left[ \left\lfloor \frac{M}{4} \right\rfloor^2 - m \right] & M \equiv 2 \pmod{4} \end{array} \right. 
$$

where $K$ is the number of signatures, $L$ is the signature length, $M = \max\{K, L\}$, and $m = \min\{K, L\}$. The subscript "$B$" in $SB$ identifies a binary signature set $SB \in \{\pm 1\}^{L \times K}$, and $[x]$ is the integer that is closest to $x$ that is less than or equal to $x$ and greater or equal than $x$, respectively.

In this paper, we consider quaternary signature sets $S_Q \triangleq \{s_1, s_2, \ldots, s_K\} \in \sqrt{L} \{\pm 1, \pm j\}^{L \times K}$, $j \triangleq \sqrt{-1}$, where the subscript "$Q$" in $S_Q$ stands for quaternary. Since binary signature sets are special cases of quaternary signature sets ($\{\pm 1\} \subset \{\pm 1, \pm j\}$), for any $K$ and $L$ any achievable lower bound on the TSC of $S_Q$ lies between the Welch bound and the Karystinos-Pados bound. Thus, using (2), whenever $M \equiv 0 \pmod{4}$ the bound on TSC of any set $S_Q$ is

$$
\text{TSC}(S_Q) \geq \left\{ \begin{array}{ll} K, & K \leq L \text{ and } L \equiv 0 \pmod{4} \\ K^2/L, & K > L \text{ and } L \equiv 0 \pmod{4} \end{array} \right. 
$$

In the rest of this section, we derive new bounds on the TSC of quaternary signature sets for all possible combinations of the values of $K$ and $L$ when $M \equiv 1 \pmod{2}$ and $M \equiv 2 \pmod{4}$. Then, via quaternary Hadamard matrix transformations, we design optimal quaternary signature sets that achieve the new bounds.

A. Underloaded system ($K \leq L$)

Theorem 1 below provides new lower bounds on the TSC of any signature set $S_Q$ when $K \leq L$ (underloaded systems) and $\max\{K, L\} = L$ is not a multiple of 4.

**Theorem 1:** Let $S_Q$ be an arbitrary quaternary signature set $S_Q \triangleq \{s_1, s_2, \ldots, s_K\} \in \sqrt{L} \{\pm 1, \pm j\}^{L \times K}$, $j \equiv \sqrt{-1}$, $K \leq L$. Then,

$$
\text{TSC}(S_Q) \geq \left\{ \begin{array}{ll} \frac{K(K-1)}{L}, & L \equiv 1 \pmod{2} \\ \frac{K^2}{L}, & L \equiv 0 \pmod{2} \end{array} \right. 
$$

**Proof:** The TSC of $S_Q$ can be expressed as

$$
\text{TSC}(S_Q) = K + \sum_{m=1}^{K} \sum_{m \neq n} |\bar{s}_m \bar{s}_n|^2 
$$

where the double-summation term is the TSC between different signatures in $S_Q$ (if this term has zero value, i.e. all pairs of signatures have zero cross-correlation, the lower bound of TSC($S_Q$) reduces to the Welch bound). To obtain a lower bound on the double-summation term in (5), we consider the set $C$ of all non-ordered pairs of signatures $\{s_m, s_n\}$, $m \neq n$, with non-zero cross correlation, i.e. $C(\{s_1, s_2, \ldots, s_K\}) \triangleq \{\{s_m, s_n\} \text{ such that } m \neq n \text{ and } s_m^H s_n \neq 0, m = 1, 2, \ldots, K, n = 1, 2, \ldots, K\}$. Then,

$$
\text{TSC}(S_Q) \geq K + 2 |C(S_Q)||A|^2 
$$

where $A$ is a lower bound on the cross-correlation of any two signatures in $S_Q$ (i.e., $|s_m^H s_n| \geq A \forall s_m, s_n \in S_Q$ and $|C(\cdot)|$ denotes the cardinality of the set $C(\cdot)$). Since the quaternary signature alphabet $\sqrt{\frac{1}{L}} \{\pm 1, \pm j\}$ is closed under multiplication and conjugation (denoted by "$^*$"), signature cross-correlations can be expressed as

$$
s_m^H s_n = \sum_{i=1}^{L} s_m^i s_n^i = a + b \left( \frac{1}{L} \right) + c \left( \frac{1}{L} \right) + d \left( \frac{1}{L} \right) 
$$

for some integers $a, b, c, d$ such that $a + b + c + d = L$. Then, $|s_m^H s_n| = \sqrt{L} |(a - b)^2 + (c - d)^2|$. If $L \equiv 1 \pmod{2}$, $a + b + c + d \equiv 1 \pmod{2}$ which implies that $(a - b)^2 + (c - d)^2 \equiv 1 \pmod{2}$ and since $(a - b)^2 + (c - d)^2 \geq 0$, we have $(a - b)^2 + (c - d)^2 \geq 1$. Thus, $|s_m^H s_n| \geq \frac{1}{L}$ for any $s_m, s_n \in S_Q$. We conclude that if $L \equiv 1 \pmod{2}$ then the cross-correlation value between any two signatures in $S_Q$ is non-zero ($|A| = \frac{1}{L}$), therefore $|C(S_Q)| = \left( \frac{L}{2} \right) = K(K-1)/2$. On the other hand, if $L \equiv 0 \pmod{2}$ there may be signature pairs in $S_Q$ that exhibit zero cross-correlation.

The new bounds on the TSC of quaternary signature sets for underloaded systems ($K \leq L$) are summarized in Table I. Table I can also be viewed as proof that when the signature length is not even, no orthogonal quaternary signature set exists.

B. Overloaded system ($K > L$)

Let $d_l \triangleq \{s_1(l), s_2(l), \ldots, s_K(l)\}^T \in \sqrt{L} \{\pm 1, \pm j\}^K$ denote the transpose of the $l$th row, $l = 1, 2, \ldots, L$, of the signature matrix $S_Q$. Due to the “row-column equivalence” [18],

$$
\text{TSC}(S_Q) = \sum_{m=1}^{K} \sum_{n=1}^{L} |s_m^H s_n|^2 = \sum_{l=1}^{L} \sum_{r=1}^{L} |d_l^H d_r|^2. 
$$

Therefore, we can proceed with the calculation of TSC($S_Q$) as follows

$$
\text{TSC}(S_Q) = \frac{K^2}{L} + \sum_{l=1}^{L} \sum_{r=1, r \neq l}^{L} |d_l^H d_r|^2 = \frac{K^2}{L} + 2 |C(\{d_1, d_2, \ldots, d_L\})||d_l^H d_r|^2. 
$$
By Theorem 1, we obtain that

\[
TSC(S_Q) \geq \begin{cases} 
\frac{K^2}{L}, & K \equiv 0 \pmod{2} \\
\frac{K^2 - 1}{L} + \frac{L - 1}{L}, & K \equiv 1 \pmod{2}.
\end{cases}
\] (9)

The new bounds on the TSC of quaternary signature sets for overloaded \((K > L)\) systems are summarized in Table II.

We observe that when \(M \equiv \max\{K, L\} \equiv 0 \pmod{4}\) or \(1 \pmod{2}\) \((\text{multiple of four or odd})\) the TSC lower bounds for binary signature sets in Tables I and II are identical to the tight binary Karystinos-Pados (KP) bounds [13], while for \(M \equiv 2 \pmod{4}\), the lower bounds of TSC \((S_Q)\) in Table I and Table II are less than the corresponding KP bounds for binary signature sets. We also recall that \(K = L \equiv 1 \pmod{4}\) is an open standing problem (the only one) [13]-[15] in optimal binary signature set design and KP-bound-equality sets may in fact exist only for values \(K = L = 2x(x + 1) + 1, x = 1, 2, \ldots \) (i.e. \(K = L = 5, 13, 25, 41, 61, \ldots \)). We conclude that we can benefit in TSC by moving from the binary to the quaternary alphabet if \(\max\{K, L\}/2\) is an odd integer or \(K = L \neq 2x(x + 1) + 1, x = 1, 2, \ldots \). Particularly, when \(\max\{K, L\}/2\) is odd, the reduction in TSC is approximately

\[
\frac{M}{L} = \frac{K^2 - 1}{L} + \frac{L - 1}{L}.
\]

Moreover, for any \(K \) and \(L\), half of the minimum-TSC quaternary sets reach the Welch bound while only a quarter of the minimum-TSC binary sets do. To illustrate the potential reduction in TSC by quaternary designs, in Fig. 1 we consider systems with four different signature lengths \(L = 34, 46, 54, 66\) and plot TSC \((S) - K\) \((\text{i.e. total squared cross-correlation})\) for both binary and quaternary sets as a function of the number of signatures \(K\). To materialize the potential TSC/multiplexing improvements we need designs that meet the corresponding new bounds in Tables I and II with equality as described in the following subsection.

C. Design of Minimum TSC Quaternary Signature Sets

Our designs of optimal quaternary signature sets are based on transformations of quaternary Hadamard matrices as in [13].

\textbf{Definition 1:} (Quaternary Hadamard matrix)

Let \(H_Q\) be an \(N\)-order square matrix over the quaternary alphabet, i.e. \(H_Q \in \{\pm 1, \pm j\}^{N \times N}, j \equiv \sqrt{-1}, \) \(N > 0\). \(H_Q\) is a quaternary Hadamard matrix if \(H_QH_Q^T = N1_N\) where \(1_N\) is the \(N \times N\) identity matrix.

Tables I and II indicate that when \(M = \max\{K, L\}\) is even, the lower bound on the TSC of quaternary signature sets is equal to the real/complex Welch bound and can be achieved by any set \(S_Q\) that has orthogonal rows when \(K \geq L\) or orthogonal columns when \(K < L\). If \(M\) is not even, our lower bounds on the TSC of quaternary signature sets in (4), (9) are strictly larger than the Welch bound which implies that there is no quaternary matrix \(S_Q\) that has orthogonal rows when \(K \geq L\) or orthogonal columns when \(K < L\). In other words, a necessary condition for a quaternary Hadamard matrix to exist is that its size is even\(^1\); equivalently, if \(M\) is not even, then quaternary Hadamard matrices do not exist.

As is the case for binary Hadamard matrices, there is no universal procedure to generate quaternary Hadamard matrices for all even orders. Binary Hadamard matrices can be considered as a special case of quaternary Hadamard matrices. The generation of binary Hadamard matrices of orders that are multiples of four has been well studied. To generate a quaternary Hadamard matrix with order that is a multiple of four, we may multiply by \(\pm j\) any column or row of a binary Hadamard matrix of the same order. Unfortunately, it is not easy to generate a quaternary Hadamard matrix of even order \(N\) that is not a multiple of four. Below, we propose two alternative methods for this task. First, we suggest a modified version of the analytical procedure of Dijâ [25]-[27] that generates \((N - 2)^2\) polynomial equations. Our modification incorporates constraints that restrict the phase of the matrix elements to be in the set \(\{0, \pi/2, \pi, 3\pi/2\}\). Then, the solutions of the constrained system of \((N - 2)^2\) polynomial equations can be used as the entries of a quaternary Hadamard matrix. We note that solving such a constrained system of \((N - 2)^2\) polynomial equations is, in general, a computationally complex process and no specific algorithm is available in the literature for this task.

On the other hand, exhaustive search may be thought of as a method to return all quaternary Hadamard matrices of order \(N\). We understand, however, that the complexity of exhaustive search (which is equal to \(4^{N^2}\)) is prohibitively high\(^2\) even for

\(^1\)A necessary condition for a binary Hadamard matrix to exist is that its size is a multiple of four, except for the trivial cases of size one or two.

\(^2\)The complexity may be less if only one quaternary Hadamard matrix needs to be found.
moderate values of $N$. The second method that we suggest herein has significantly less computational complexity (equal to $4^N$) than exhaustive search. Our method is based on the following lemma [28], [29].

**Lemma 1:** If $A_1, A_2$ are two circulant matrices such that $A_1, A_2 \in \{\pm 1, \pm j\}^{N \times N}$, $N \in \mathbb{N}$, and $A_1, A_2$ satisfy

$$A_1 A_1^H + A_2 A_2^H = N I_N$$

(10)

where $I_N$ is an $N \times N$ identity matrix, then the construction

$$H_Q = \begin{bmatrix} A_1 & A_2 \\ A_2^H & -A_1^H \end{bmatrix}$$

is an $N$-order quaternary Hadamard matrix.

As an example, if $A_1 = \begin{bmatrix} j & 1 & 1 \\ 1 & j & 1 \\ 1 & 1 & j \end{bmatrix}$ and $A_2 = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$, then $H_Q = \begin{bmatrix} j & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & j & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ is a quaternary Hadamard matrix. There are only $4^N$ distinct $N \times N$ circulant matrices over the quaternary alphabet and each of them can be identified by its first row only. If two $N \times N$ circulant matrices $A_1$ and $A_2$ that satisfy (10) exist, they can be found by examining all $4^N$ possible pairs of circulant matrices. Then, a quaternary Hadamard matrix with order $N$ can be generated by $A_1$ and $A_2$ as given by Lemma 1. Examples of the first rows of $A_1$ and $A_2$-type matrices for different values of $N$ are given in Table III. Additional quaternary Hadamard matrices generated by this method can be found in [30].

In the rest of this section we present a sufficient condition under which the new TSC lower bounds of Tables I and II become tight. Then, we outline a design procedure of quaternary signature sets that achieve the bounds.

**Proposition 1:** Set $N \triangleq 2 [\max \{K, L\}/2]$ and $P \triangleq 2 [\max \{K, L\}/2]$. If there exists a quaternary Hadamard matrix of size $N$, then for any $L$ there exists a quaternary signature matrix $S_Q = [s_1, s_2, \ldots, s_K] \in \sqrt{N} \{\pm 1, \pm j\}^{L \times K}$ that achieves the TSC lower bound in Table I or II. If there exists a quaternary Hadamard matrix of size $P$, then there exists a quaternary signature matrix $S_Q = [s_1, s_2, \ldots, s_K] \in \sqrt{L} \{\pm 1, \pm j\}^{L \times K}$ with $K \neq L$ that achieves the TSC lower bound in Table I or II.

For overloaded systems, $K \leq L$, let $N = 2 [L/2]$ and generate an $N$-order quaternary Hadamard matrix $H_Q$. Either $L = N$ or $L = N - 1$. If $L = N$, then a quaternary signature set $S_Q$ can be formed by selecting and normalizing by $\frac{1}{\sqrt{N}}$ any $K$ columns of $H_Q$; if $L = N - 1$, then we first truncate $H_Q$ by one row and then form $S_Q$ by selecting and normalizing by $\frac{1}{\sqrt{L}}$ any $K$ columns from the truncated matrix. For overloaded systems, $K \geq L$, let $N = 2 [K/2]$ and generate an $N$-order quaternary Hadamard matrix $H_Q$. Then, $K = N$ or $K = N - 1$. If $K = N$, we may choose any $L$ rows of $H_Q$ and normalize them by $\frac{1}{\sqrt{N}}$, this is our $S_Q$. If $K = N - 1$, we may proceed by truncating $H_Q$ by one column and then form $S_Q$ by choosing and normalizing by $\frac{1}{\sqrt{L}}$ any $L$ rows of the truncated matrix.

By Proposition 1, a minimum-TSC quaternary signature set can also be designed based on a $P = 2 [\max \{K, L\}/2]$ order quaternary Hadamard matrix if it exists. Since $P = N$ when $\max \{K, L\} \equiv 0 \pmod{2}$, we focus on the case $\max \{K, L\} \equiv 1 \pmod{2}$ and $K \neq L$. For overloaded systems, $K < L$, $L \equiv 1 \pmod{2}$, and $P = 2 [L/2] = L - 1$. Generate an $(L - 1)$-order quaternary Hadamard matrix $H_Q$. To form $S_Q$, we first select any $K$ columns of $H_Q$, then insert an arbitrary row vector $v_1^T \in \{\pm 1, \pm j\}^{1 \times K}$, and finally normalize all columns by $\frac{1}{\sqrt{P}}$. For overloaded systems, $K > L$, $L \equiv 1 \pmod{2}$, and $P = 2 [K/2] = K - 1$. Generate a $(K - 1)$-order quaternary Hadamard matrix $H_Q$. To form $S_Q$ we may proceed by choosing any $L$ rows of $H_Q$, inserting an arbitrary column vector $v_2 \in \{\pm 1, \pm j\}^{L \times 1}$, and finally normalizing all rows by $\frac{1}{\sqrt{P}}$.

Fig. 2 summarizes the quaternary signature set design procedure described above in the form of a flow chart subject to the existence of a quaternary Hadamard matrix of order $N = 2 [\max \{L, K\}/2]$ or $P = 2 [\max \{L, K\}/2]$. We can show that the TSC of sets $S_Q$ designed by this procedure is exactly equal to the corresponding new bounds in Tables I or II and thus the produced quaternary signature sets are TSC-optimal.

As an illustrative example, in Fig. 3 we give a TSC-optimal quaternary signature set for an overloaded system with signature length $L = 13$ and $K = 22$ signatures. Another example of optimal design with $L = K = 9$ is shown in Fig. 4. These optimal sets were obtained directly by the design procedure of Fig. 2.

**TABLE III EXAMPLES OF $A_1, A_2$ CIRCULANT MATRICES**

<table>
<thead>
<tr>
<th>$\frac{1}{N} = 3$</th>
<th>$\frac{1}{N} = 5$</th>
<th>$\frac{1}{N} = 7$</th>
<th>$\frac{1}{N} = 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1 A_1^H + A_2 A_2^H = N I_N$</td>
<td>$[j - 1 - 1</td>
<td>j]$</td>
<td>$[1 - 1 - 1</td>
</tr>
<tr>
<td>$A_1 A_1^H + A_2 A_2^H = N I_N$</td>
<td>$[j - 1 - 1</td>
<td>-1]$</td>
<td>$[j - 1 -1</td>
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<td>$A_1 A_1^H + A_2 A_2^H = N I_N$</td>
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<td>-1]$</td>
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<td>$[j - 1 - 1</td>
<td>-1]$</td>
<td>$[j - 1 -1</td>
</tr>
</tbody>
</table>

**III. MAXIMUM SQUARED CORRELATION (MSC) OF MINIMUM-TSC QUANTERNARY SIGNATURE SETS**

Let $S_Q = [s_1, s_2, \ldots, s_K]_{L \times K}$ be an underloaded, $K \leq L$, signature matrix with quaternary normalized signatures $s_k \in \{\pm 1, \pm j\}$.
about the MSC of underloaded minimum-TSC quaternary signatures. By the proof of Theorem 1 in Section II, we can obtain that the maximum squared correlation of signatures. We recall that the maximum squared magnitude among all inner products between distinct signatures is lower-bounded as follows:

$$\frac{1}{\sqrt{L}} \{ \pm 1, \pm j \}^L, \quad k = 1, 2, \ldots, K.$$  

We recall that the maximum squared correlation (MSC) of a signature set is the maximum squared magnitude among all inner products between distinct signatures. By the proof of Theorem 1 in Section II, we can obtain that the maximum squared correlation of $S_Q$, denoted by $\text{MSC}(S_Q)$, is lower-bounded as follows:

$$\text{MSC}(S_Q) = \max_{m \neq n} |s_m^H s_n|^2 \geq \begin{cases} 0, & L \equiv 0 \pmod{2} \\ \frac{1}{L^2}, & L \equiv 1 \pmod{2}. \end{cases}$$  

(11)

The following two Propositions summarize our findings about the MSC of underloaded minimum-TSC quaternary signature sets. The proof is obtained directly from the material in Section II and is, therefore, omitted.

**Proposition 2:** Let $S_Q \in \frac{1}{\sqrt{L}} \{ \pm 1, \pm j \}^{L \times K}, \quad 1 < K \leq L$, be a quaternary signature matrix that achieves the corresponding TSC lower bound in Table I. Then,

(i) $\text{MSC}(S_Q) = 0$, if $L \equiv 0 \pmod{2}$;

(ii) $\text{MSC}(S_Q) = \frac{1}{L^2}$, if $L \equiv 1 \pmod{2}$.

**Proposition 3:** An underloaded quaternary signature set achieves the lower bound on TSC in Table I if and only if it achieves the lower bound on MSC in (11).

We conclude that the MSC of minimum-TSC quaternary underloaded sets is less than the MSC of minimum-TSC.
Fig. 4. Optimal quaternary signature set with signature length \( L = 9 \) and \( K = 9 \) signatures.

Binary set by \( \frac{4}{L} \) when \( L \equiv 2(\text{mod} 4) \) and \( K > 2 \). Most importantly, for all \( K, L \) with \( K \leq L \), the minimum-TSC quaternary signature sets obtained in Section II are doubly optimal (subject to the existence of a quaternary Hadamard matrix of size \( 2^{\lceil \log_4 L \rceil} \)). They exhibit minimum both TSC and MSC at the same time. Therefore, when we design quaternary signature sets (Fig. 2) we can focus on minimizing TSC only and can rest assured that MSC will also be minimized. It is interesting to note that the equivalence between TSC and MSC optimization is not true, in general, for binary sets\(^3\) [16].

IV. TOTAL ASYMPTOTIC EFFICIENCY (TAE) OF MINIMUM-TSC QUATERNARY SIGNATURE SETS

The TAE of a complex-valued signature matrix \( S = [s_1, \ldots, s_K] \), \( s_k \in \mathbb{C}^L \), \( \|s_k\| = 1 \), \( k = 1, 2, \ldots, K \), is equal to the determinant of the signature cross-correlation matrix, 

\[
\text{TAE}(S) = |S^H S| \leq 1.
\]

Since \( S^H S \) is rank-deficient and \( \text{TAE}(S) = 0 \) when \( K > L \) (overloaded system), we only consider the underloaded case. \( \text{TAE}(S) \) achieves the unit upper bound if \( S \) has orthogonal columns. However, it has been an open question whether tightness is maintained when \( S \) is quaternary, that is \( s_k \in \{\pm 1, \pm j\}^L, \) \( k = 1, 2, \ldots, K \). In this section, we obtain closed form expressions for the TAE of minimum-TSC quaternary signature sets for all \( K \leq L \). Our developments are based on the proposition that we state below and prove in the Appendix.

**Proposition 4:** Let \( S_Q \in \frac{1}{L} \{\pm 1, \pm j\}^{L \times K} \), \( K \leq L \), be a quaternary signature matrix that achieves the corresponding TSC lower bound in Table I and \( [S_Q^H S_Q]_{mn} \) denotes the \((m, n)\)th element of \( S_Q^H S_Q \), \( m = 1, 2, \ldots, K \), \( n = 1, 2, \ldots, K \). Then, \( S_Q^H S_Q \) has following properties:

(i) \( \text{If } L \equiv 0(\text{mod} 2), \) \( S_Q^H S_Q = I_K \);

(ii) \( \text{if } L \equiv 1(\text{mod} 2), \) \( \text{then } S_Q^H S_Q = 1 \) and \( [S_Q^H S_Q]_{mn} = \frac{1}{L} \{\pm 1, \pm j\}, \) \( m \neq n, m = 1, 2, \ldots, K, \)

(iii) \( \text{if } L \equiv 1(\text{mod} 2), \) \( \text{there exists a quaternary Hadamard matrix } H_Q \) of size \( L + 1 \), we can obtain a minimum-TSC signature set which has \( [S_Q^H S_Q]_{mn} = -\frac{1}{L}, \) \( m \neq n, m = 1, 2, \ldots, K, \) \( n = 1, 2, \ldots, K \); and

(iv) \( \text{if } L \equiv 1(\text{mod} 4) \) and \( K \leq L - 1, \) there exists a quaternary Hadamard matrix \( H_Q \) of size \( L - 1 \) and \( K \leq L - 1, \)

we can obtain a minimum-TSC signature set which has \( [S_Q^H S_Q]_{mn} = \frac{1}{L}, \) \( m \neq n, m = 1, 2, \ldots, K, \) \( n = 1, 2, \ldots, K \).

Based on the above proposition, the TAE of an underloaded minimum-TSC quaternary signature set can be derived and the findings are presented in the form of a proposition given below. The proof is given in the Appendix.

**Proposition 5:** Let \( S_Q \in \frac{1}{L} \{\pm 1, \pm j\}^{L \times K}, K \leq L, \) be a quaternary signature matrix that achieves the corresponding TSC lower bound in Table I. Then,

1. \( \text{TAE}(S_Q) = 1, \) if \( L \equiv 0(\text{mod} 2); \)
2. \( \left(\frac{L + 1}{L}\right)^2 \leq \text{TAE}(S_Q) \leq \left(\frac{L - K}{L}\right)^2, \) if \( L \equiv 1(\text{mod} 2). \)

V. SUM CAPACITY OF MINIMUM-TSC QUATERNARY SIGNATURE SETS

The sum capacity \( C_{\text{sum}} \) of a multiple-access communication channel is the maximum sum of user transmission rates at which reliable decoding at the receiver end is possible [2], [22], [23]. In a synchronous code-division multiple-access system that employs an \( L \times K \) complex-valued signature matrix \( S = [s_1, s_2, \ldots, s_K] \), \( s_k \in \mathbb{C}^L, \) \( \|s_k\| = 1 \), \( k = 1, 2, \ldots, K \), for transmissions over a common additive white Gaussian noise (AWGN) channel, the received data vector is of the form \( r = \sum_{k=1}^{K} d_k s_k + n \) where \( d_k \in \mathbb{C}, k = 1, 2, \ldots, K \), is the \( k \)th user transmitted symbol (complex in general) and \( n \) is a zero-mean complex Gaussian vector with auto-covariance matrix \( N_0 I_L \). If \( E[|d_k|^2] = E, k = 1, 2, \ldots, K \), it is known [2], [22] that

\[
C_{\text{sum}} \triangleq \log_2 \left[ 1 + \gamma S^H S \right] = \log_2 \left[ \frac{1}{K} + \gamma S^H S \right] \quad (12)
\]

\( K \) is a zero-mean complex Gaussian vector with auto-covariance matrix \( N_0 I_L \). If \( E[|d_k|^2] = E, k = 1, 2, \ldots, K \), it is known [2], [22] that

\[
C_{\text{sum}} \triangleq \log_2 \left[ 1 + \gamma S^H S \right] = \log_2 \left[ \frac{1}{K} + \gamma S^H S \right] \quad (12)
\]
where \( \gamma \triangleq \frac{E}{N_0} \) is the received signal-to-noise ratio (SNR) of each user signal and \( I_L, L_K \) are the size-\( L \) and size-\( K \) identity matrices. It is also well known that the sum capacity is bounded as follows \([2], [7], [19]\)

\[
0 \leq C_{\text{sum}}(S) \leq \begin{cases} 
K \log_2(1 + \gamma), & K \leq L \\
L \log_2(1 + \frac{K}{L} \gamma), & K \geq L.
\end{cases} \tag{13}
\]

While the upper bound in (13) is tight for real/complex-valued signature sets for any \( K, L \), it has been shown in \([16]\) that tightness is not always maintained if \( S \) is binary. In this section, we consider minimum-TSC quaternary signature sets \( S_Q \) and obtain closed-form expressions for \( C_{\text{sum}} \) for any \( K, L \). Our developments are presented in the form of a proposition given below. The proof is given in the Appendix.

**Proposition 6:** Let \( S_Q \in \mathbb{F}^L \{\pm 1, \pm j\}^{L \times K} \) be a quaternary signature matrix that achieves the corresponding TSC lower bound in Table I or Table II. Then,

A) if \( K \leq L \) (underloaded system)

(i) \( C_{\text{sum}}(S_Q) = K \log_2(1 + \gamma) \), if \( L \equiv 0 \pmod{2} \);

(ii) \( (K - 1) \log_2(1 + \frac{L-1}{L} \gamma) + \log_2(1 + \frac{L-K+1}{L} \gamma) \leq C_{\text{sum}}(S_Q) \leq (K - 1) \log_2(1 + \frac{K}{L} \gamma) + \log_2(1 + \frac{L-K+1}{L} \gamma) \), if \( L \equiv 1 \pmod{2} \). The lower bound is tight if there exists a quaternary Hadamard matrix of size \( L+1 \), while the upper bound is tight if \( K \leq L-1 \) and there exists a quaternary Hadamard matrix of size \( L-1 \).

B) if \( K \geq L \) (overloaded system)

(i) \( C_{\text{sum}}(S_Q) = L \log_2(1 + \frac{K}{L} \gamma) \), if \( K \equiv 0 \pmod{2} \);

(ii) \( (L - 1) \log_2(1 + \frac{K+1}{L} \gamma) + \log_2(1 + \frac{K-L+1}{L} \gamma) \leq C_{\text{sum}}(S_Q) \leq (L - 1) \log_2(1 + \frac{K}{L} \gamma) + \log_2(1 + \frac{K-L+1}{L} \gamma) \), if \( K \equiv 1 \pmod{2} \). The lower bound in (ii) is tight if there exists a quaternary Hadamard matrix of size \( K+1 \) while the upper bound is tight if \( L \leq K-1 \) and there exists a quaternary Hadamard matrix of size \( K-1 \).

Comparing Proposition 6 with expression (13) for real/complex-valued sets, we see that minimum-TSC quaternary signature sets meet the upper bound in (13) only if \( L \equiv 0 \pmod{2} \) for underloaded systems or \( K \equiv 0 \pmod{2} \) for overloaded systems. In addition, by Proposition 6, when \( L \equiv 1 \pmod{2} \) for underloaded systems or \( K \equiv 1 \pmod{2} \) for overloaded systems and \( K \neq L \), there exist quaternary minimum-TSC sets that do not exhibit maximum sum capacity. Thus, minimum-TSC and maximum-\( C_{\text{sum}} \) criteria are not equivalent, in general, for quaternary sets for all \( K, L \). In particular, by Proposition 6, design module B of Fig. 2 produces \( C_{\text{sum}} \)-optimal minimum-TSC designs, while module A produces minimum-TSC designs that are not \( C_{\text{sum}} \)-optimal.

Furthermore, comparing Proposition 6 with Proposition 2 in \([16]\) for binary sets, we notice that minimum-TSC quaternary signature sets have higher sum-capacity than minimum-TSC binary signature sets when \( L \equiv 2 \pmod{4} \) for underloaded systems and \( K \equiv 2 \pmod{4} \) for overloaded systems. More importantly, similar to the TAE metric, \( C_{\text{sum}} \)-optimal minimum-TSC quaternary signature sets can be produced by module B of Fig. 2, while the design of minimum-TSC binary sets that maximize \( C_{\text{sum}} \) is an open problem.

To visualize the theoretical developments of Proposition 6 on the sum capacity of quaternary signature sets, we consider the relative sum-capacity-loss expression

\[
\Delta(S) \triangleq 1 - \frac{C_{\text{sum}}(S)}{C_{\text{sum}}^*} \tag{14}
\]

where \( C_{\text{sum}}^* \) is the sum capacity of a real/complex-valued Welch-bound-equality (WBE) signature set of the same size as \( S \). In Fig. 6, we plot the sum-capacity-loss \( \Delta(S) \) of minimum-TSC quaternary sets as a function of \( K \) for a common received SNR per user \( \gamma = 12 \) dB and four different signature length values \( L = 31, 32, 33, \) and \( 34 \). Whenever \( C_{\text{sum}} \) of minimum-TSC quaternary sets is multi-valued, we use the maximum \( C_{\text{sum}} \) value (module-B produced set). For fair comparison, maximum \( C_{\text{sum}} \) values are also used for the minimum-TSC binary sets when their corresponding \( C_{\text{sum}} \) is multi-valued\(^3\) \([16]\). We observe that minimum-TSC quaternary sets exhibit rather negligible sum-capacity-loss for almost all \( K, L \) in comparison with WBE real/complex-valued sets. In addition, the sum-capacity-loss of quaternary minimum-TSC sets is quite less than the sum-capacity loss of binary minimum-TSC sets when \( K \) is near \( L \). In Fig. 7, we repeat the same study as in Fig. 6 for \( L = 63, 64, 65, \) and \( 66 \). Similar conclusions can be drawn. It can be argued that sum-capacity-wise it is not worth raising the code-division alphabet size above four for any \( K, L \), since the sum-capacity-loss of minimum-TSC quaternary sets already approaches zero rather closely.

\(^3\)The sum-capacity-loss study in \([16]\) used instead the smallest \( C_{\text{sum}} \) value when multiple values exist among min-TSC binary sets of a given \((K, L)\) size.

\[\text{VI. CONCLUSIONS}\]

In an effort to gain better understanding of the theoretical intricacies of finite-alphabet code-division multiplexing, we examined the following four signature performance metrics: Total squared correlation (TSC), maximum squared correlation (MSC), total asymptotic efficiency (TAE), and sum capacity (\( C_{\text{sum}} \)). In this paper, we derived new bounds on the TSC of quaternary signature sets for both underloaded and overloaded systems.
the number of signatures $K$ is less than or equal to the signature length $L$ and have maximum sum-capacity for any $K$, $L$. Interestingly, for quaternary (and binary [16]) signature sets, there exist $K$, $L$ values for which different metrics are optimized by different sets. Our studies showed that the sum-capacity loss of the minimum-TSC quaternary signature sets is negligible in comparison with minimum-TSC real/complex-alphabet (Welch-bound-equality) sets and quite smaller than that exhibited by minimum-TSC binary signature sets.

**APPENDIX**

**PROOF OF PROPOSITION 4**

The proof of parts (i) and (ii) can be obtained directly from the proof of Theorem 1 and is omitted here. With respect to part (iii), we recall that if the rows and columns of a quaternary Hadamard matrix are permuted or any row or column is multiplied by $-1$ or $\pm j$, the Hadamard orthogonality property is retained. Hence, we can always arrange one row or one column of a quaternary Hadamard matrix to have only $+1$ entries. If there exists a quaternary Hadamard matrix $H_Q$ of size $L + 1$ and $L \equiv 1(\text{mod } 2)$, a minimum-TSC signature set can be obtained by taking $K$ columns from $H_Q$ and removing one row which contains only $+1$ entries. After normalization, the cross-correlation matrix of the created minimum-TSC signature set is

$$S_Q^H S_Q = \frac{L + 1}{L} I_K - \frac{1}{L} 1_K 1_K^T$$

where $1_K$ is the $K$-dimensional all-one column vector. With respect to part (iv), if there exists a quaternary Hadamard matrix $H_Q$ of size $L - 1$ and $K \leq L - 1$, a minimum-TSC signature set can be obtained by appending an all-one row $1_{L - 1}$ to $H_Q$ and taking $K$ columns. After normalization, the cross-correlation matrix of the created minimum-TSC signature set is

$$S_Q^H S_Q = \frac{L - 1}{L} I_K + \frac{1}{L} 1_K 1_K^T.$$  

**PROOF OF PROPOSITION 5**

(i) When $L \equiv 0(\text{mod } 2)$ and $S_Q$ achieves the TSC lower bound in Table I, by Proposition 4, part (i), we obtain TAE($S_Q$) = $|S_Q^H S_Q| = |I| = 1$.

(ii) By Proposition 4, $|S_Q^H S_Q|_{m,n} = 1$ and $|S_Q^H S_Q|_{m,n} = \frac{1}{L}$, $m \neq n$, $m = 1, 2, \ldots, K$, $n = 1, 2, \ldots, K$. Then, by Lemma 2 of [16], we obtain that

$$\left(1 + \frac{1}{L}\right)^{K-1}(1 - (K - 1)\frac{1}{L}) \leq |S_Q^H S_Q|$$

$$\leq (1 - \frac{1}{L})^{K-1} \left(1 + (K - 1)\frac{1}{L}\right).$$

Expression (17) leads to the bounds on TAE as they appear in Proposition 5. If there exists a quaternary Hadamard matrix $H_Q$ of size $L + 1$, by (15) we can obtain a minimum-TSC quaternary signature set which has

$$|S_Q^H S_Q| = \left|\frac{L + 1}{L} I_K - \frac{1}{L} 1_K 1_K^T\right|$$

$$= \left(\frac{L + 1}{L}\right)^K \left(L - K + 1\right).$$


and this reaches the lower bound in Proposition 5. If there exists a quaternary Hadamard matrix $H_Q$ of size $L-1$, by (16) we can obtain a minimum-TSC quaternary set with TAE

$$ |S_Q^H S_Q| = \left( \frac{L-1}{L} \right)^{K} \left( \frac{L+K-1}{L-1} \right) $$

(19)

and this is the upper bound value in Proposition 5. ■

PROOF OF PROPOSITION 6

Part A

(i) If $L \equiv 0 (\bmod 2)$ and $S_Q$ achieves the TSC lower bound in Table I, it has orthogonal columns, i.e. $S_Q^H S_Q = I_K$.

Therefore,

$$ C_{sum}(S_Q) = \log_2 |I_K + \gamma S_Q^H S_Q| $$

$$ = \log_2 |1 + \gamma| |I_K| $$

$$ = K \log_2 (1 + \gamma). $$

(20)

(ii) By Proposition 4, the minimum-TSC quaternary set $S_Q$ has following properties: 1) $|I_K + \gamma S_Q^H S_Q|_{mm} = 1 + \gamma, m = 1, 2, \ldots, K$; 2) $|I_K + \gamma S_Q^H S_Q|_{mn} = \tau, m \neq n, m = 1, 2, \ldots, K, n = 1, 2, \ldots, K.$ Then, Lemma 2 of [16] implies that the determinant of $I_K + \gamma S_Q^H S_Q$ is bounded as follows:

$$ (1 + \gamma + \tau)^{(K-1)(1 + \gamma - (K-1)\tau) \leq I_K + \gamma S_Q^H S_Q \leq (1 + \gamma - \tau)^{(K-1)(1 + \gamma + (K-1)\tau)}. $$

(21)

Therefore, $C_{sum}(S_Q) = \log_2 |I_K + \gamma S_Q^H S_Q|$ is bounded as

$$ (K-1)\log_2 (1 + \frac{L+1}{L} \gamma) + \log_2 (1 + \frac{L-K+1}{L} \gamma) \leq C_{sum}(S_Q) \leq (K-1)\log_2 (1 + \frac{L+\gamma}{L} \gamma) + \log_2 (1 + \frac{L+K-1}{L} \gamma). $$

(22)

If there exists a quaternary Hadamard matrix $H_Q$ of size $L+1$, by Proposition 4, part (ii), we can obtain a minimum-TSC quaternary set that satisfies (16). Therefore,

$$ C_{sum}(S_Q) = \log_2 |I_K + \gamma S_Q^H S_Q| $$

$$ = \log_2 \left| \left( 1 + \frac{L+1}{L} \right) I_K - \gamma \frac{1}{L} 1_K 1_K^T \right| $$

$$ = (K-1)\log_2 \left( 1 + \frac{L+1}{L} \gamma \right) + \log_2 \left( 1 + \frac{L-K+1}{L} \gamma \right) $$

(23)

which is equal to the lower bound in Proposition 6, Part A(ii).

If there exists a quaternary Hadamard matrix $H_Q$ of size $L-1$ and $K \leq L-1$, we can obtain a minimum-TSC quaternary set that satisfies (16) and by similar to (23) derivation we can evaluate $C_{sum}$ as follows:

$$ C_{sum}(S_Q) = \log_2 |I_K + \gamma S_Q^H S_Q| $$

$$ = (K-1)\log_2 \left( 1 + \frac{L-1}{L} \gamma \right) $$

$$ + \log_2 \left( 1 + \frac{L+1}{L} \gamma \right) $$

(24)

which is the upper bound in Proposition 6, Part A(ii).

Part B

Set $D \triangleq \sqrt{\frac{K}{Q}} S_Q^H$ Then

$$ C_{sum}(S_Q) = \log_2 |I_L + \gamma S_Q S_Q^H| $$

$$ = \log_2 \left| I_L + \frac{K}{L} D^H D \right| $$

$$ = \log_2 \left| I_K + \frac{K}{L} D D^H \right|. $$

(25)

$D \in \mathbb{Q}^{K \times L}$ can be viewed as a signature matrix with $L$ unit-norm quaternary signatures of length $K \geq L$. Therefore, $C_{sum}(S_Q)$ at SNR $\gamma$ equals $C_{sum}(D)$ at SNR $\sqrt{\gamma} \frac{K}{L}$ where $S_Q$ is the overloaded and $D$ is the corresponding underloaded set. We can show that if TSC($S_Q$) achieves the TSC lower bound for overloaded sets in Table II, then TSC(D) achieves the TSC lower bound for underloaded sets in Table I. Hence, we can apply our results in Part A of Proposition 6 to $D$ and obtain the $C_{sum}(S_Q)$ expressions in all cases of Proposition 6, Part B, directly.

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